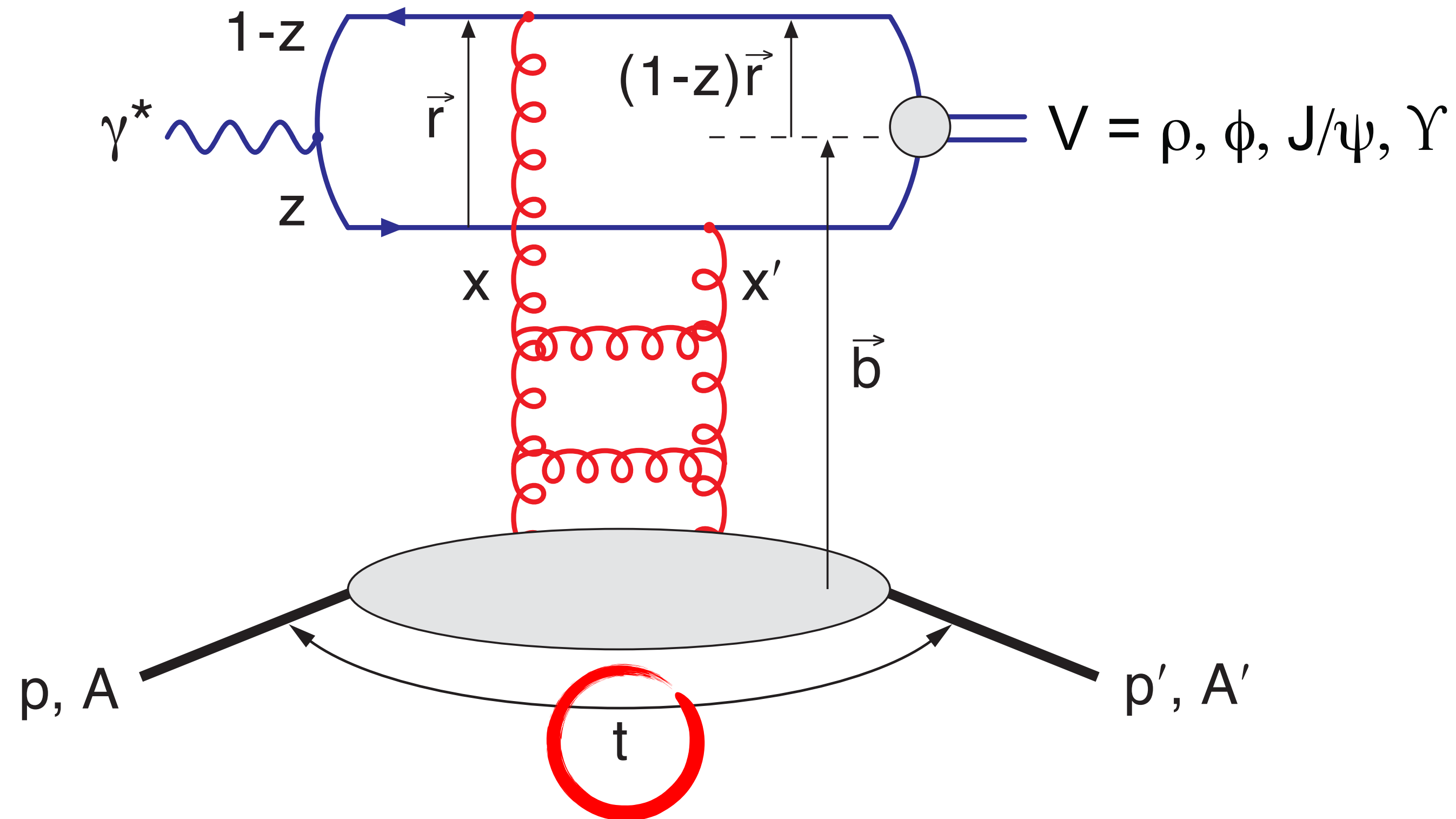


# On the Calculation of $t$



*Thomas Ullrich*  
*Exclusive WG*

*September 11, 2020*

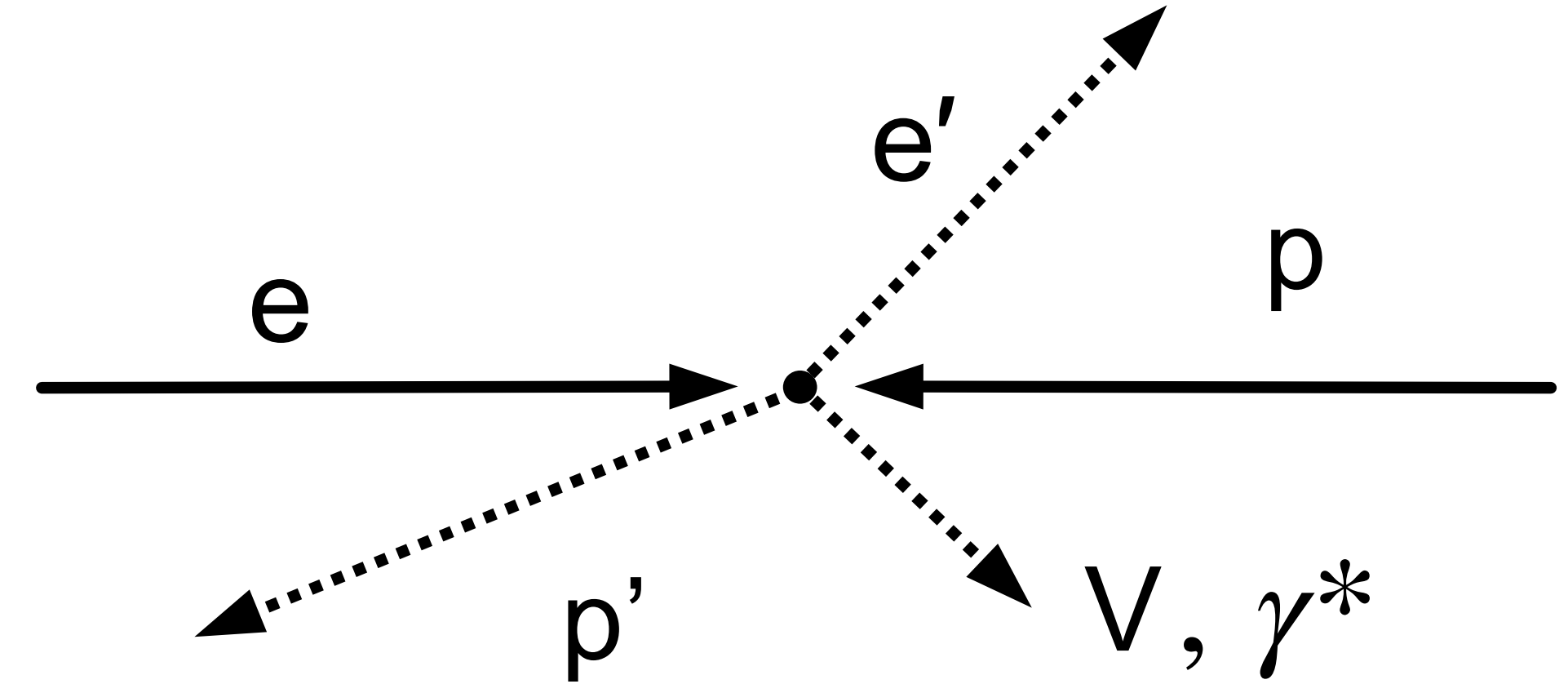
# e+p

---

In e+p we can follow the definition of  $t$ :

$$t = (p_A - p_{A'})^2$$

$p_A$  is known (beam) and  $p_{A'}$  is measured by forwards proton spectrometers (Roman Pots etc)



How well that ultimately works in terms of  $\sigma_t/t$  one has to see.

