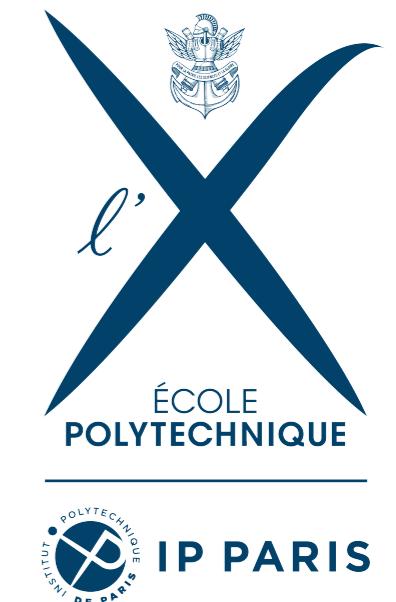


Quarkonium production and gluon TMDs at the EIC

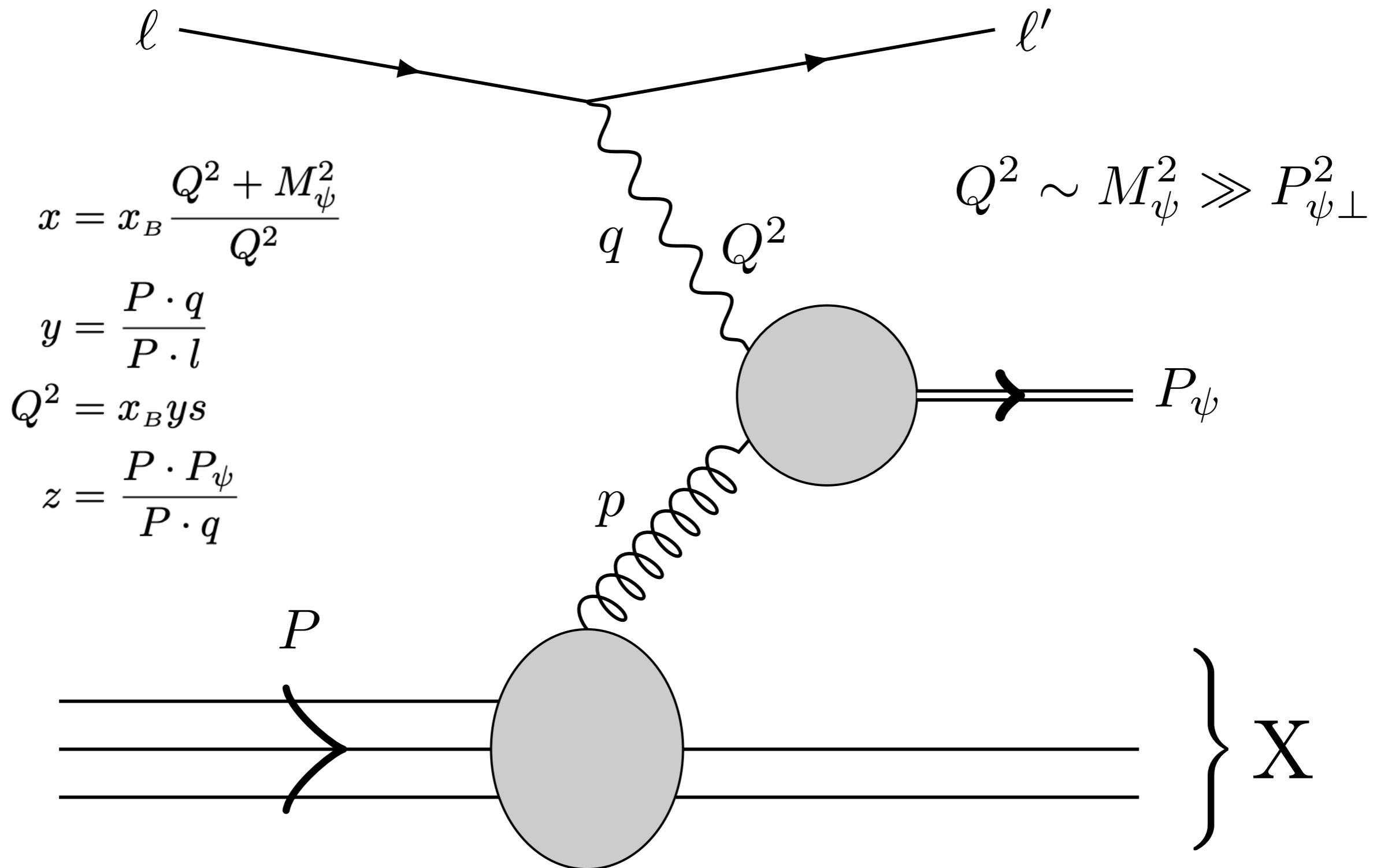
Pieter Taels

CPHT, École polytechnique, France

EIC opportunities for SnowMass
January 26th 2021

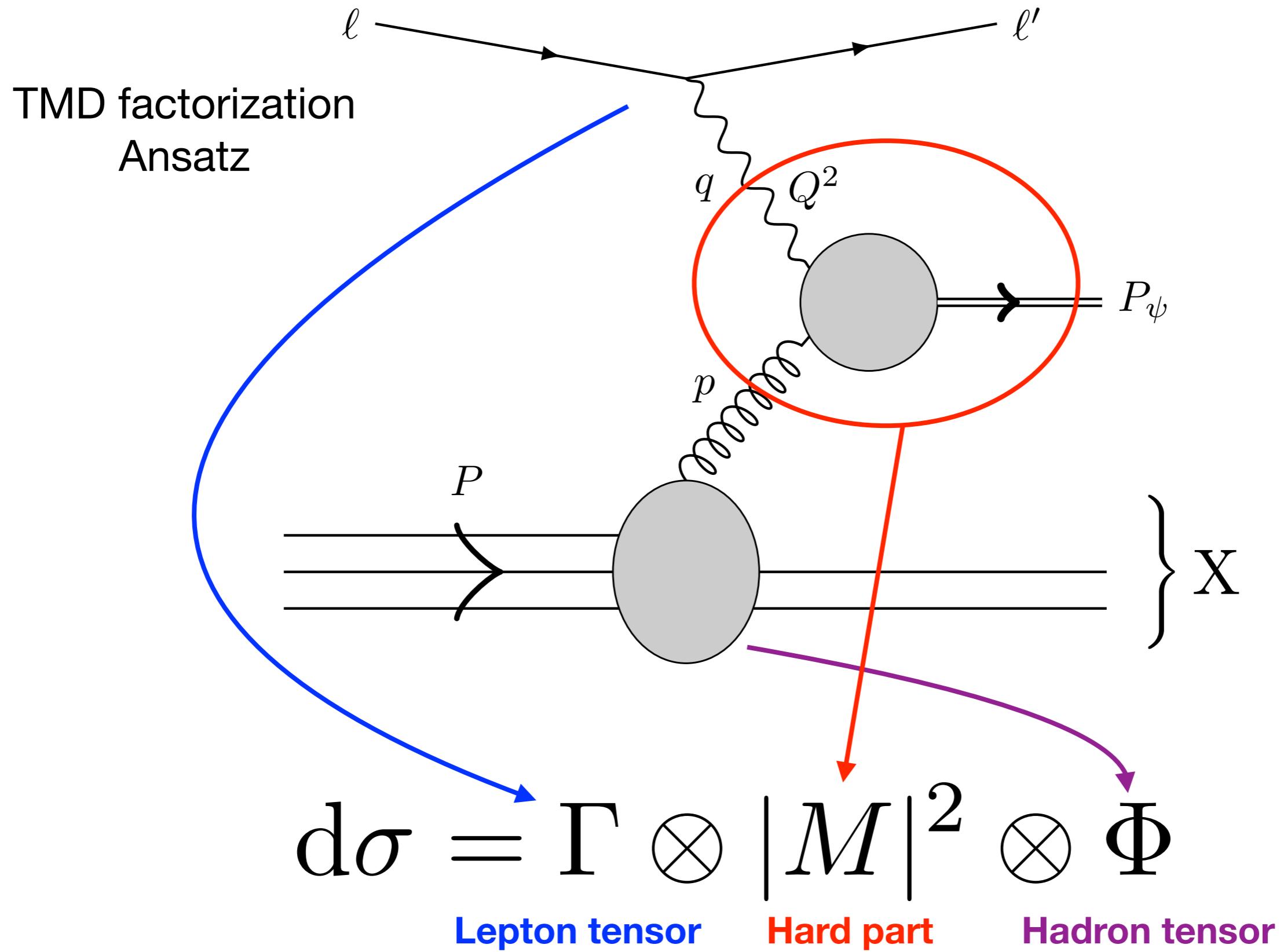


$$\ell + p \rightarrow \ell + J/\psi + X$$



Rajesh, Kishore, Mukherjee (2018)
 Bacchetta, Boer, Pisano, PT (2018)

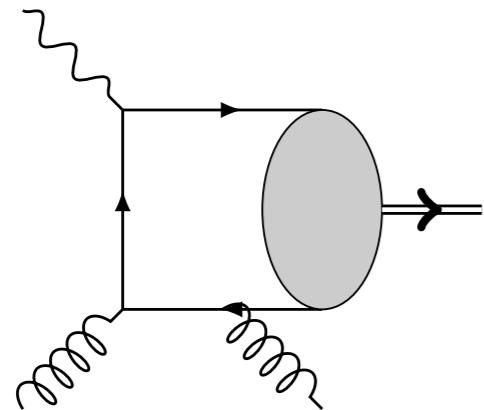
$$\ell + p \rightarrow \ell + J/\psi + X$$



Hard part: non-relativistic QCD (NRQCD)

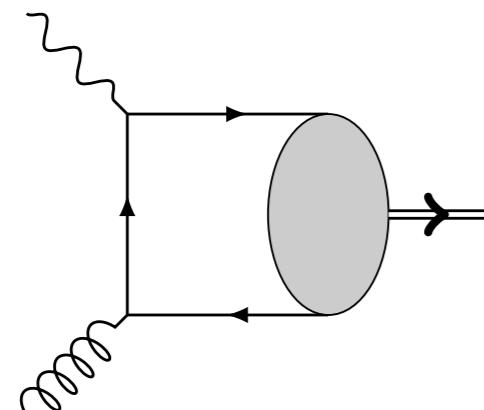
Dependance on nonperturbative long-distance matrix elements (LDMEs)

CS mechanism: colorless bound state in quantum numbers of J/ψ



$$\langle \mathcal{O}_1^{J/\psi}(^3S_1) \rangle \sim \frac{\alpha_s}{\pi} v^0$$

CO mechanism: $Q\bar{Q}$ pair in some colored excited state, hadronizes later



$$\begin{aligned} \langle \mathcal{O}_8^{J/\psi}(^1S_0) \rangle &\sim v^3 \\ \langle \mathcal{O}_8^{J/\psi}(^3P_J) \rangle &\sim v^4 \end{aligned} \quad \frac{\alpha_s}{\pi v^3} \sim \frac{1}{2}$$

In DIS for large z and $Q^2 \geq 4 \text{ GeV}^2$, CO mechanism is dominant

Bodwin, Braaten and Lepage (1995)
Fleming, Mehen (1998)

