

Characterization of the transition region in the QCD phase diagram

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COLLABORATION

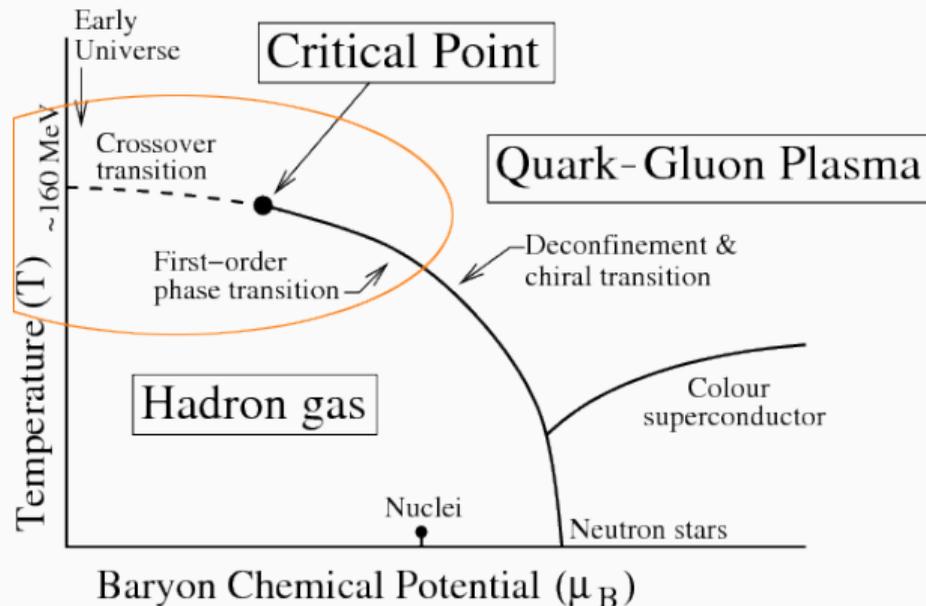


Premise

Phase diagram of QCD has a very rich structure, still awaiting unambiguous confirmation

The “transition region” of QCD

- ★ Transition between confined, hadronic and deconfined, partonic matter
- ★ Accessible to heavy-ion collisions (HIC)
→ Beam Energy Scan
- ★ Scene of (pseudo)-critical and critical phenomena
- ★ Many phenomenological consequences!



I studied this region under several aspects and with different methods

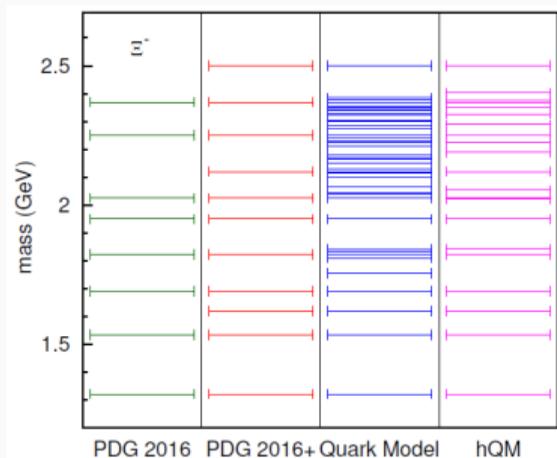
Among the questions I tackled in my thesis

- #1 What are the effective degrees of freedom in proximity of the transition?
- #2 What happens at freeze-out in HICs? Is the freeze-out temperature unique?
- #3 If the crossover transition become first order at larger μ_B – i.e., there is a critical point (CP) – what is its effect on the equation of state (EoS)?
- #4 And on observable quantities – e.g., baryon number fluctuations?
- #5 What is the effect of additional conserved charges (BQS) on the equation of state?