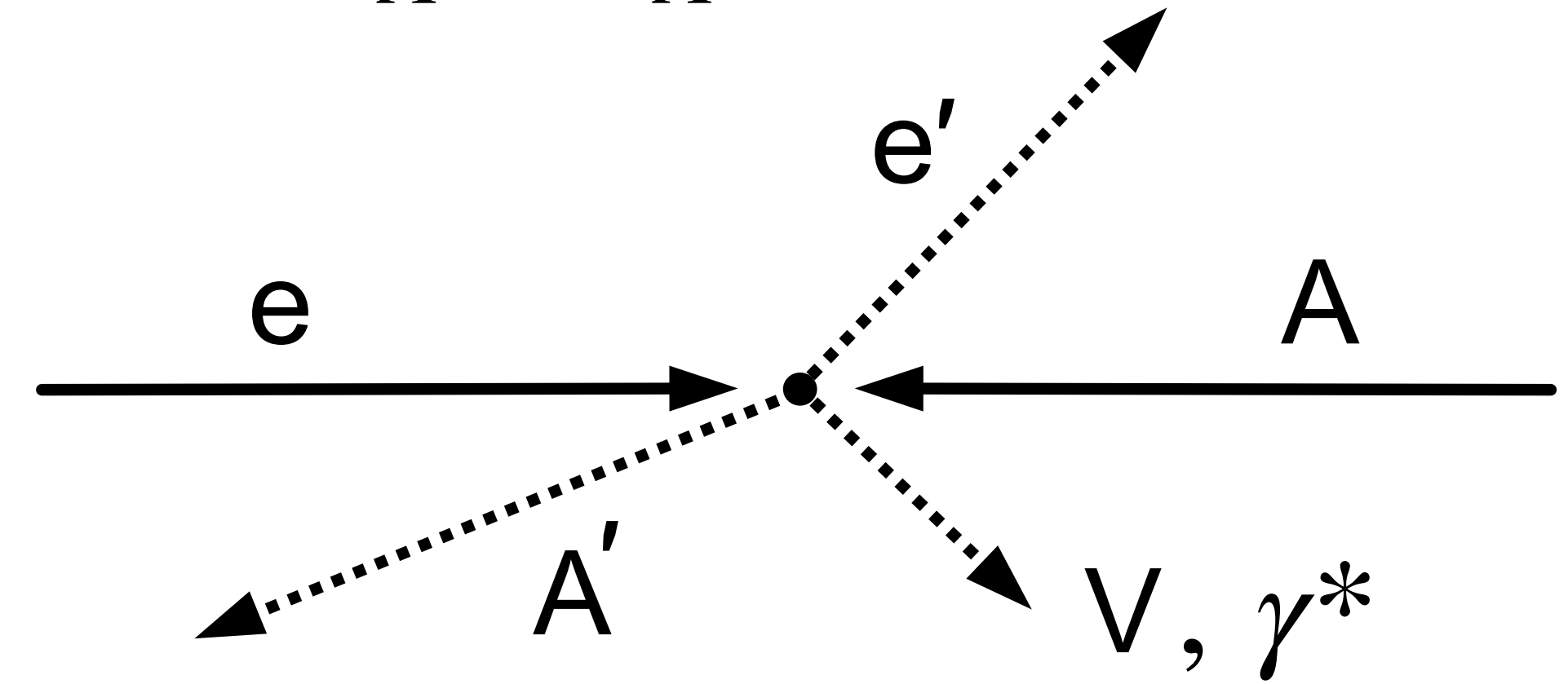
In any case alternative methods should be considered either to improve the precision or for systematic cross-checks.

# $e+A$

In  $e+A$  we *cannot* measure  $p_{A'}$ :

- coherent:  $t$  kick not big enough to get heavy ions out of the beam pipe
- incoherent: unlikely we can measure all fragments and reconstruct the whole ion and its momentum.

$$t = (p_A - p_{A'})^2$$



In general  $t$  cannot be measured w/o knowing  $p_{A'}$  except in exclusive vector meson production:

$$e + A \rightarrow e' + A' + V$$

since 4-momenta from  $e, A, e'$  and  $V$  are known

# Method E

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One can directly calculate  $t$  as:

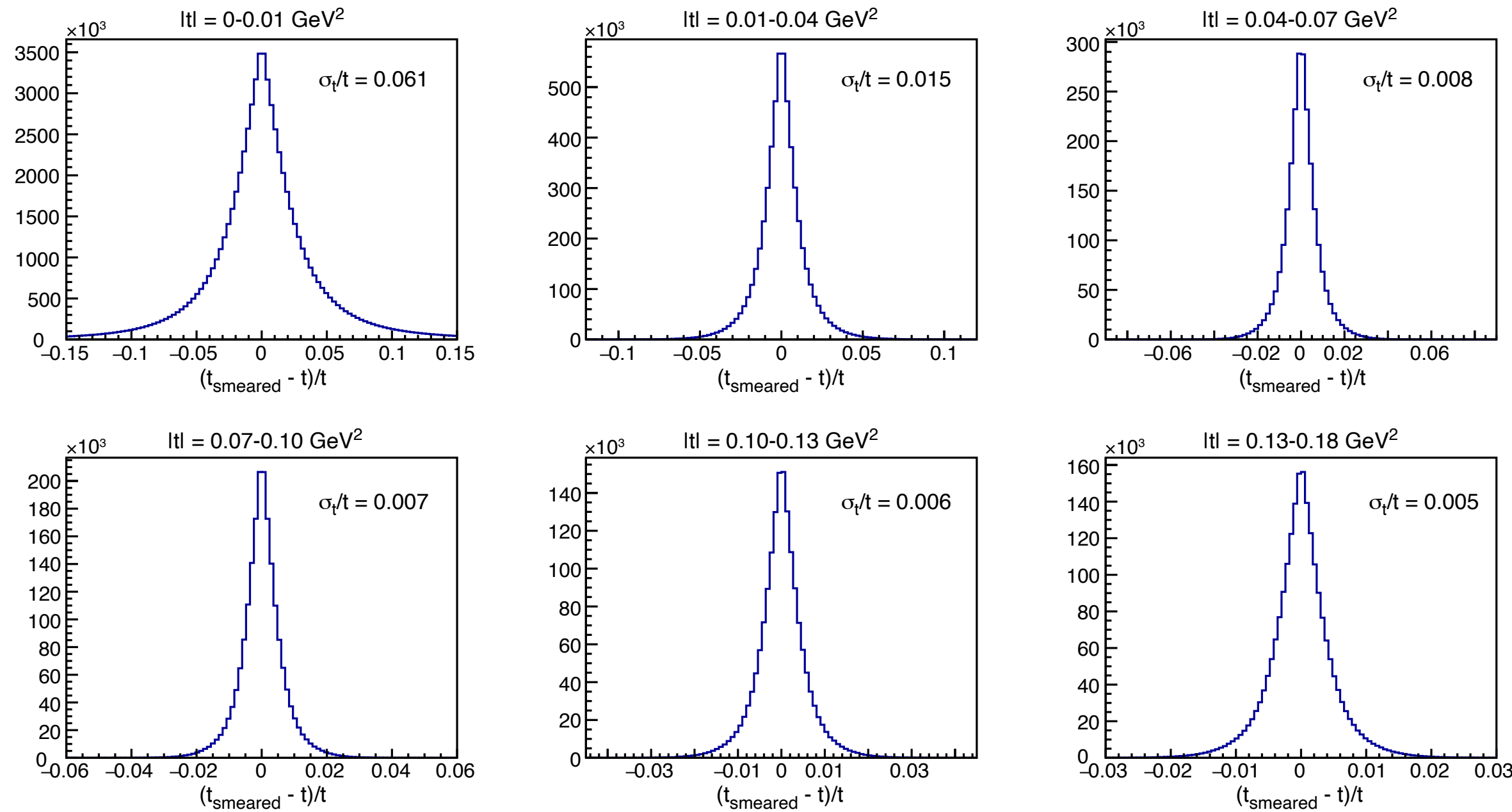
$$t = (p_A - p_{A'})^2 = (p_V + p_{e'} - p_e)^2$$

we call this method E (exact)