Hadron tensor

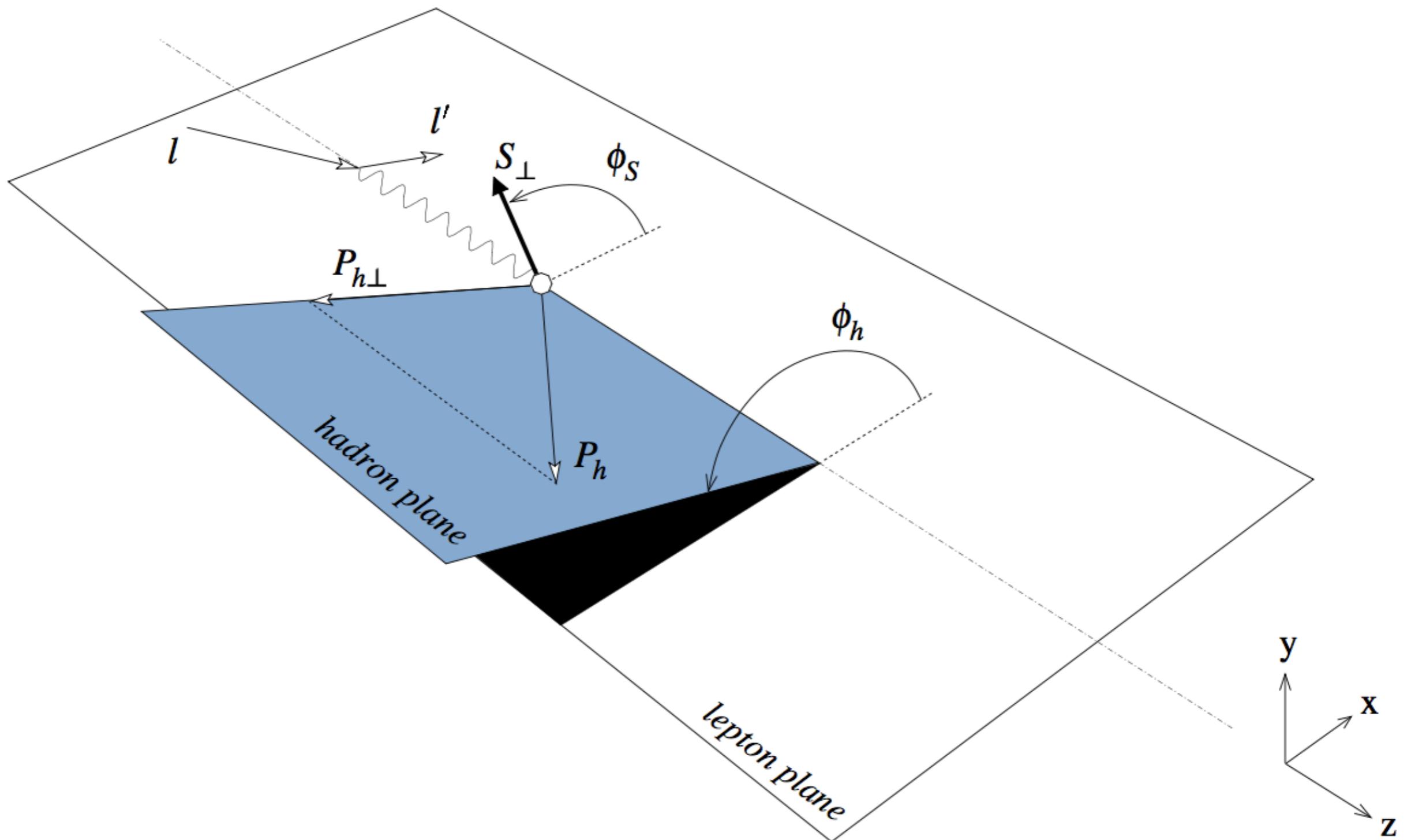
Sivers function circularly polarized gluons

$$\Gamma_T^{\mu\nu}(x, \mathbf{p}_T^2) = \frac{x}{2} \left\{ g_T^{\mu\nu} \frac{\epsilon_T^{\rho\sigma} p_{\perp\rho} S_{\perp\sigma}}{M_p} f_{1T}^{\perp g}(x, \mathbf{p}_T^2) + i \epsilon_T^{\mu\nu} \frac{p_{\perp} \cdot S_{\perp}}{M_p} g_{1T}^g(x, \mathbf{p}_T^2) \right. \\ \left. + \frac{p_{\perp\rho} \epsilon_T^{\rho\{\mu} p_{\perp}^{\nu\}}}{2M_p^2} \frac{p_{\perp} \cdot S_{\perp}}{M_p} h_{1T}^{\perp g}(x, \mathbf{p}_T^2) - \frac{p_{\perp\rho} \epsilon_T^{\rho\{\mu} S_{\perp}^{\nu\}} + S_{\perp\rho} \epsilon_T^{\rho\{\mu} p_{\perp}^{\nu\}}}{4M_p} h_{1T}^g(x, \mathbf{p}_T^2) \right\}$$

All of the Weizsäcker-Williams type

Mulders & Rodrigues (2001); Meissner, Metz & Goeke (2007)

Reference frame



Bacchetta, D'Alesio, Diehl & Miller (2004)

Cross section

$$\frac{d\sigma}{dy dx_B d^2 \mathbf{q}_T} \equiv d\sigma^U(\phi_T) + d\sigma^T(\phi_T, \phi_S)$$

$$d\sigma^U = \mathcal{N} \left[A^U f_1^g(x, \mathbf{q}_T^2) + \frac{\mathbf{q}_T^2}{M_p^2} B^U h_1^{\perp g}(x, \mathbf{q}_T^2) \cos 2\phi_T \right]$$

↓ ↓

unpolarized linearly polarized T even

Sivers function

T odd → only CO!

$$d\sigma^T = \mathcal{N} |\mathbf{S}_T| \frac{|\mathbf{q}_T|}{M_p} \left\{ A^T f_{1T}^{\perp g}(x, \mathbf{q}_T^2) \sin(\phi_S - \phi_T) \right.$$

$$+ B^T \left[h_1^g(x, \mathbf{q}_T^2) \sin(\phi_S + \phi_T) - \frac{\mathbf{q}_T^2}{2M_p^2} h_{1T}^{\perp g}(x, \mathbf{q}_T^2) \sin(\phi_S - 3\phi_T) \right] \left. \right\}$$