#1 Hadronic spectrum: QCD partial pressures

To determine optimal hadronic spectrum, consider both known and predicted states

Predicted states largely outnumber
experimentally confirmed ones

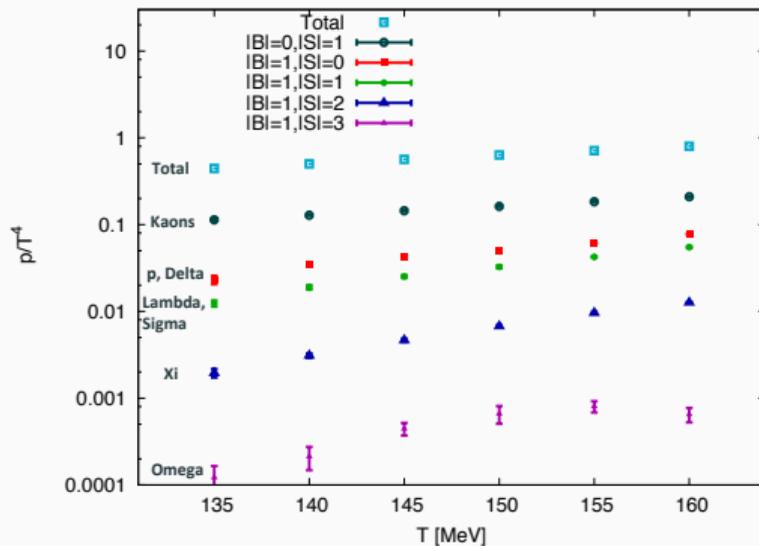


Compare HRG to lattice QCD using different
lists with increasing number of states

Alba, PP *et al.*, PRD 96 (2017), 034517

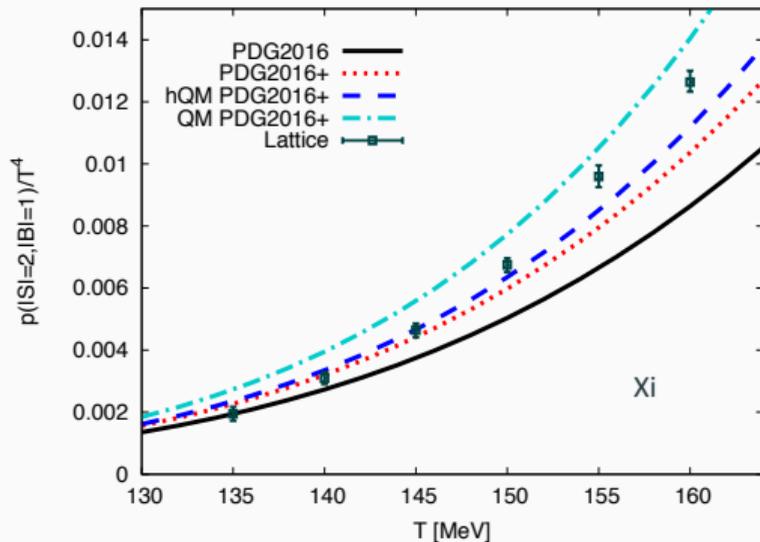
Decomposition in **partial pressures**:

$$P(T, \hat{\mu}_B, \hat{\mu}_S) = P_{00}^{BS} + P_{10}^{BS} \cosh(\hat{\mu}_B) + P_{01}^{BS} \cosh(\hat{\mu}_S) \\ + P_{11}^{BS} \cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\ + P_{13}^{BS} \cosh(\hat{\mu}_B - 3\hat{\mu}_S)$$