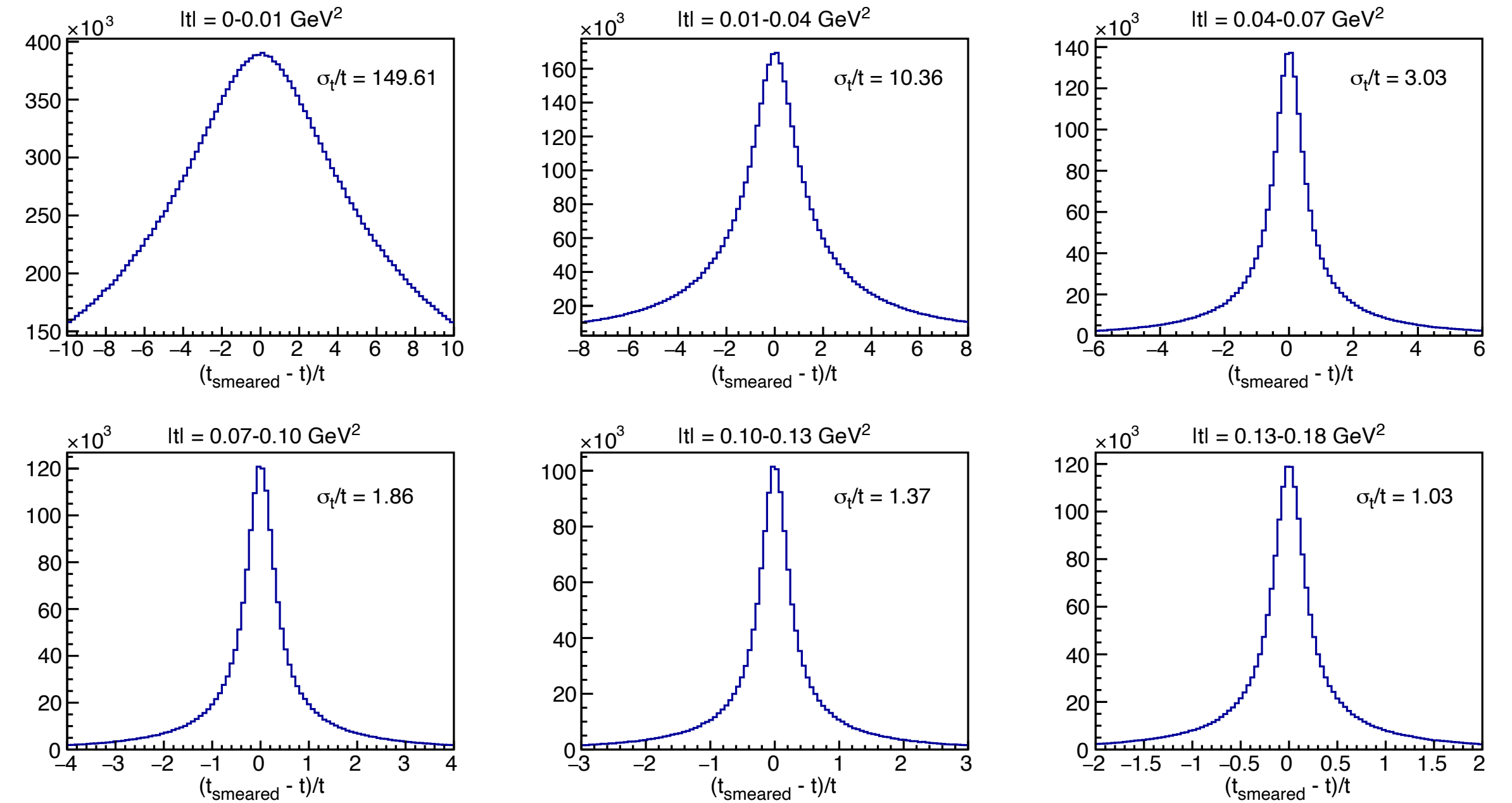
- In absence of any distortions (e.g. MC) this method delivers the true  $t$

# Method E

Sensitivity to beam effects:  $\sigma_t/t = (t_{\text{measured}} - t_{\text{true}})/t_{\text{true}}$



EIC beam divergence



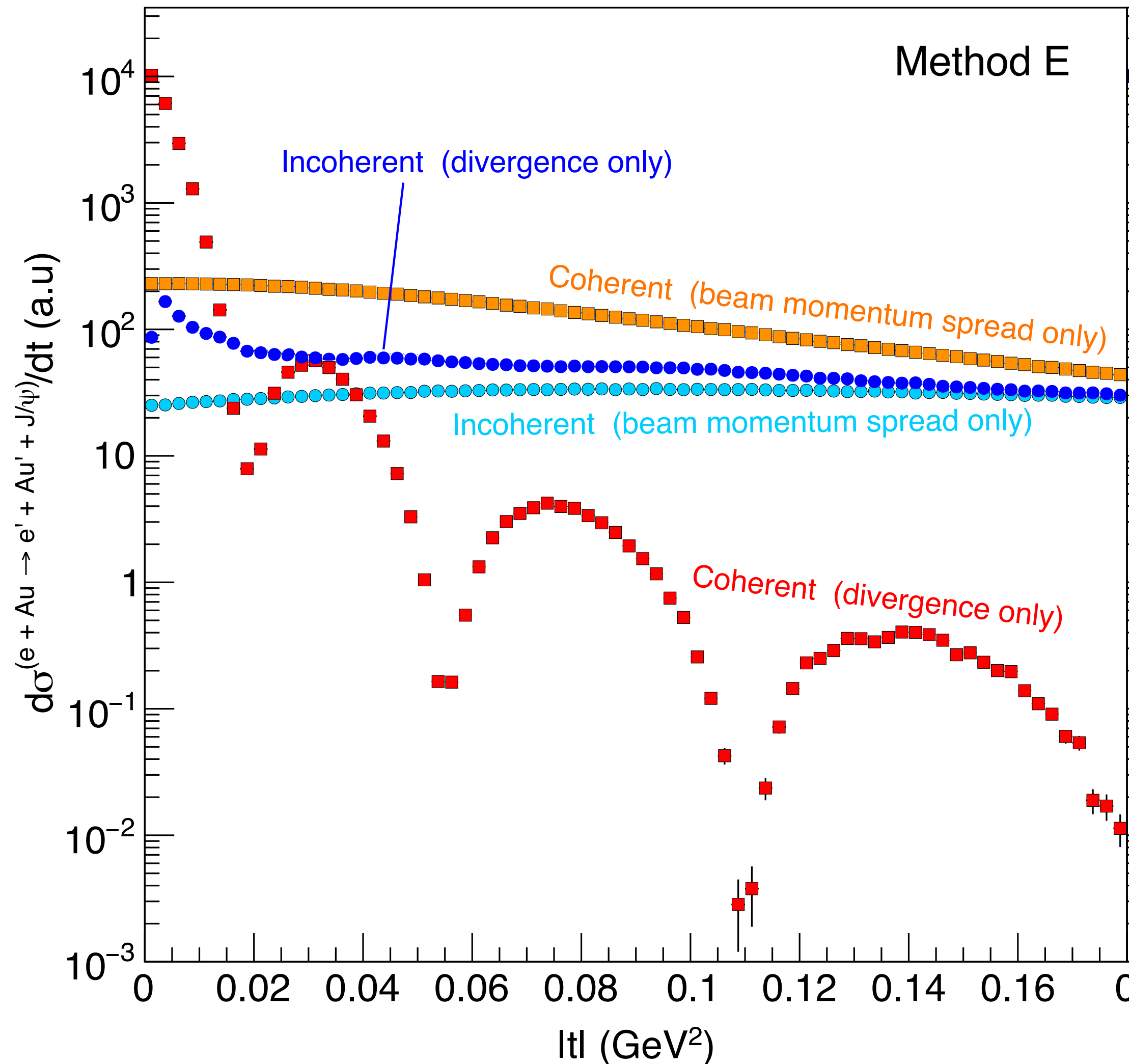
EIC beam momentum spread

- Beam divergence affects little:  $\sigma_t/t \sim 6\%$  to  $0.5\%$
- Beam momentum spread is devastating:  $\sigma_t/t \sim 15000\%$  to  $103\%$

# Method E

Effect on  $d\sigma/dt$ :

$$t = (p_V + p_{e'} - p_e)^2$$



Why does it fail:

Have to subtract large incoming and large outgoing momenta to get the "longitudinal part" of  $t$ . So a small error/smearing/inaccuracy in these has enormous effect on  $t$

# Method A

---

Approximate method:

Rely only on the transverse momenta of the vector meson and the scattered electron ignoring all longitudinal momenta. Therefore beam momentum fluctuations do not enter the calculations. This method was extensively used at HERA in diffractive vector meson studies.

$$t = \left[ \vec{p}_T(e') + \vec{p}_T(V) \right]^2$$

- This formula is valid only for small  $t$  and small  $Q^2$ . It also performs better for lighter vector mesons such as  $\phi$  and  $\rho$ . In what follows we refer to this method as method A.

