Mapping out the QCD phase diagram

Agnieszka Sorensen (UCLA/LBNL)

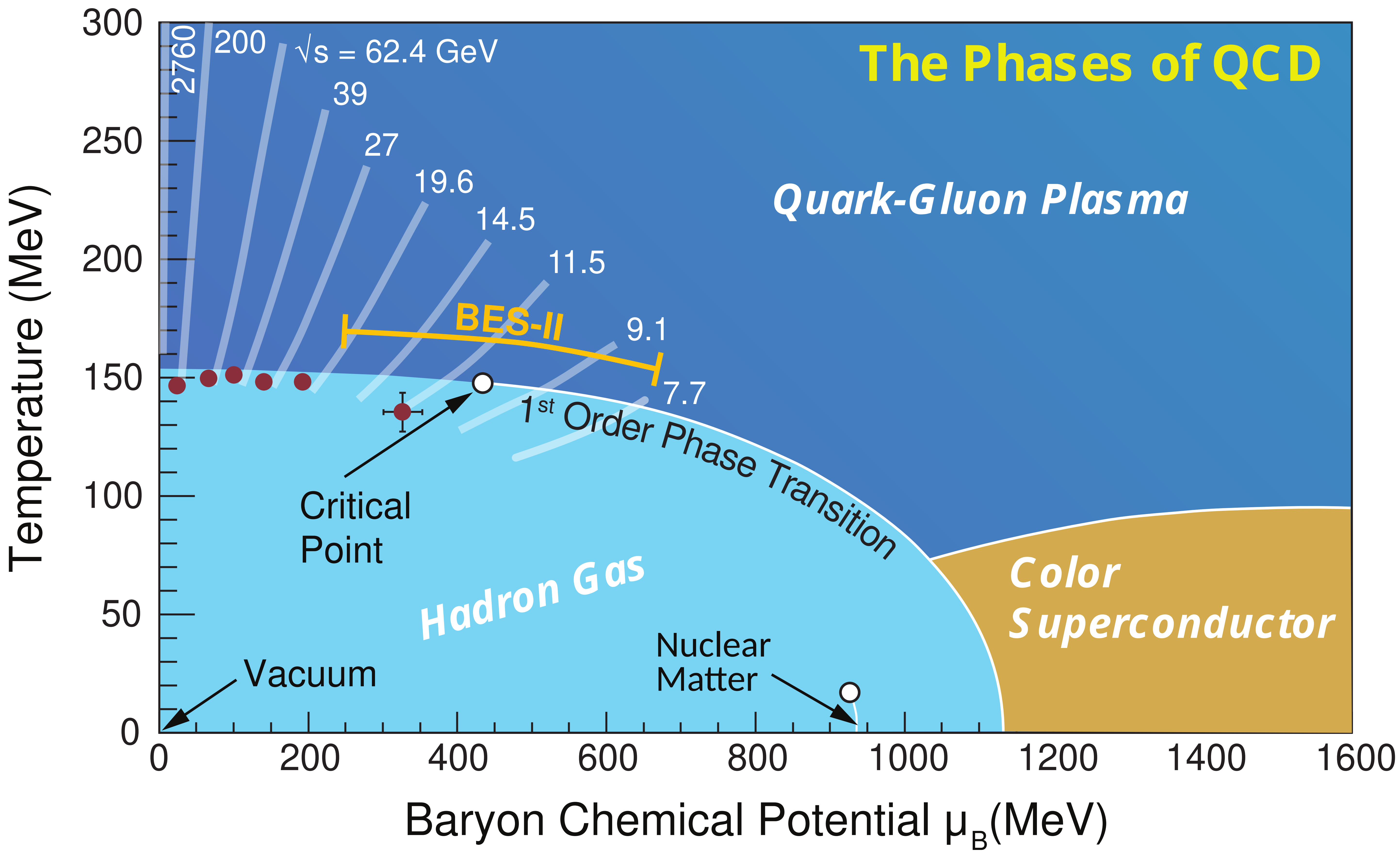
in collaboration with Volker Koch (LBNL)

Mapping out the QCD phase diagram

Outline

- The search for the QCD critical point and related observables
- Experimental data and the role of simulations
- Critical behavior in hadronic transport

The conjectured phase diagram of QCD



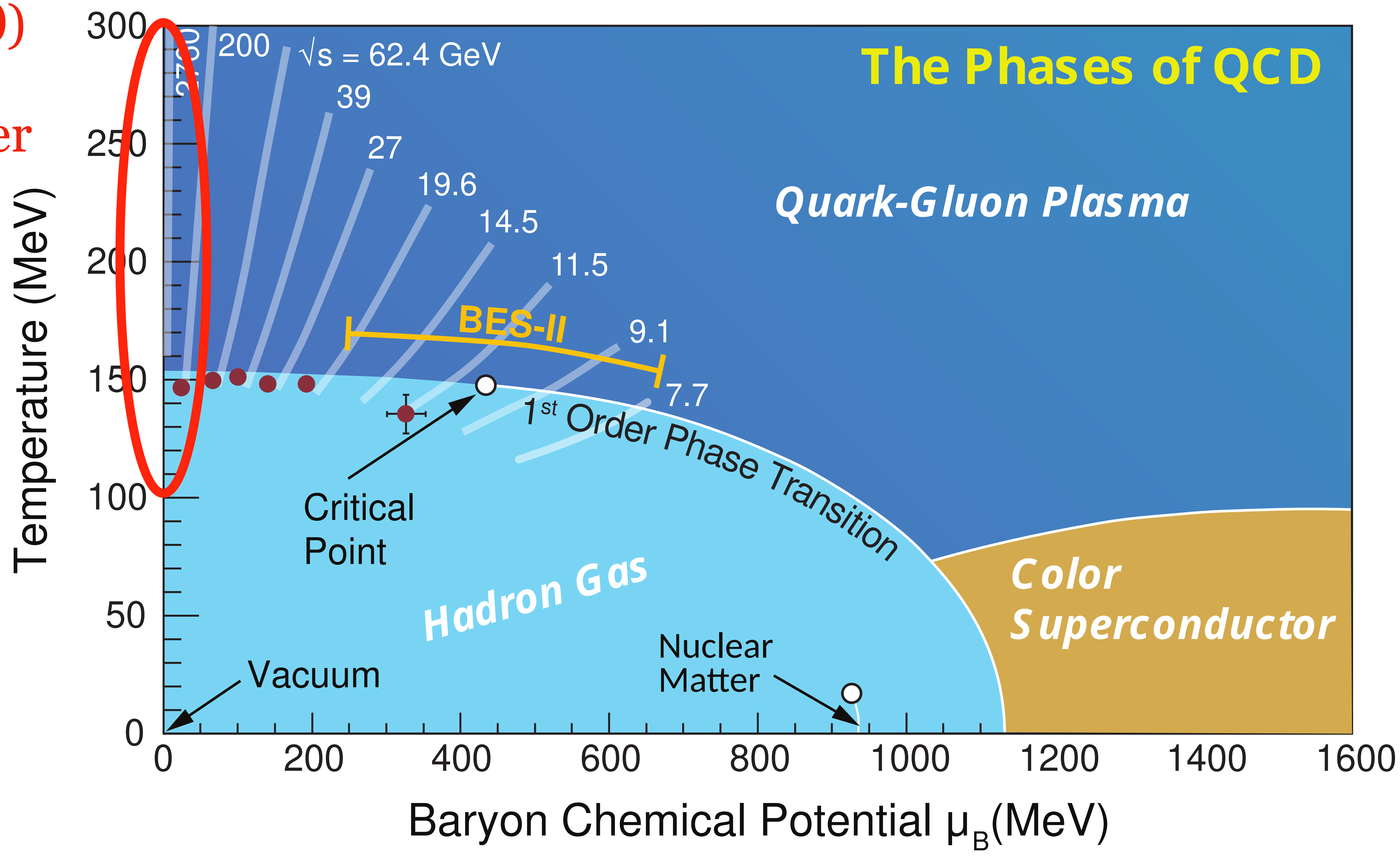
The conjectured phase diagram of QCD

… and what we **really** know about it

LQCD EOS ($\mu_B = 0$)

finite m_q = crossover

pseudocritical temperature $T_{pc} \simeq 150$ MeV



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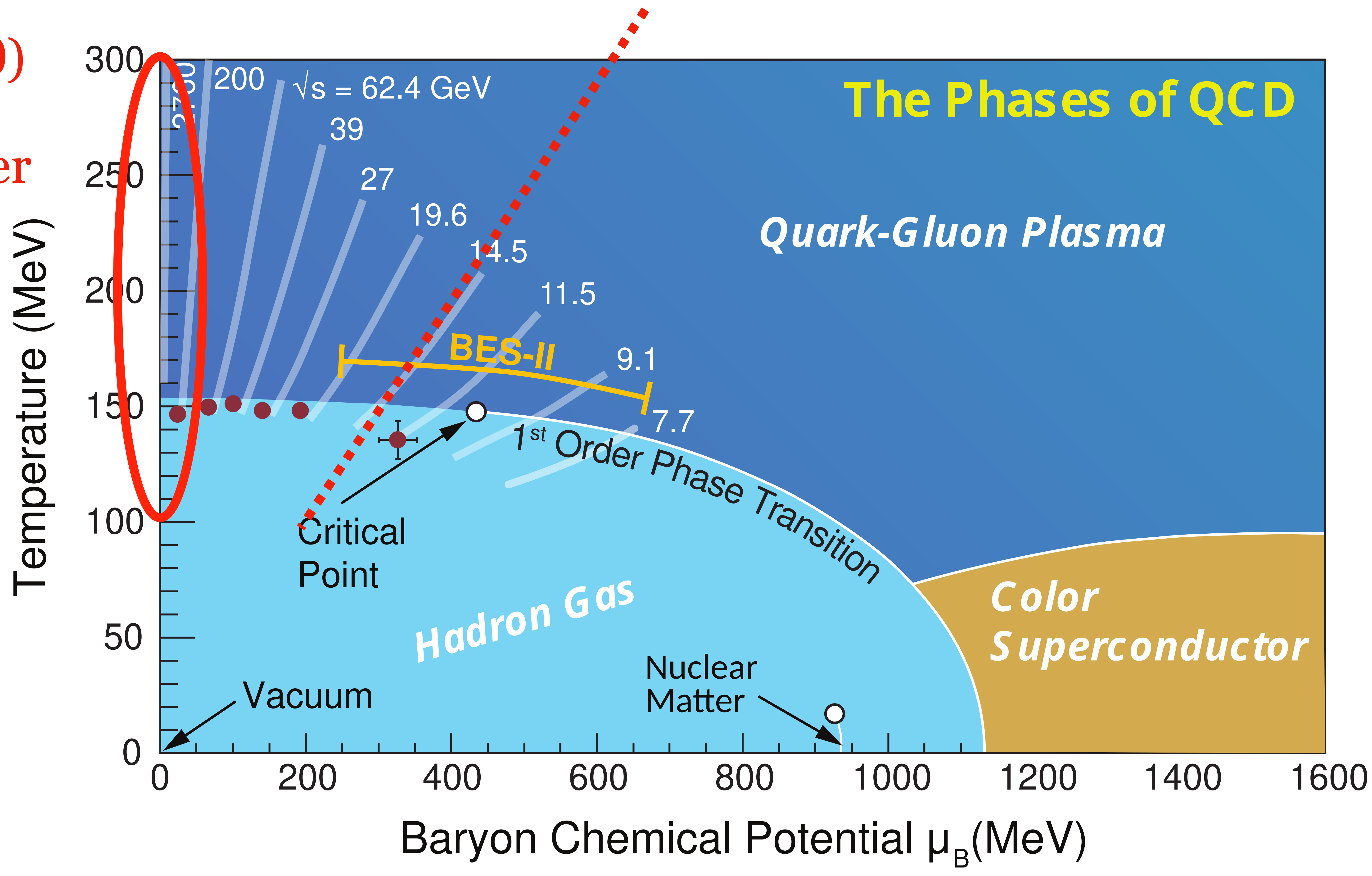

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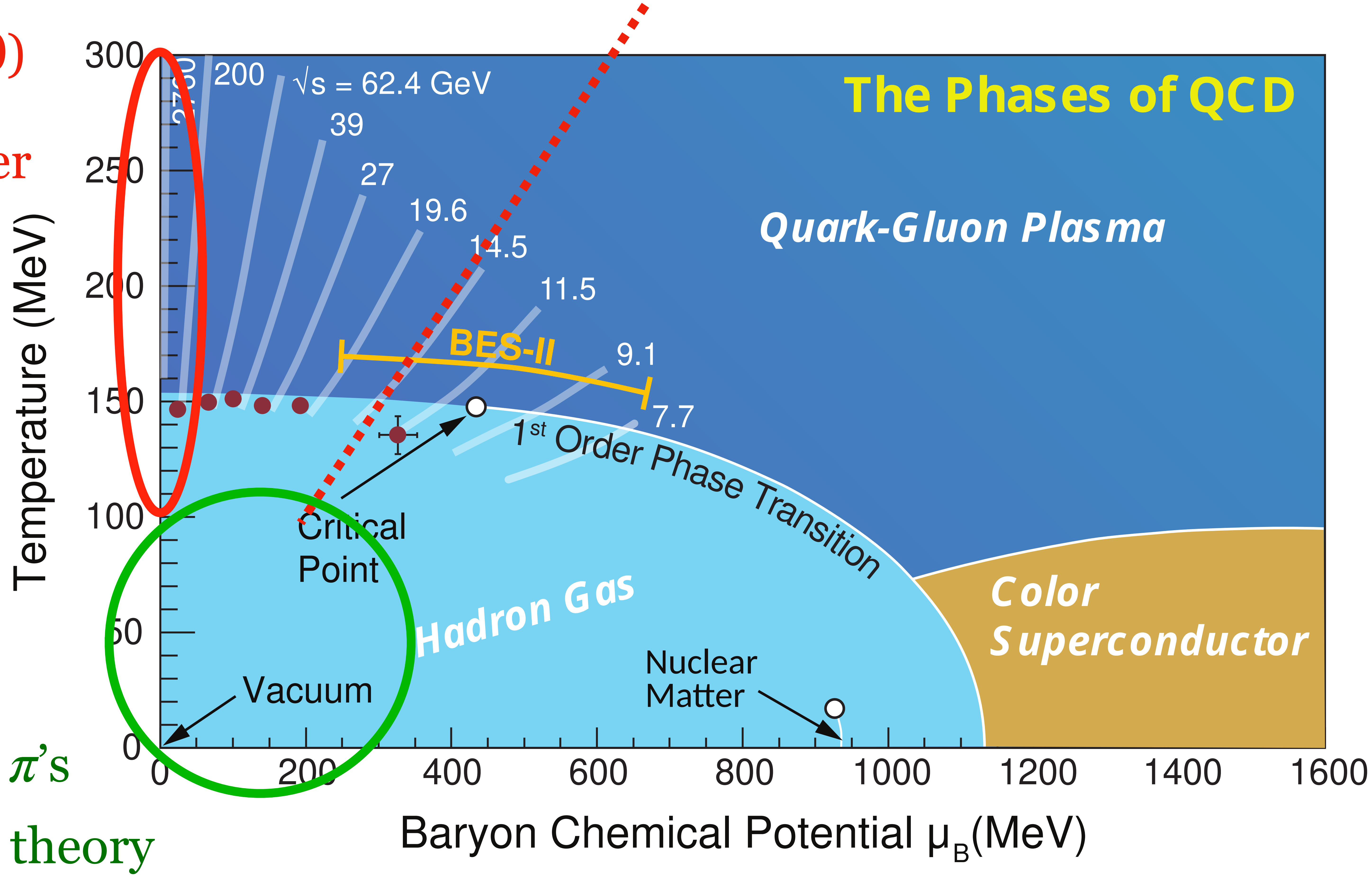
weakly coupled hadron gas, mostly π's

chiral perturbation theory + cross sections from experiment



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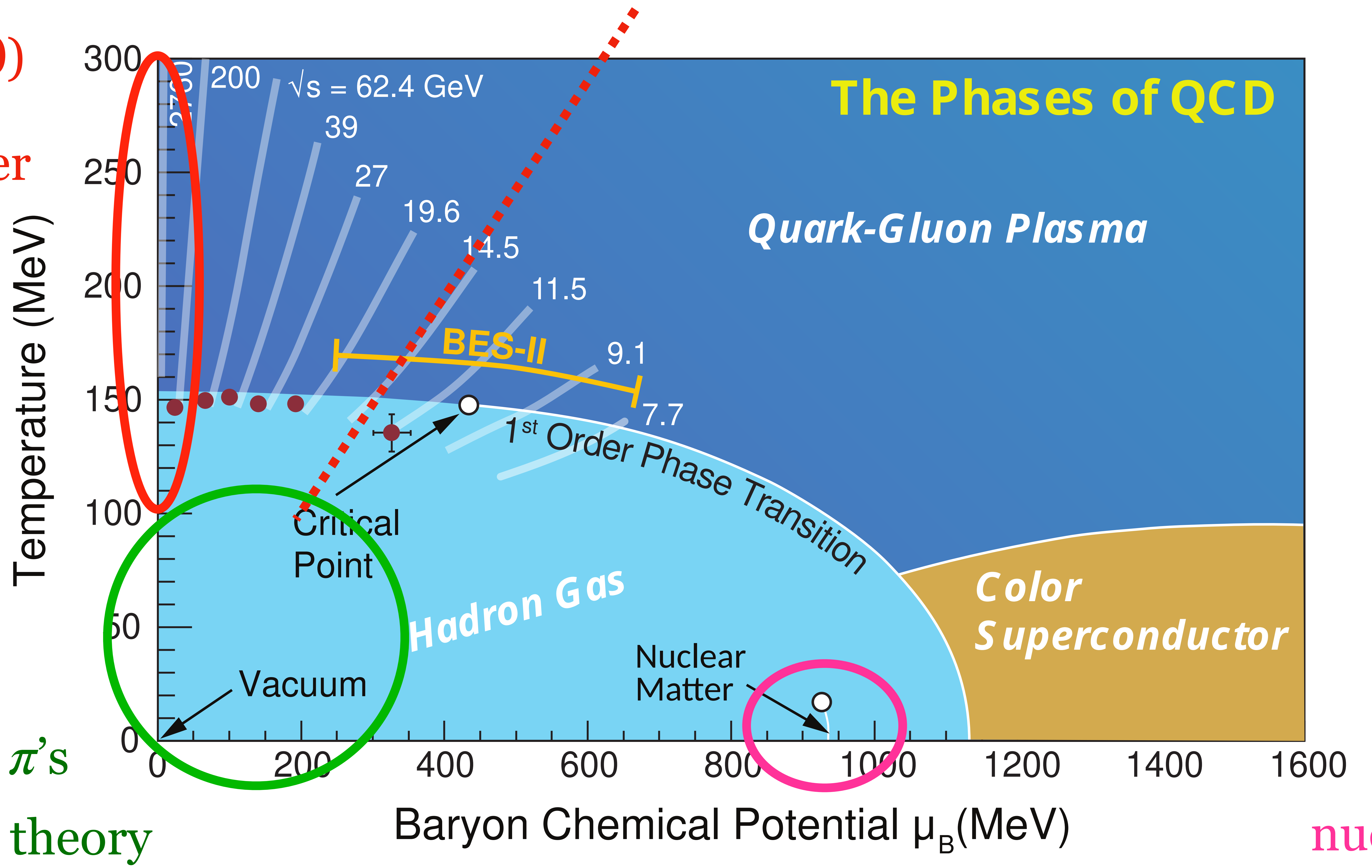

