New Physical Phenomena Associated with Vorticity and Spin in Heavy-Ion Collisions

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N. Weickgenannt, E. S., X.-l. Sheng, Q. Wang, and D. H. Rischke, arXiv:2005.01506 (2020)

E. S., and N. Weickgenannt, arXiv:2007.00138 (2020)

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

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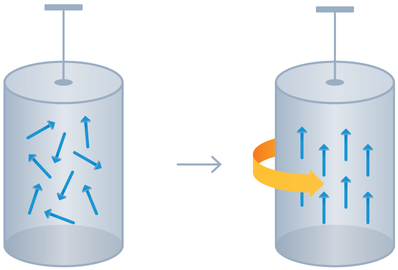
Outline

- Vorticity and spin physics in heavy-ion collisions
- Quantum kinetic theory
- Spin hydrodynamics
- Polarization observable in heavy-ion collisions

Rotation and polarization

- Condensed matter: **Barnett effect**

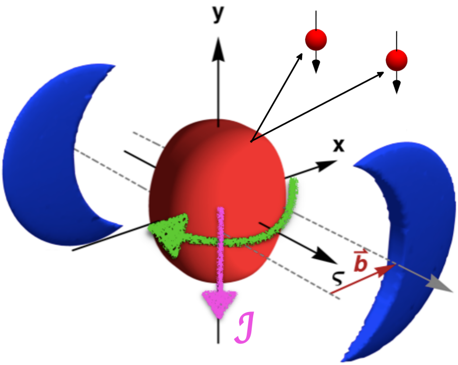


Picture by Andrey Sadofyev

Ferromagnet gets magnetized when it rotates

Polarization effects through rotation in heavy-ion collisions? Yes!

Noncentral heavy-ion collisions

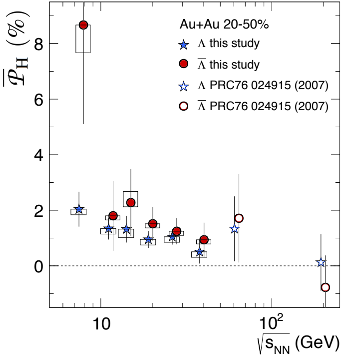


Picture from W. Florkowski, R. Ryblewski and A. Kumar, Prog. Part. Nucl. Phys. **108**, 103709 (2019)

Noncentral nuclear collisions $\Rightarrow$ Large global angular momentum
$\Rightarrow$ Vorticity of hot and dense matter $\Rightarrow$ particle polarization along vorticity

Experimental observation - Global Λ polarization

- Polarization along global angular momentum



L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

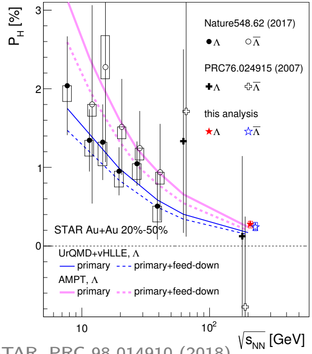
- Weak decay: $\Lambda \to p + \pi^-$ angular distr.: $dN/d\cos\theta = \frac{1}{2}(1 + \alpha|\vec{\mathcal{P}}_H|\cos\theta)$
- Quark-gluon plasma is the "most vortical fluid ever observed"

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})k_B T/\hbar \approx (9+1) \times 10^{21}\mathrm{s}^{-1}$$

Great Red Spot of Jupiter $10^{-4}\,\mathrm{s}^{-1}$, Turbulent flow superfluid He-II $150\,\mathrm{s}^{-1}$, Superfluid nanodroplets $10^{7}\,\mathrm{s}^{-1}$

Experiments vs theory: Λ polarization

Global - along J



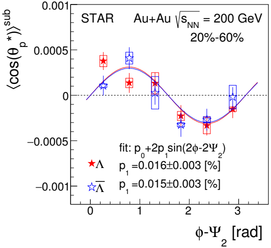
STAR, PRC 98 014910 (2018)

$$\Pi^{\mu}(x,p) \propto (1-n_F)\epsilon^{\mu\nu\rho\tau} p_{\nu} \varpi_{\rho\tau}$$