# Method A

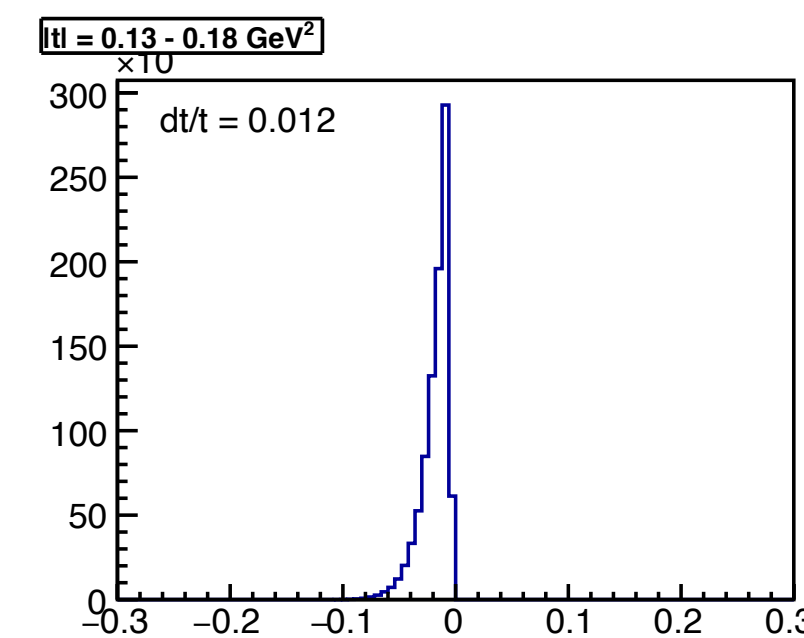
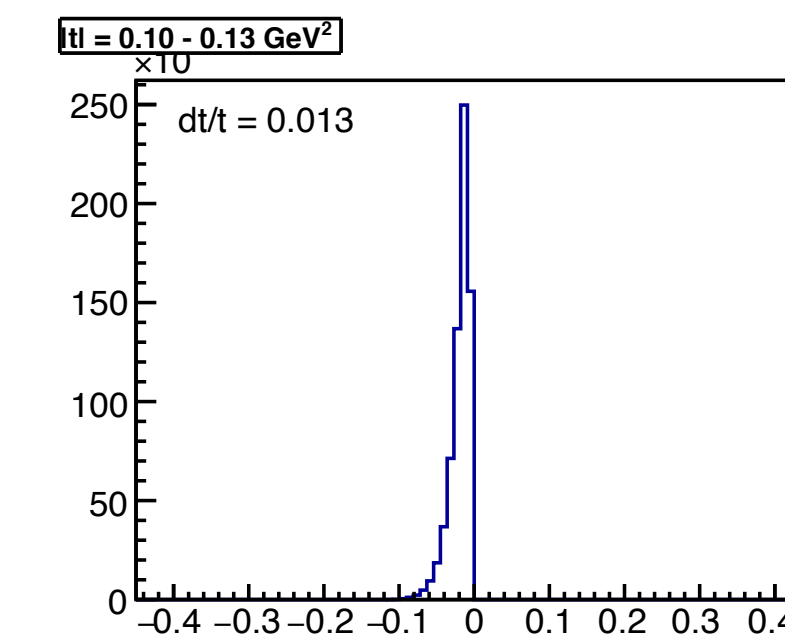
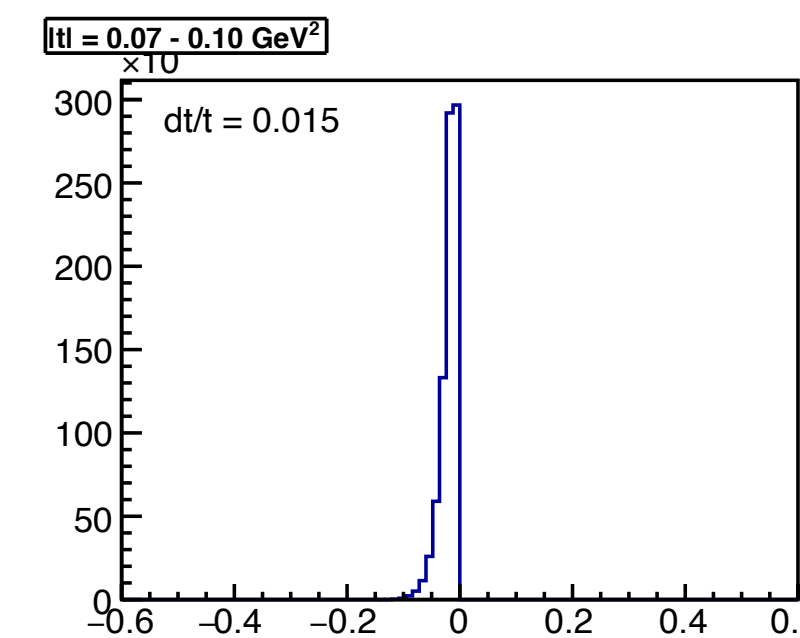
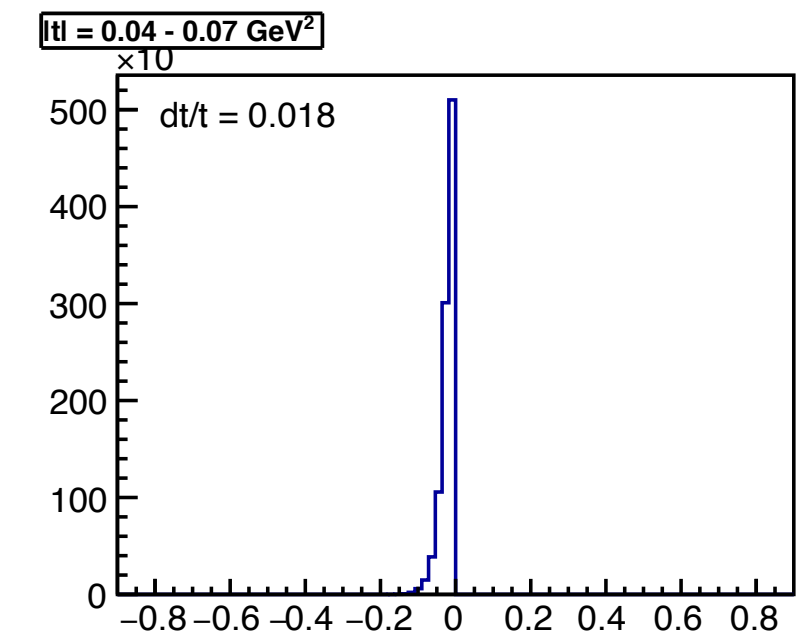
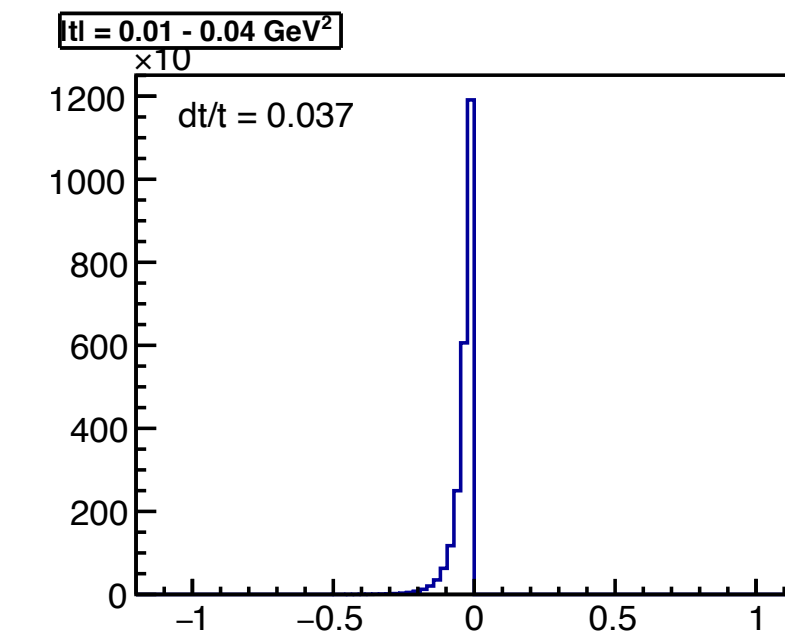
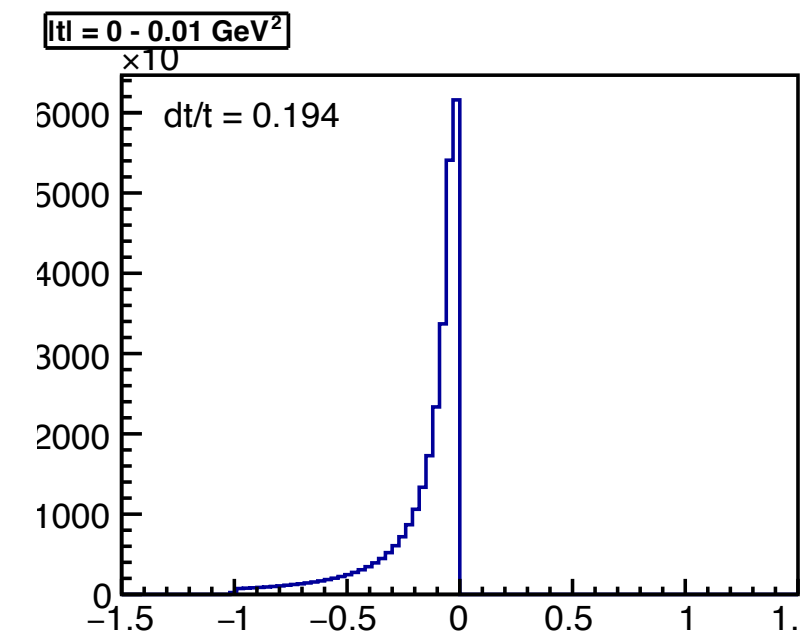
## Downside:

- Even absence of any distortions (e.g. MC) this method us underestimating the true  $t$ , although the difference is minimal

- ▶ Offset is largest at  $Q^2 = 1-2 \text{ GeV}^2$  with around 2% and decreases towards larger  $Q^2$  to 1% at  $Q^2 = 9-10 \text{ GeV}^2$ . The offset is absent for photoproduction ( $Q^2 < 0.01 \text{ GeV}^2$ ).

- ▶ For  $1 < Q^2 < 10 \text{ GeV}^2$  and including the offset we obtain  $\sigma_t/t$  resolutions (r.m.s.) of 10%  $t < 0.01 \text{ GeV}^2$ , 1.8 % at  $t = 0.10 \text{ GeV}^2$ , and 1.6% at  $t = 0.16 \text{ GeV}^2$ . In photoproduction we observe no  $t$  smearing except at the lowest  $t$  ( $t < 0.01 \text{ GeV}^2$ ) of 1.3%.

$$1 < Q^2 < 10 \text{ GeV}^2$$

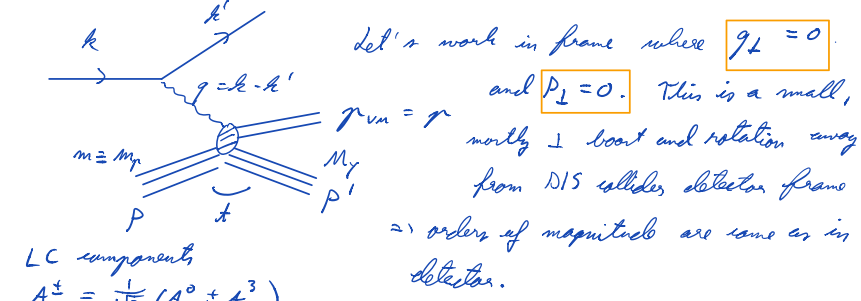




# A New Approach: Method L

Triggered by T. Lappi (Notes from March 18, 2020)

Thoughts on measuring  $M_Y$  keep forget-ness!  
(Brought by T.U.'s presentation for EIC 4R, discussion with M)



LC components  
 $A^\pm = \frac{1}{\sqrt{2}}(A^0 \pm A^3)$   
 $p = \begin{pmatrix} \frac{m^+}{2p^-} \\ p^- \\ 0_\perp \end{pmatrix}, p' = \begin{pmatrix} \frac{M_Y}{2(1-\tilde{x})p^-} \\ (1-\tilde{x})p^- \\ -p'_\perp \end{pmatrix}$   
 $q = \begin{pmatrix} q^+ \\ \frac{Q^2}{2q^+} \\ 0_\perp \end{pmatrix}, V.M.: p' = \begin{pmatrix} \frac{M_Y + p'_\perp}{2p^+} \\ p^+ \\ p'_\perp \end{pmatrix}$

Basic DIS variables  
 $x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2q^+p^-} = \frac{Q^2}{2p^+p^-} = \frac{Q^2}{W^2 - m^2 + Q^2}$   
 $W^2 = (p+q)^2 = m^2 - Q^2 + 2p \cdot q = m^2 - Q^2 + \frac{Q^2}{x} = m^2 + Q^2 \frac{1-x}{x}$

From this we really need  $q^+p^-$ , these are the variables we think we know from the incoming beam and the scattered electron.

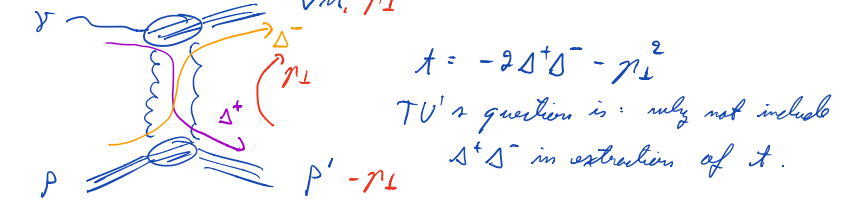
$2p \cdot q = 2p^+q^- = 2p^+ \frac{Q^2}{2p^+p^-} \Rightarrow (2p^+q^-)^2 = (2p^+p^-)^2 \frac{Q^4}{4p^+p^-} = 0$   
 $\Rightarrow (2p^+q^-)^2 - (2p^+p^-)^2 \frac{Q^4}{4p^+p^-} = 0 \Rightarrow 2p^+q^- = p^+p^- \sqrt{4p^+p^- - Q^2/m^2}$   
 $\Rightarrow 2p^+q^- = 2p^+p^- \cdot \frac{1}{2} \left(1 + \sqrt{1 + \frac{Q^2/m^2}{(p^+p^-)^2}}\right) \approx 2p^+p^- \left(1 + \frac{Q^2/m^2}{4(p^+p^-)^2}\right)$   
 So  $2p^+q^- \approx 2p^+p^- \left(1 + \frac{m^2 Q^2}{4(p^+p^-)^2}\right) \rightarrow$  Small target mass corrections, but really  $2p^+q^- \approx W^2$ .

Usually one would define  $x_p = \frac{q \cdot (P-p')}{q \cdot P}$ . My  $\tilde{x}$  is an approximation:  
 $x_p = \frac{q \cdot (P-p')}{q \cdot P} = \frac{q^+P^- - \frac{Q^2}{4q^+p^-} (m^2 - \frac{M_Y^2}{1-\tilde{x}})}{q^+P^- - \frac{Q^2}{4q^+p^-} m^2} \approx \tilde{x}$ , since

$x \approx \frac{Q^2}{2q^+p^-} \ll 1$  and  $q^+p^-$  is large,  $x$  small  
 Conservation of 4-momentum gives:  
 $P^- = (1-\tilde{x})P^- + \frac{M_Y^2 + p'_\perp}{2p^+} \Rightarrow \tilde{x} = \frac{M_Y^2 + p'_\perp}{2q^+p^-} \ll 1$ ,  
 since  $P^-$  is big,  $q^+ \approx q^+$  and  $p^- \approx p^- \Rightarrow M_Y^2 + p'_\perp$ .

