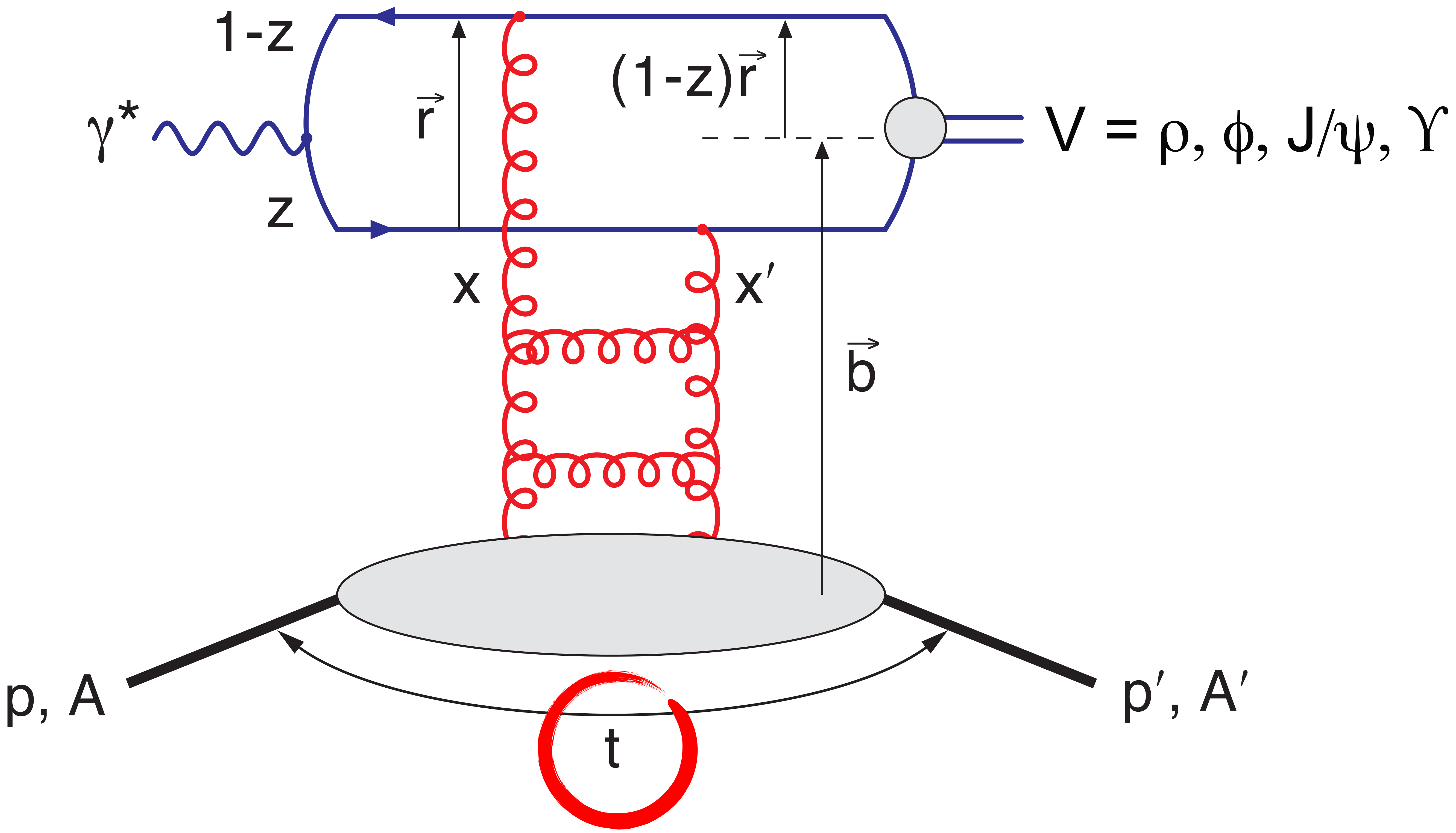
# On the Calculation of *t*





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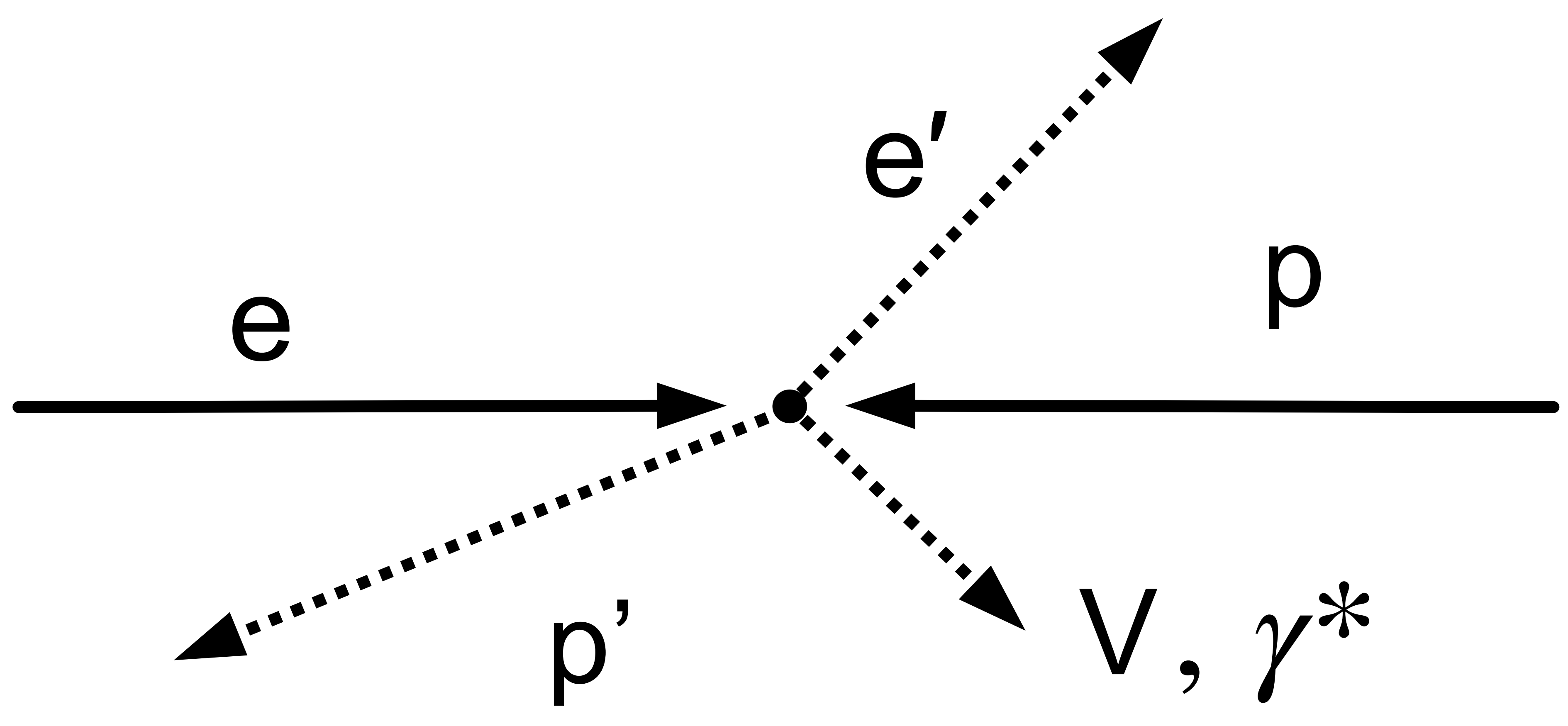
# e+p

In e+p we can follow the definition of *t*:

$$t = (p_A - p_{A'})^2$$




$p_A$ is known (beam) and $p_{A'}$ is measured by forwards proton spectrometers (Roman Pots etc)

How well that ultimately works in terms of $\sigma_t/t$ one has to see.