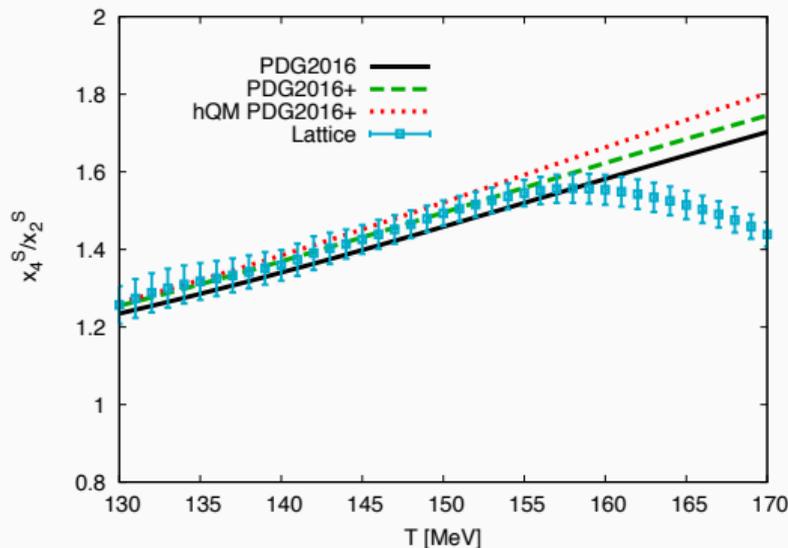


#1 Hadronic spectrum: QCD partial pressures

Compare partial pressures from lattice QCD and HRG for the different lists



Family by family, the analysis tells us where hadronic states might be missing

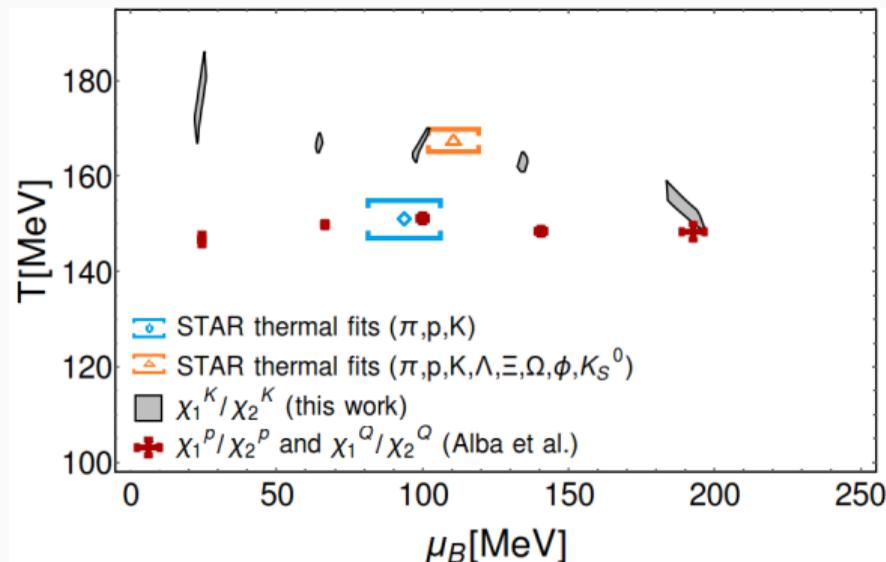


- PDG *established* states seems to be insufficient
- Predicted states seem to be too many

Conclusion: the optimal list is given by PDG states, *including* not-well-established ones
→ **PDG2016+ list**

#2 Freeze-out temperature: strange vs light

- I analyzed STAR BES results on net-kaon fluctuations to extract freeze-out parameters
- Compared with previous results from net-proton and net-charge fluctuations



Conclusion: A clear separation in the freeze-out temperature is seen!

#3 EoS with a critical point - I

The ingredients

- #1** Current knowledge of EoS at finite μ_B from lattice QCD:

$$P_{\text{QCD}}(T, \mu_B) = T^4 \sum_n c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

- #2** Expected critical behavior: 3D Ising model universality class

The goal

Build an EoS matching lattice QCD and containing the correct critical behavior

The strategy

- I.** Implement scaling behavior of 3D Ising model
- II.** Map 3D Ising model onto QCD
- III.** Enforce critical EoS matching to lattice QCD
- IV.** Reconstruct full thermodynamics



#3 EoS with a critical point - II

I. Scaling EoS in 3D Ising

Parametrize Ising variables (r, h)

$$M = M_0 R^{\beta} \theta$$

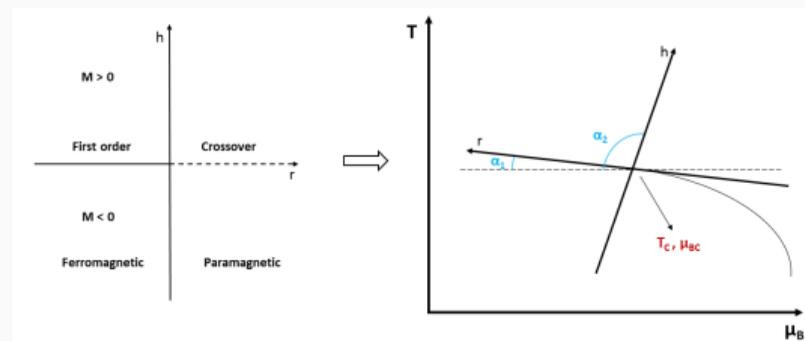
$$h = h_0 R^{\beta\delta} \tilde{h}(\theta)$$