↓ ↓

linearly polarized linearly polarized

Azimuthal asymmetries

probe *rations* of gluon TMDs

$$A^{W(\phi_S, \phi_T)} \equiv 2 \frac{\int d\phi_S d\phi_T W(\phi_S, \phi_T) d\sigma(\phi_S, \phi_T)}{\int d\phi_S d\phi_T d\sigma(\phi_S, \phi_T)}$$

...we have:

$$\int d\phi_S d\phi_T d\sigma(\phi_S, \phi_T) = (2\pi)^2 \mathcal{N} A^U f_1^g(x, \mathbf{q}_T^2)$$

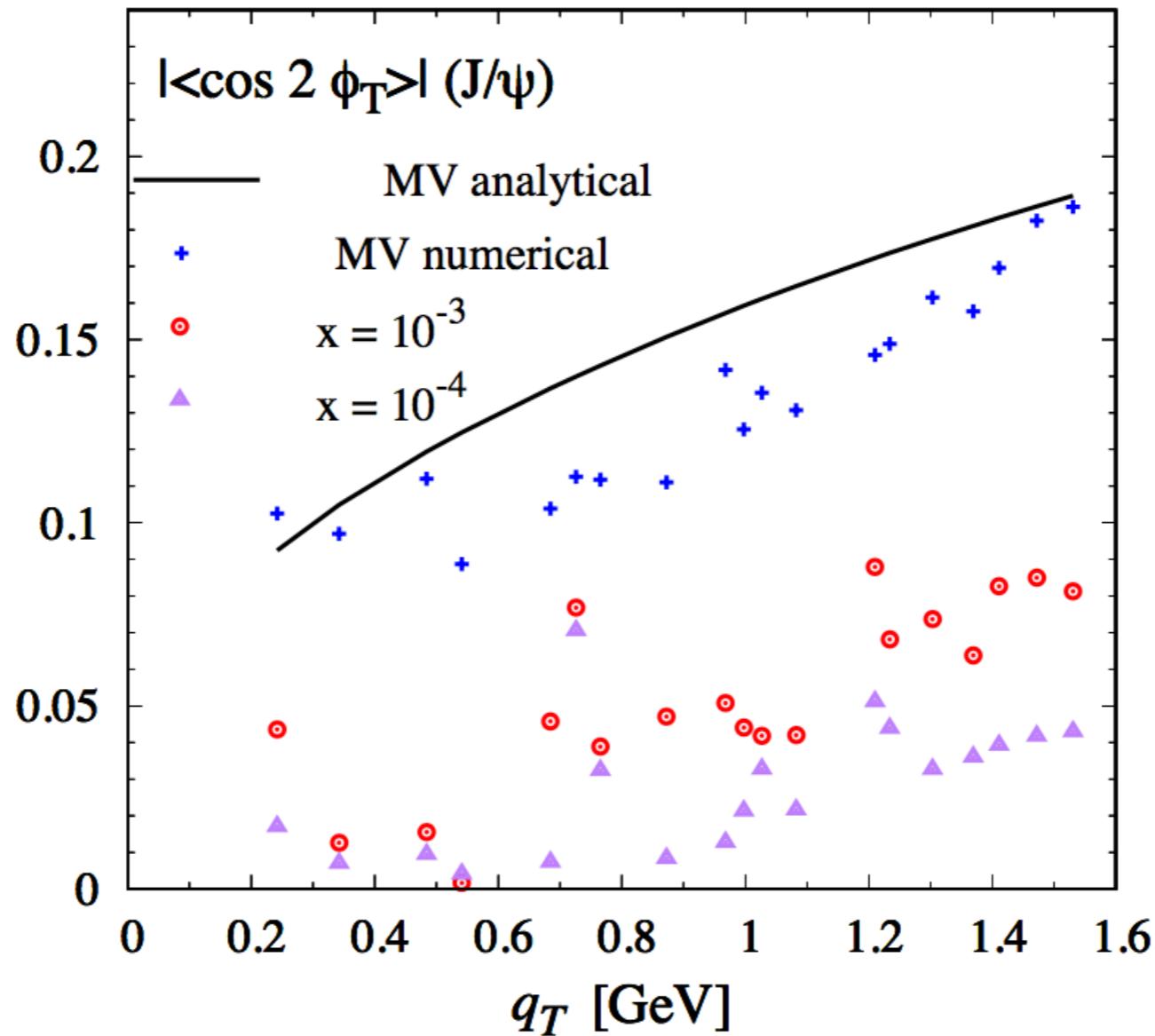
$$A^{\cos 2\phi_T} = H(y, M_\psi, Q) \frac{\mathbf{q}_T^2}{M_p^2} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$A^{\sin(\phi_S - \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$A^{\sin(\phi_S + \phi_T)} = H(y, M_\psi, Q) \frac{|\mathbf{q}_T|}{M_p} \frac{h_1^g(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$A^{\sin(\phi_S - 3\phi_T)} = -H(y, M_\psi, Q) \frac{|\mathbf{q}_T|^3}{2M_p^3} \frac{h_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

$$|\langle \cos 2\phi_T \rangle|(J/\psi)$$



$$Q = M_{J/\psi}$$

$$y = 0.1$$

$$\langle \cos 2\phi_T \rangle = H(y, Q^2, M_Q) \times \frac{\mathbf{q}_T^2}{2M_p^2} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)}$$

Bacchetta, Boer, Pisano, PT (2018)
 gluon TMDs from Marquet, Roiesnel, PT (2018)
 LDMEs from Chao, Ma, Shao, Wang, Zhang (2012)

$\ell + p \rightarrow \ell + J/\psi + X$:
matching of collinear and TMD calculation

Notion of LDMEs needs to be extended to small transverse momenta (cfr. fragmentation functions).