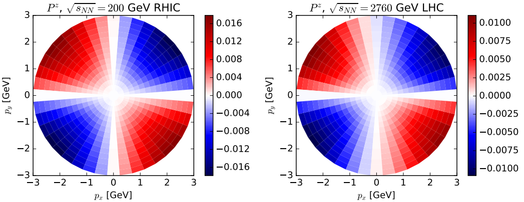
$$\varpi_{\rho\tau} = -\frac{1}{2}(\partial_{\rho}\beta_{\tau} - \partial_{\tau}\beta_{\rho})$$

Becattini et al An. Phys. (2013)

Longitudinal - along beam axis



J. Adam *et al.* [STAR Collaboration], PRL 123, 132301 (2019)



F. Becattini, I Karpenko, PRL **120**, 012302

- Theory assumes local equilibrium of spin degrees of freedom
- "Sign problem" between theory and experiments for longitudinal polarization!

Does spin play a dynamical role in hydro?

- Relativistic hydrodynamics is a good effective theory: $\partial_\mu T^{\mu\nu} = 0$

Goal: Relativistic hydrodynamics (classical) with spin (quantum) as dynamical variable

W. Florkowski, B. Friman, A. Jaiswal, and E. S., Phys. Rev. C **97**, no. 4, 041901 (2018)
W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. S., Phys. Rev. D **97**, no. 11, 116017 (2018)
W. Florkowski, F. Becattini, and E. S., Acta Phys. Polon. B **49**, 1409 (2018)
F. Becattini, W. Florkowski, E. S., Phys. Lett. B **789**, 419 (2019)
S. Bhadury, W. Forkowski, A. Jaiswal, A. Kumar, R. Ryblewski, 2002.03937, 2008.10976 (2020)
S. Shi, C. Gale, S. Jeong, 2008.08618 (2020)

Starting point: Kinetic theory from quantum field theory

N. Weickgenannt, X.l. Sheng, E. S., Q. Wang, and D. H. Rischke, Phys. Rev. D **100**, no. 5, 056018 (2019)
N. Weickgenannt, E. S., X.l. Sheng, Q. Wang, and D. H. Rischke, 2005.01506 (2020)

- Alternative approaches: Lagrangian formulation, entropy current
  D. Montenegro, L. Tinti, G. Torrieri, PRD 96, 056012 (2017)
  D. Montenegro, G. Torrieri, 2004.10195 (2020)
  K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, PLB795, 100 (2019)
  K. Fukushima, S. Pu, 2010.01608 (2020)

Orbital-to-spin angular momentum conversion

Our results:

- How do we describe the orbital-to-spin angular momentum conversion in kinetic theory?

  Nonlocal particle scatterings (finite impact parameter)

- And in hydrodynamics?

  Antisymmetric part of energy-momentum tensor

Nonrelativistic kinetic theory

Kinetic Theory for a Dilute Gas of Particles with Spin

S. Hess and L. Waldmann

Institut für Theoretische Physik der Universität Erlangen-Nürnberg, Erlangen

(Z. Naturforschg. **21 a**, 1529—1546 [1966]; received 6 April 1966)

The kinetic theory of particles with spin previously developed for a Lorentzian gas is extended to the case of a pure gas. In part A the transport (Boltzmann) equation for the one particle distribution operator is stated and discussed (conservation laws, H-theorem). A magnetic field acting on the magnetic moment of the particles is incorporated throughout. In part B the pertaining linearized collision operator and certain bracket expressions linked with this operator are considered. Part C deals with the expansion of the distribution operator and of the linearized transport equation with respect to a complete set of composite irreducible tensors built from the components of particle velocity and spin. Thus, the distribution operator is replaced by a set of tensors depending only on time and space-coordinates. The physical meaning of these tensors (expansion coefficients) is invoked. They obey a set of coupled first-order differential equations (transport-relaxation equations). The reciprocity relations for the relaxation matrices are stated. Finally a detailed discussion of angular momentum conservation is given.