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nuclear critical point

extrapolations of well-tested nuclear forces
+ experiments on nuclear fragmentation

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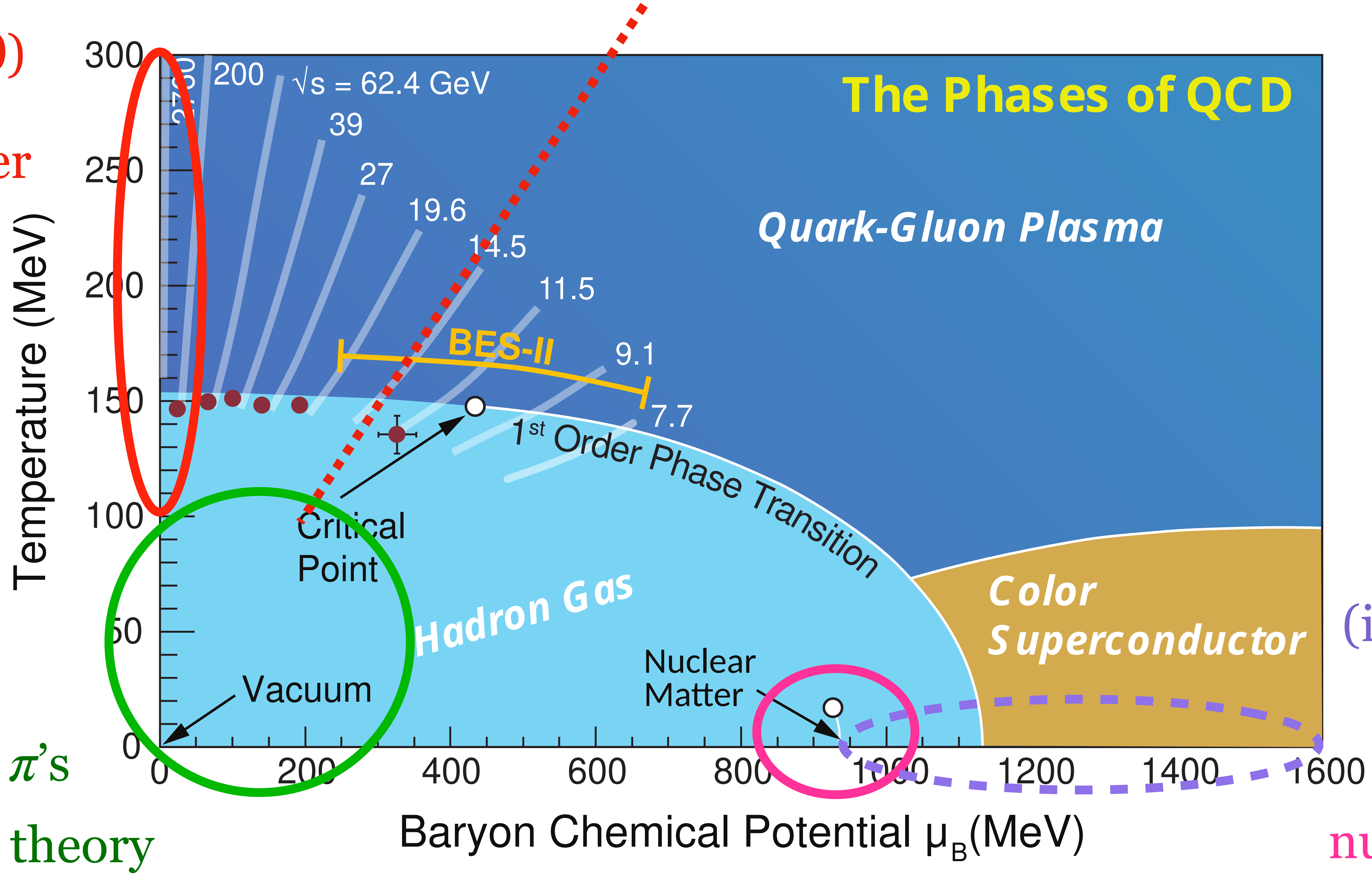


weakly coupled hadron gas, mostly π's

chiral perturbation theory
+ cross sections from experiment

neutron stars (isospin asymmetric nuclear matter)

nuclear critical point

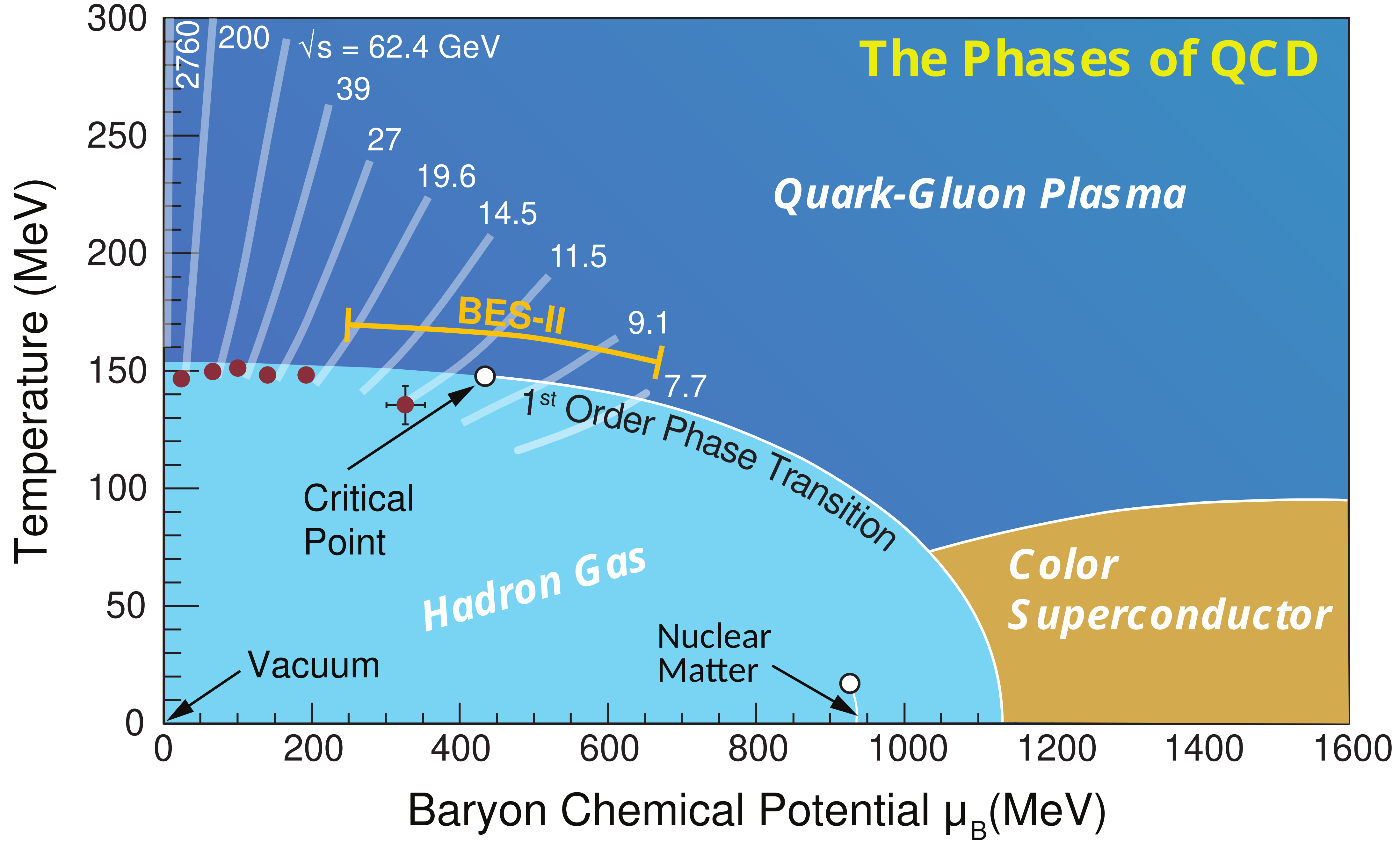
extrapolations of well-tested nuclear forces
+ experiments on nuclear fragmentation

Why do we think we may see something interesting in BES-II?

Model calculations predict a
1st order phase transition at large $n_B \Longleftrightarrow$ large μ_B

If this is the case, the QCD phase diagram contains a critical point

How to create systems probing different points in the phase diagram?



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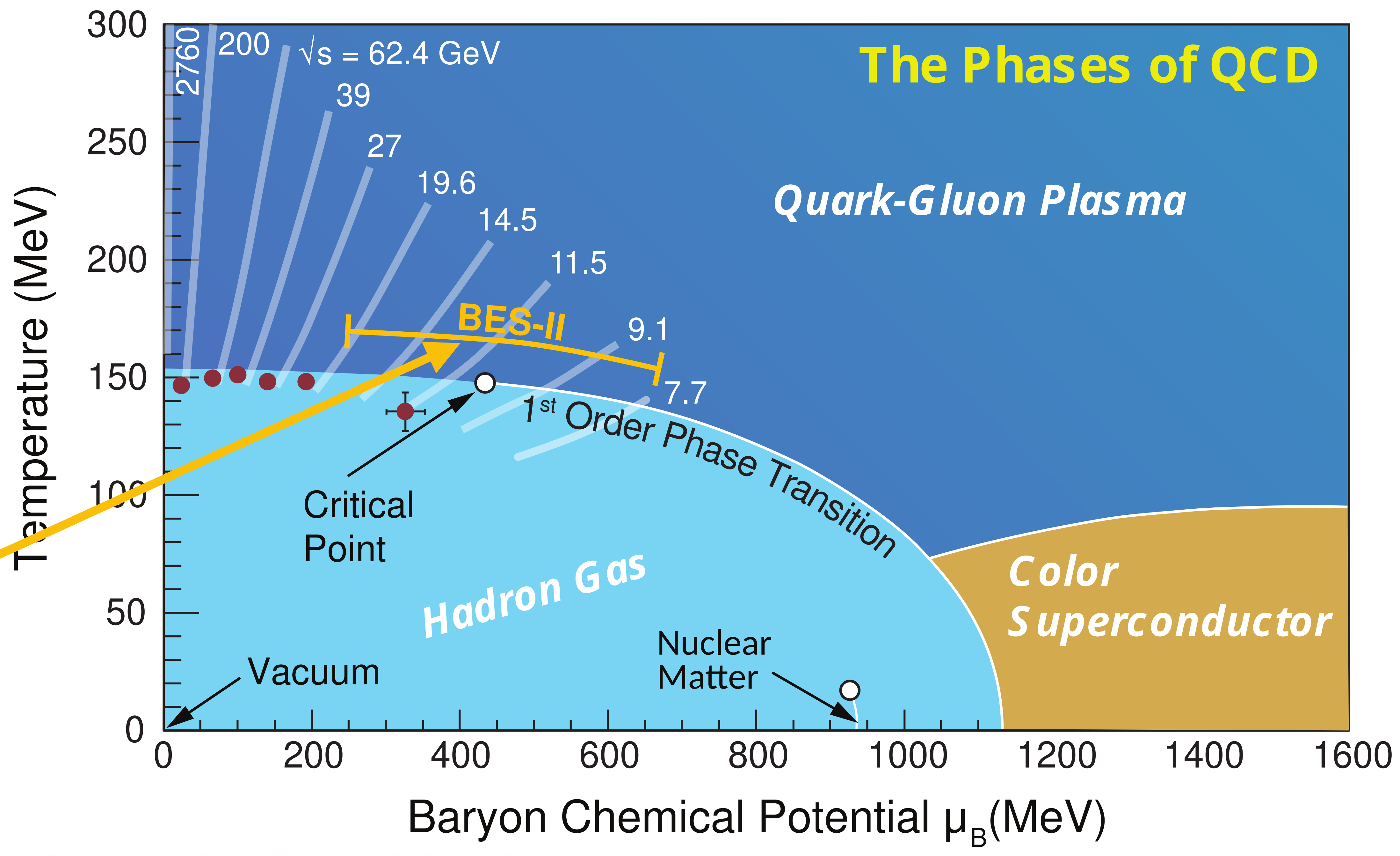
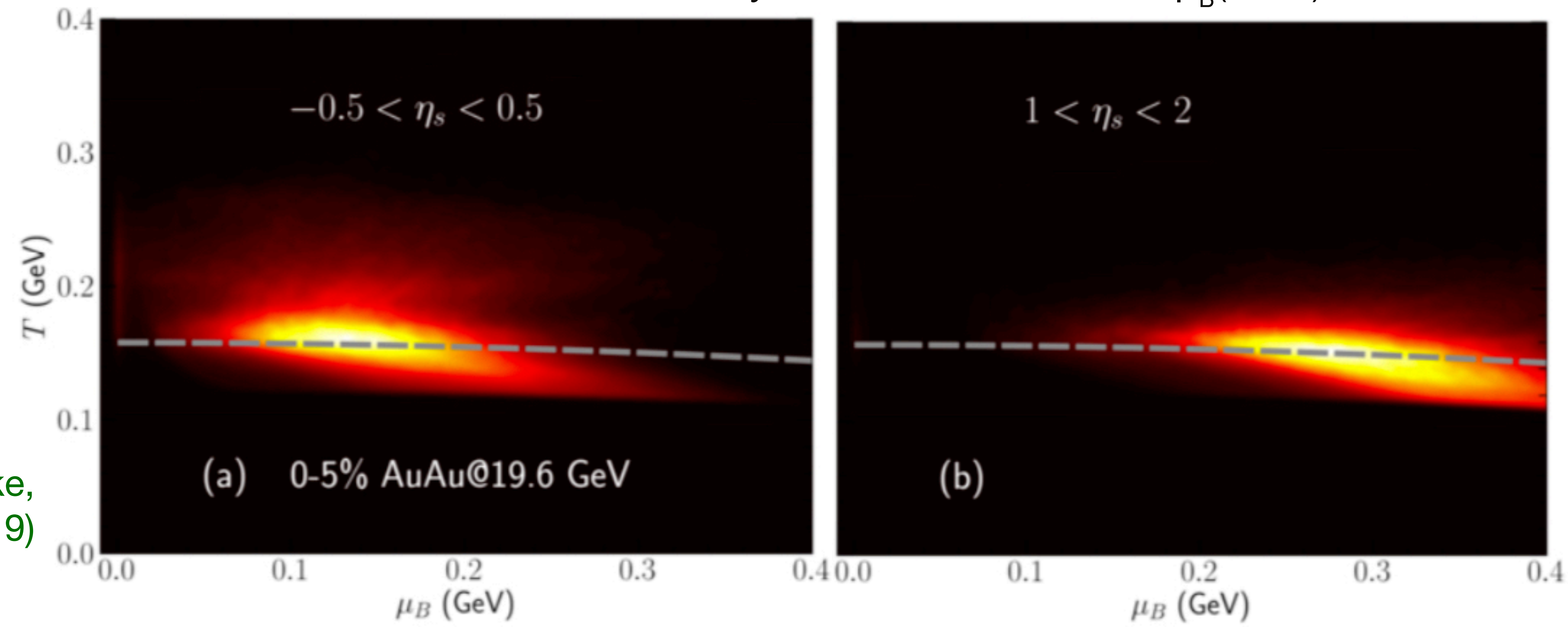


Low collision energies
(down to $\sqrt{s} = 3$ GeV for fixed-target)
= finite net baryon density at midrapity

Higher rapidity window
(ITPC upgrade:
rapidity coverage from 1.0 to 1.5)

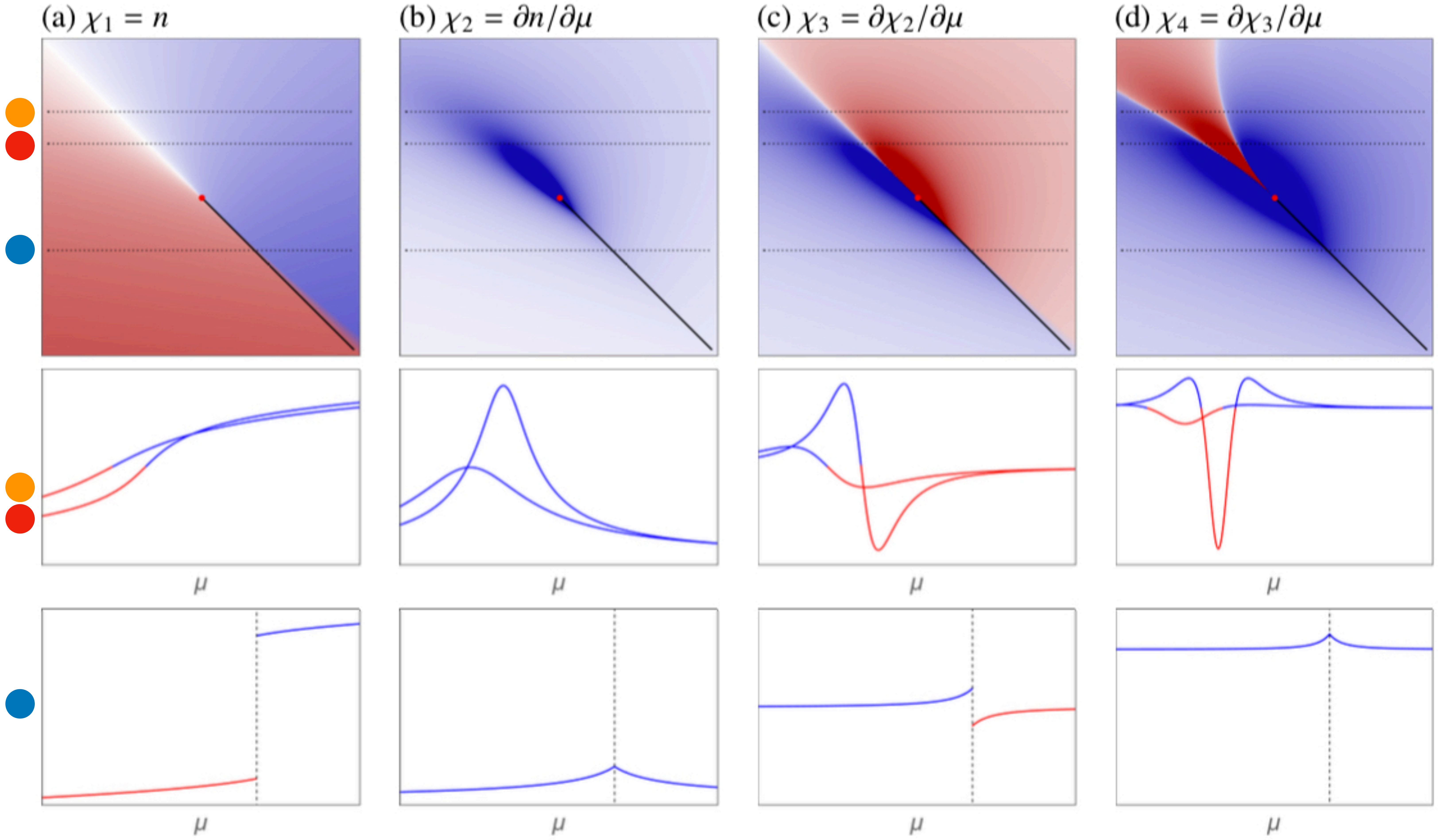


C. Shen and B. Schenke,
Nucl. Phys. A **982**, 411-414 (2019)

What are possible signatures of a critical point?