If μ is the hard scale, collinear factorization holds when

$$\mu \sim P_{\psi\perp} \gg M_p .$$

TMD factorization is applicable when $\mu \gg P_{\psi\perp} \gtrsim M_p$.

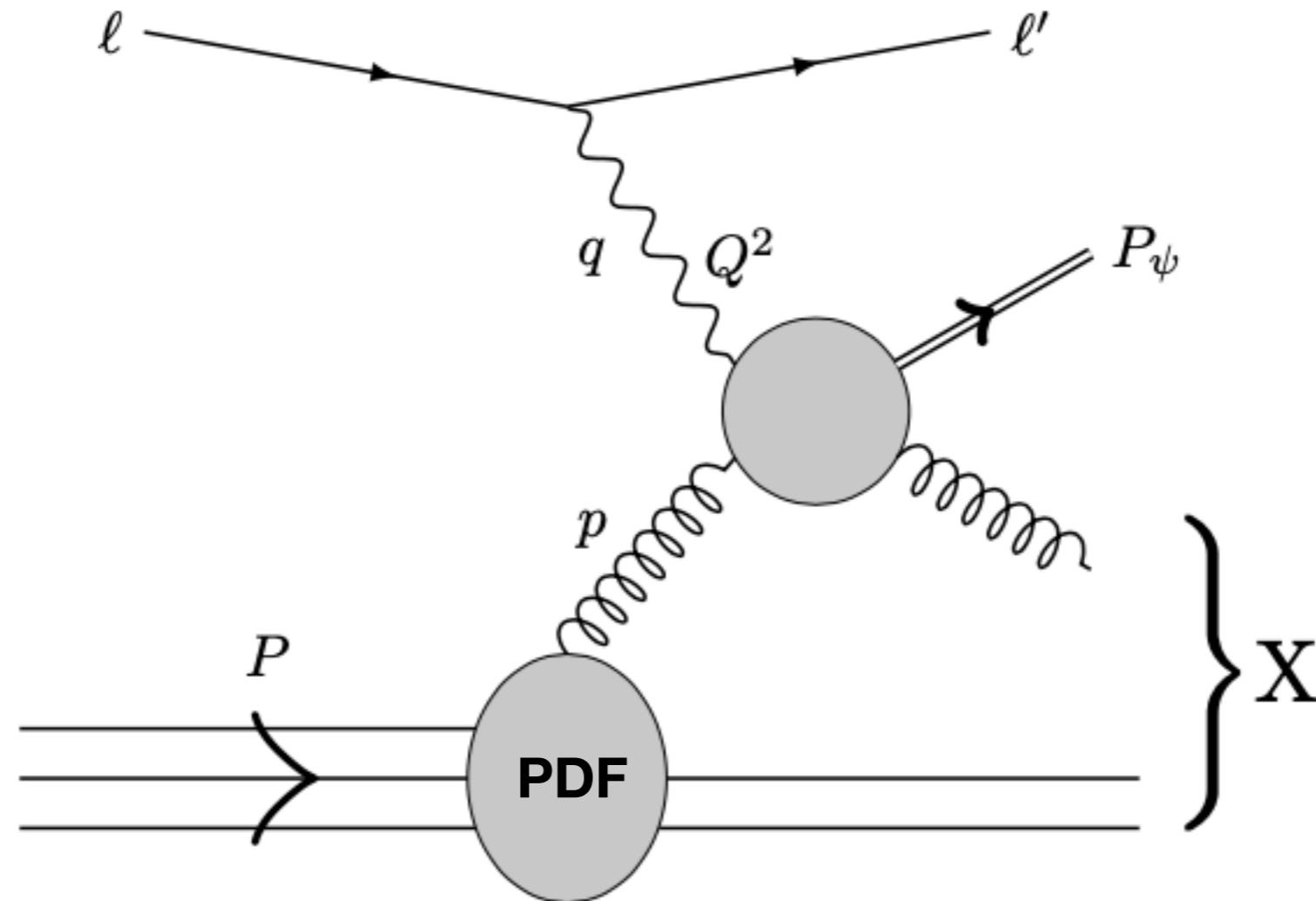
The two calculations should match in the region where both frameworks are valid, i.e. when $\mu \gg P_{\psi\perp} \gg M_p$.

Strategy: assume TMD factorization, and study which conditions follow from the matching requirement.

$\ell + p \rightarrow \ell + J/\psi + X$:
matching of collinear and TMD calculation

In the collinear regime: $\mu \sim P_{\psi\perp} \gg M_p$ with $\mu = \sqrt{Q^2 + M_\psi^2}$