One usually (at small  $x$ ) uses the eikonal approx, where  $q^+ = q^+ \rightarrow \tilde{x} \approx \frac{M_Y^2}{2q^+p^-} \approx x \frac{m^2}{Q^2}$ , but this cannot be used for  $Q^2=0$  (this: neglecting target mass and taking  $x \approx \frac{Q^2}{2q^+p^-}$ . Should use  $W^2$  instead)

Pomeron 4-momentum flow  
 For both  $M_Y$  and  $t$  (exact kinematics vs. only  $\perp$  moment)  
 it is crucial to understand the 'pomeron' 4-momentum.  
 It has 4 components, denote it  $(-\Delta^+, \Delta^-, p_\perp)$



Let's calculate:  $\Delta^- = \frac{M_Y^2 + p'_\perp}{2p^+} + \frac{Q^2}{2q^+} = p^- - (1-\tilde{x})p^- = \tilde{x}p^-$   
 (a+b gave us  $\tilde{x}$ )

The other component, similarly 2 options  
 $\Delta^+ = q^+ - p^+ = \frac{M_Y^2}{2(1-\tilde{x})p^-} - \frac{m^2}{2p^-}$

If you only want to measure VM and not target remnants (i.e. no Roman Pots) you use  $a$  and  $c$ . This is TU's 'exact method'. To see what happens we need to discuss several things

- Target breakup: in general  $M_Y^2 \geq m^2$ , with  $M_Y^2 = m^2 + \delta m^2$  coherent and  $M_Y^2 > m^2$  incoherent. Let's parametrize  $M_Y^2 = m^2 + \delta m^2$ .  $\delta m^2$  is something we can constrain or even measure with forward detectors. This affects the  $\Delta^+$ -momentum of the pomeron:  $\Delta^+ \approx \frac{m^2}{2p^-} \left(1 + \tilde{x} + \frac{\delta m^2}{m^2} + O(\tilde{x} \frac{\delta m^2}{m^2})\right) - 1 \approx \frac{m^2}{2p^-} \left(\tilde{x} + \frac{\delta m^2}{m^2} + \dots\right)$

- Eikonal approximation: one usually assumes, at high energy, that  $q^+ = q^+$ . This is the eikonal approximation. Eg IPsat assumes this, and thus strictly speaking always has  $\Delta^+ = 0$  and  $t$  given only by transverse momenta. Also for the coherent process  $q^+ - q^+ \sim m^2$ , and one often neglects target mass corrections. But in fact we use from  $c = d$  that this cannot be exactly true, but, even for  $\delta m^2 = 0$ , we have  $\Delta^+ = q^+ - p^+ \approx \frac{m^2}{2p^-} \tilde{x}$

- Electron beam energy: if there is a dispersion in the electron beam energy, this leads to an uncertainty in  $q^+$ , likewise experimental uncertainties in measuring  $k'$  and  $p'$  (from V.M. decay leptons). There are small effects, but they are important, because they are corrections to  $\Delta^+$  which is very very small.

The other component  $\Delta^-$  is easier. Even without detecting  $p'$  it is  $\Delta^- = \frac{M_Y^2 + p'_\perp}{2p^+} + \frac{Q^2}{2q^+}$ . Here, unlike in  $\Delta^+ = q^+ - p^+$ , there is no cancellation between different sign terms. Thus we can here safely replace  $q^+ \approx q^+$  and get  $\Delta^- \approx \frac{Q^2}{2q^+} \left(1 + \frac{M_Y^2 + p'_\perp}{Q^2}\right) = x p^- \left(1 + \frac{x m^2}{Q^2}\right)^{-1} \left(1 + \frac{M_Y^2 + p'_\perp}{Q^2}\right)$  (negligible, unless  $Q^2=0$ )

Now the longitudinal contribution to  $t$  is ( $p^-$  cancels)  
 $2\Delta^+\Delta^- = m^2 x \left(\tilde{x} + \frac{\delta m^2}{m^2} + \dots\right) \left(1 + \frac{M_Y^2 + p'_\perp}{Q^2}\right)$

(recall  $\tilde{x} = \frac{M_Y^2 + p'_\perp}{2q^+p^-} \approx \frac{M_Y^2 + p'_\perp}{2q^+p^-} \approx \frac{M_Y^2 + p'_\perp}{Q^2} x$ )

For photoproduction we write in terms of  $W^2 \approx 2q^+p^-$   
 $\tilde{x} \approx \frac{M_Y^2 + p'_\perp}{W^2}, \Delta^- \approx \frac{M_Y^2 + p'_\perp}{2q^+} = \frac{M_Y^2 + p'_\perp}{W^2} p^-$  and  
 $2\Delta^+\Delta^- = m^2 \left(\frac{M_Y^2 + p'_\perp}{W^2} + \frac{\delta m^2}{m^2} + \dots\right) \frac{M_Y^2 + p'_\perp}{W^2}$

To summarize: how to measure  $t$

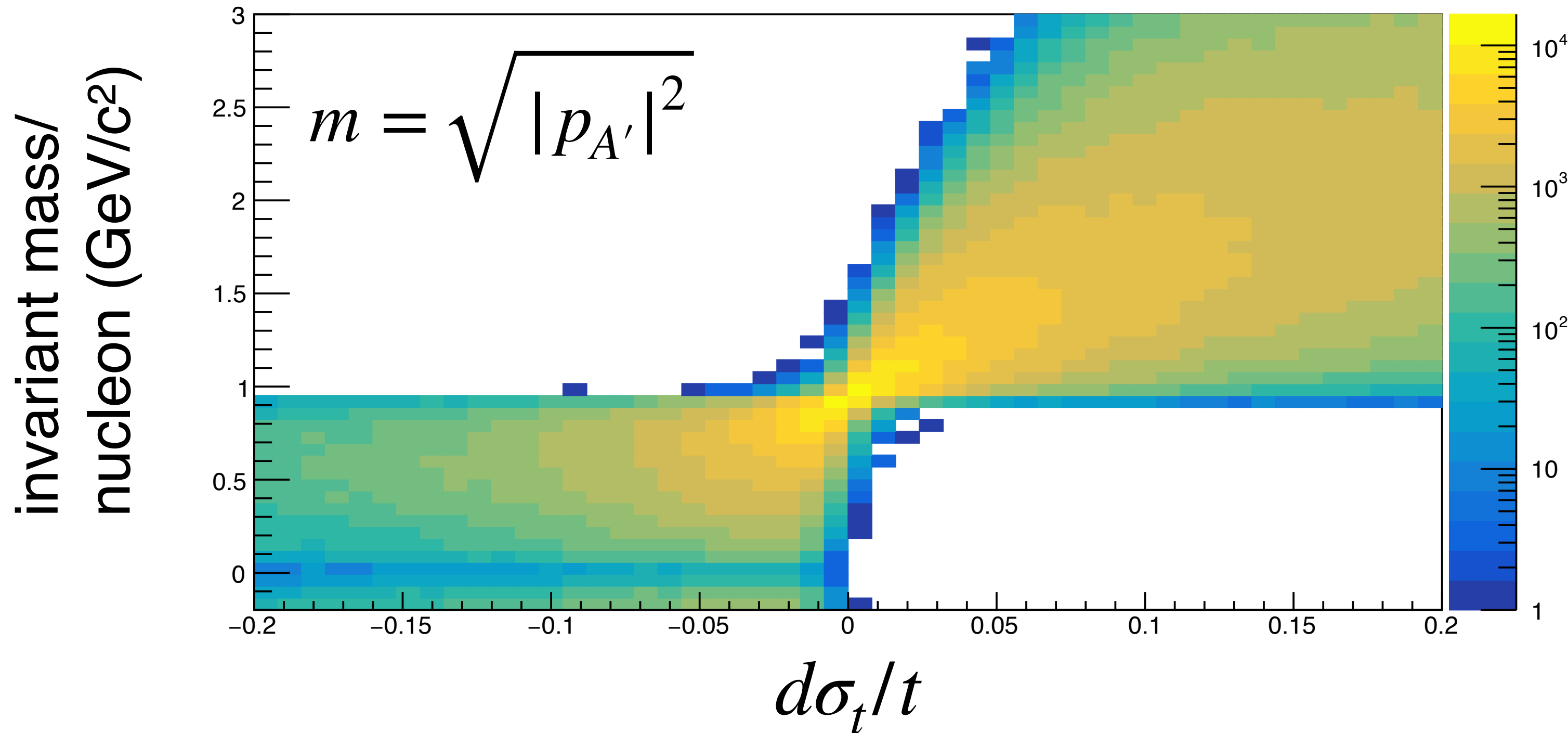
- An uncertainty in the beam electron  $k^+$  propagates into an uncertainty in the pomeron  $\Delta^+$ . Even a small uncertainty is serious, because this quantity is very small:  $\Delta^+ \approx \frac{m^2}{2p^-} \tilde{x}$  for coherent production.
- We have to veto incoherent events anyway. If we can turn this into an upper limit on the target fragment mass  $M_Y^2 = m^2 + \delta m^2$  with a small  $\delta m^2$ , we can turn this into an upper limit on the longitudinal contribution to  $t$ :  $2\Delta^+\Delta^- = m^2 x \left(\tilde{x} + \frac{\delta m^2}{m^2} + \dots\right) \left(1 + \frac{M_Y^2 + p'_\perp}{Q^2}\right)$ . So even a relatively inaccurate measurement of  $M_Y$  can give a better constraint on  $t$  than knowing  $k^+$

Measuring  $M_Y$   
 The breakup mass  $M_Y$  distribution depends on soft confinement scale physics. One would ask if it can be measured at EIC or HERA. The problem is the converse of the above: since an unprecise knowledge of  $M_Y$  is more constraining than the incoming electron energy, it follows that  $q^+$  and  $p^+$ , i.e. incoming, outgoing and decay leptons would need to be known extremely accurately to measure  $M_Y$  from them. I think this is hopeless: you can only get to  $M_Y$  with final detectors measuring the breakup system.