Enhanced fluctuations of conserved charges in the vicinity of a CP

A. Bzdak *et al.*, "Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan", Physics Reports 853 (2020) 1-87"



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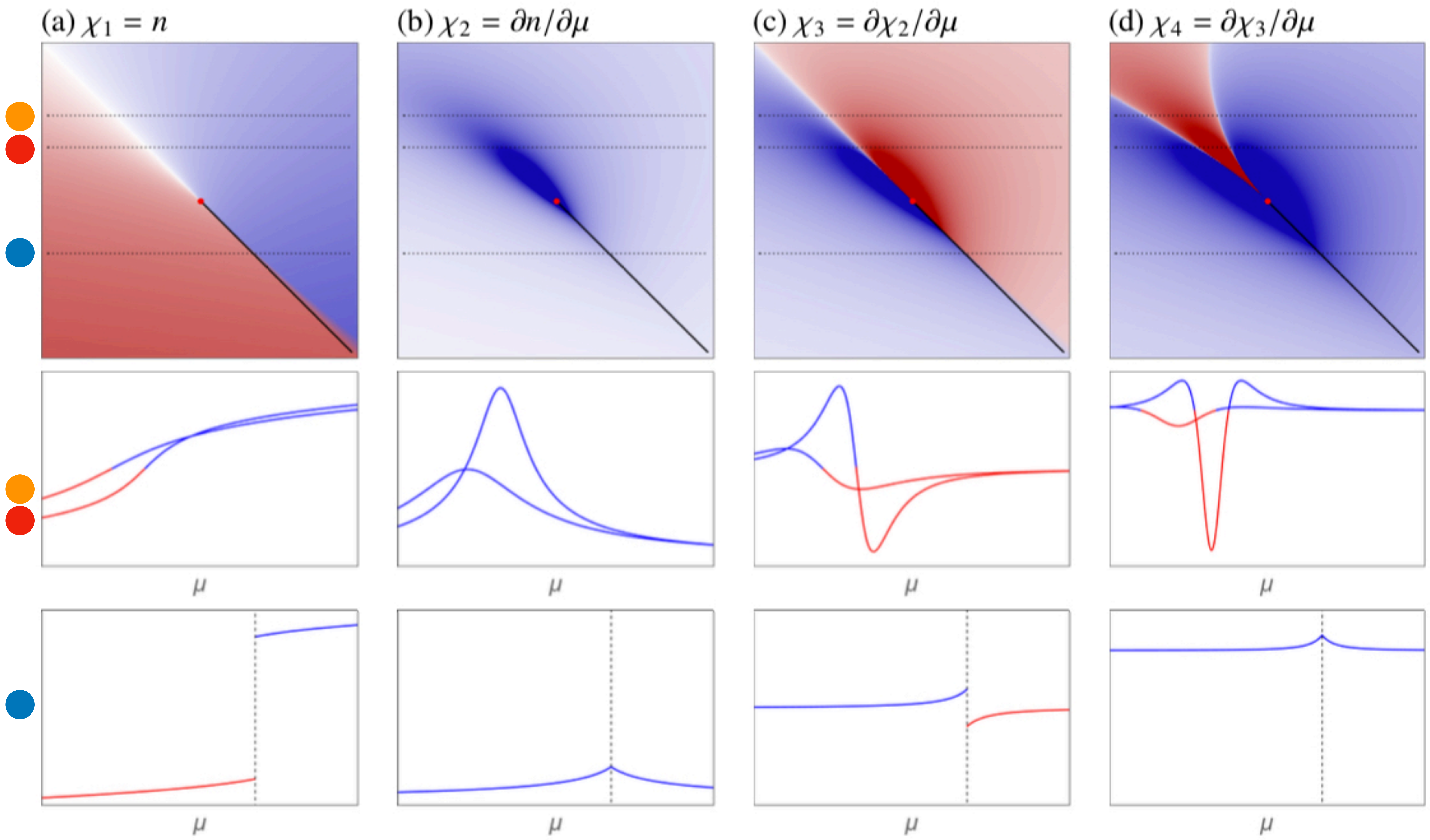

susceptibilities χ_j and cumulants κ_j:

$$\kappa_j = VT^{j-1}\chi_j = VT^{j-1}\frac{\partial^j P}{\partial\mu_B^j}$$

cumulant ratios:

$$\frac{\kappa_2}{\kappa_1}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

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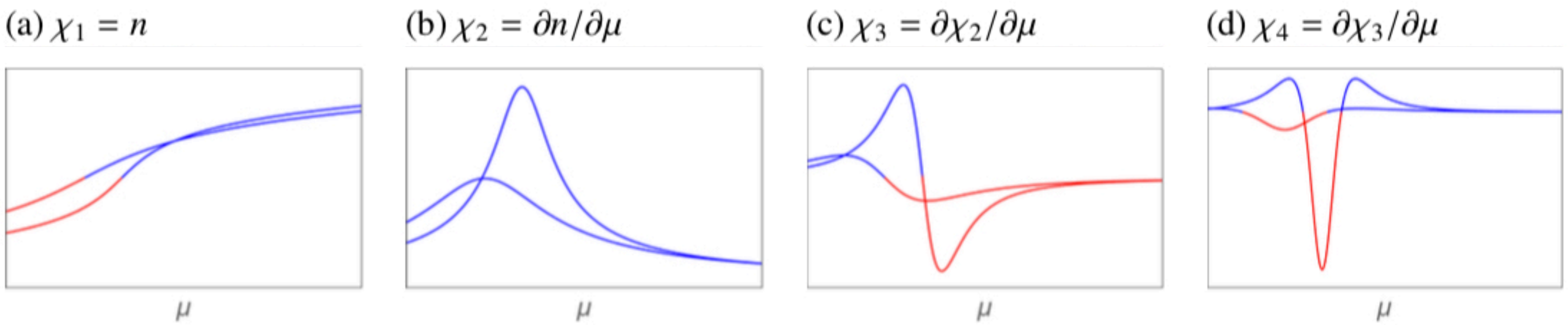

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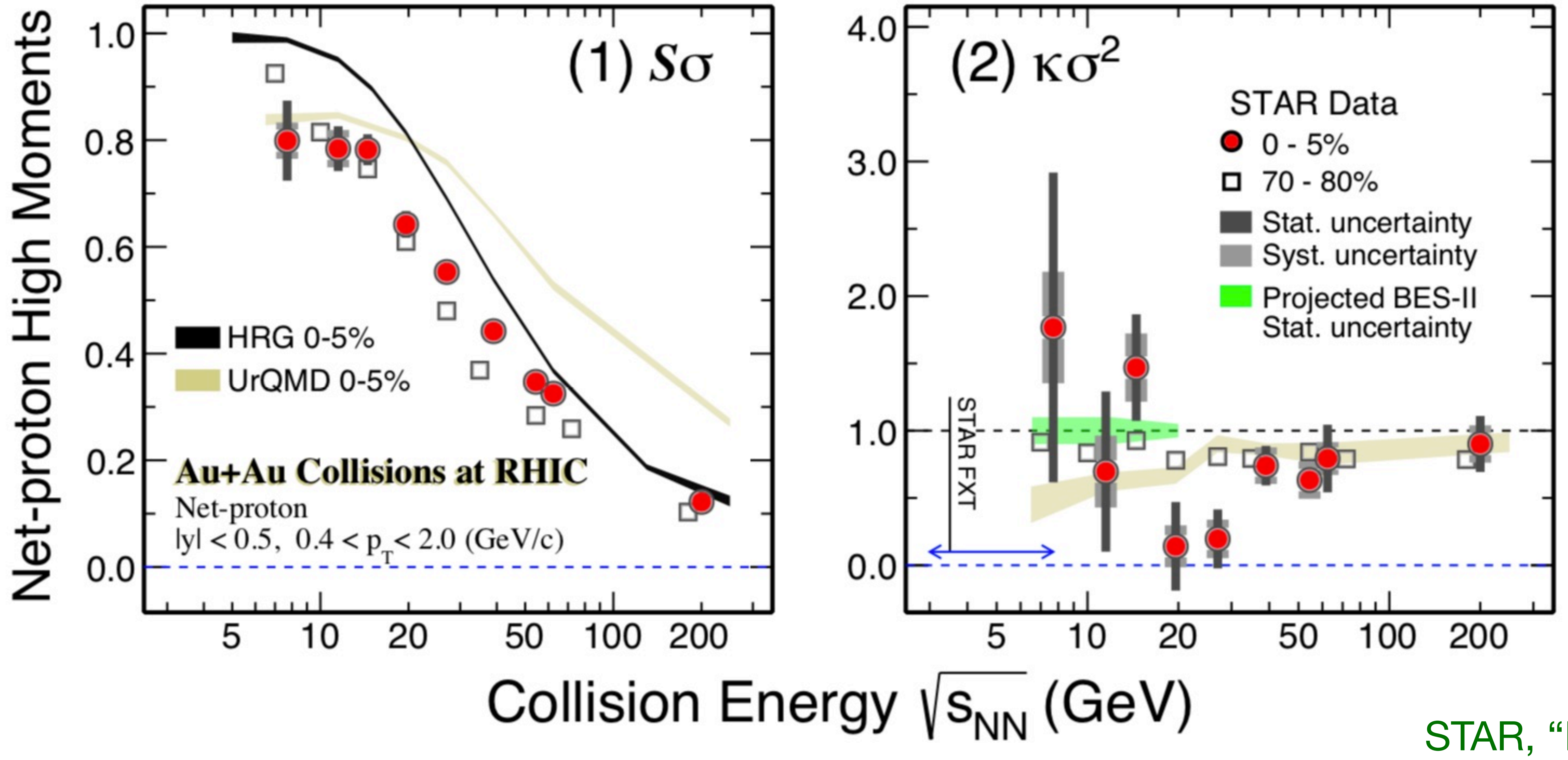

μ_B →

← $\sqrt{s}$

STAR, “Net-proton number fluctuations and the Quantum Chromodynamics critical point,'' arXiv:2001.02852

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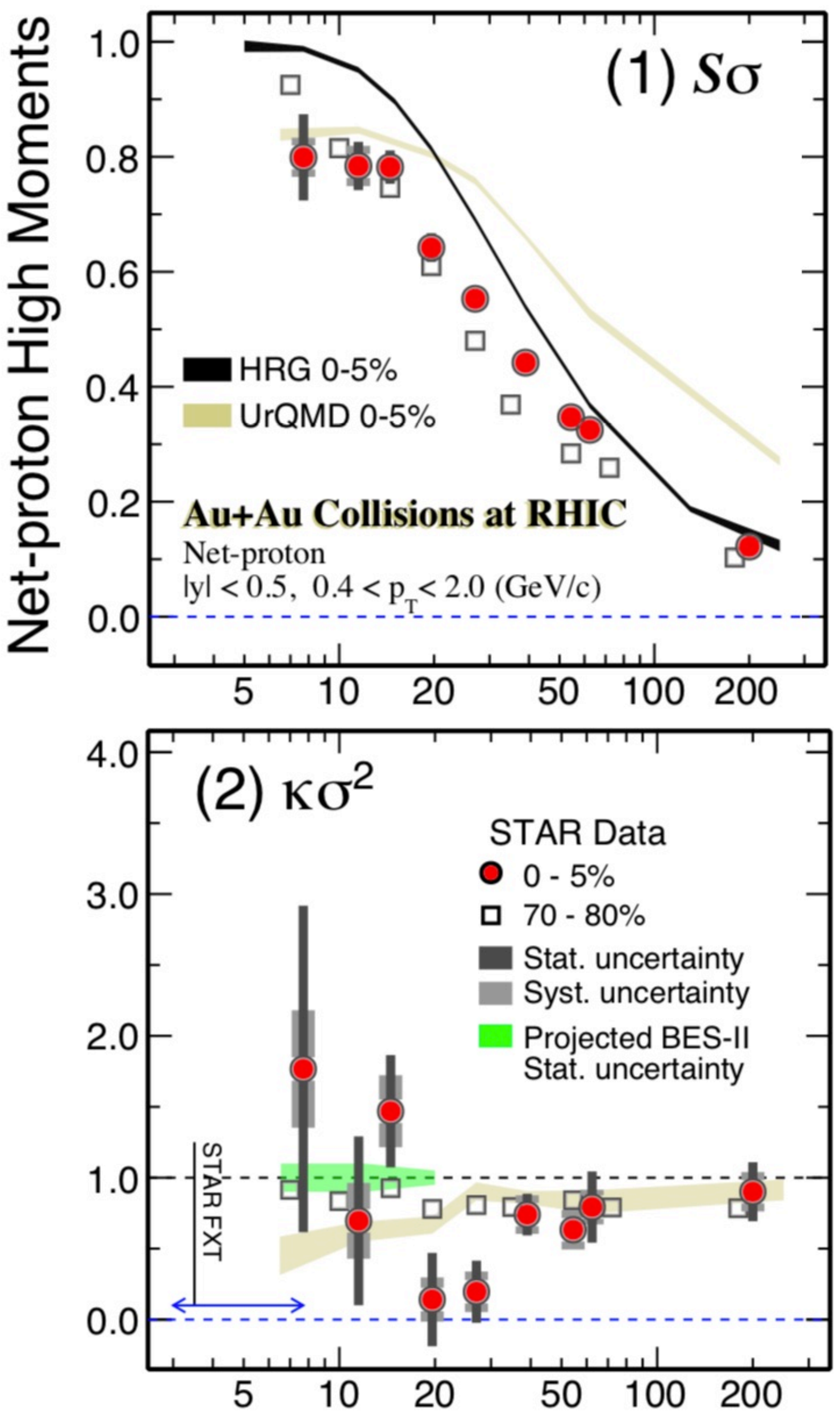
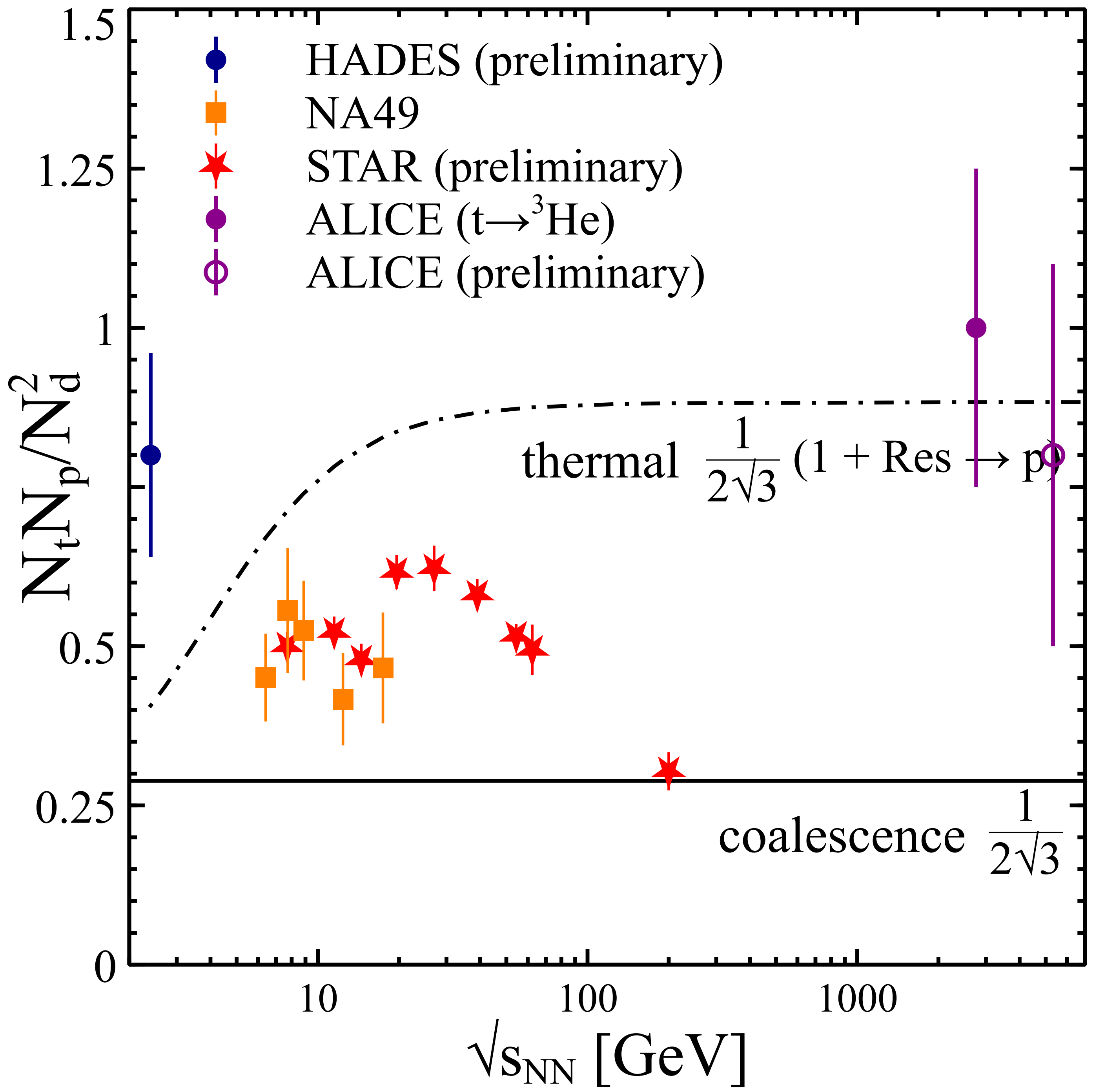
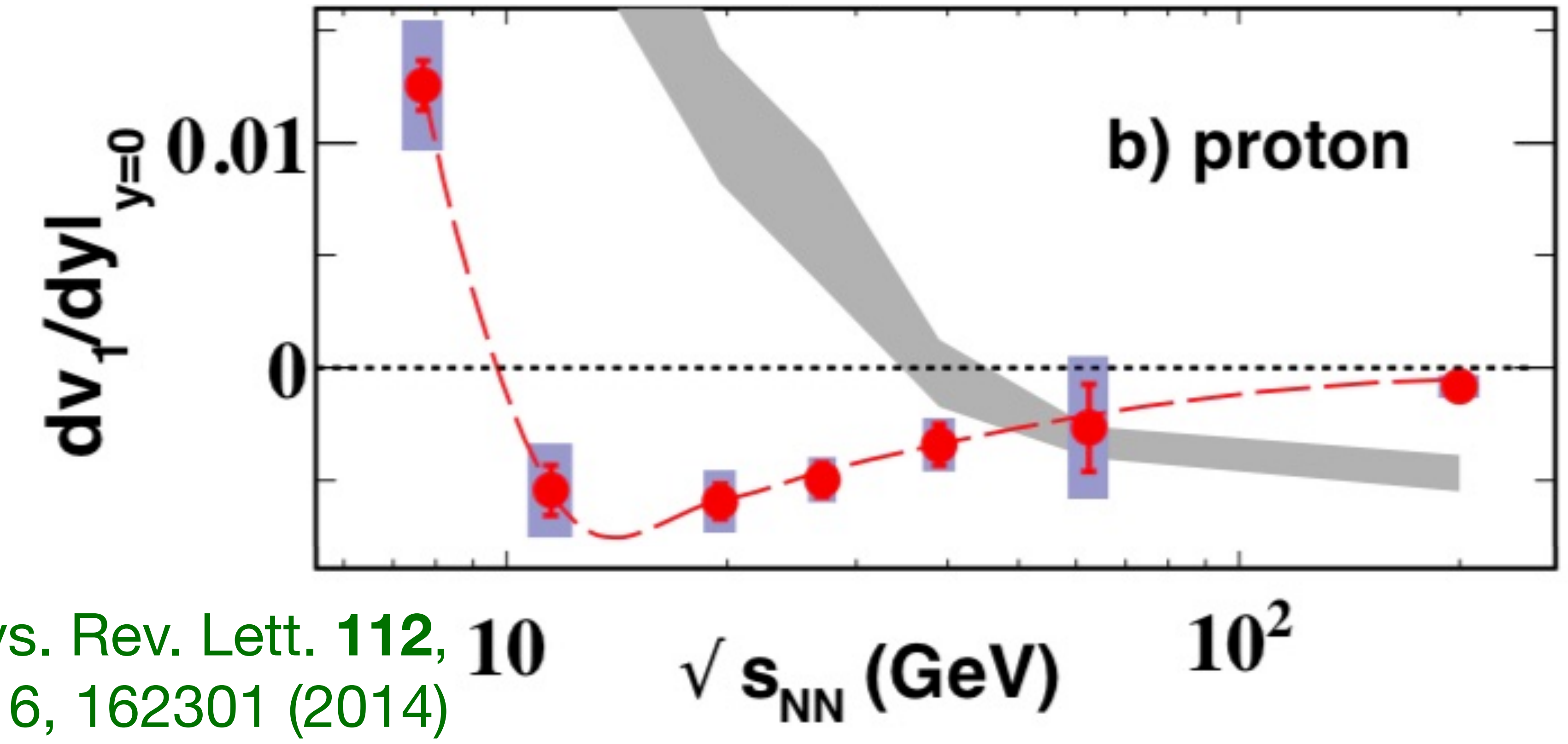