$P_{\psi\perp}$ is generated by recoil off hard parton

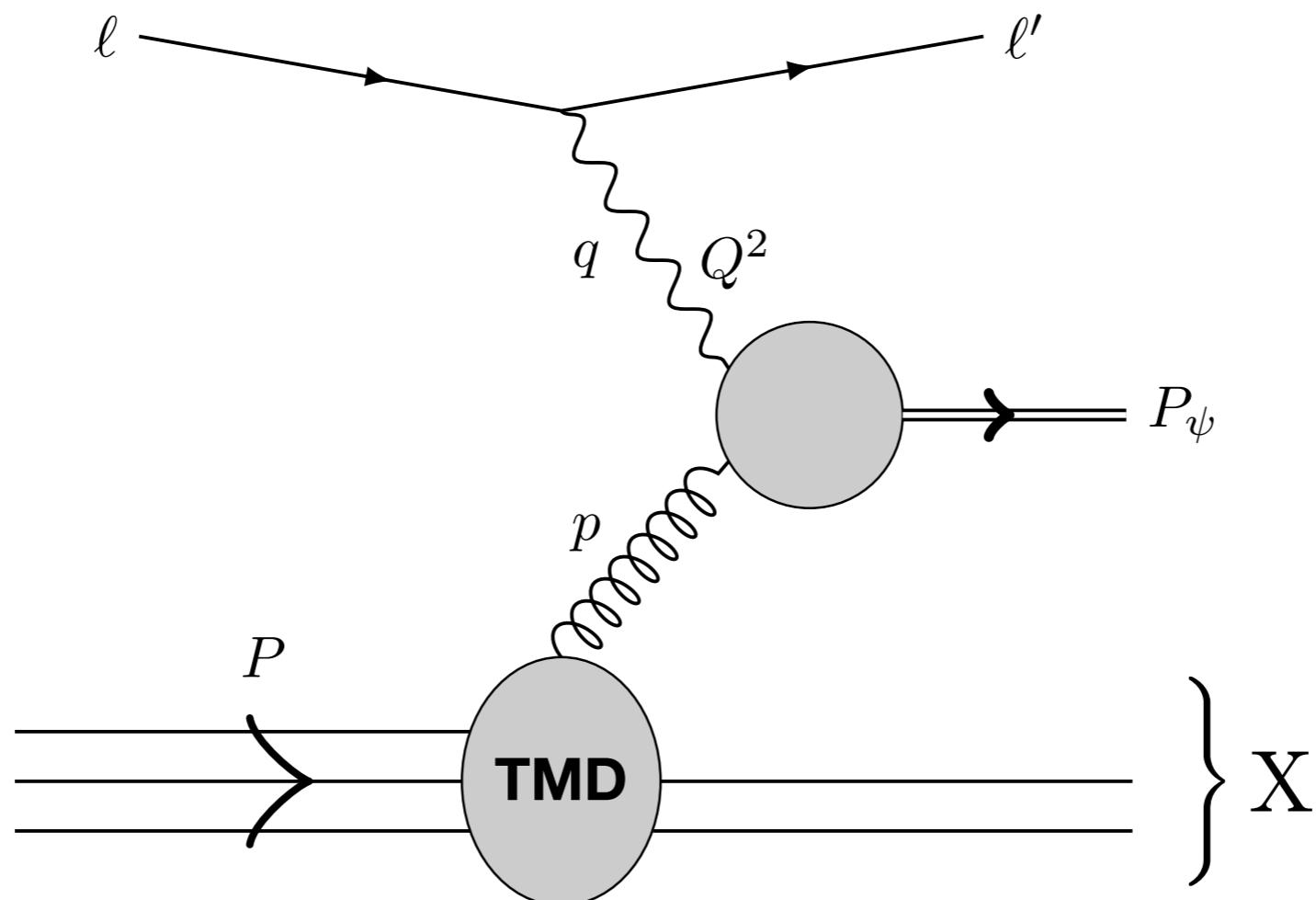


$$\ell + p \rightarrow \ell + J/\psi + X:$$

matching of collinear and TMD calculation

In the TMD regime: $\mu \gg P_{\psi\perp} \gtrsim M_p$

$P_{\psi\perp}$ stems from intrinsic transverse momentum in target, or from soft emissions



$\ell + p \rightarrow \ell + J/\psi + X$:
matching of collinear and TMD calculation

If TMD factorization holds, both the collinear and the TMD calculation should match in overlapping kinematic region $\mu \gg P_{\psi\perp} \gg M_p$

From collinear calculation: hard emission is absorbed into DGLAP

From TMD calculation: high- p_\perp tail also matches to DGLAP

Collins, Soper & Sterman (1985, 1989)
Bacchetta, Boer, Diehl & Mulders (2008)
Collins (2011)
Bacchetta, Bozzi, Echevarria, Pisano, Prokudin & Radici (2019)

$\ell + p \rightarrow \ell + J/\psi + X$:
 matching of collinear and TMD calculation

Collinear calculation at small q_T :

$$\frac{d\sigma}{dy dx_B dz dq_T^2 d\phi_\psi} = \frac{\alpha^2}{y} \left[\frac{1+(1-y)^2}{Q^2} F_{UU,T} + 4(1-y) F_{UU,L} + (1-y) \cos 2\phi_\psi F_{UU}^{\cos 2\phi_\psi} \right]$$

transverse γ^* **longitudinal γ^*** **lin. pol. γ^***

TMD calculation at high q_T :

$$\frac{d\sigma}{dy dx_B dz dq_T^2 d\phi_\psi} = \frac{\alpha^2}{y} \left[\frac{1 + (1-y)^2}{Q^2} \mathcal{F}_{UU,T} + 4(1-y) \mathcal{F}_{UU,L} + (1-y) \cos 2\phi_\psi \mathcal{F}_{UU}^{\cos 2\phi_\psi} \right] \delta(1-z)$$

In kinematic region of overlap:

$$\mathcal{F}_{UU,T} = F_{UU,T} - \sigma_{UU,T} C_A \ln\left(\frac{Q^2 + M_\psi^2}{q_T^2}\right)$$

$$\mathcal{F}_{UU,L} = F_{UU,L} - \sigma_{UU,L} C_A \ln\left(\frac{Q^2 + M_\psi^2}{q_T^2}\right)$$

$$\mathcal{F}_{UU}^{\cos 2\phi_\psi} = F_{UU}^{\cos 2\phi_\psi}$$

$\ell + p \rightarrow \ell + J/\psi + X$:
 matching of collinear and TMD calculation