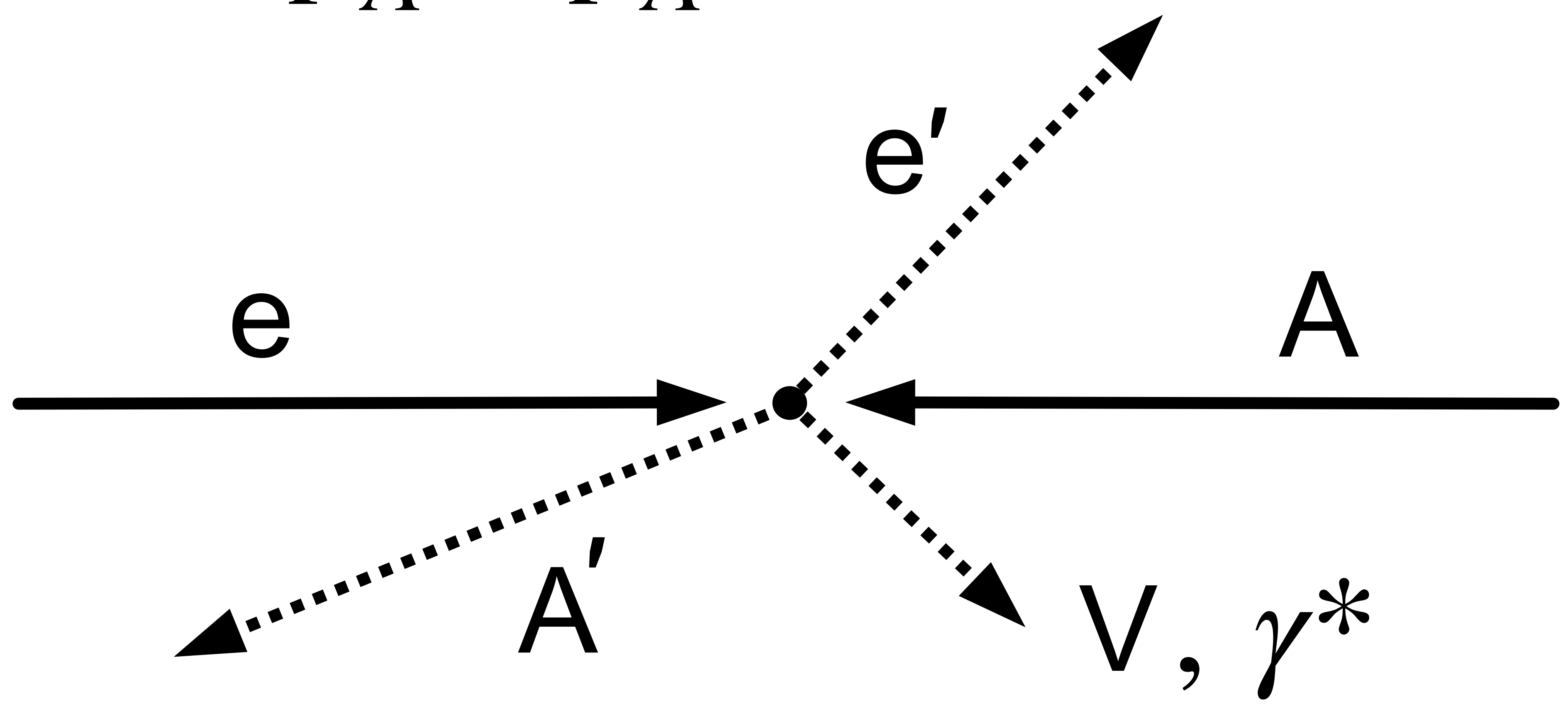
In any case alternative methods should be considered either to improve the precision or for systematic cross-checks.

# e+A

In e+A we *cannot* measure $p_{A'}$:

- coherent: *t* kick not big enough to get heavy ions out of the beam pipe
- incoherent: unlikely we can measure all fragments and reconstruct the whole ion and its momentum.

$t = (p_A - p_{A'})^2$




In general *t* cannot be measured w/o knowing $p_{A'}$ except in exclusive vector meson production:

$$e + A \rightarrow e' + A' + V$$

since 4-momenta from $e, A, e'$ and $V$ are known

# Method E

One can directly calculate $t$ as:

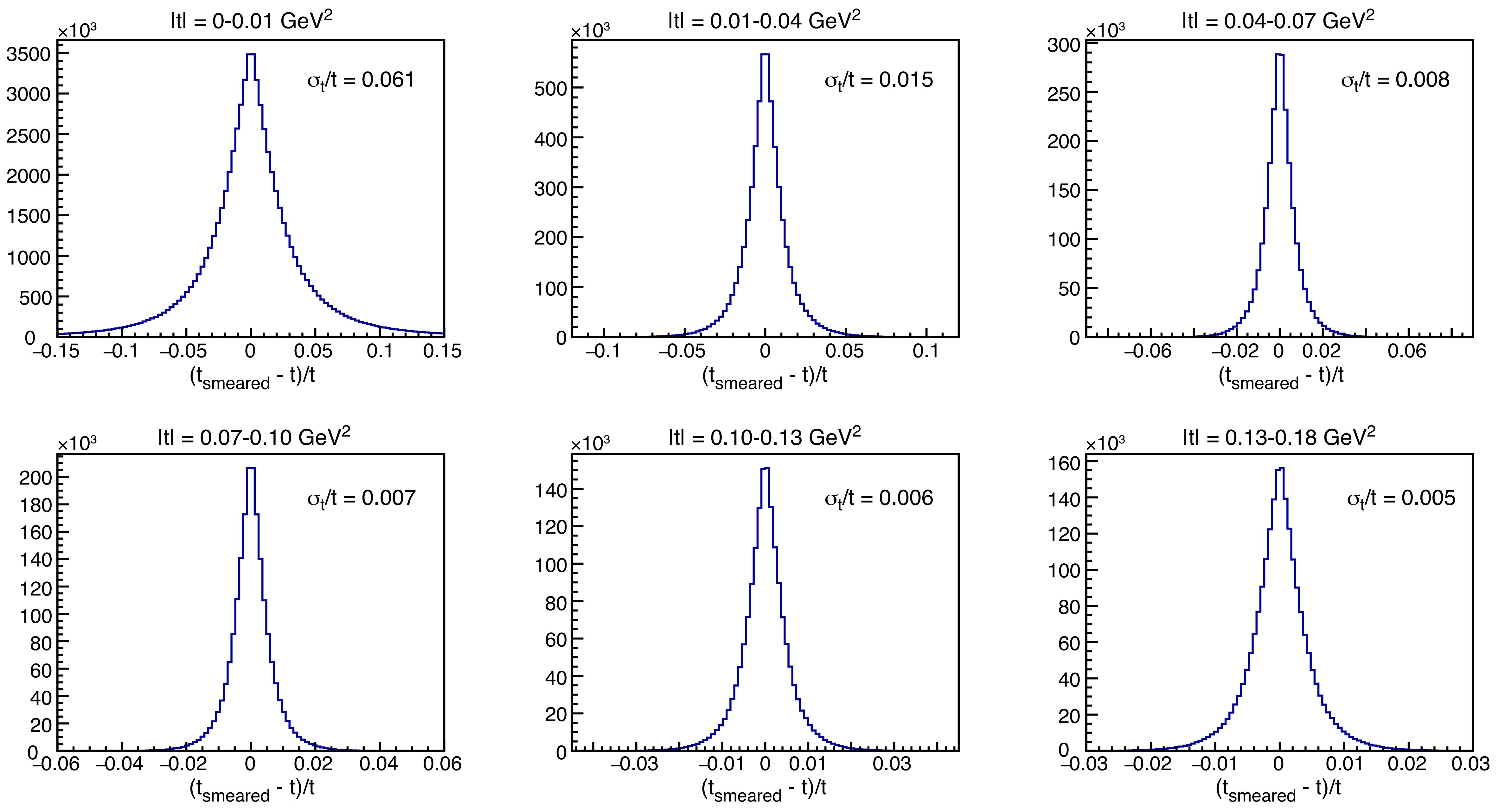
$$t = (p_A - p_{A'})^2 = (p_V + p_{e'} - p_e)^2$$

we call this method E (exact)