$$r = R(1 - \theta^2)$$

II. Ising-to-QCD mapping

$$\frac{T - T_C}{T_C} = \mathbf{w} (r \rho \sin \alpha_1 + h \sin \alpha_2)$$

$$\frac{\mu_B - \mu_{BC}}{T_C} = \mathbf{w} (-r \rho \cos \alpha_1 - h \cos \alpha_2)$$



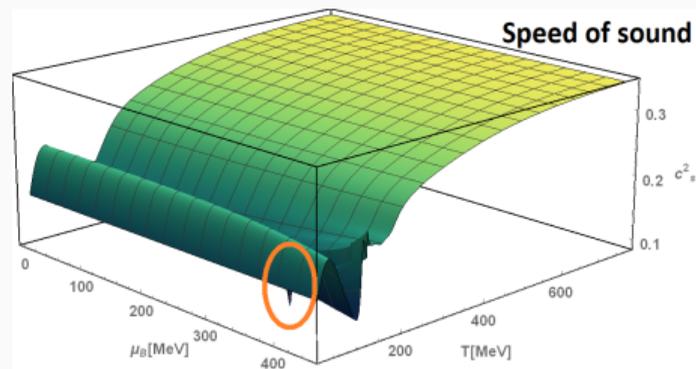
III. Matching with lattice QCD

Matching to lattice QCD at the level of baryon fluctuations:

$$T^4 c_n^{LAT}(T) = T^4 c_n^{N-Is}(T) + T_C^4 c_n^{Is}(T)$$

IV. Reconstruct thermodynamics

$$P(T, \mu_B) = T^4 \sum_n c_n^{N-Is}(T) \left(\frac{\mu_B}{T}\right)^n + T_C^4 P^{Is}(T, \mu_B)$$



The code is public!

► [BEST repo](#)

#4 EoS with a critical point - Kurtosis of B

Refined earlier predictions for $\chi_4^B(\sqrt{s})$ by including **sub-leading divergences** at the CP

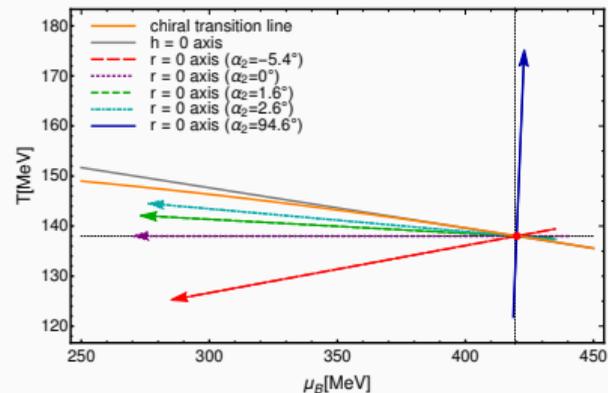
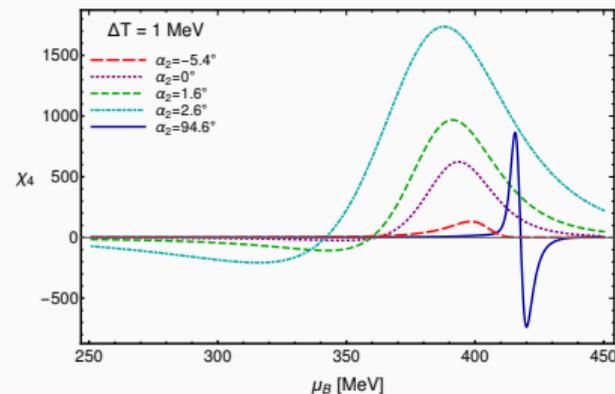
$$\chi_4^{\text{Ising}}(r, h) \quad \Leftrightarrow \quad \chi_4^B(T, \mu_B)$$