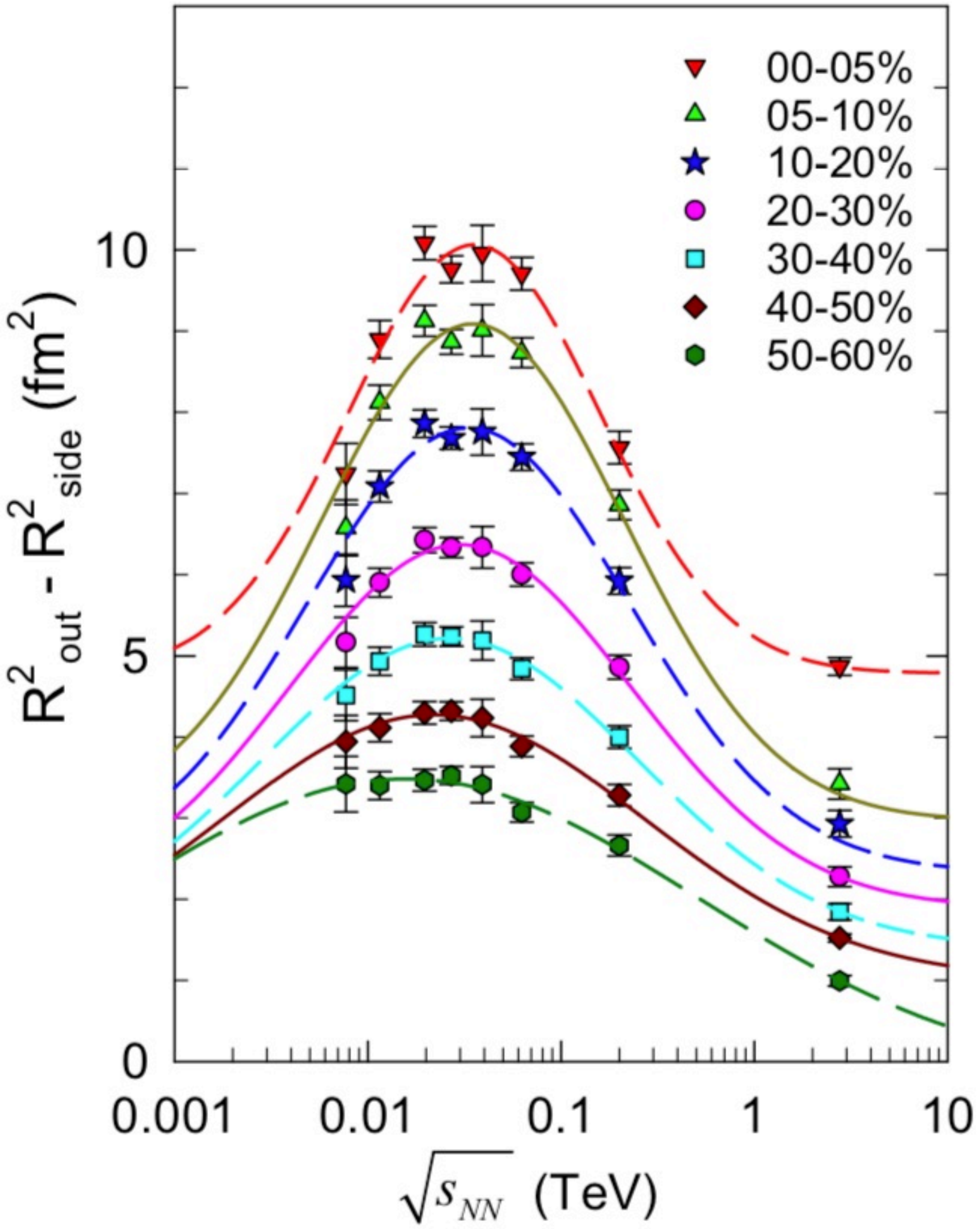
D. Oliinychenko, arXiv:2003.05476





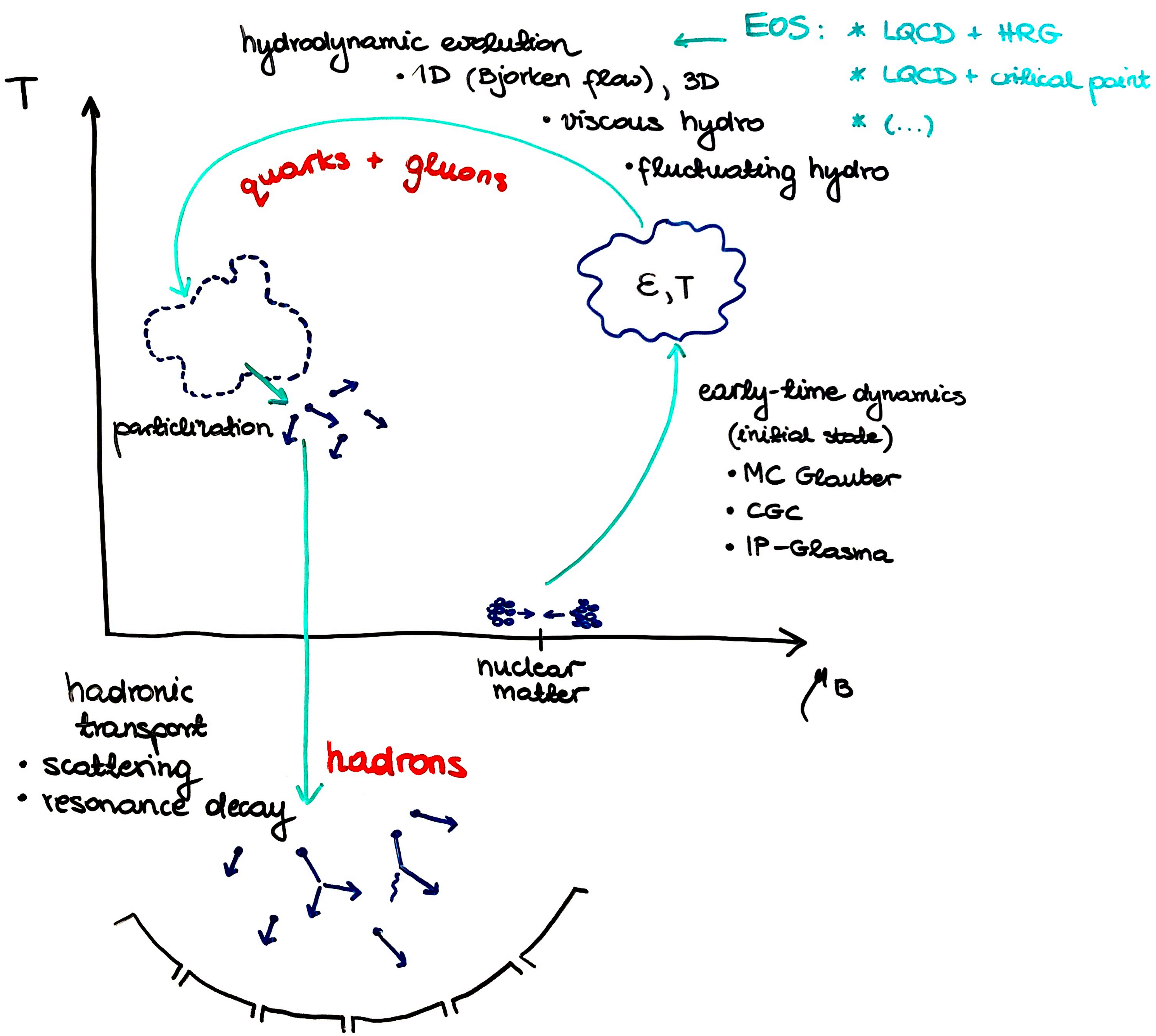
STAR, Phys. Rev. Lett. **112**, no.16, 162301 (2014)

R. A. Lacey, Phys. Rev. Lett. **114**, no.14, 142301 (2015)



We need comparison with simulations

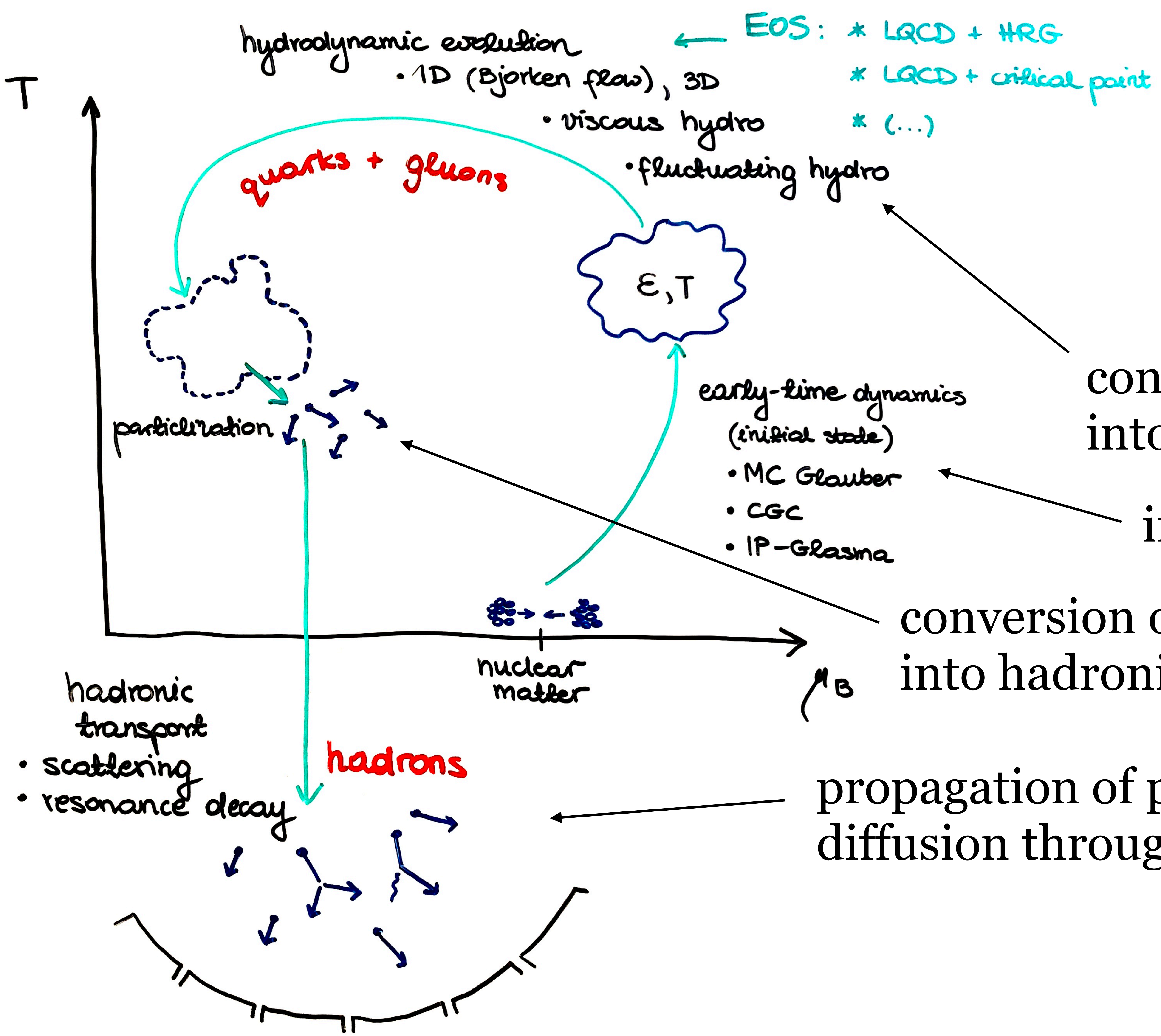
Multi-stage simulations of heavy-ion collisions



We need a simulation framework designed for studying critical behavior

BEST COLLABORATION

Multi-stage simulations of heavy-ion collisions



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conversion of position anisotropies into momentum anisotropies, EOS

initial fluctuations

conversion of fluid degrees of freedom into hadronic ones

+ Bayesian analysis!

propagation of particles to kinetic freeze-out, diffusion through scatterings and decays

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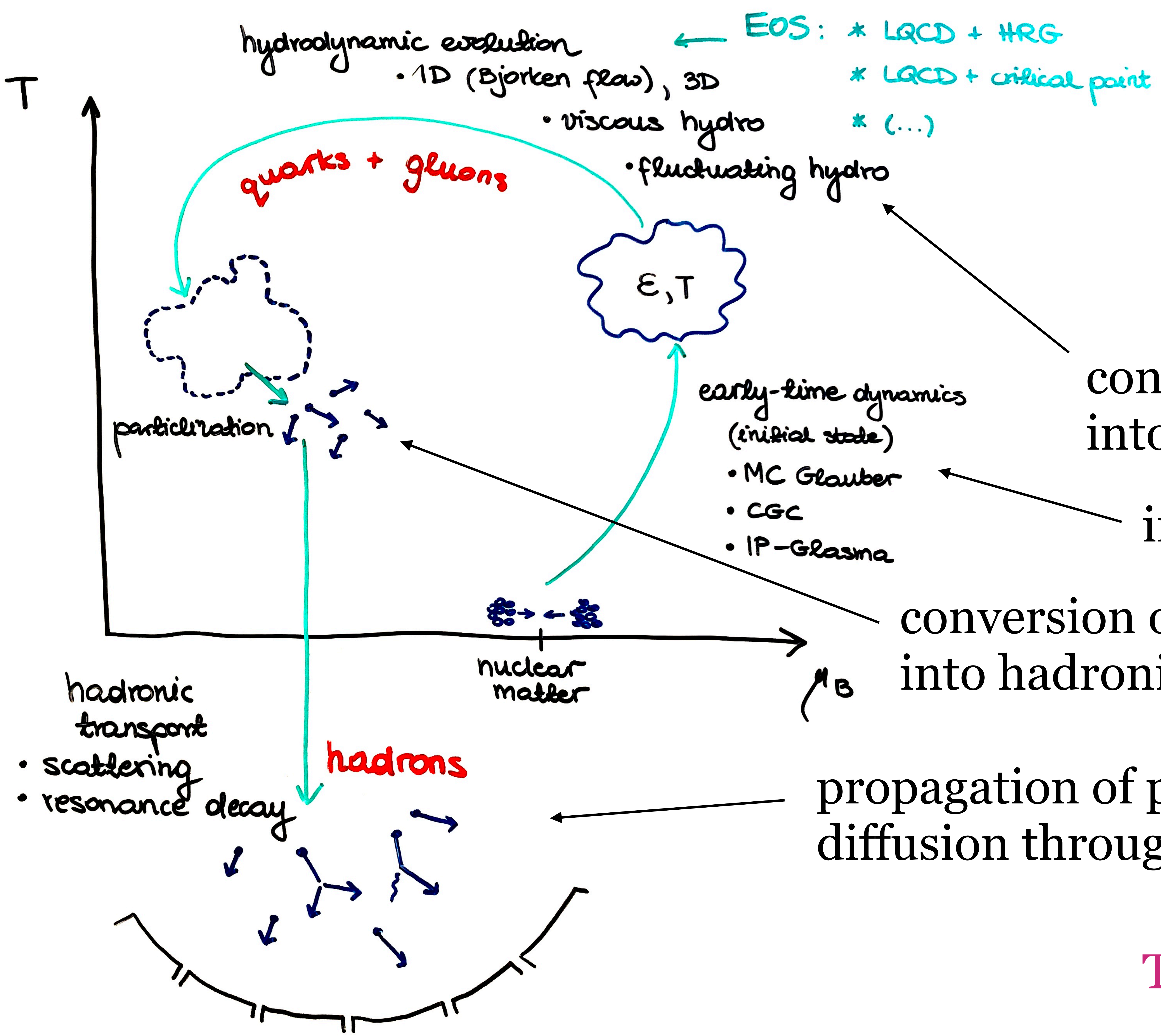

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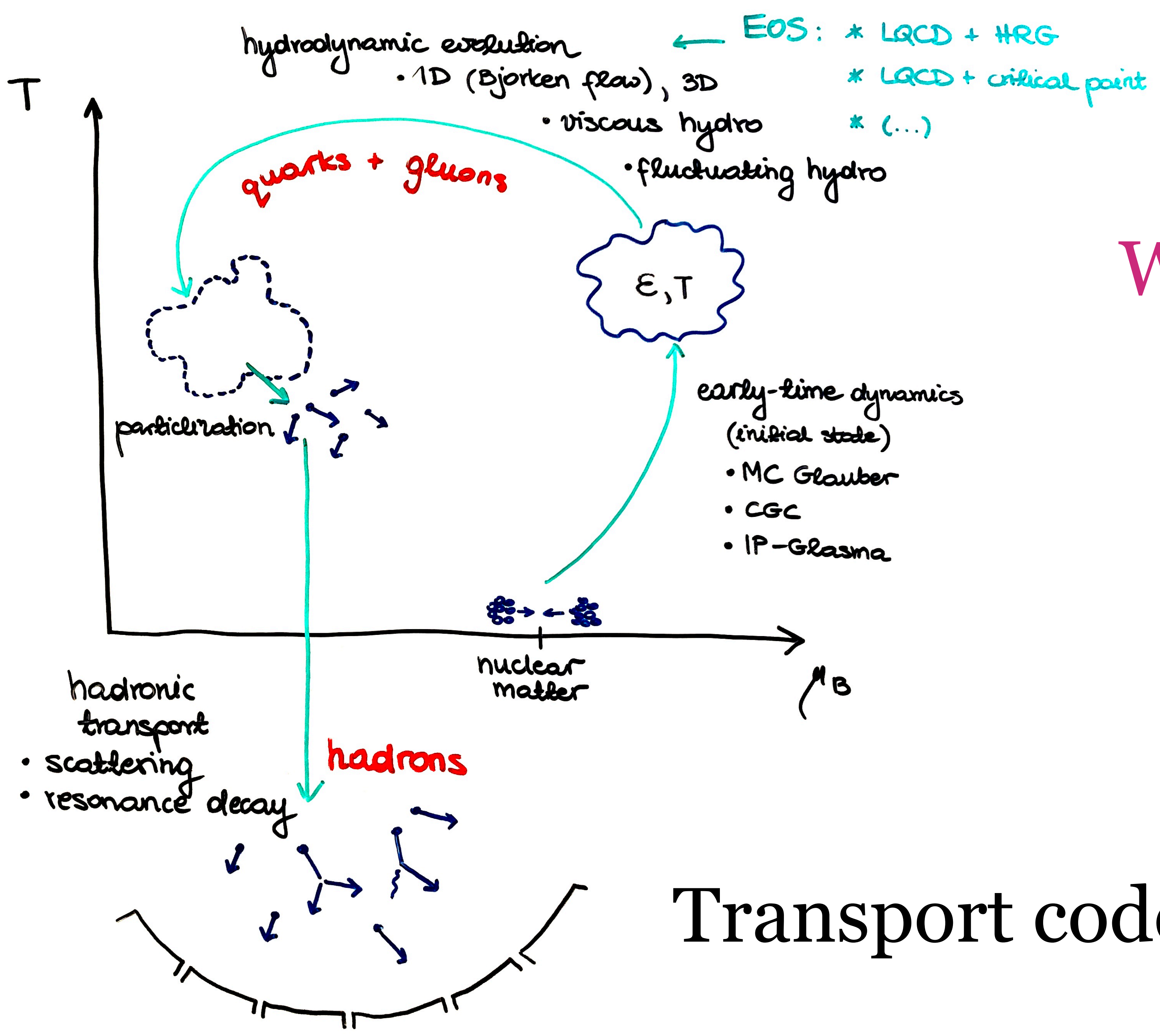
The role of nuclear interactions in hadronic afterburners is largely unexplored!