- "There is another effect which we cannot describe with a local collision operator: the orientation of the spin by a local or uniform rotation of the system (Barnett effect)"
- **Caveat**: Nonrelativistic Barnett effect $\Longrightarrow$ Magnetization
  Heavy-ion collisions $\Longrightarrow$ Spin polarization without magnetization
  (Spin particles + Spin antiparticles)

Wigner function

$$W(x,p) = \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p\cdot y} \langle : \bar{\Psi}(x+\frac{y}{2})\Psi(x-\frac{y}{2}) : \rangle$$

- Dirac equation $\Longrightarrow$ Equation of motion for Wigner function
  H.-Th. Elze, M. Gyulassy, and D. Vasak, Ann. Phys. 173 (1987) 462
  de Groot, van Leeuwen, van Weert, Relativistic Kinetic Theory. Principles and Applications

$$\left[\gamma\cdot\left(p + i\frac{\hbar}{2}\partial\right) - m\right] W(x,p) = \hbar \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p\cdot y} \left\langle : \rho(x-\frac{y}{2})\bar{\psi}(x+\frac{y}{2}) : \right\rangle = \hbar\mathcal{C}$$

$\rho = -(1/\hbar)\partial\mathcal{L}_I/\partial\bar{\psi}$, $\mathcal{L}_I$ = interaction Lagrangian

- $\Longrightarrow$ Boltzmann equation and on-shell modification

$$p\cdot\partial W(x,p) = C \qquad \left(p^2 - m^2 - \frac{\hbar^2}{4}\partial^2\right) W(x,p) = \hbar\delta M$$

- **Idea**: Find approximate solution by expanding in powers of $\hbar$
  and truncate at first order, e.g.

$$W = W^{(0)} + \hbar W^{(1)} + \mathcal{O}(\hbar^2)$$

Calculating the Wigner function

- Clifford decomposition

$$W = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)$$

- Determine $\mathcal{V}^\mu$ and $\mathcal{A}^\mu$ from equations of motion
- Assumption: polarization effects at least $\mathcal{O}(\hbar)$

$$\mathcal{V}^\mu = \frac{1}{m}p^\mu\bar{\mathcal{F}} + \mathcal{O}(\hbar^2), \qquad \bar{\mathcal{F}} \equiv \mathcal{F} - \frac{\hbar}{m^2}p^\mu \mathrm{ReTr}(\gamma_\mu\mathcal{C})$$

- Transport equations:

$$p\cdot\partial\bar{\mathcal{F}} = m\,C_F, \qquad p\cdot\partial\mathcal{A}^\mu = m\,C_A^\mu$$

with $C_F = 2\mathrm{Im}\,\mathrm{Tr}(\mathcal{C})$, $C_A^\mu \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\nu \mathrm{Im}\,\mathrm{Tr}(\sigma_{\alpha\beta}\mathcal{C})$

Spin in phase space

- In order to account for spin dynamics enlarge phase space
  J. Zamanian, M. Marklund, and G. Brodin, NJP 12, 043019 (2010)
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)
- Introduce new phase-space variable $\mathfrak{s}^\mu$

$$\mathfrak{f}(x, p, \mathfrak{s}) \equiv \frac{1}{2}\left[\bar{\mathcal{F}}(x, p) - \mathfrak{s} \cdot \mathcal{A}(x, p)\right]$$

- Obtain $\bar{\mathcal{F}}$ and $\mathcal{A}^\mu$ via

$$\bar{\mathcal{F}} = \int dS(p)\, \mathfrak{f}(x, p, \mathfrak{s}) \qquad \mathcal{A}^\mu = \int dS(p)\, \mathfrak{s}^\mu \mathfrak{f}(x, p, \mathfrak{s})$$

  with $dS(p) \equiv \frac{\sqrt{p^2}}{\sqrt{3}\pi} d^4\mathfrak{s}\, \delta(\mathfrak{s}^2 + 3)\delta(p \cdot \mathfrak{s})$
- Boltzmann equation

$$p \cdot \partial\, \mathfrak{f}(x, p, \mathfrak{s}) = m\, \mathfrak{C}[\mathfrak{f}]$$

$$\mathfrak{C}[\mathfrak{f}] \equiv \frac{1}{2}(C_F - \mathfrak{s} \cdot C_A)$$

- All dynamics in one scalar equation!