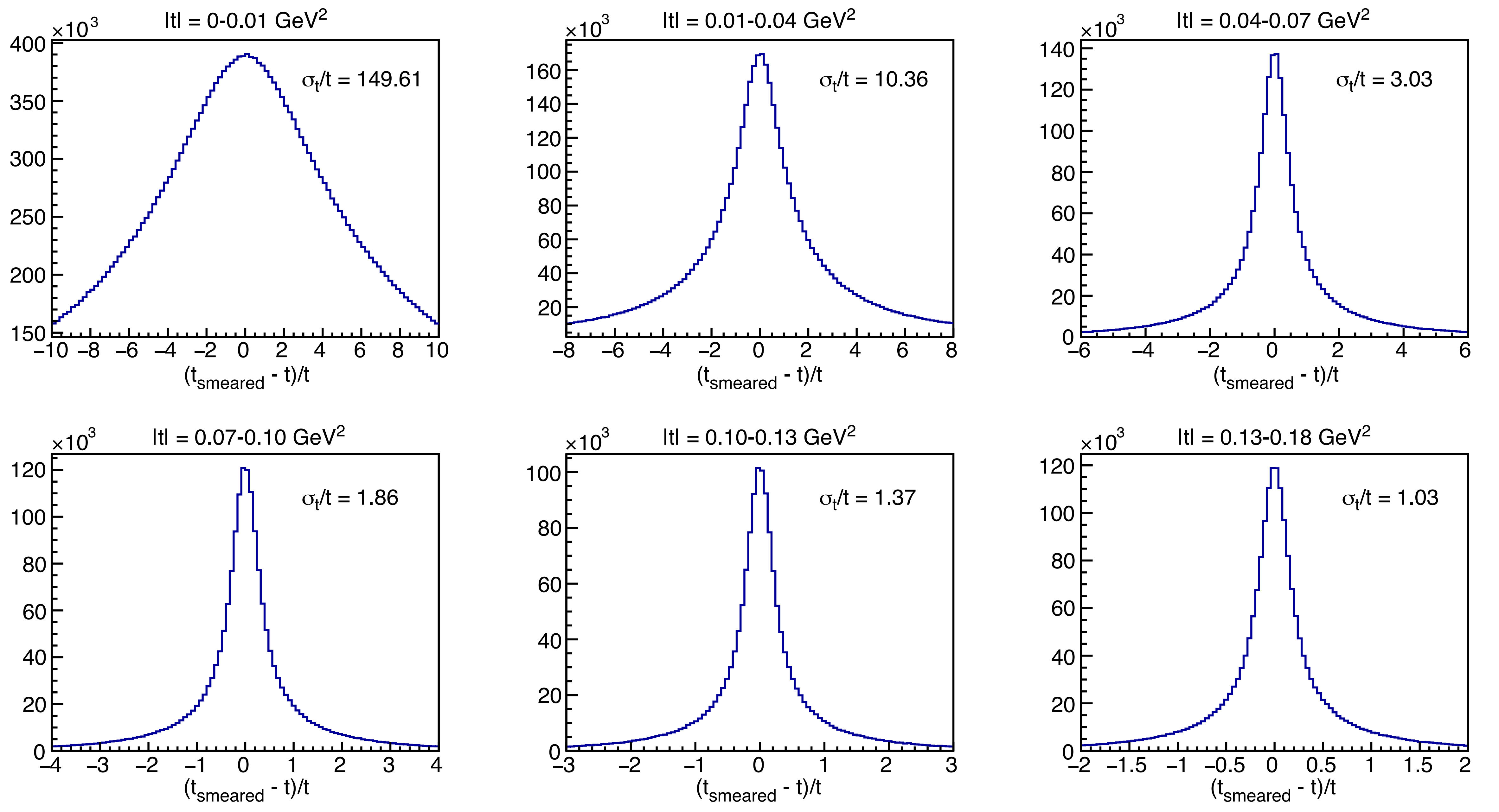
- In absence of any distortions (e.g. MC) this method delivers the true $t$

# Method E

Sensitivity to beam effects: $\sigma_t/t = (t_{\mathrm{measured}} - t_{\mathrm{true}})/t_{\mathrm{true}}$




EIC beam divergence

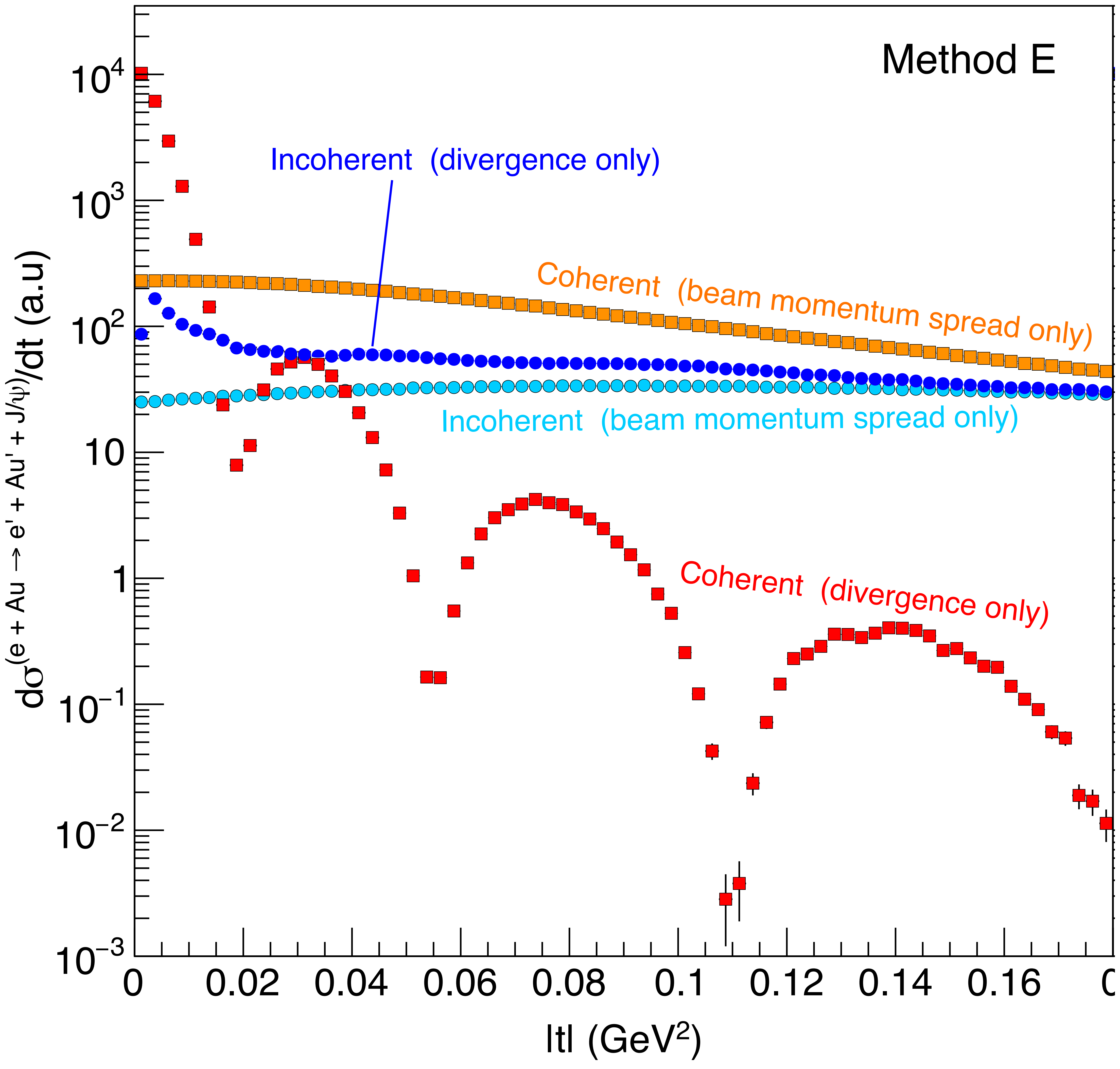



EIC beam momentum spread

- Beam divergence affects little: $\sigma_t/t$ ~ 6% to 0.5%
- Beam momentum spread is devastating: $\sigma_t/t$ ~ 15000% to 103%

# Method E

Effect on $d\sigma/dt$:




$$t = (p_V + p_{e'} - p_e)^2$$

Why does it fail:

Have to subtract large incoming and large outgoing momenta to get the "longitudinal part" of *t*. So a small error/smearing/inaccuracy in these has enormous effect on *t*

# Method A

Approximate method:

Rely only on the transverse momenta of the vector meson and the scattered electron ignoring all longitudinal momenta. Therefore beam momentum fluctuations do not enter the calculations. This method was extensively used at HERA in diffractive vector meson studies.