Hadronic transport with mean-field potentials



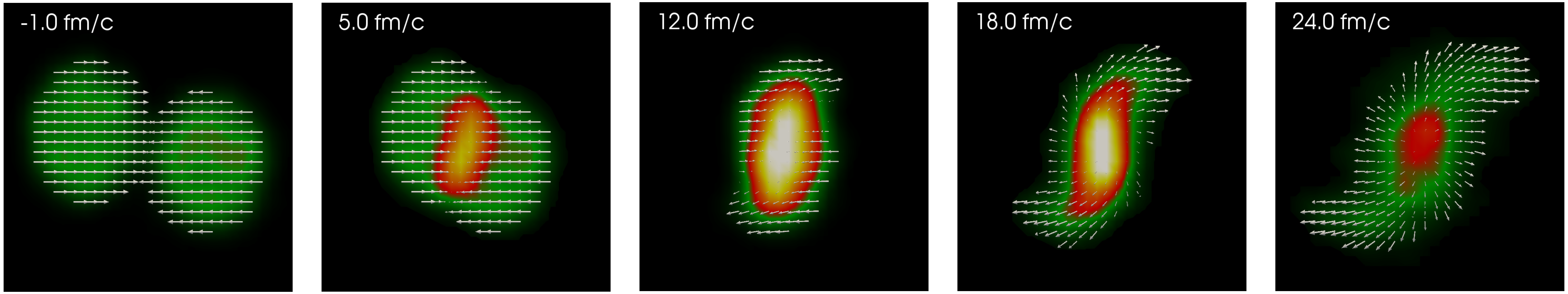
Hadronic phase is ever more important for low energies: how is it affected by the QCD equation of state (EOS)?

We need hadronic transport that is sensitive to critical phenomena!

Goal: construct an EOS with parameterizable interactions + relativistic EOMs that can be implemented in a transport code.
Important: numerical efficiency!

Transport code of choice: SMASH

H. Petersen, D. Oliinychenko, M. Mayer, J. Staudenmaier, S. Ryu, "SMASH – A new hadronic transport approach," Nucl. Phys. A **982**, 399 (2019)



Relativistic density functional with vector-density—based interactions

G. Baym and S. A. Chin, "Landau Theory of Relativistic Fermi Liquids," Nucl. Phys. A **262**, 527 (1976)

Energy density of the system:

$$\mathcal{E} = \mathcal{E}[f_{\mathbf{p}}] = \int \frac{d^3p}{(2\pi)^3}\, \epsilon_{kin}\, f_{\mathbf{p}} \;+\; \sum_{i=1}^{N} C_i \left(j_\mu j^\mu\right)^{\frac{b_i}{2}-1} \left[j^0 j^0 - g^{00}\left(\frac{b_i - 1}{b_i}\right) j_\lambda j^\lambda \right]$$

← like Skyrme energy, but Lorentz covariant

$$\mathcal{E}\Big|_{\textsf{rest frame}} = \int \frac{d^3p}{(2\pi)^3}\, \epsilon_{kin}\, f_{\mathbf{p}} + \sum_{i=1}^{N} \frac{C_i}{b_i} n_B^{b_i}$$

← mean field interactions parameterized by C_i and b_i

Quasiparticle energy:

$$\varepsilon_{\mathbf{p}} \equiv \frac{\delta \mathcal{E}[f_{\mathbf{p}}]}{\delta f_{\mathbf{p}}} = \epsilon_{kin} \;+\; \sum_{i=1}^{N} C_i n_B^{b_i - 1}$$

Equations of motion:

$$\frac{dx^i}{dt} \equiv -\frac{\partial \varepsilon_{\mathbf{p}}}{\partial p_i}\,, \qquad \frac{dp^i}{dt} \equiv \frac{\partial \varepsilon_{\mathbf{p}}}{\partial x_i}$$

← input to transport code

Relativistic DF: two 1st order phase transitions

Systems with two 1st order phase transitions: nuclear and "quark/hadron", or "QGP-like"

- Degrees of freedom: nucleons
- "Quark" matter coexists with dense nuclear matter, not vacuum
- 4 interactions terms = 8 parameters to fix:

Pressure: $$P = g\int \frac{d^3p}{(2\pi)^3}\, T\, \ln\left[1 + e^{-\beta(\varepsilon_{\mathbf{p}}-\mu_B)}\right] + \sum_{i=1}^{N=4} C_i \frac{b_i - 1}{b_i} n_B^{b_i}$$

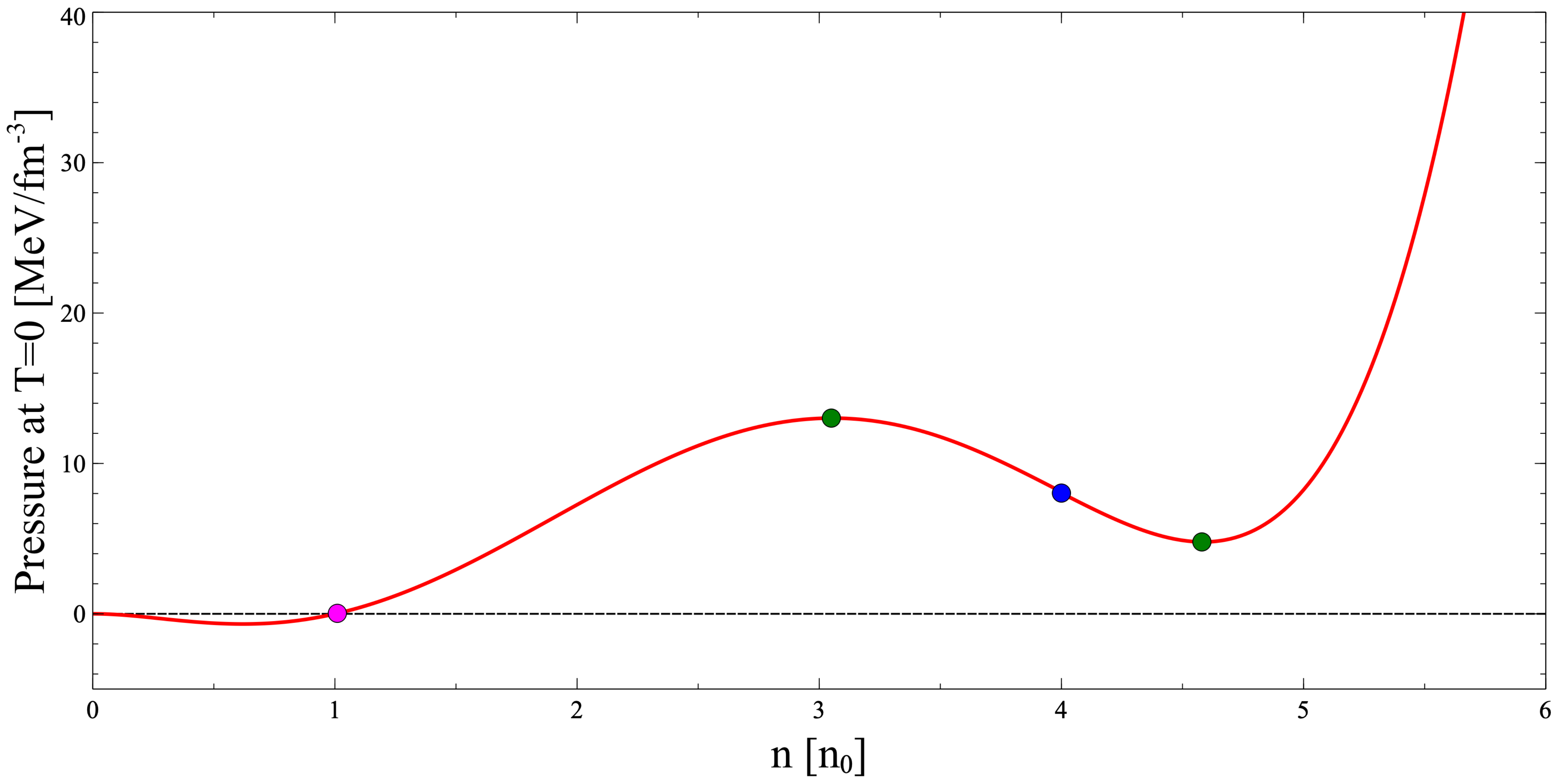
C_i and b_i are fitted to reproduce:

$n_0 = 0.160\ \mathrm{fm}^{-3}$, $E_B = -16.3$ MeV

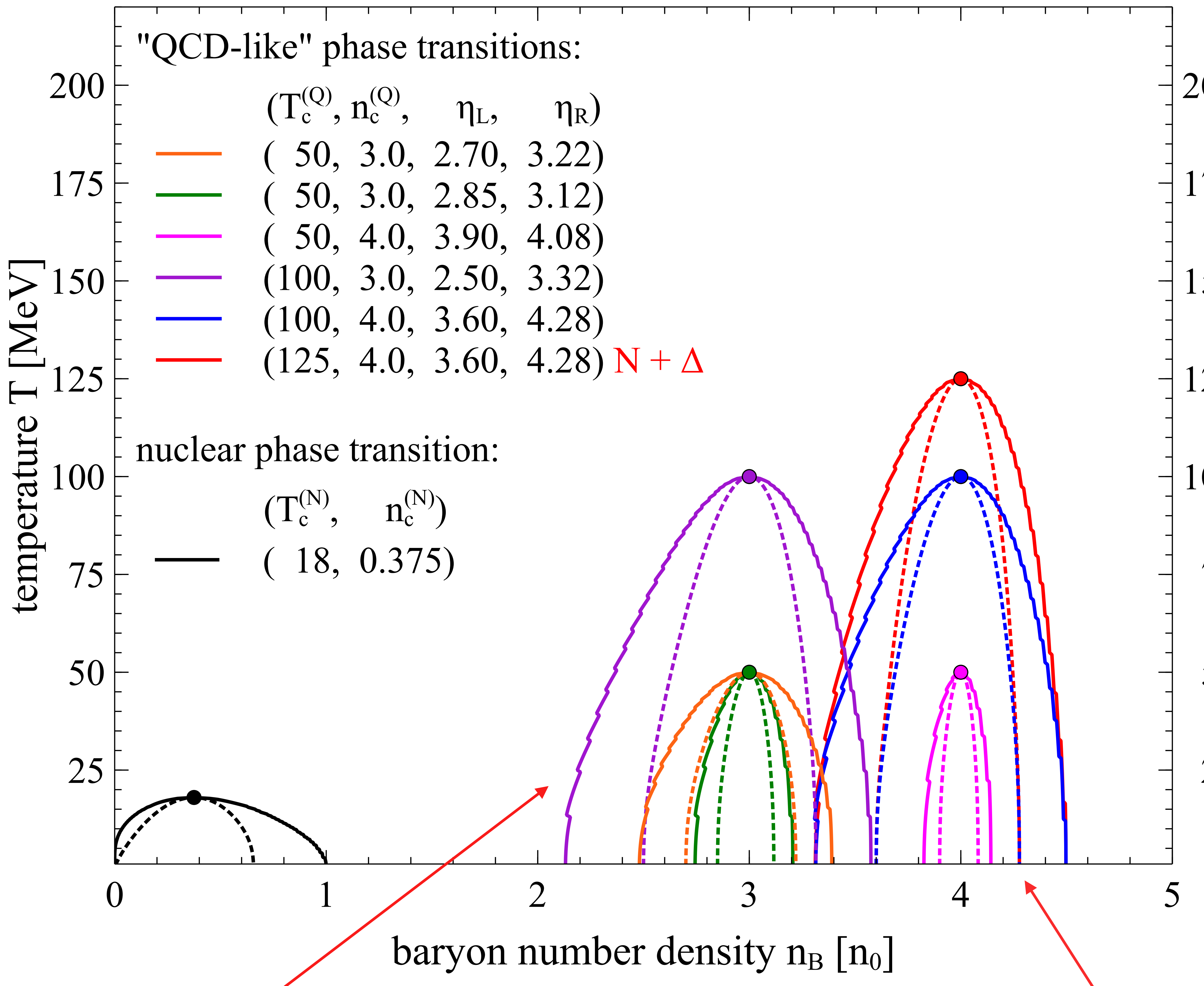
$T_c^{(N)} = 18$ MeV, $n_c^{(N)} = 0.375 n_0$