Method is based on method E but overcomes several of its shortcomings. It is, however, strictly **only applicable for coherent events**. While in method E we are not using any information about the target nucleus at all, in method L we make use of the fact that the longitudinal momentum has to get transferred to the target due to 4-momentum conservation.

# Method L

- Calculate  $A'$  4-momentum:  $p_{A'} = p_A - (p_V + p_{e'} - p_e)$



- Any smearing of the longitudinal momentum difference will change the invariant mass of the target
- For coherent events this essentially indicates the failure of method E due to beam and detector smearing effects or that the event was mischaracterized as coherent  $\implies$  **Important analysis/cross-check tool**

# Method L

- How the method works

- ▶ Calculate p of outgoing A':  $p_{A'} = p_A - (p_V + p_{e'} - p_e)$

- ▶ Express and correct the outgoing nucleus in light cone variables:

- ◉  $p_{A'}^+ = E_{A'} + p_{z,A'}$

- ◉  $p_{T,A'}^2 = p_{x,A'}^2 + p_{y,A'}^2$

- ◉  $p_{A'}^- = (M_A^2 + p_{T,A'}^2) / p_{A'}^+$  where  $p_{A'}^-$  is now modified by using the true mass  $M_A^2$ .

- ▶ The corrected 4-momentum of the outgoing nuclei is now

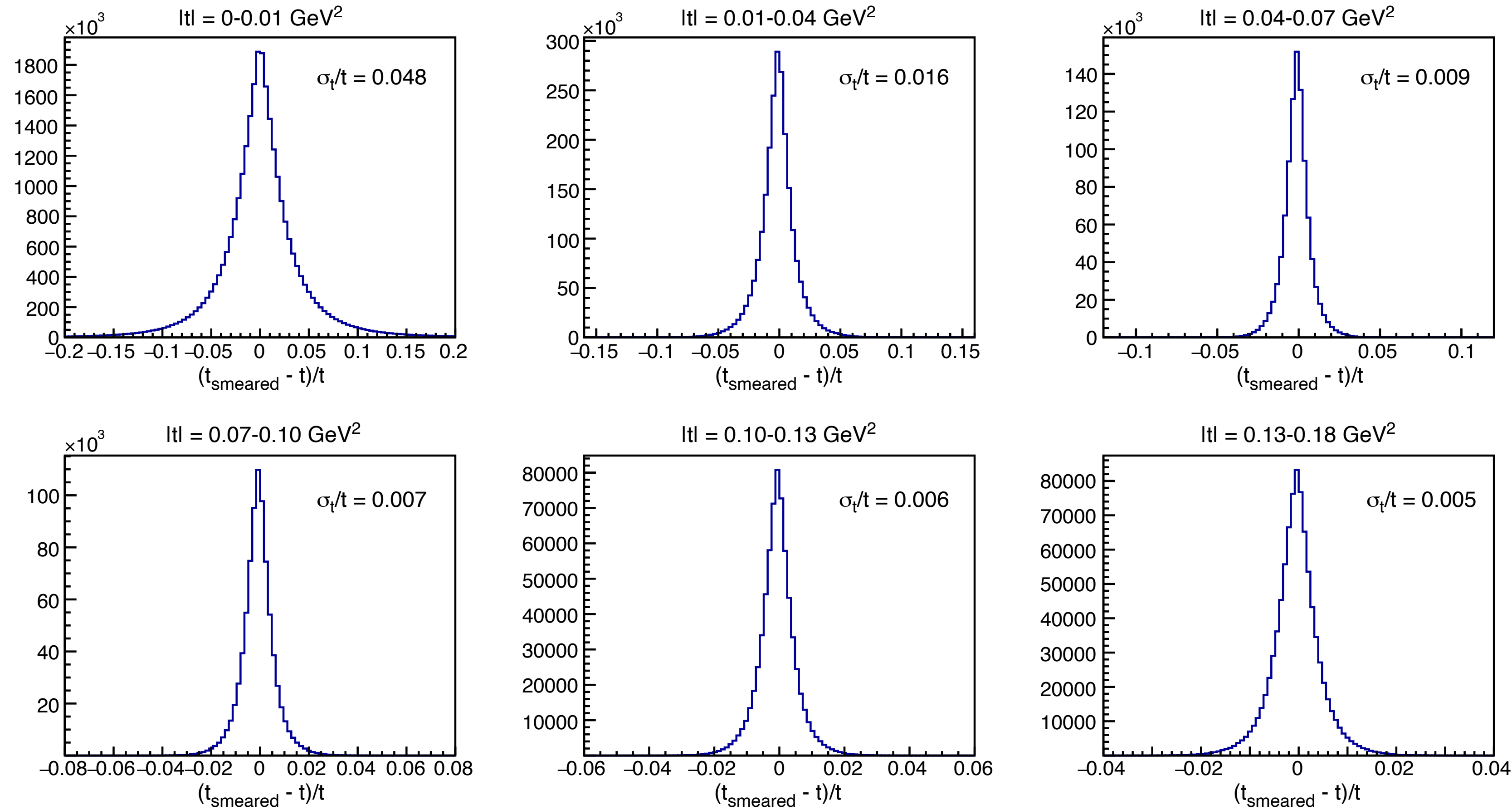
$$p_{A'}^{\text{corr}} = \left[ p_{x,A'}, p_{y,A'}, (p_{A'}^+ - p_{A'}^-) / 2, (p_{A'}^+ + p_{A'}^-) / 2 \right]$$

- ▶ In short, you are using the true invariant mass of the nucleus to compensate the smearing in the larger component of the electron 4-momentum by **modifying  $E_{A'}$  and  $p_{z,A'}$  simultaneously**.