$$t = \left[\overrightarrow{p}_T(e') + \overrightarrow{p}_T(V)\right]^2$$

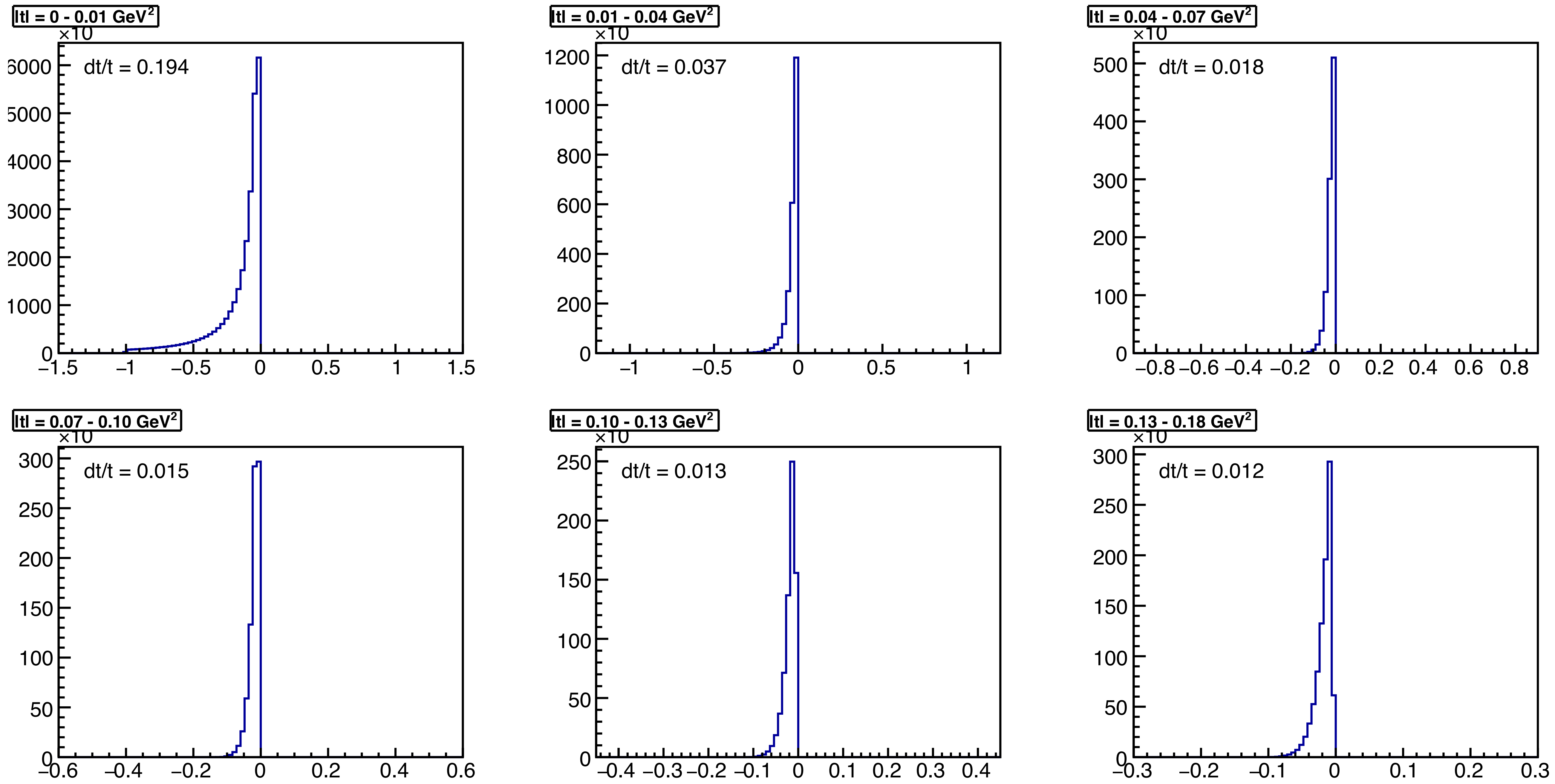
- This formula is valid only for small *t* and small $Q^2$. It also performs better for lighter vector mesons such as $\phi$ and $\rho$. In what follows we refer to this method as method A.

# Method A

Downside:

- Even absence of any distortions (e.g. MC) this method us underestimating the true *t*, although the difference is minimal
  - Offset is largest at $Q^2$ =1-2 GeV$^2$ with around 2% and decreases towards larger $Q^2$ to 1% at $Q^2$ = 9-10 GeV$^2$. The offset is absent for photoproduction ($Q^2$ < 0.01 GeV$^2$).