Nonlocal collisions

- Expand collision term up to **first order in $\hbar$-gradients**

$$\mathfrak{C}[\mathfrak{f}] = \mathfrak{C}_l[\mathfrak{f}] + \hbar\,\mathfrak{C}_{nl}[\mathfrak{f}]\ .$$

Local contribution + **Nonlocal** contribution

- Long calculation $\Longrightarrow$ Intuitive result in low-density approximation:

$$\begin{aligned}\mathfrak{C}[\mathfrak{f}] = &\int d\Gamma_1 d\Gamma_2 d\Gamma'\,\mathcal{W}\,[\mathfrak{f}(x+\Delta_1, p_1, \mathfrak{s}_1)\\ &\times\mathfrak{f}(x+\Delta_2, p_2, \mathfrak{s}_2) - \mathfrak{f}(x+\Delta, p, \mathfrak{s})\mathfrak{f}(x+\Delta', p', \mathfrak{s}')]\\ &+\int d\Gamma_2\, dS_1(p)\,\mathfrak{W}\,\mathfrak{f}(x+\Delta_1, p, \mathfrak{s}_1)\mathfrak{f}(x+\Delta_2, p_2, \mathfrak{s}_2)\end{aligned}$$

$d\Gamma \equiv d^4p\, dS(p)$

- Structure: Momentum and spin exchange + Spin exchange only
- Nonlocal Collisions $\Longrightarrow$ Displacement Δ
- $\mathcal{W}$, $\mathfrak{W}$ vacuum transition probabilities, depend on phase-space spins

Condition for $\mathfrak{C}[\mathfrak{f}] = 0 \Longrightarrow$ **Global equilibrium**

Equilibrium distribution function

- **Equilibrium condition:** Collision term has to vanish
- Ansatz for distribution function
  F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, AP. 338, 32 (2013)
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$\mathfrak{f}_{eq}(x,p,\mathfrak{s}) = \frac{m}{(2\pi\hbar)^3} \exp\left[-\beta(x)\cdot p + \frac{\hbar}{4}\Omega_{\mu\nu}(x)\Sigma_{\mathfrak{s}}^{\mu\nu}\right]\delta(p^2 - M^2)$$

  - $\beta^\mu = \frac{u^\mu}{T}$ - Lagrange multiplier for 4-momentum conservation
  - Spin potential $\Omega^{\mu\nu}$ - Lagrange multiplier for **total** angular momentum conservation
  - M - mass possibly modified by interactions
  - Spin-dipole-moment tensor

$$\Sigma_{\mathfrak{s}}^{\mu\nu} \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$$

- Insert into $\mathfrak{C}[\mathfrak{f}]$ and expand up to first order in $\hbar$

Equilibrium conditions

$$\mathfrak{C}[\mathfrak{f}_{eq}] = -\int d\Gamma' d\Gamma_1 d\Gamma_2\, \widetilde{\mathcal{W}}\, e^{-\beta\cdot(p_1+p_2)}$$
$$\times \left[\partial_\mu\beta_\nu \left(\Delta_1^\mu p_1^\nu + \Delta_2^\mu p_2^\nu - \Delta^\mu p^\nu - \Delta'^\mu p'^\nu\right) - \frac{1}{2}\Omega_{\mu\nu}\frac{\hbar}{2}\left(\Sigma_{\mathfrak{s}_1}^{\mu\nu} + \Sigma_{\mathfrak{s}_2}^{\mu\nu} - \Sigma_{\mathfrak{s}}^{\mu\nu} - \Sigma_{\mathfrak{s}'}^{\mu\nu}\right)\right]$$
$$-\int d\Gamma_2\, dS_1(p) dS'(p_2)\, \mathfrak{W}\, e^{-\beta\cdot(p+p_2)}$$
$$\times \left\{\partial_\mu\beta_\nu \left[(\Delta_1^\mu - \Delta^\mu)p^\nu + (\Delta_2^\mu - \Delta'^\mu)p_2^\nu\right] - \frac{1}{2}\Omega_{\mu\nu}\frac{\hbar}{2}(\Sigma_{\mathfrak{s}_1}^{\mu\nu} + \Sigma_{\mathfrak{s}_2}^{\mu\nu} - \Sigma_{\mathfrak{s}}^{\mu\nu} - \Sigma_{\mathfrak{s}'}^{\mu\nu})\right\}.$$