Therefore, assuming both TMD factorization and enforcing the matching:

$$\begin{aligned}\mathcal{F}_{UU,T} &= \sum_n \mathcal{H}_{UU,T}^{[n]} \mathcal{C}[f_1^g \Delta^{[n]}](x, \mathbf{q}_T^2) \\ \mathcal{F}_{UU,L} &= \sum_n \mathcal{H}_{UU,L}^{[n]} \mathcal{C}[f_1^g \Delta^{[n]}](x, \mathbf{q}_T^2) \\ \mathcal{F}_{UU}^{\cos 2\phi_\psi} &= \sum_n \mathcal{H}_{UU,}^{[n], \cos 2\phi_\psi} \mathcal{C}[w h_1^{\perp g} \Delta^{[n]}](x, \mathbf{q}_T^2)\end{aligned}$$

we have to introduce the following ‘smearing’ function:

$$\Delta^{[n]}(\mathbf{k}_T^2, \mu^2) = \frac{\alpha_s}{2\pi^2 \mathbf{k}_T^2} C_A \langle 0 | \mathcal{O}_c(n) | 0 \rangle \ln \frac{\mu^2}{\mathbf{k}_T^2}$$

Boer, D’Alesio, Murgia, Pisano, PT (2020)
 Echevarria (2019); Fleming, Makris, Mehen (2019)

$\ell + p \rightarrow \ell + J/\psi + X$:
matching of collinear and TMD calculation

Matching perfectly works (at LO) if we expand the LDMEs as follows:

$$\Delta^{[n]}(\mathbf{k}_T^2, \mu^2) = \frac{\alpha_s}{2\pi^2 \mathbf{k}_T^2} C_A \langle 0 | \mathcal{O}_c(n) | 0 \rangle \ln \frac{\mu^2}{\mathbf{k}_T^2}$$

This is exactly the perturbative part of the LDME *shape functions* introduced by our colleagues from SCET !

Important step towards proof of TMD factorization of single-inclusive quarkonium production.

At least for ${}^1S_0^{[8]}$ and ${}^3P_J^{[8]}$ states, the transverse momentum dependence of the shape function is the same.

Boer, D'Alesio, Murgia, Pisano, PT (2020)
Echevarria (2019); Fleming, Makris, Mehen (2019)

Conclusions & outlook

Leptoproduction of J/ψ (+jet) at the Electron-Ion Collider seems a very promising process to probe gluon TMDs. Factorization Ansatz is reasonable and corroborated by SCET studies and matching. We need dedicated fit at low transverse momentum of the shape functions.

Other possible channels:

$ep \rightarrow J/\psi \text{ jet } X$

D'Alesio, *et al.* (2020); Kishore *et al.* (2020)

$pp \rightarrow J/\psi \gamma X$

den Dunnen, Lansberg, Pisano, Schlegel (2014)

$pp \rightarrow J/\psi \ell^+ \ell^- X$

Lansberg, Pisano, Schlegel (2017)

$pp \rightarrow J/\psi J/\psi X$

Scarpa, Boer, *et al.* (2020)

Extracting LDMEs from comparison of open charm and quarkonium
see Boer, Pisano, PT, *in preparation*

Thanks for your attention !

Backup slides: SIDIS

Primordial example: semi-inclusive DIS (SIDIS)

$$Q^2 = - (e - e')^2$$

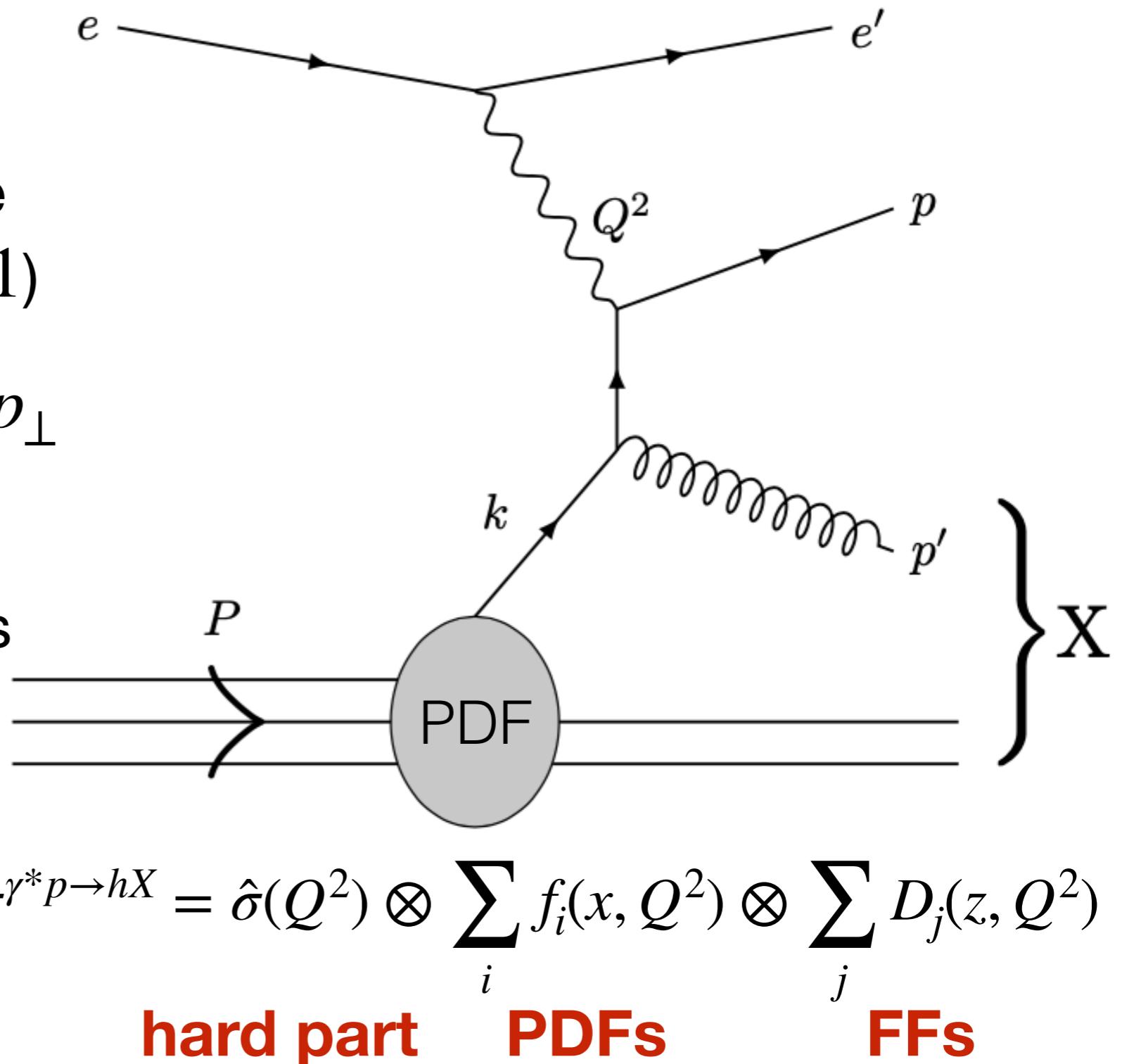
is the hard scale which allows for a perturbative description ($\alpha_s(Q^2) \ll 1$)