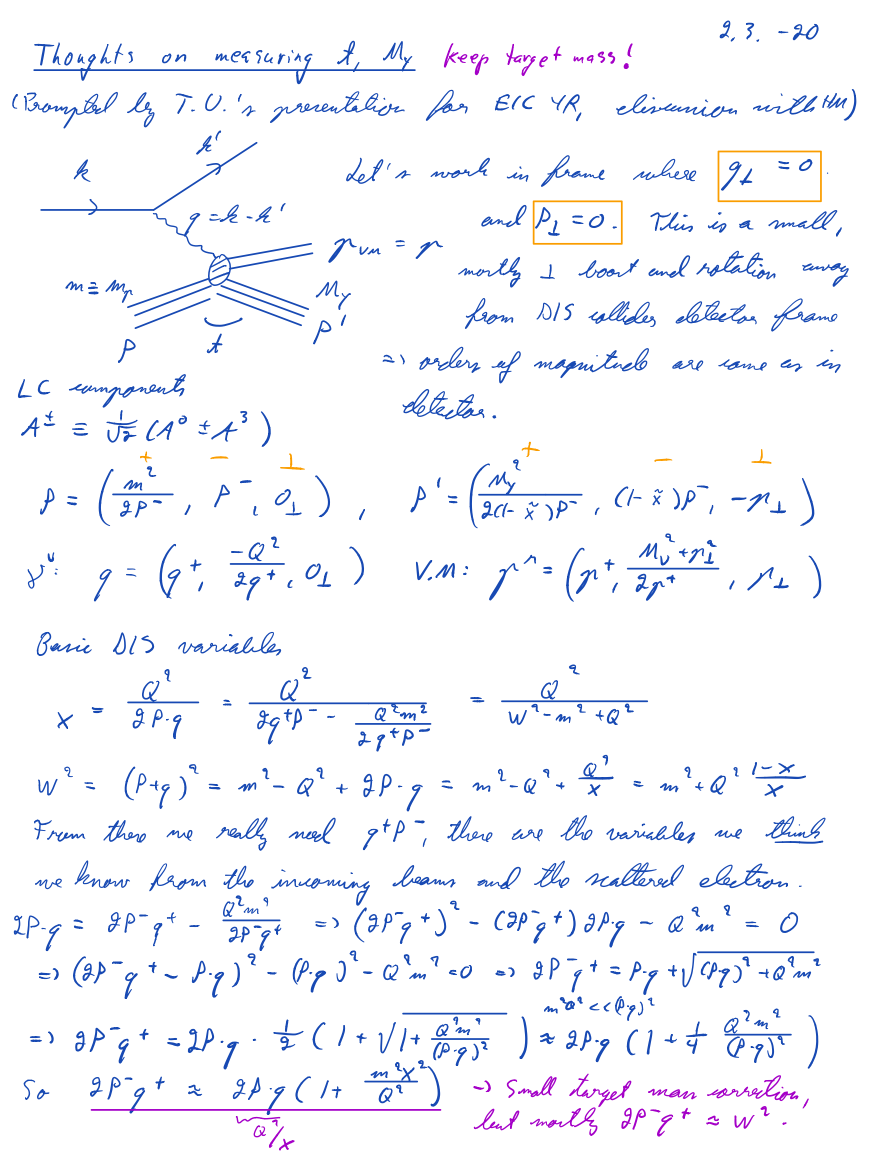
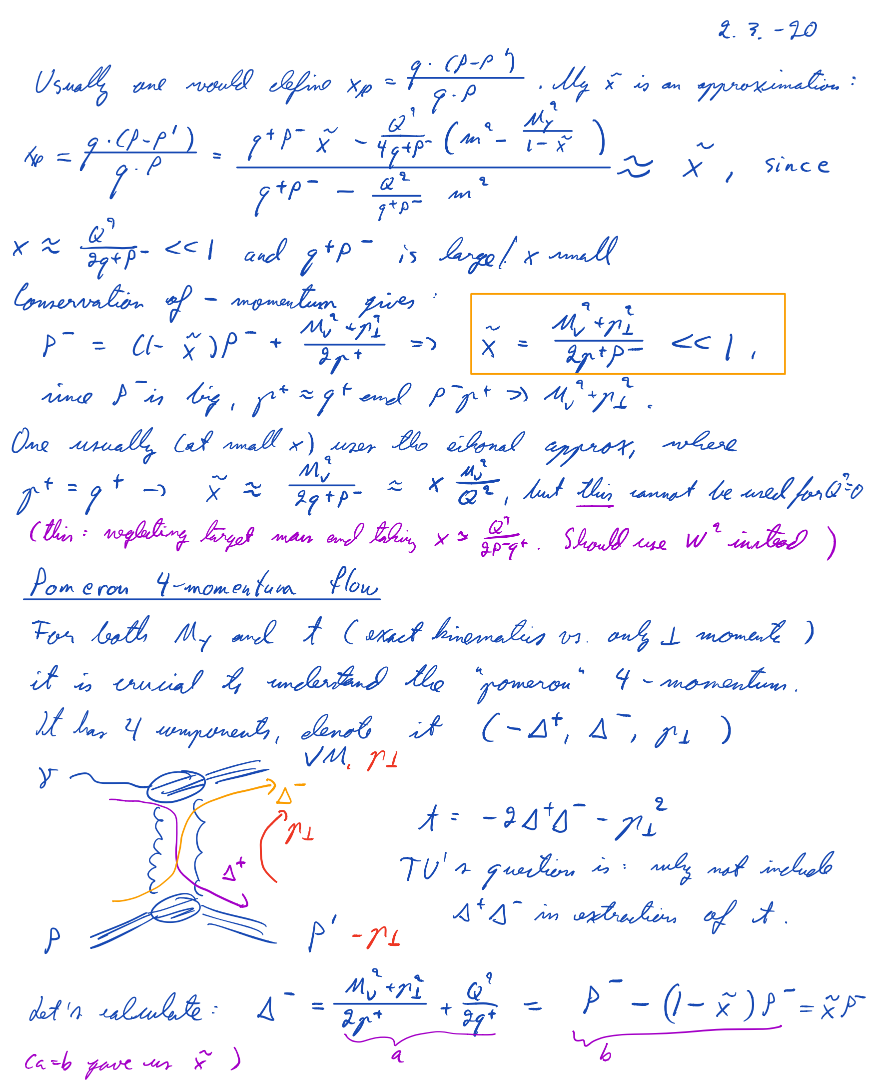
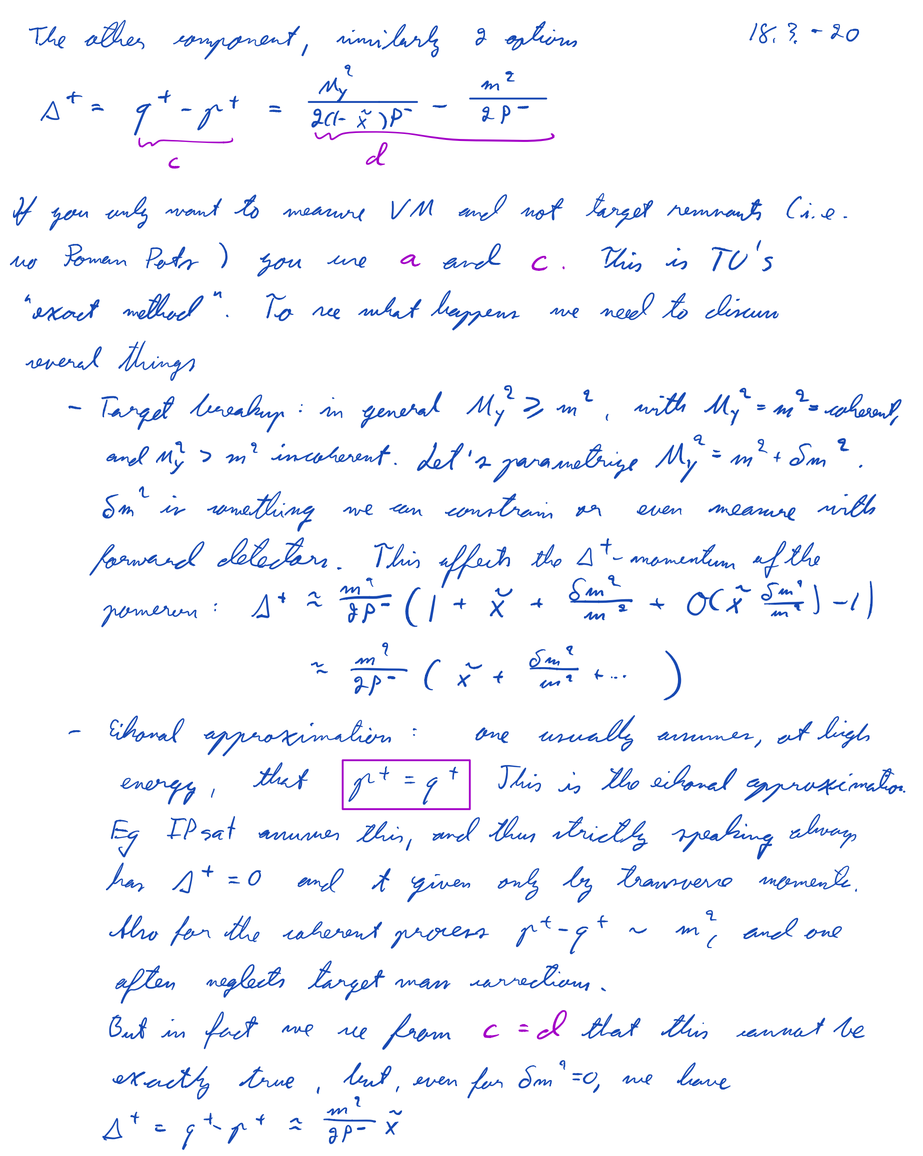
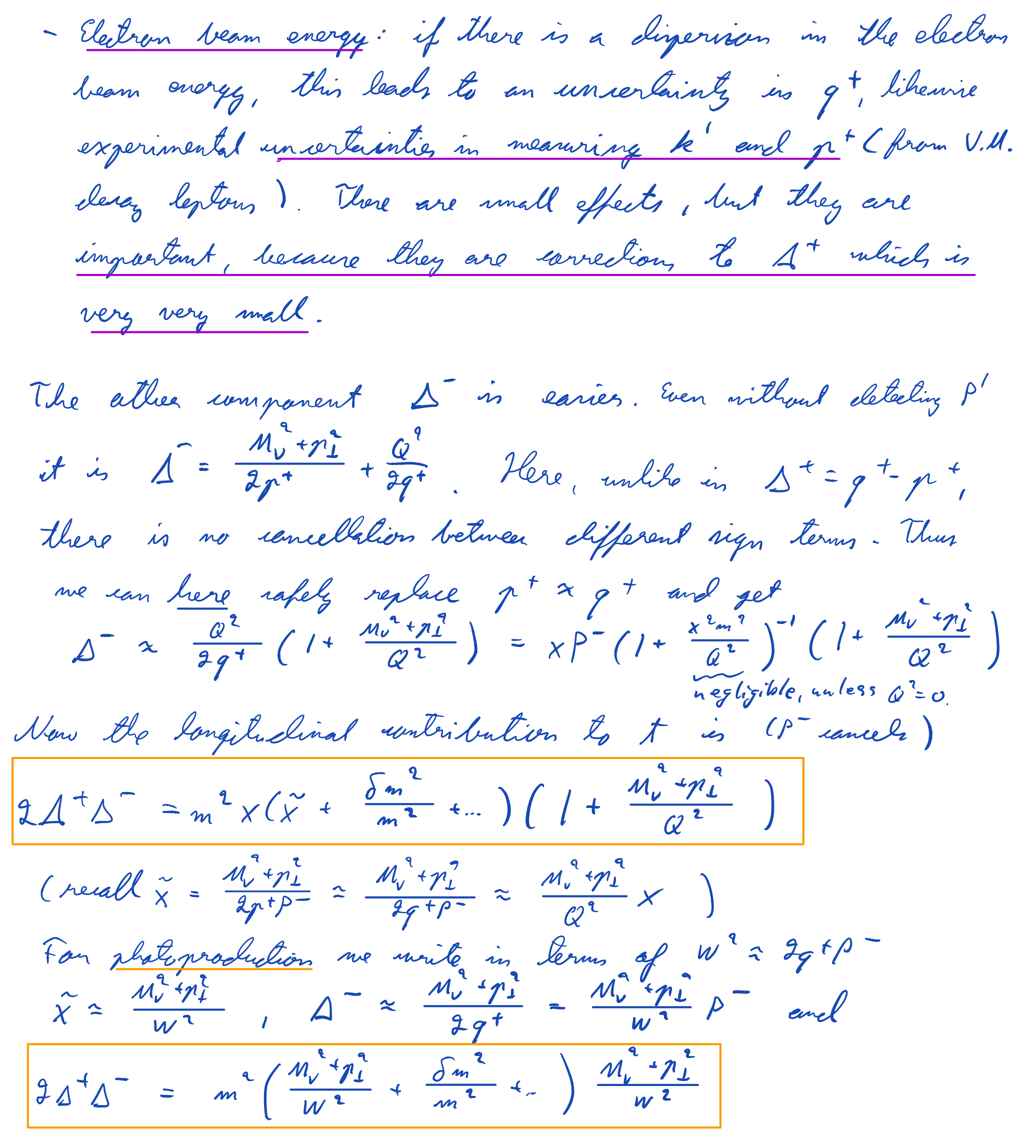
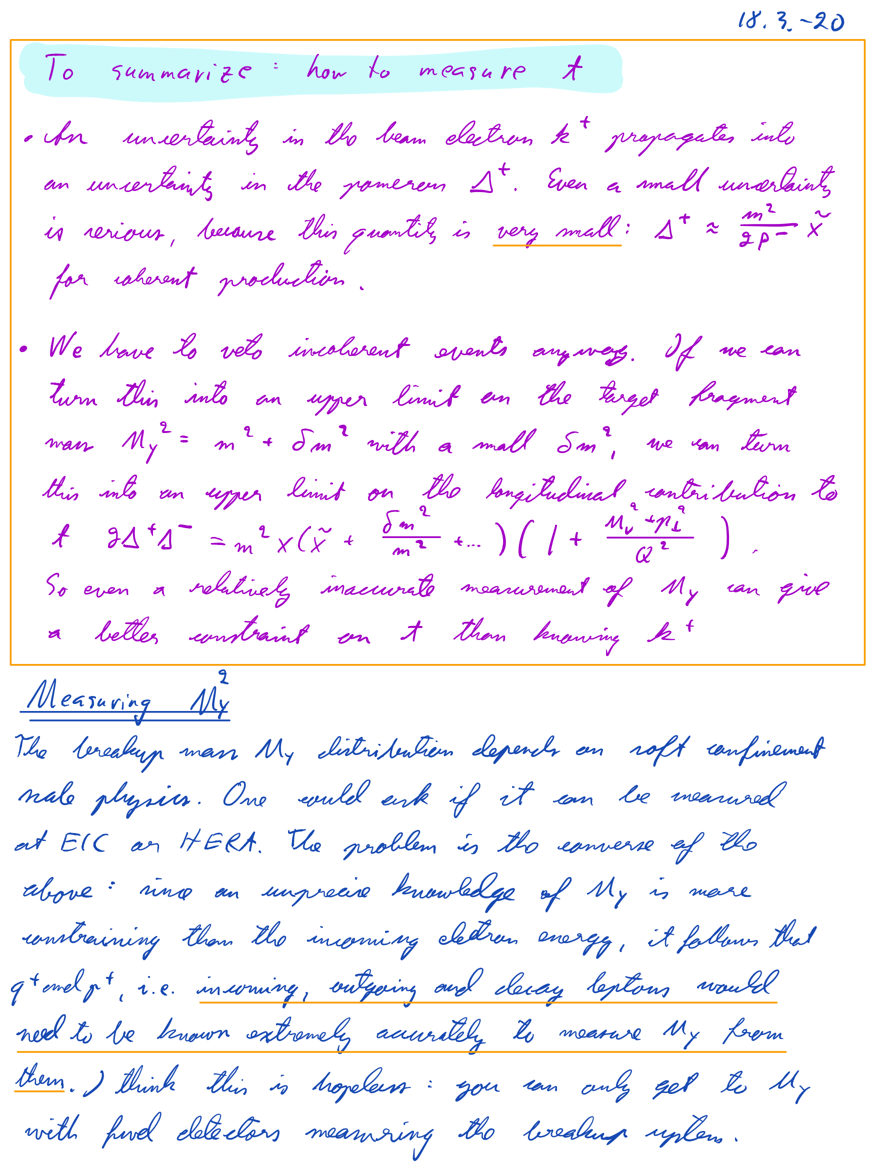
1< $Q^2$ < 10 GeV$^2$