- Conservation of total angular momentum (orbital+spin) in a collision

$$j^{\mu\nu} = \Delta^\mu p^\nu - \Delta^\nu p^\mu + \frac{\hbar}{2}\Sigma_{\mathfrak{s}}^{\mu\nu}$$

- Conditions for vanishing collision term

$$\partial_\mu\beta_\nu + \partial_\nu\beta_\mu = 0$$
$$\Omega_{\mu\nu} = \varpi_{\mu\nu} \equiv -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu) = \text{const.}$$

Global equilibrium!

Spin hydrodynamics

- **Hydrodynamic densities:**
  Energy-momentum tensor $T^{\lambda\nu}$
  Total angular momentum tensor

$$J^{\lambda,\mu\nu} \equiv x^{\mu} T^{\lambda\nu} - x^{\nu} T^{\lambda\mu} + \hbar S^{\lambda,\mu\nu}$$

  Additional dynamical tensor: Spin tensor $S^{\lambda,\mu\nu}$

- 10 equations of motion: 4 usual hydro + 6 due to total angular momentum conservation

$$\partial_{\mu} T^{\mu\nu} = 0 \qquad \hbar\, \partial_{\lambda} S^{\lambda,\mu\nu} = T^{[\nu\mu]}$$

  Antisymmetric part: $T^{[\nu\mu]} = T^{\nu\mu} - T^{\mu\nu}$

- 10 unknowns: 4 + 6 additional independent fields (spin potential)

$$\beta^{\mu} = \frac{u^{\mu}}{T} \qquad \Omega^{\mu\nu}$$

  Plus dissipative quantities

W. Florkowski, B. Friman, A. Jaiswal, and E. S., PRC **97**, no. 4, 041901 (2018)
W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. S., PRD **97**, no. 11, 116017 (2018)
W. Florkowski, F. Becattini, and E. S., APB **49**, 1409 (2018)
W. Florkowski, F. Becattini, and E. S., PLB **789**, 419 (2019)
K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, PLB795, 100 (2019)

Pseudo-gauge transformations

Densities are NOT uniquely defined $\Longrightarrow$ Relocalization

F. W. Hehl, Rept. Math. Phys. 9, 55 (1976), F. Becattini, W. Florkowski, and E. S., PLB 789, 419 (2019), E. S. and N. Weickgenannt, 2007.00138 (2020)

$$T'^{\,\mu\nu}(x) = T^{\mu\nu}(x) + \tfrac{1}{2}\partial_\lambda \left[\Phi^{\lambda,\,\mu\nu}(x) + \Phi^{\mu,\,\nu\lambda}(x) + \Phi^{\nu,\,\mu\lambda}(x)\right]$$
$$S'^{\,\lambda,\,\mu\nu} = S^{\lambda,\,\mu\nu}(x) - \Phi^{\lambda,\,\mu\nu}(x) + \partial_\rho Z^{\mu\nu,\lambda\rho}(x)$$

$\Phi^{\lambda,\,\mu\nu} = -\Phi^{\lambda,\,\nu\mu}$, $Z^{\mu\nu,\lambda\rho} = -Z^{\nu\mu,\lambda\rho} = -Z^{\mu\nu,\rho\lambda}$

- Leave global charges invariant

$$P^\mu = \int d^3\Sigma_\lambda\, T^{\lambda\mu}(x) \qquad J^{\mu\nu} = \int d^3\Sigma_\lambda\, J^{\lambda,\,\mu\nu}(x)$$

  Σ_λ - Hypersurface

- Conservation laws: $\partial_\mu T'^{\mu\nu} = 0$, $\partial_\lambda S'^{\lambda,\mu\nu} = T'^{[\nu\mu]}$
- **Canonical choice**: apply Noether's theorem to Dirac Lagrangian
  $\rightarrow$ **Problem:** does not lead to a covariant description of spin
- **Solution**: apply Noether's theorem to Klein-Gordon Lagrangian for spinors
  J. Hilgevoord, S. Wouthuysen, NP 40 (1963) 1