- ▶ Now simply:  $t_{\text{corr}} = \left| p_A - p_{A'}^{\text{corr}} \right|^2$

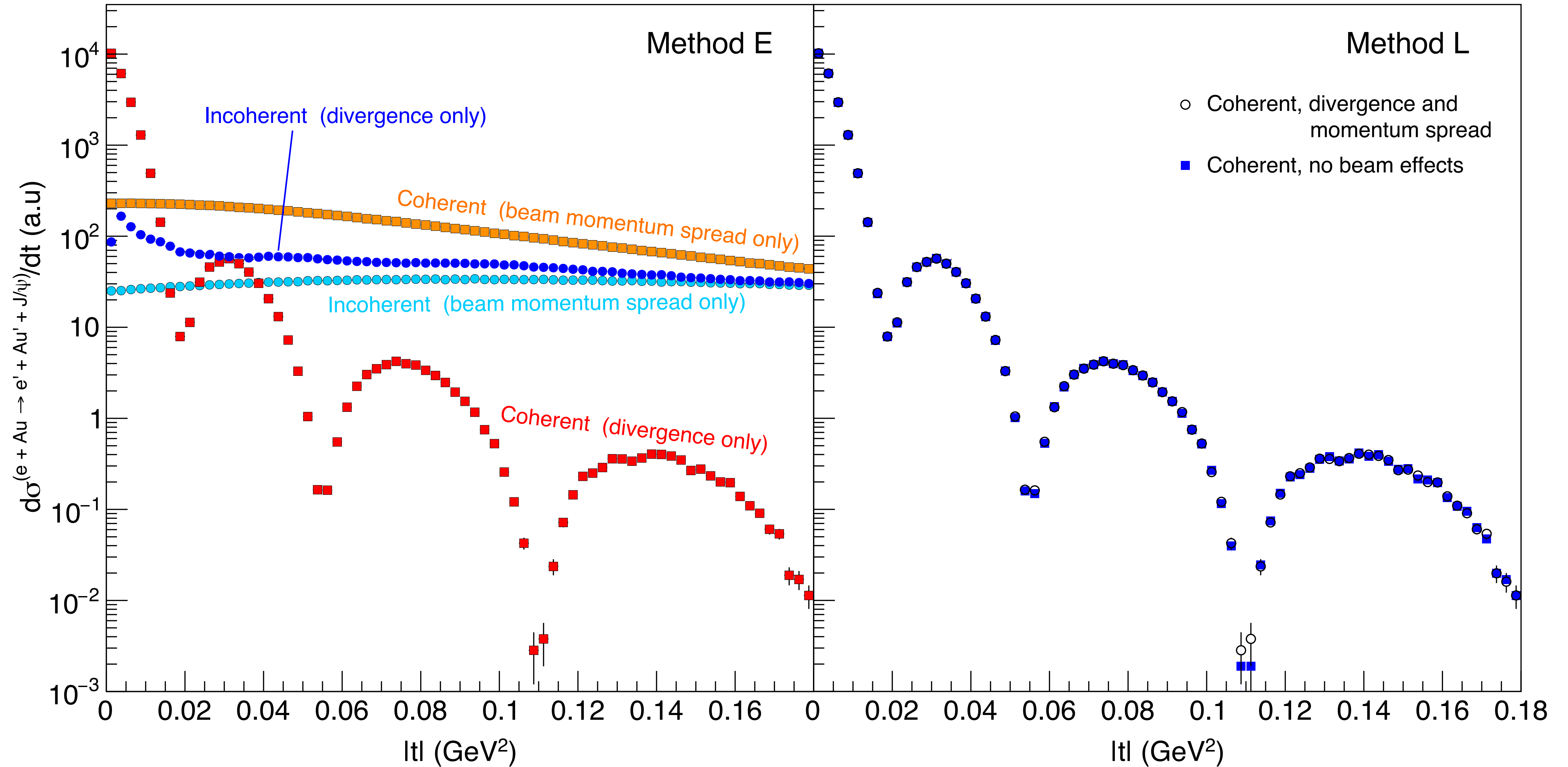
# Method L

All beam effects on



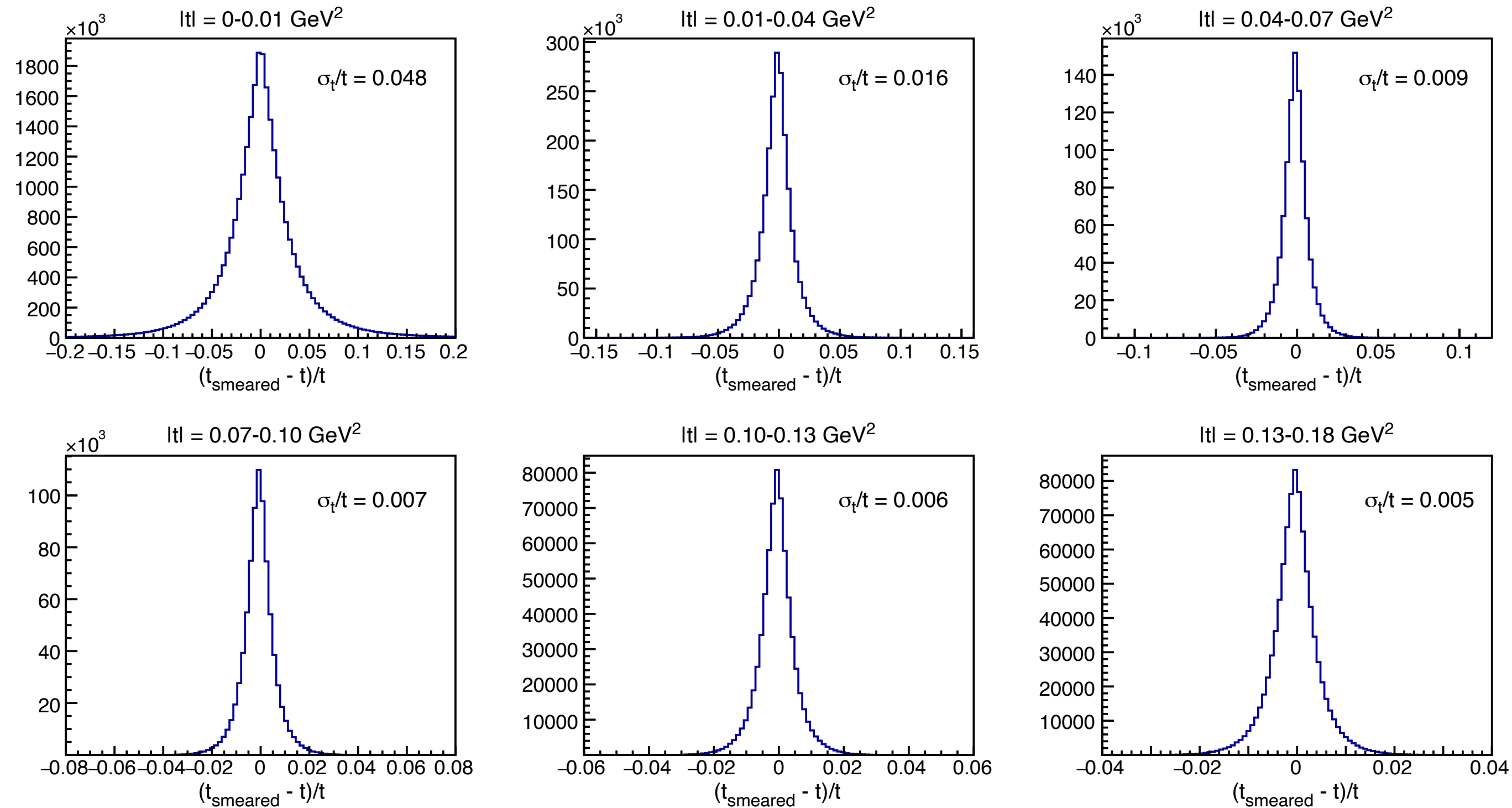
method	effect	$t$ -range ( $\text{GeV}^2$ )					
		0-0.1	0.1-0.4	0.04 - 0.07	0.07 - 0.10	0.10 - 0.13	0.13 - 0.18
E	beam divergence	0.061	0.015	0.008	0.007	0.006	0.005
E	beam mom. spread	149.61	10.36	3.03	1.86	1.37	1.03
L	divergence & mom. spread	0.048	0.016	0.009	0.007	0.006	0.005

# Method L



# Method L

- Method L with beam effects and nominal  $p_T$  resolution



Method L and A give similar  $t$  resolutions

measurement precision term for barrel (backward) (%)	MS term for barrel (backward) (%)	$t$ -range ( $\text{GeV}^2$ )					
		0-0.1	0.1-0.4	0.04 - 0.07	0.07 - 0.10	0.10 - 0.13	0.13 - 0.18
0.05 (0.1)	1.0 (2.0)	4.58	0.45	0.25	0.19	0.16	0.14
0.1 (0.2)	1.0 (2.0)	4.71	0.46	0.25	0.20	0.17	0.14
0.025 (0.05)	1.0 (2.0)	4.54	0.45	0.24	0.19	0.16	0.14
0.05 (0.1)	0.5 (2.0)	3.53	0.38	0.21	0.17	0.14	0.12
0.05 (0.1)	0.5 (1.0)	1.29	0.22	0.12	0.10	0.08	0.07
0.05 (0.1)	0.5 (0.5)	0.78	0.16	0.09	0.07	0.06	0.05
0.05 (0.1)	0.25 (0.5)	0.49	0.12	0.07	0.05	0.05	0.04
0.05 (0.1)	0.25 (0.25)	0.36	0.09	0.05	0.04	0.04	0.03

# Summary

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- Method E fails in the presence of beam momentum resolution
- Method L is an extension and a huge improvement
  - ▶ Only applicable for coherent events
  - ▶ We confirmed in simulations that all results obtained by method L for coherent processes are identical or very similar to that of method A in the studies discussed below.
- Ultimately, in the actual analysis once the EIC is realized, both methods (A and L) should be carefully compared and studied.
  - ▶ For coherent processes method L is likely the better choice as it does not rely on any approximations
  - ▶ For incoherent processes method A is the only option available