- For 1< $Q^2$ < 10 GeV$^2$ and including the offset we obtain $\sigma_t/t$ resolutions (r.m.s.) of 10% t < 0.01 GeV$^2$, 1.8 % at t = 0.10 GeV$^2$, and 1.6% at t = 0.16 GeV$^2$. In photoproduction we observe no t smearing except at the lowest t (t < 0.01 GeV$^2$) of 1.3%.

# A New Approach: Method L

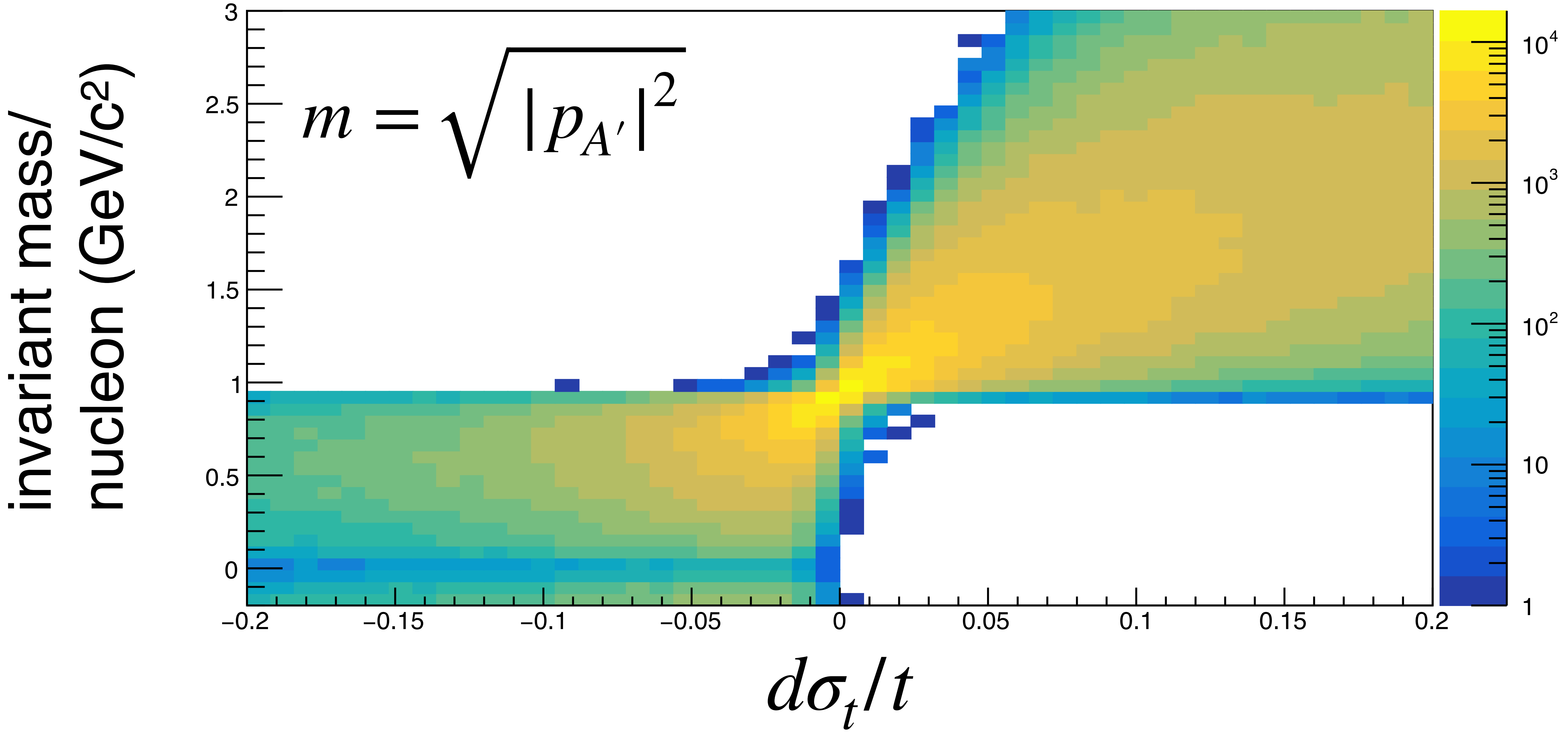
Triggered by T. Lappi (Notes from March 18, 2020)

Method is based on method E but overcomes several of its shortcomings. It is, however, strictly only applicable for coherent events. While in method E we are not using any information about the target nucleus at all, in method L we make use of the fact that the longitudinal momentum has to get transferred to the target due to 4-momentum conservation.