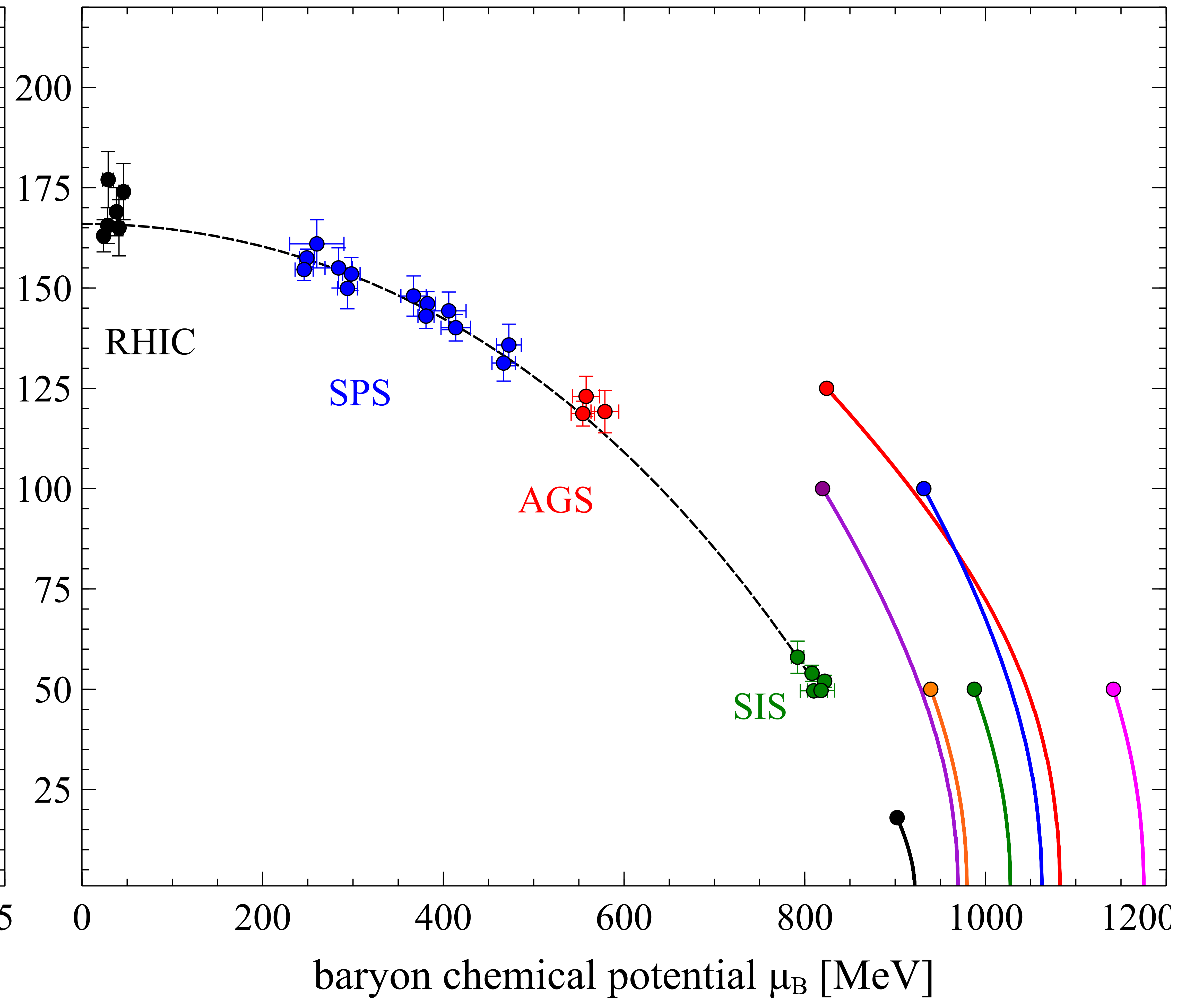
$T_c^{(Q)} = ?$, $n_c^{(Q)} = ?$

$\eta_L = ?$, $\eta_R = ?$



Phase diagram in the (T, n_B) and (T, μ_B) plane





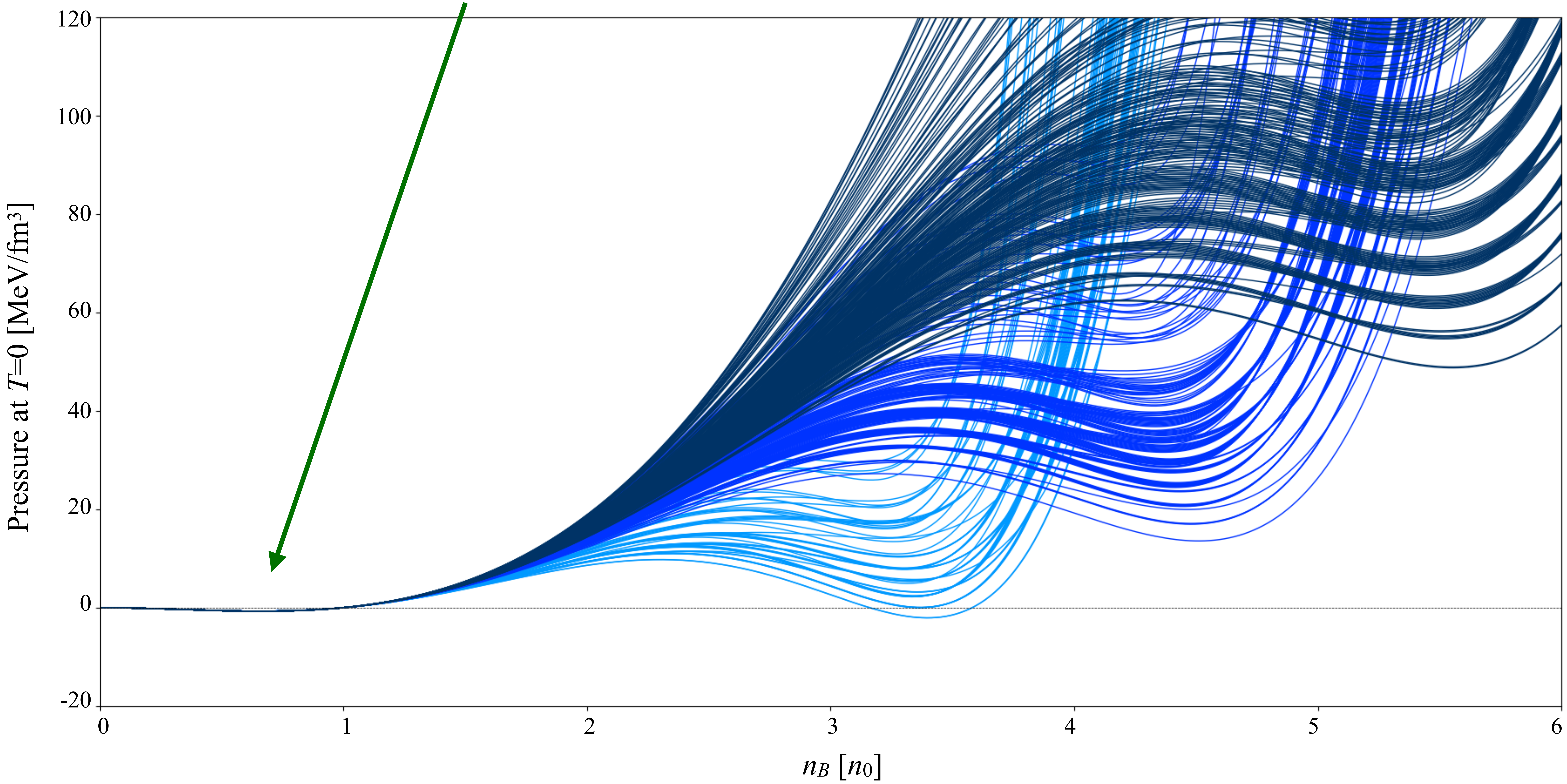
solid lines = coexistence regions

dashed lines = spinodal regions

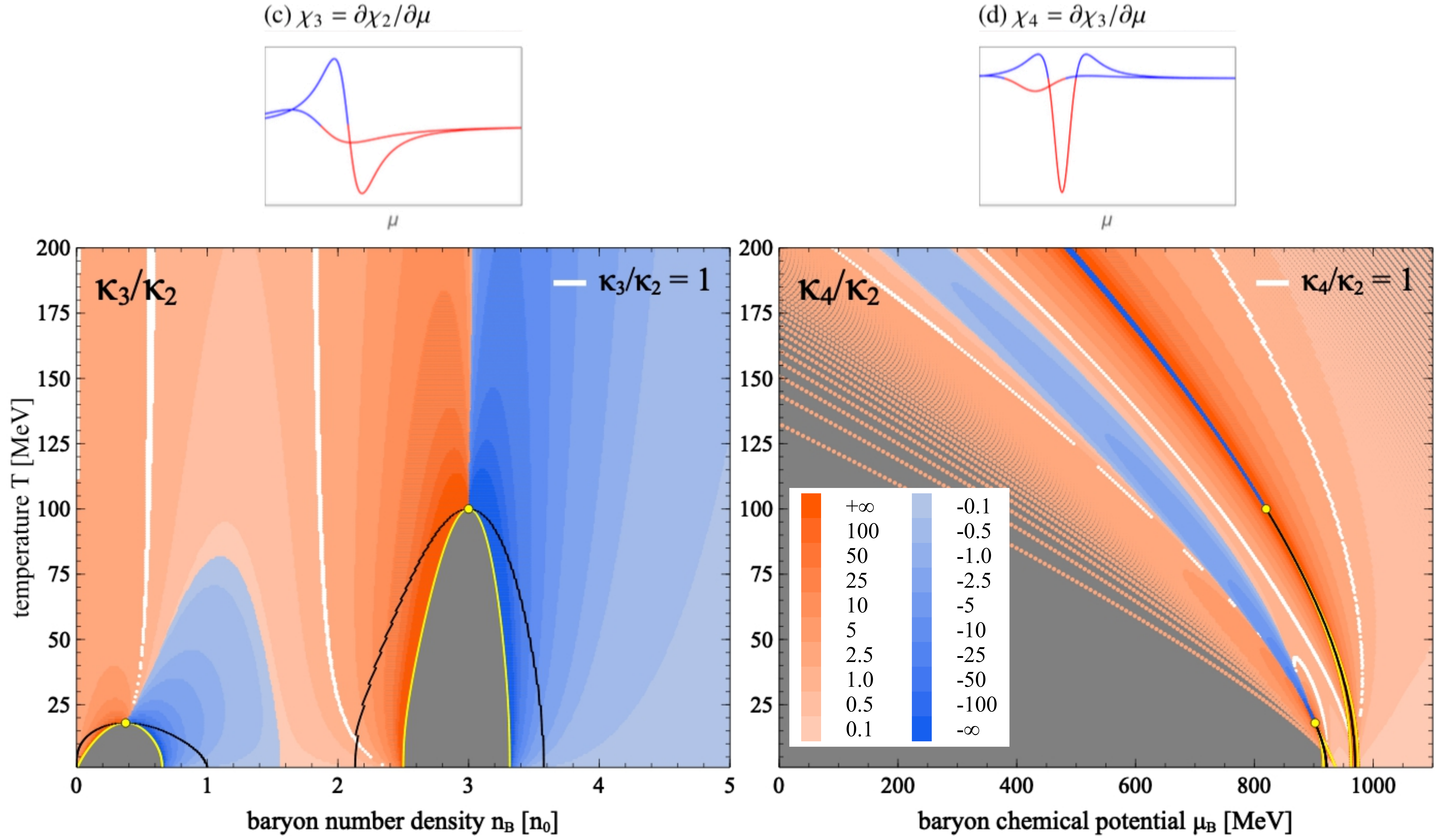
* J. Cleymans, H. Oeschler, K. Redlich and S. Wheaton, "Comparison of chemical freeze-out criteria in heavy-ion collisions," Phys. Rev. C **73**, 034905 (2006)

Relativistic DF: two 1st order phase transitions

Properties of ordinary nuclear matter are well known, but few constraints for $n_B \geq 1.5 n_0$



Cumulants diagrams in the (T, n_B) and (T, μ_B) plane

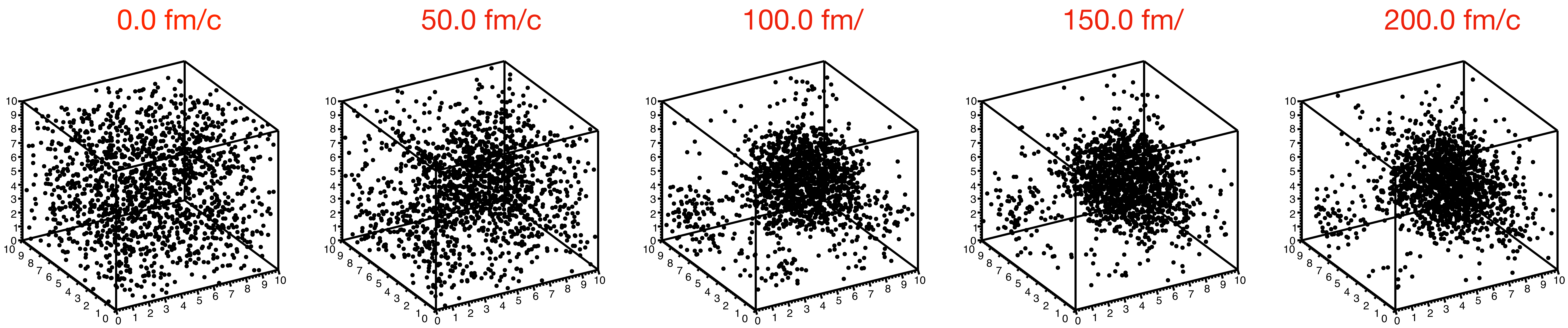


SMASH: spinodal region of the nuclear liquid-gas phase transition

We simulate dense nuclear matter in a box with periodic boundary conditions = “infinite matter”

Initialization inside the spinodal region: nuclear spinodal decomposition occurs.

Nuclear matter is well described!



L = 10 fm, $n_B = 0.25 n_0$, T = 1 MeV, $N_{test} = 200$ ← **Test particles are *necessary* for the density calculation**

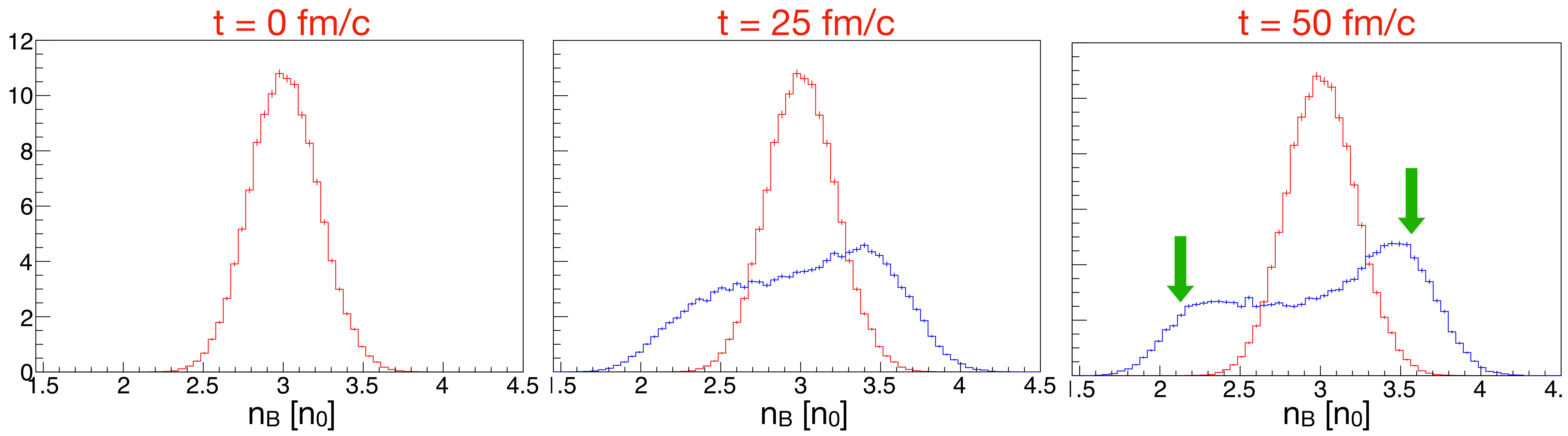
No scattering or decays

SMASH: spinodal region of the “GQP-like” phase transition

T = 1 MeV $n_B = 3n_0$

Cell (bin) length L = 2 fm

The distribution becomes bimodal as the system separates!



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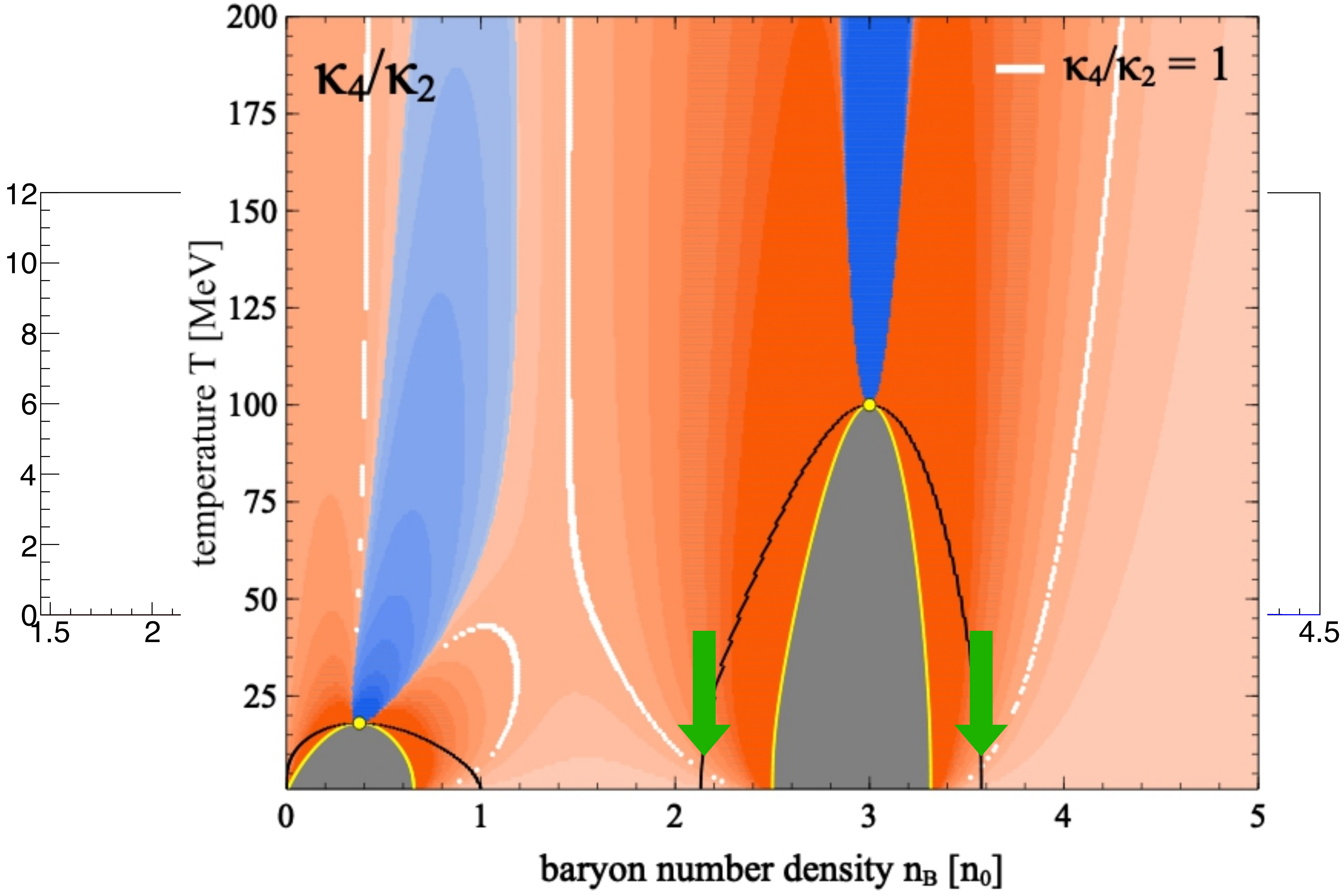
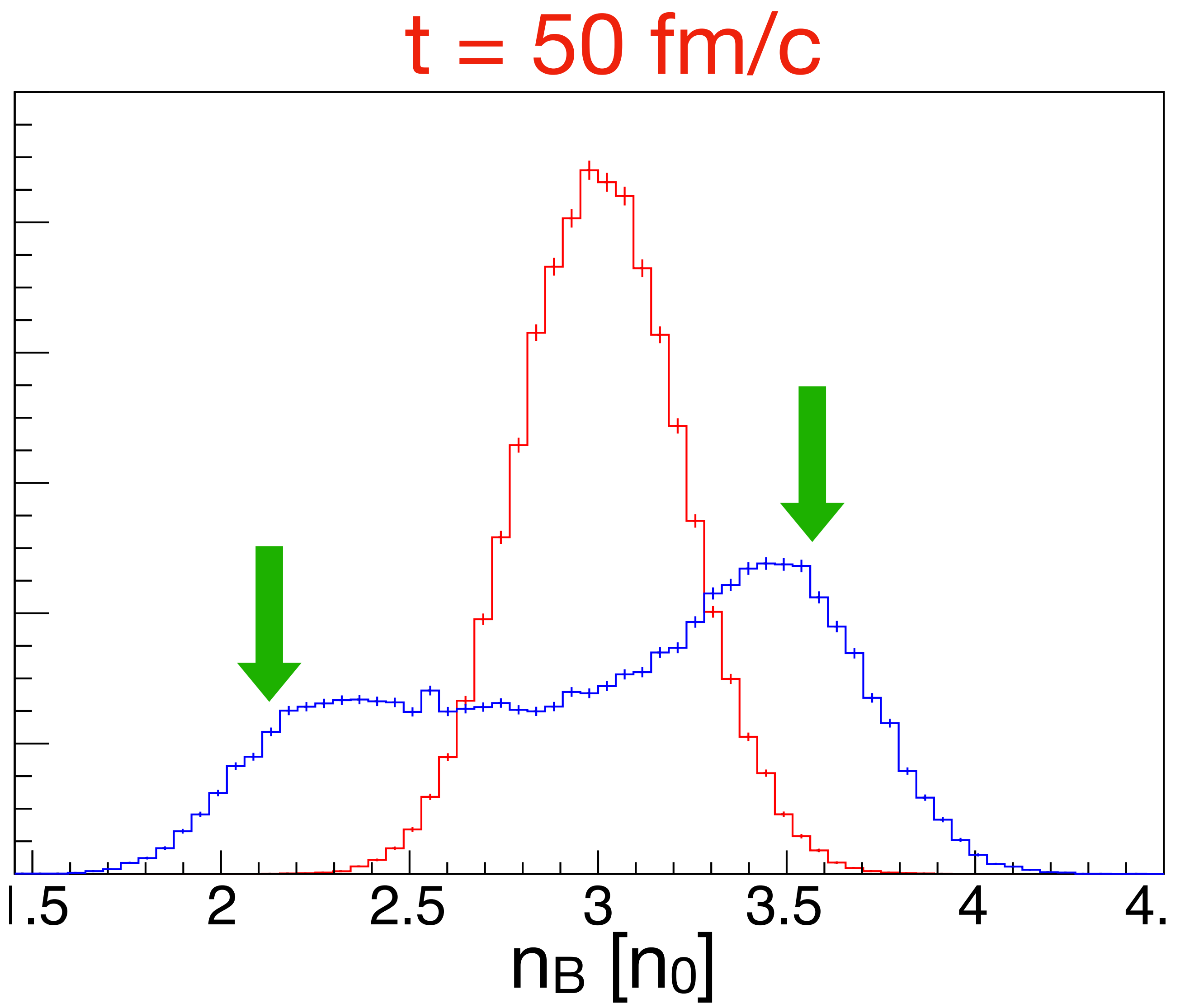