Spin hydrodynamics with HW currents

- From kinetic theory

$$T^{\mu\nu}_{\rm HW} = \int dP\, dS(p)\, p^\mu p^\nu f(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)$$

$$S^{\lambda,\mu\nu}_{\rm HW} = \int dP\, dS(p) p^\lambda \left(\frac{1}{2}\Sigma^{\mu\nu}_{\mathfrak{s}} - \frac{\hbar}{4m^2} p^{[\mu}\partial^{\nu]}\right) f(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)$$

- Equations of motion

$$\partial_\mu T^{\mu\nu}_{\rm HW} = \int d\Gamma\, p^\nu\, \mathfrak{C}[\mathfrak{f}] = 0$$

$$\hbar\, \partial_\lambda S^{\lambda,\mu\nu}_{\rm HW} = \int d\Gamma\, \frac{\hbar}{2}\Sigma^{\mu\nu}_{\mathfrak{s}}\, \mathfrak{C}[\mathfrak{f}] = T^{[\nu\mu]}_{\rm HW}$$

- Energy-momentum conserved in a collision

Spin not conserved in nonlocal collisions $\Longrightarrow T^{[\nu\mu]}_{\rm HW} \neq 0$
$\Longrightarrow$ Conversion between spin and orbital angular momentum

- $T^{[\nu\mu]}_{\rm HW} = 0$: (i) for local collisions (spin is collisional invariant)
  (ii) in **global equilibrium** ($\mathfrak{C}[\mathfrak{f}] = 0$)

- Nonlocal collisions away from global equilibrium $\Longrightarrow$ Dissipative dynamics
- Correct nonrelativistic limit

Hess, Waldmann (1966); G. Lukaszewicz, Micropolar Fluids, Theory and Applications (1999)

Fluid gets polarized through rotation!

Polarization observable in heavy-ion collisions

- Polarization vector for particle with momentum p^μ (e.g. Λ-hyperon)

F. Becattini, 2004.04050; E.S., N. Weickgenannt, 2007.00138; L. Tinti, W. Florkowski, 2007.04029

$$\Pi_\mu(p) = -\frac{\hbar}{2m}\epsilon_{\mu\nu\alpha\beta}p^\nu \frac{\int d\Sigma_\lambda s_{\text{HW}}^{\lambda,\alpha\beta}(x,p)}{\int d\Sigma_\lambda p^\lambda \mathcal{F}}$$

$S_{\text{HW}}^{\lambda,\alpha\beta}(x) = \int d^4p\, s_{\text{HW}}^{\lambda,\alpha\beta}(x,p) = \int d^4p\, p^\lambda \mathcal{S}^{\alpha\beta}(x,p)$
Σ_λ - Hypersurface

- **Equilibrium**

$$\Pi_\mu(p) = -\frac{\hbar}{4m}\epsilon_{\mu\nu\alpha\beta}p^\nu \frac{\int d\Sigma_\lambda p^\lambda f(1-f)\varpi^{\alpha\beta}}{\int d\Sigma_\lambda p^\lambda f}$$

$\varpi^{\alpha\beta} = -\frac{1}{2}(\partial^\alpha\beta^\beta - \partial^\beta\beta^\alpha)$ - Thermal vorticity
f - Distribution function

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Annals. Phys. 338, 32 (2013)

What are nonequilibrium effects on $\Pi_\mu(p)$?

Conclusions

Summary

- Derivation of **nonlocal** collisions from quantum field theory
  - Vanishing collision term $\Longrightarrow$ **global** equilibrium
- Spin hydrodynamics with Hilgevoord-Wouthuysen pseudo-gauge
  - **Antisymmetric** part of energy-momentum tensor
    $\Longrightarrow$ Orbital-to-spin angular momentum conversion
    $\Longrightarrow$ **Vanishes** with **local collisions** or in **global equilibrium**
  - Local collisions $\Longrightarrow$ Ideal spin hydrodynamics possible
    Nonlocal collisions $\Longrightarrow$ Always **dissipative** dynamics

Outlook

- Derive second-order dissipative spin hydrodynamics
- Study nonequilibrium and spin hydrodynamic effects on polarization vector
- Possible explanation to "sign problem"?