# Method L

- Calculate $A'$ 4-momentum: $p_{A'} = p_A - (p_V + p_{e'} - p_e)$ ← method E




- Any smearing of the longitudinal momentum difference will change the invariant mass of the target

- For coherent events this essentially indicates the failure of method E due to beam and detector smearing effects or that the event was mischaracterized as coherent $\Longrightarrow$ Important analysis/cross-check tool

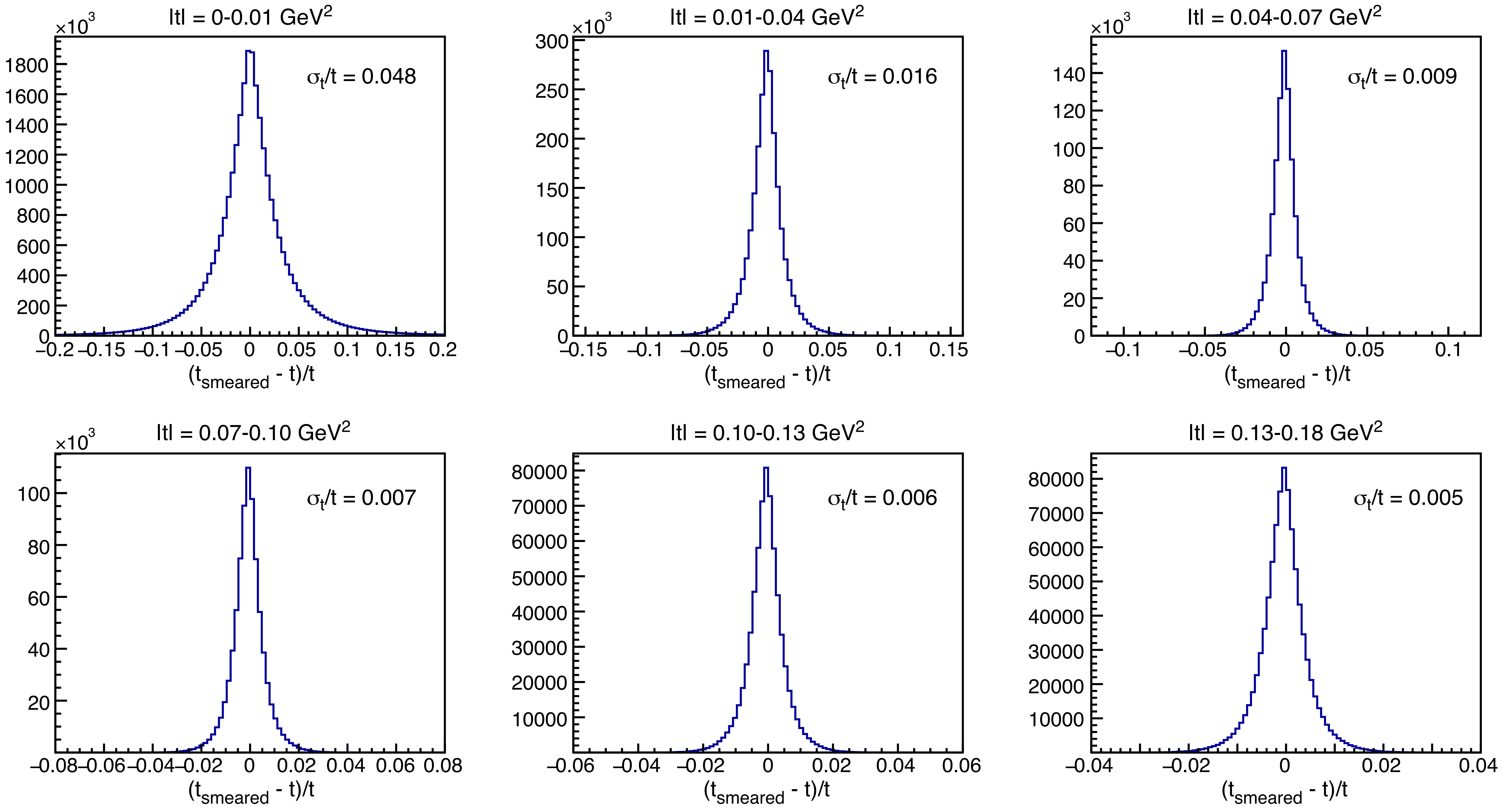
# Method L

- How the method works
  - Calculate p of outgoing A': $p_{A'} = p_A - (p_V + p_{e'} - p_e)$
  - Express and correct the outgoing nucleus in light cone variables:
    - $p^+_{A'} = E_{A'} + p_{z,A'}$
    - $p^2_{T,A'} = p^2_{x,A'} + p^2_{y,A'}$
    - $p^-_{A'} = (M^2_A + p^2_{T,A'})/p^+_{A'}$ where $p^-_{A'}$ is now modified by using the true mass $M^2_A$.
  - The corrected 4-momentum of the outgoing nuclei is now

$$p^{\text{corr}}_{A'} = \left[ p_{x,A'},\ p_{y,A'},\ (p^+_{A'} - p^-_{A'})/2,\ (p^+_{A'} + p^-_{A'})/2 \right]$$

  - In short, you are using the true invariant mass of the nucleus to compensate the smearing in the larger component of the electron 4-momentum by modifying $E_{A'}$ and $p_{z,A'}$ simultaneously.
  - Now simply: $t_{\text{corr}} = \left| p_A - p^{\text{corr}}_{A'} \right|^2$