Due to the mixing angles α_1, α_2 one has:

$$\partial_{\mu_B} \neq \partial_h$$

$$\partial_{\mu_B} \propto \sin_1 \partial_h + \sin_2 \partial_r$$

The leading contribution is suppressed except in parametrically narrow window



Mroczek, PP *et al.*, 2008.04022 [nucl-th]

Conclusion: a dip in $\chi_4^B(\sqrt{s})$ is not an unambiguous signature of critical point

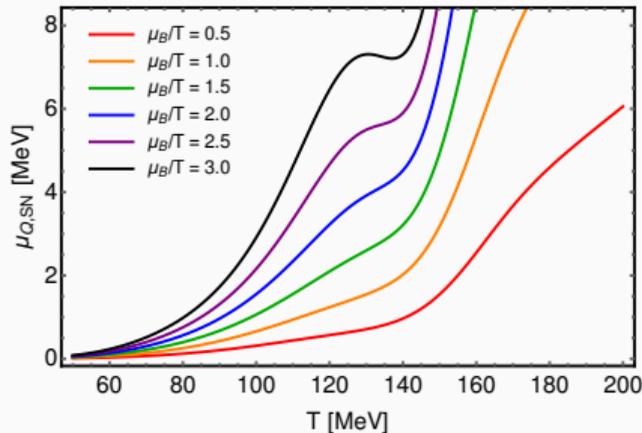
#5 EoS with three conserved charges (BQS)

I produced a full 4D EoS depending on (T, μ_B, μ_Q, μ_S) , using recent lattice QCD results for χ_{ijk}^{BQS} up to order $\mathcal{O}(\mu^4)$:

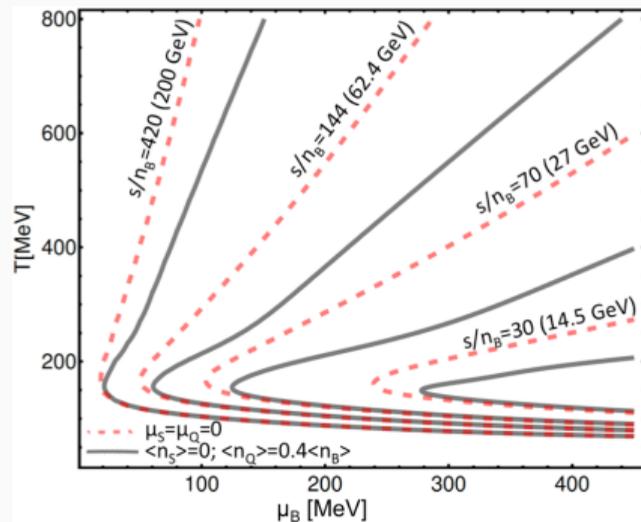
$$\frac{P(T, \vec{\mu})}{T^4} = \sum_{i,j,k} \frac{\chi_{ijk}^{BQS}(T)}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k$$

from which all thermodynamics follows.

Studied the strangeness neutral case



Isentropes show strong sensitivity:



The EoS is available [EoS BQS](#) in wide range of $T, \vec{\mu}$, and being implemented in hydro studies

Summary

- #1 The hadronic list (PDG206+) including *all* PDG states is the best compromise
Alba, PP *et al.*, PRD 96 (2017), 034517
- #2 Sensible difference in freeze-out temperature between strange (net-kaon) and light (net-proton/net-charge) fluctuation analyses Bellwied, PP *et al.*, PRC 99 (2019), 034912
- #3 I produced a family of EoS which match lattice QCD and include 3D Ising-like critical point
PP *et al.*, PRC 101 (2020), 034901
- #4 The effect of a critical point on net-B cumulants depends on mapping. A dip in the kurtosis is *not* a unique feature!
Mroczek, PP *et al.*, 2008.04022 [nucl-th]
- #5 I produced a 4-dimensional EoS for QCD including all conserved charges
Noronha-Hostler, PP *et al.*, PRC 100 (2019), 064910