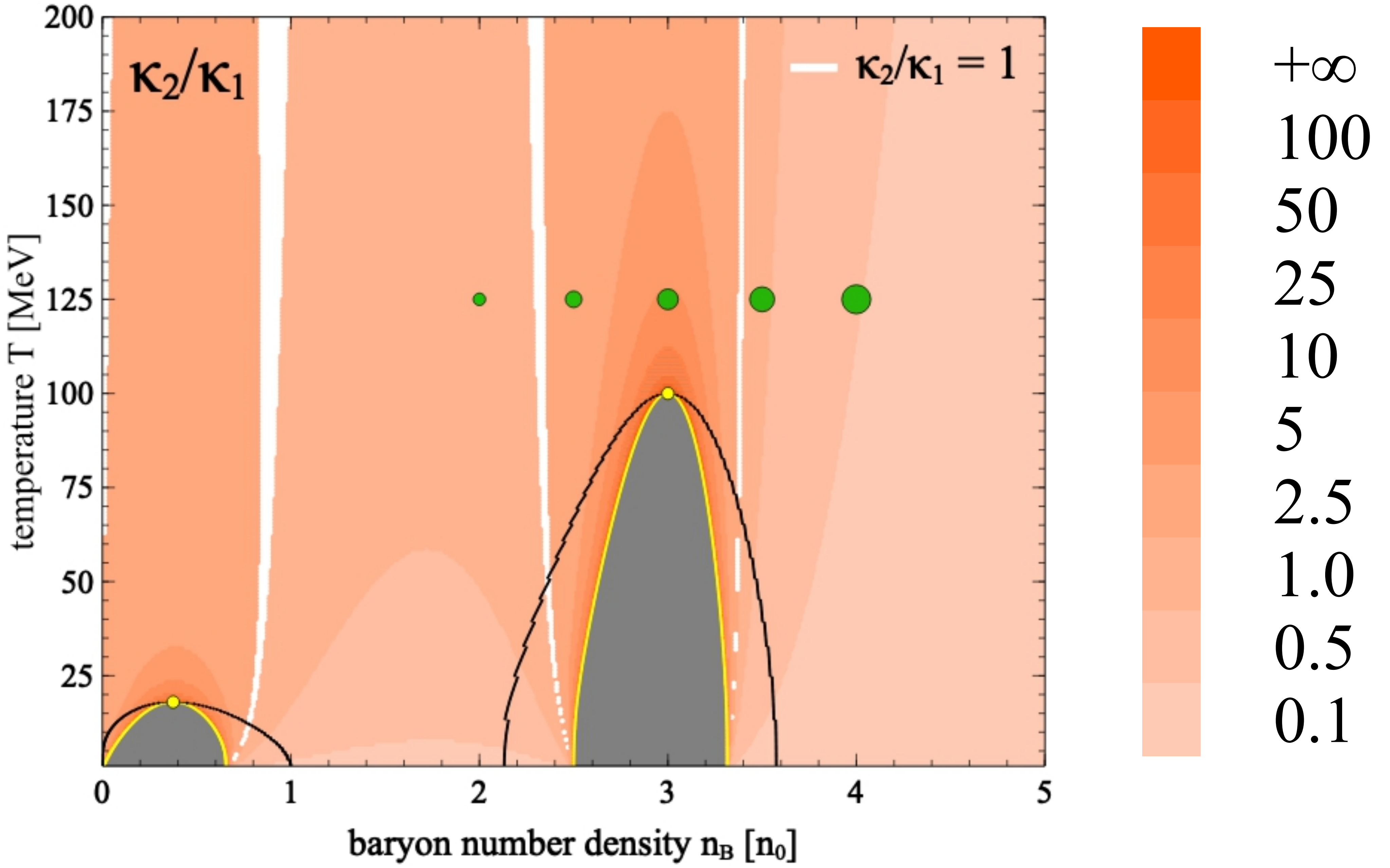
SMASH: critical behavior above the “GQP-like” phase transition

T = 125 MeV

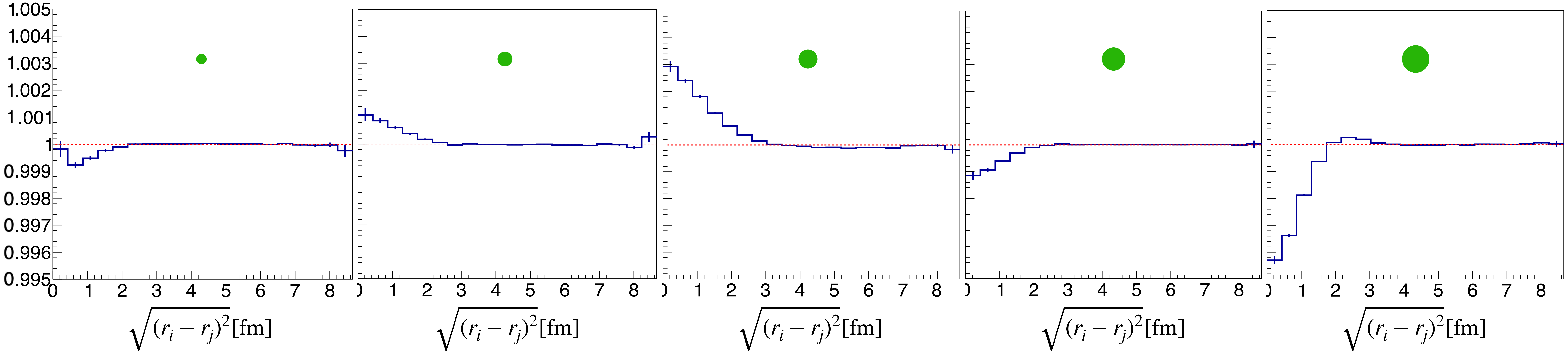
$$\frac{\kappa_2}{\kappa_1} = 1 + \frac{\int dr_1 dr_2 \ \delta\rho(r_1, r_2)}{\langle N \rangle}$$

where

$$\delta\rho(r_1, r_2) = \rho_2(r_1, r_2) - \rho_1(r_1)\rho_1(r_2)$$



Plots of $\frac{\rho_2(r_1, r_2)}{\rho_1(r_1)\rho_1(r_2)}$:



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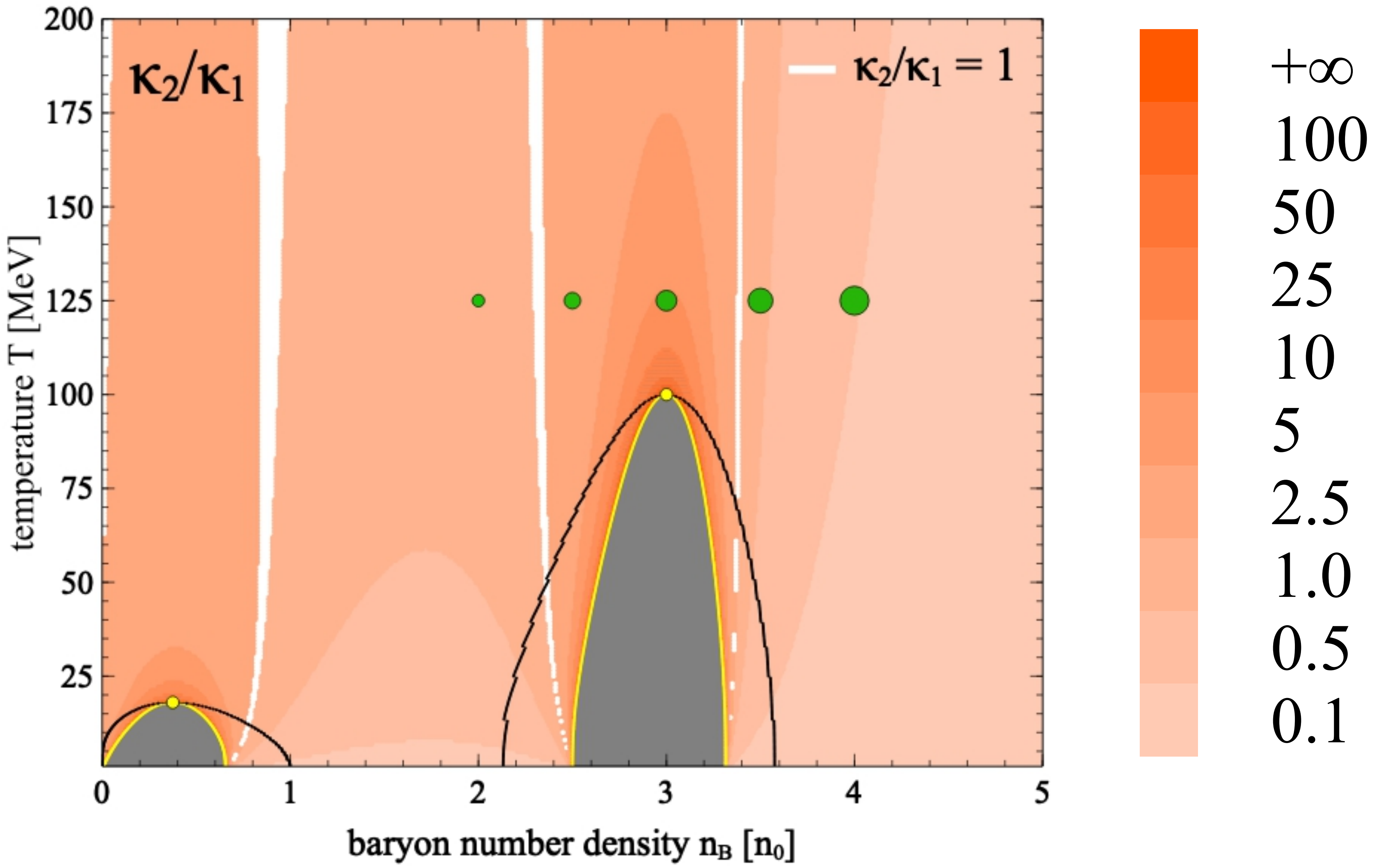

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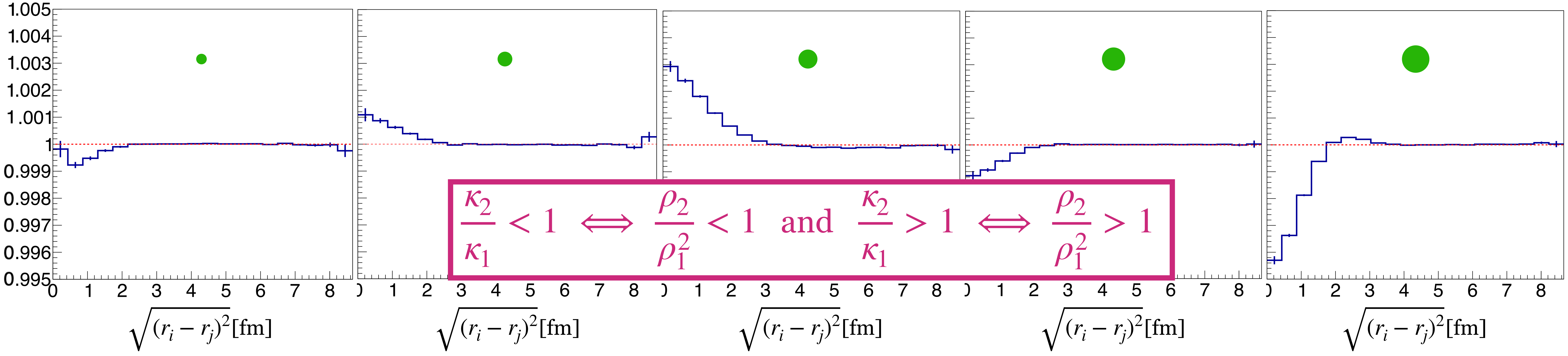

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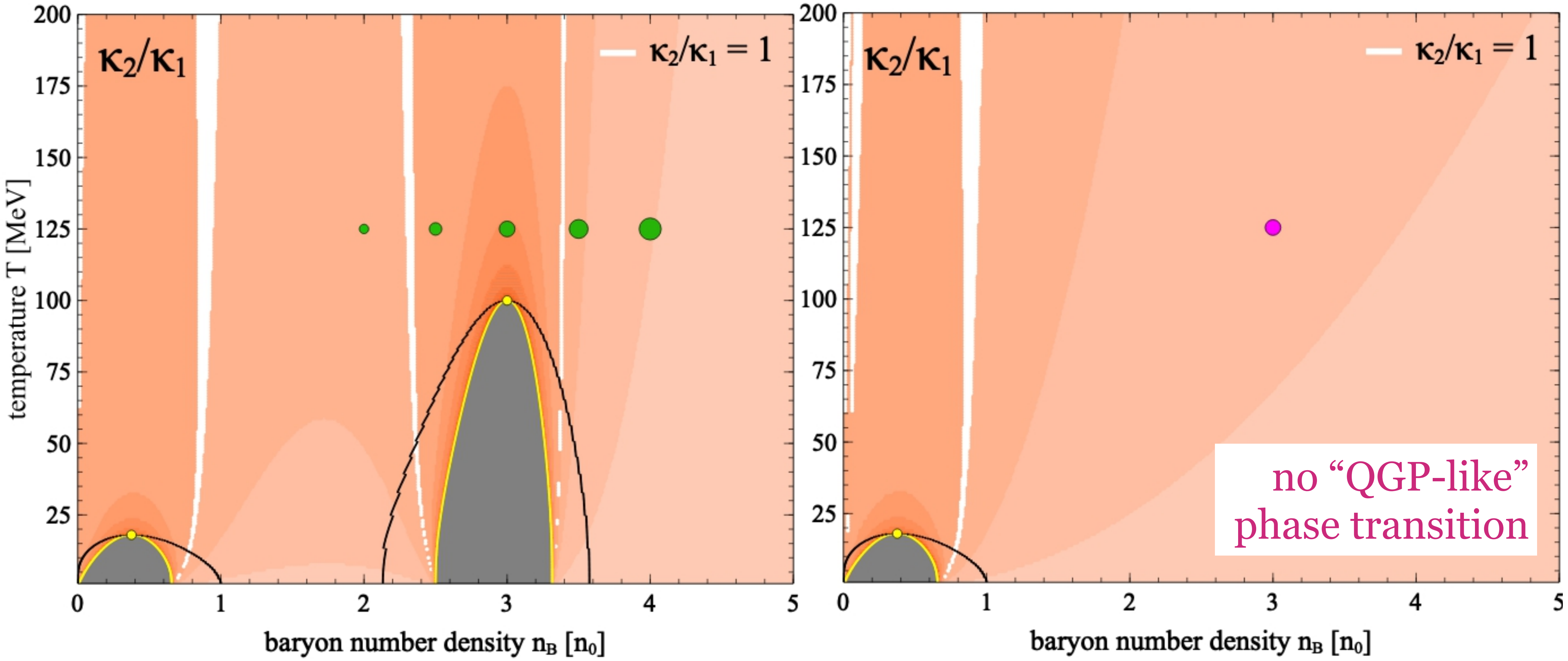

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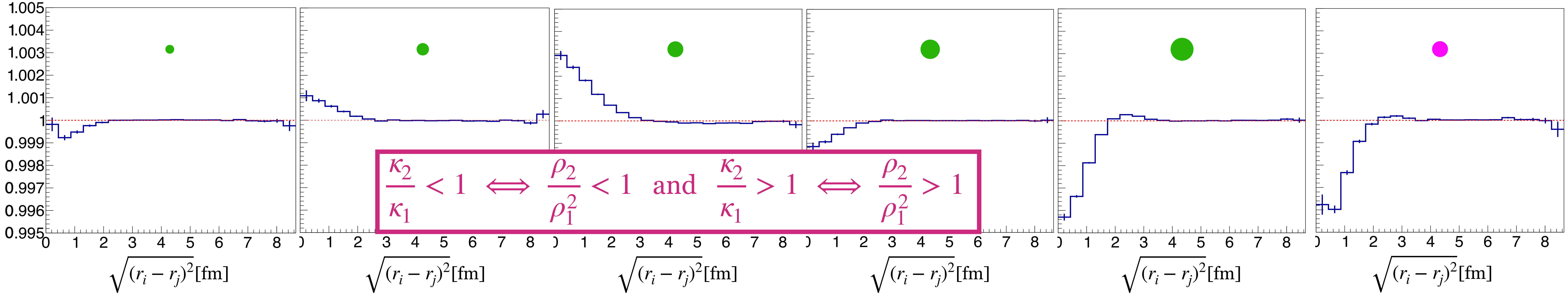

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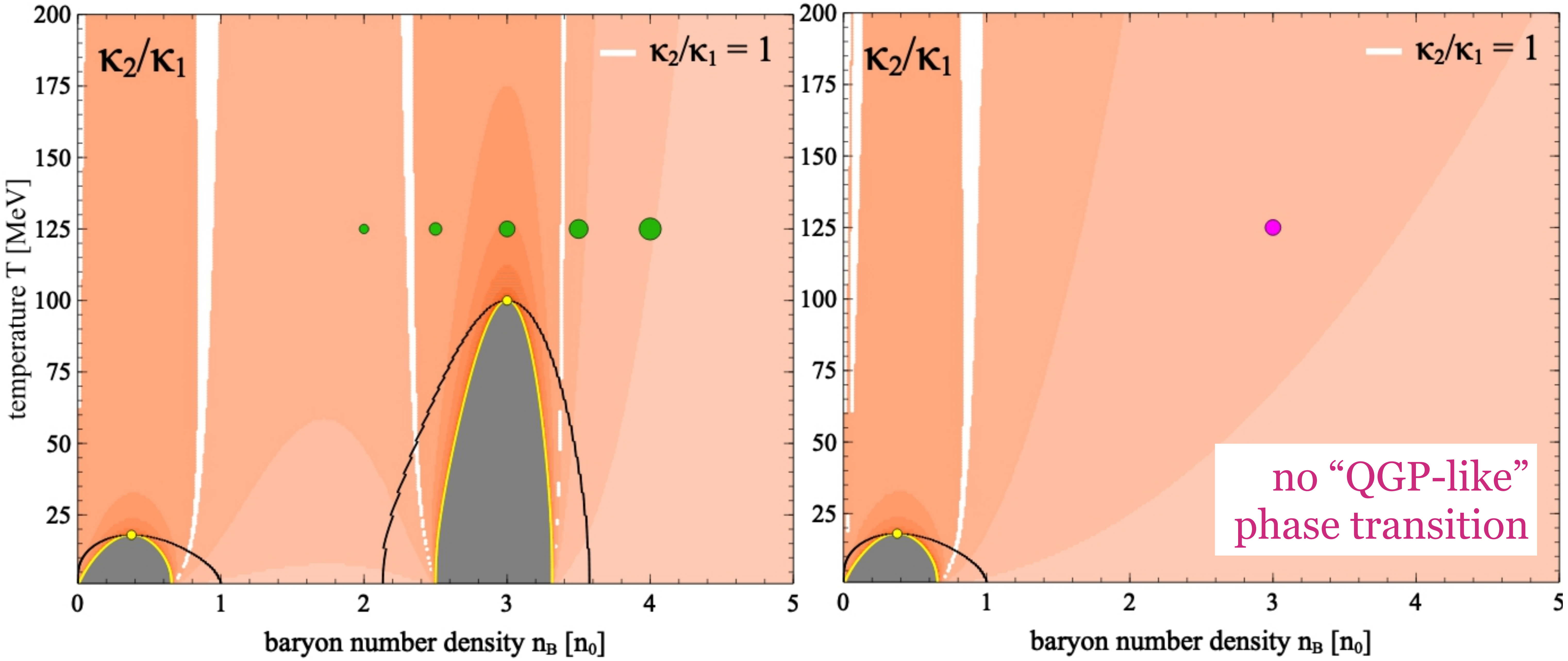