Transverse momentum p_\perp of the outgoing hadron (generated by recoil off unobserved parton) sets another scale

If $Q \sim p_\perp$, **collinear**

factorization applies: $\sigma^{\gamma^* p \rightarrow hX} = \hat{\sigma}(Q^2) \otimes \sum_i f_i(x, Q^2) \otimes \sum_j D_j(z, Q^2)$

$$x = \frac{Q^2}{2P \cdot q} \simeq \frac{k^+}{P^+}$$

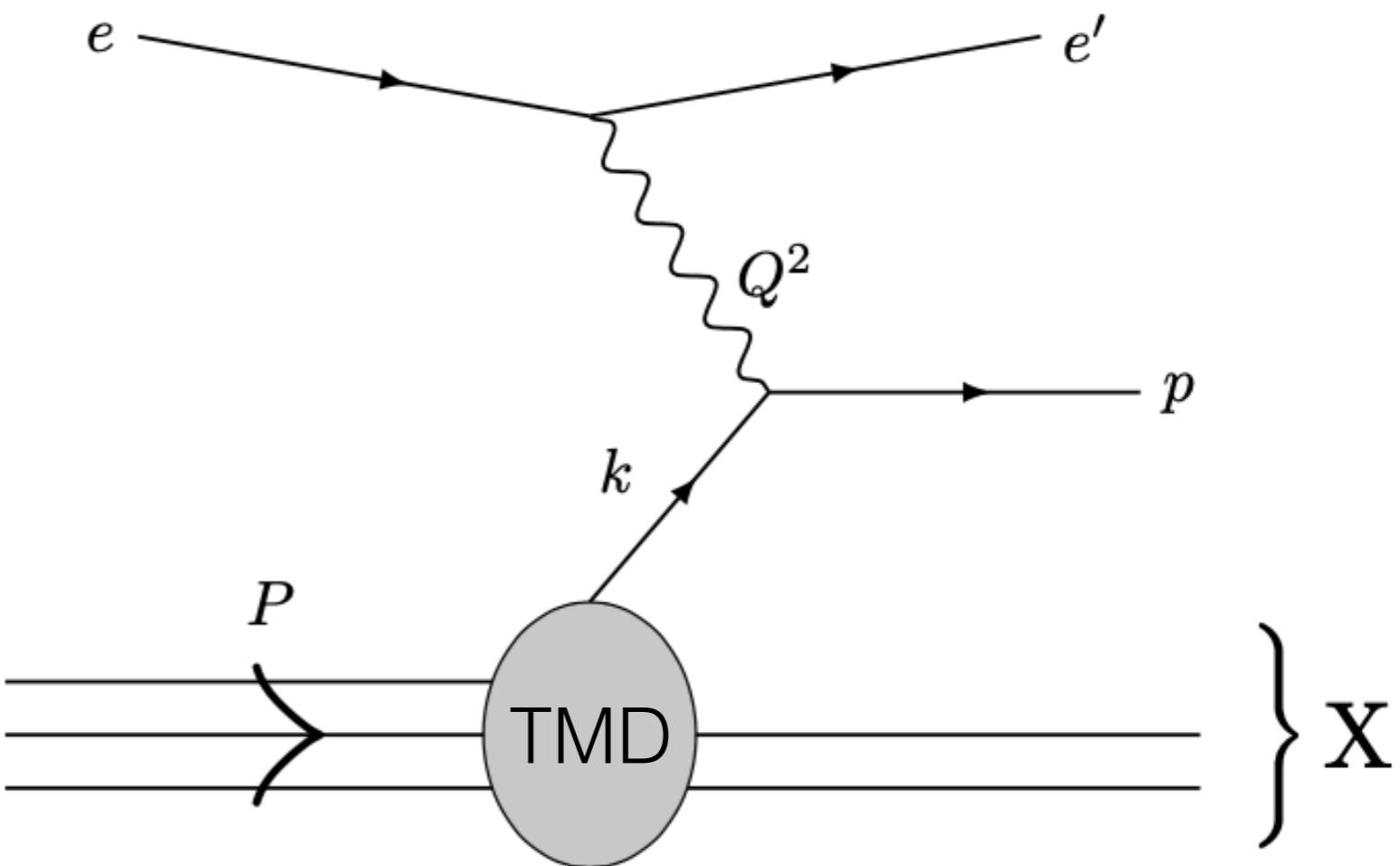


hard part PDFs FFs

Primordial example: semi-inclusive DIS (SIDIS)

What if $p_{\perp} \ll Q$? If transverse momentum is generated by soft emissions or stems from the primordial motion of the parton inside the hadron?

Need to resum large logs and introduce transverse momentum dependent PDFs (TMDs)



TMD factorization:

$$\sigma^{r^* p \rightarrow hX} = \hat{\sigma}(Q^2) \otimes \int d^2 k_{\perp} \sum_i f_i(x, k_{\perp}, Q^2) \otimes \sum_j D_j(z, p_{\perp} - k_{\perp}, Q^2)$$

hard part **TMD PDFs** **TMD FFs**