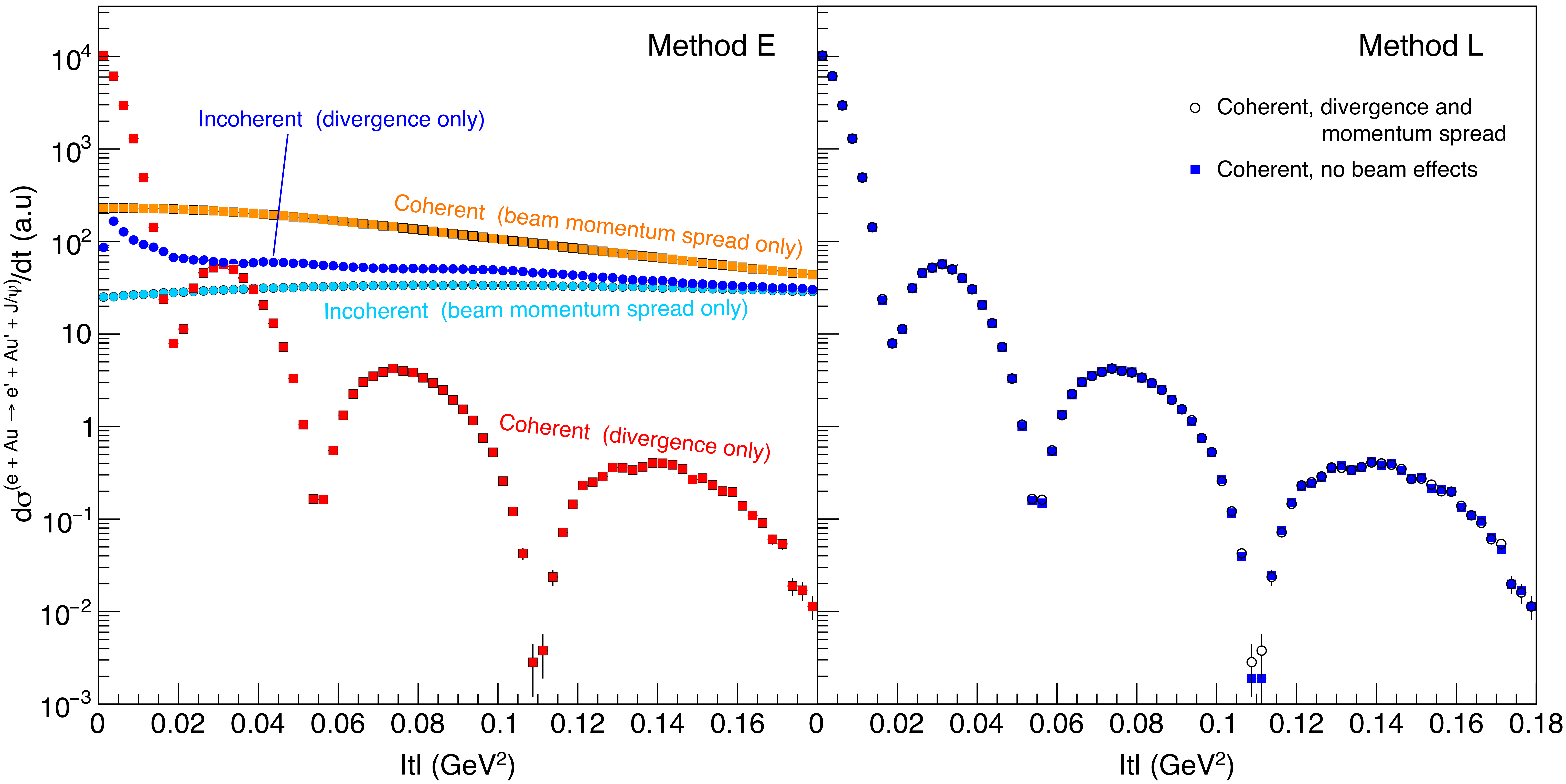
# Method L




All beam effects on

| method | effect | $t$-range (GeV$^2$) | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0-0.1 | 0.1-0.4 | 0.04 - 0.07 | 0.07 - 0.10 | 0.10 - 0.13 | 0.13 - 0.18 |
| E | beam divergence | 0.061 | 0.015 | 0.008 | 0.007 | 0.006 | 0.005 |
| E | beam mom. spread | 149.61 | 10.36 | 3.03 | 1.86 | 1.37 | 1.03 |
| L | divergence & mom. spread | 0.048 | 0.016 | 0.009 | 0.007 | 0.006 | 0.005 |

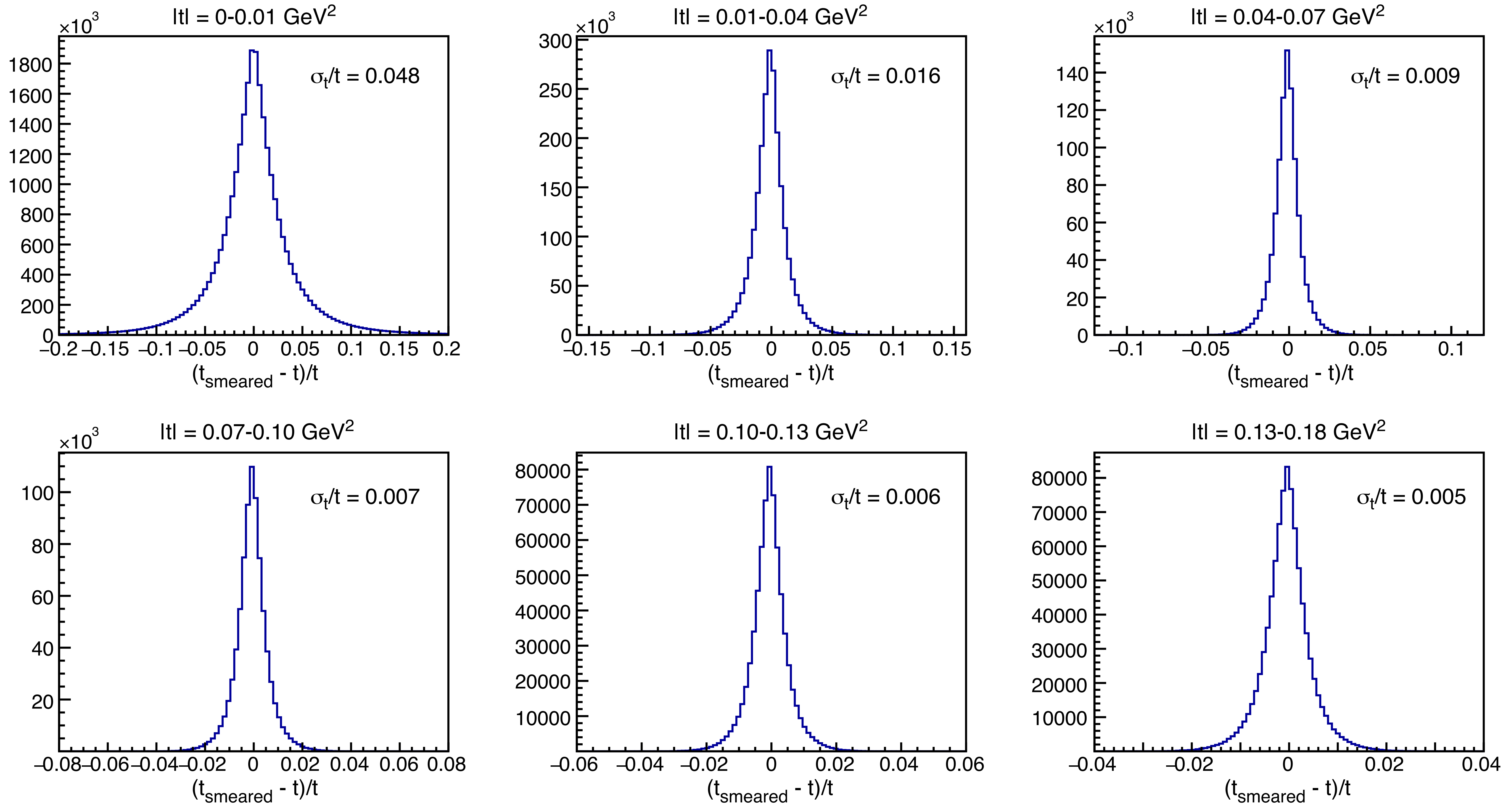
# Method L



Method E
Method L
Incoherent (divergence only)
Coherent (beam momentum spread only)
Incoherent (beam momentum spread only)
Coherent (divergence only)
Coherent, divergence and momentum spread
Coherent, no beam effects
dσ^(e + Au → e' + Au' + J/ψ)/dt (a.u)
|t| (GeV²)

# Method L

- Method L  with beam effects and nominal $p_T$ resolution




Method L and A  give similar t resolutions

| measurement precision term for barrel (backward) (%) | MS term for barrel (backward) (%) | $t$-range (GeV$^2$) | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0-0.1 | 0.1-0.4 | 0.04 - 0.07 | 0.07 - 0.10 | 0.10 - 0.13 | 0.13 - 0.18 |
| 0.05 (0.1) | 1.0 (2.0) | 4.58 | 0.45 | 0.25 | 0.19 | 0.16 | 0.14 |
| 0.1 (0.2) | 1.0 (2.0) | 4.71 | 0.46 | 0.25 | 0.20 | 0.17 | 0.14 |
| 0.025 (0.05) | 1.0 (2.0) | 4.54 | 0.45 | 0.24 | 0.19 | 0.16 | 0.14 |
| 0.05 (0.1) | 0.5 (2.0) | 3.53 | 0.38 | 0.21 | 0.17 | 0.14 | 0.12 |
| 0.05 (0.1) | 0.5 (1.0) | 1.29 | 0.22 | 0.12 | 0.10 | 0.08 | 0.07 |
| 0.05 (0.1) | 0.5 (0.5) | 0.78 | 0.16 | 0.09 | 0.07 | 0.06 | 0.05 |
| 0.05 (0.1) | 0.25 (0.5) | 0.49 | 0.12 | 0.07 | 0.05 | 0.05 | 0.04 |
| 0.05 (0.1) | 0.25 (0.25) | 0.36 | 0.09 | 0.05 | 0.04 | 0.04 | 0.03 |

# Summary

- Method E fails in the presence of beam momentum resolution
- Method L is an extension and a huge improvement
  - Only applicable for coherent events
  - We confirmed in simulations that all results obtained by method L for coherent processes are identical or very similar to that of method A in the studies discussed below.
- Ultimately, in the actual analysis once the EIC is realized, both methods (A and L) should be carefully compared and studied.
  - For coherent processes method L is likely the better choice as it does not rely on any approximations
  - For incoherent processes method A is the only option available