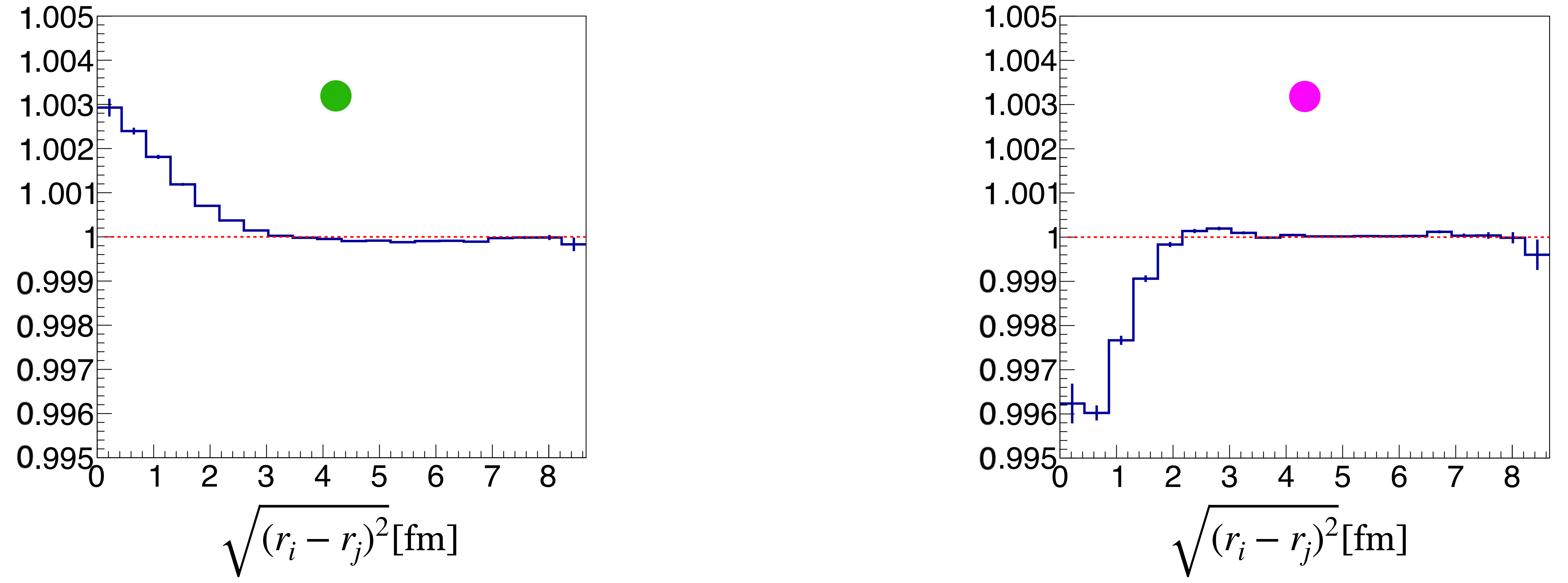

Opposite behavior of 2-particle correlations with or without the "QGP" critical point!

Summary

- Hadronic transport is sensitive to critical behavior if critical behavior is implemented
- Comparisons of different EOSs are possible
- The framework is ready to be used as the BEST afterburner!

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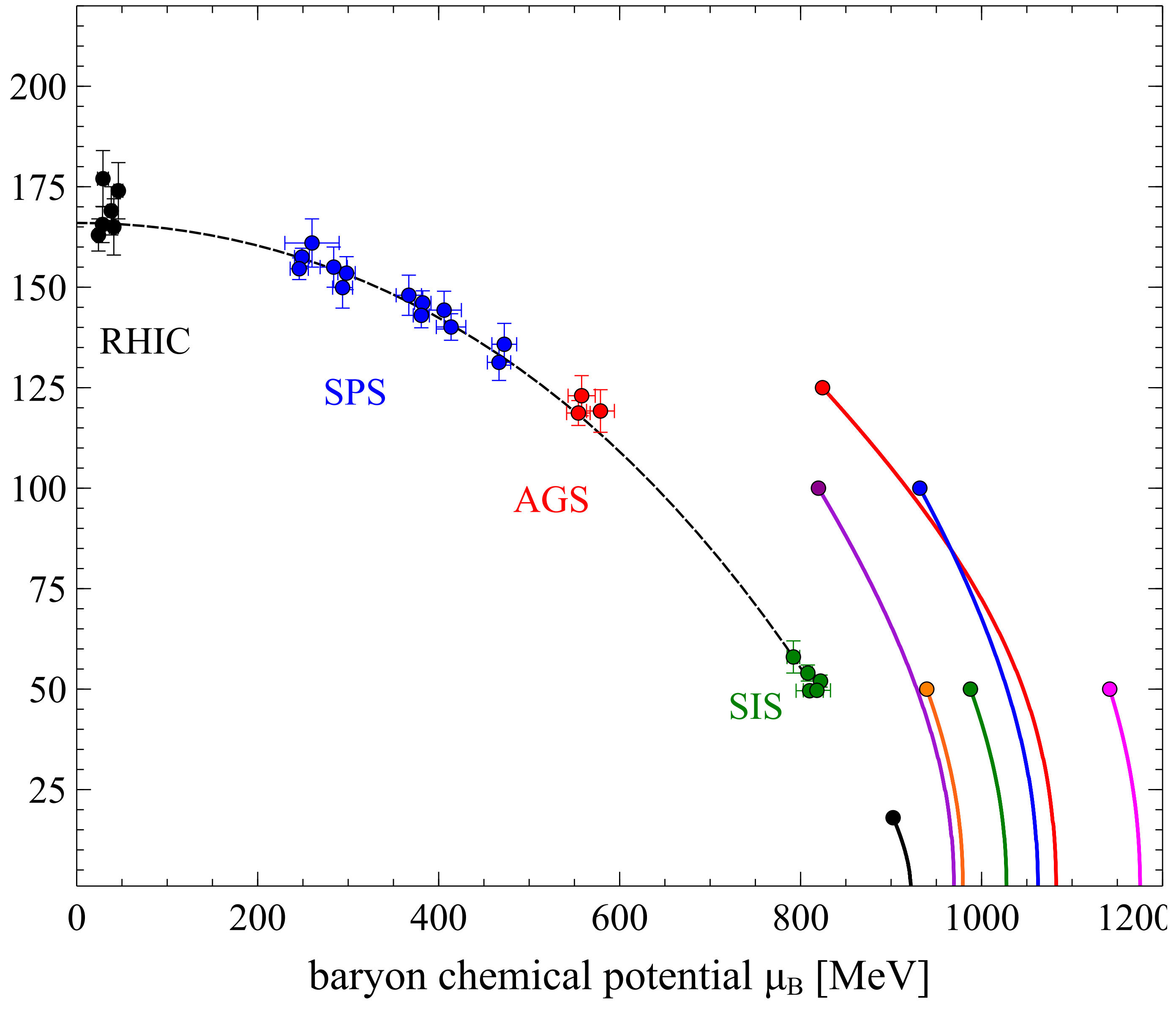
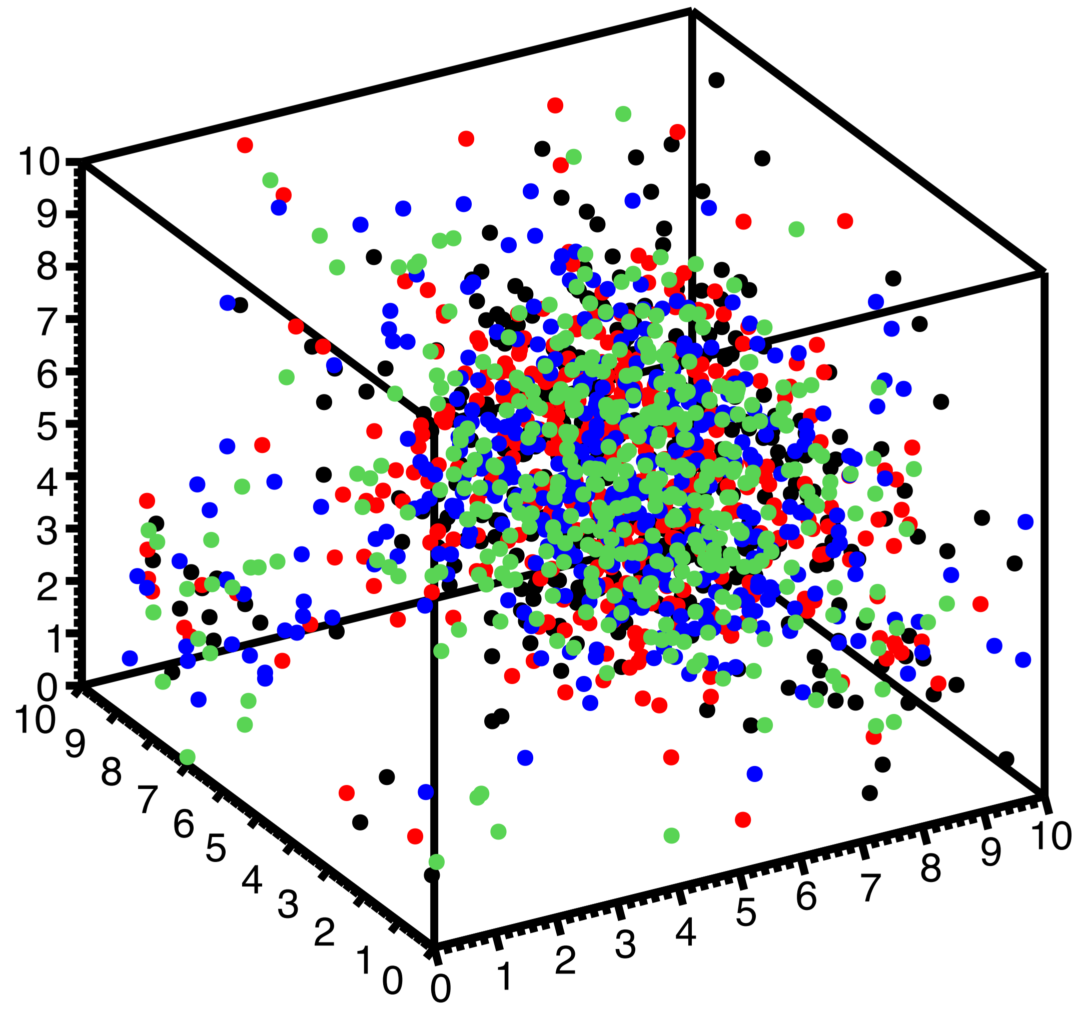
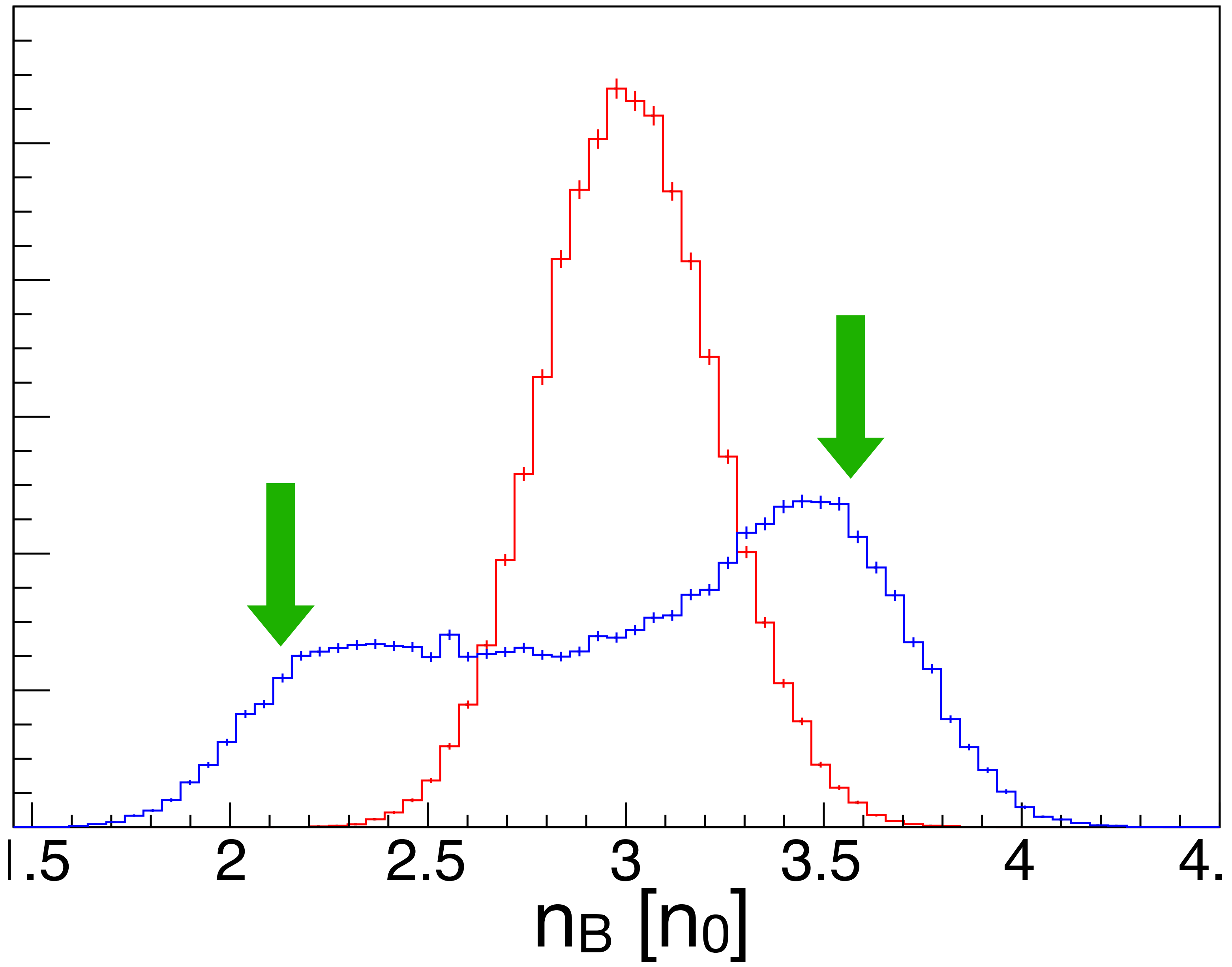
Thank you for your attention







This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under contract number DE-AC02-05CH11231.

Back-up slides

Relativistic density functional with vector-density—based interactions

G. Baym and S. A. Chin, "Landau Theory of Relativistic Fermi Liquids," Nucl. Phys. A **262**, 527 (1976)

1) The energy density of the system: $\mathscr{E} = \mathscr{E}[f_{\mathbf{p}}]$ Quasiparticle energy: $\varepsilon_{\mathbf{p}} \equiv \dfrac{\delta \mathscr{E}[f_{\mathbf{p}}]}{\delta f_{\mathbf{p}}}$

2) Collisionless Boltzmann equation: $$\frac{\partial f_{\mathbf{p}}}{\partial t} + \left(\nabla_{\mathbf{p}} \varepsilon_{\mathbf{p}}\right) \cdot \left(\nabla_{\mathbf{r}} f_{\mathbf{p}}\right) - \left(\nabla_{\mathbf{r}} \varepsilon_{\mathbf{p}}\right) \cdot \left(\nabla_{\mathbf{p}} f_{\mathbf{p}}\right) = 0$$

3) Energy-momentum tensor: $$T^{00} = \mathscr{E} \qquad T^{0i} = - \int \frac{d^3p}{(2\pi)^3}\, \varepsilon_{\mathbf{p}} \frac{\partial \varepsilon_{\mathbf{p}}}{\partial p_i}\, f_{\mathbf{p}} \qquad T^{0i} = T^{i0}$$

Transforms like a tensor should $$T^{i0} = \int \frac{d^3p}{(2\pi)^3}\, p^i\, f_{\mathbf{p}} \qquad T^{ij} = - \int \frac{d^3p}{(2\pi)^3}\, p^i \frac{\partial \varepsilon_{\mathbf{p}}}{\partial p_j}\, f_{\mathbf{p}} - \delta^{ij} \left(\mathscr{E} - \int \frac{d^3p}{(2\pi)^2}\, \varepsilon_{\mathbf{p}}\, f_{\mathbf{p}} \right)$$

4) Equations of motion: $$\frac{dx^i}{dt} \equiv - \frac{\partial \varepsilon_{\mathbf{p}}}{\partial p_i}\,, \qquad \frac{dp^i}{dt} \equiv \frac{\partial \varepsilon_{\mathbf{p}}}{\partial x_i}$$

Relativistic density functional with vector-density—based interactions

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5) Obtain pressure from the energy-momentum tensor: $P = \frac{1}{3} \sum_i T^{ii} \Big|_{\textbf{rest frame}}$

Up to this point the distribution function could have been anything.
In particular, it could be

$$f_{\mathbf{p}} = \frac{1}{V} \sum_{i=1}^{N_{part}} \delta^3\left(\mathbf{p} - \mathbf{p}_i\right)$$ as in hadronic transport.

6) 1st law of thermodynamics: $\delta\mathscr{E} = T\,\delta s + \mu_B\,\delta n_B$

Number density: $n_B = \int \frac{d^3p}{(2\pi)^3}\, f_{\mathbf{p}}$

Entropy density in a system of Fermi particles: $s = -\int \frac{d^3p}{(2\pi)^3} \left[f_{\mathbf{p}} \ln f_{\mathbf{p}} + \left(1 - f_{\mathbf{p}}\right) \ln\left(1 - f_{\mathbf{p}}\right) \right]$

$$\Rightarrow \quad f_{\mathbf{p}} = g\, \frac{1}{e^{\beta\left(\varepsilon_{\mathbf{p}} - \mu_B\right)} + 1}$$

Relativistic density functional with vector-density—based interactions

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One interaction term: $$\mathcal{E}_{(1)} = \int \frac{d^3p}{(2\pi)^3}\, \epsilon_{kin}\, f_{\mathbf{p}} \;+\; A_0 j^0 - \left(\frac{b_1 - 1}{b_1}\right) A_\lambda j^\lambda \,, \qquad b_1 \in \mathbb{R}_+$$

where $$A^\lambda(b_1; x) \equiv C_1 \left(j_\mu j^\mu\right)^{\frac{b_1}{2}-1} j^\lambda$$ ("generalized interaction current")

and $$\epsilon_{kin} = \sqrt{\left(\mathbf{p} - \mathbf{A}\right)^2 + m_N^2} \qquad \varepsilon_{\mathbf{p}} = \epsilon_{kin} + A_0$$

$$\mathbf{j} = \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p} - \mathbf{A}}{\epsilon_{kin}} f_{\mathbf{p}} \qquad j^0 = \int \frac{d^3p}{(2\pi)^3} f_{\mathbf{p}} = n_B$$

EOMs: $$\frac{dx^i}{dt} = \frac{p^i - A^i}{\epsilon_{kin}} \,, \qquad \frac{dp^i}{dt} = \frac{\left(p^k - A^k\right)}{\epsilon_{kin}} \frac{\partial A_k}{\partial x_i} + \frac{\partial A_0}{\partial x_i}$$

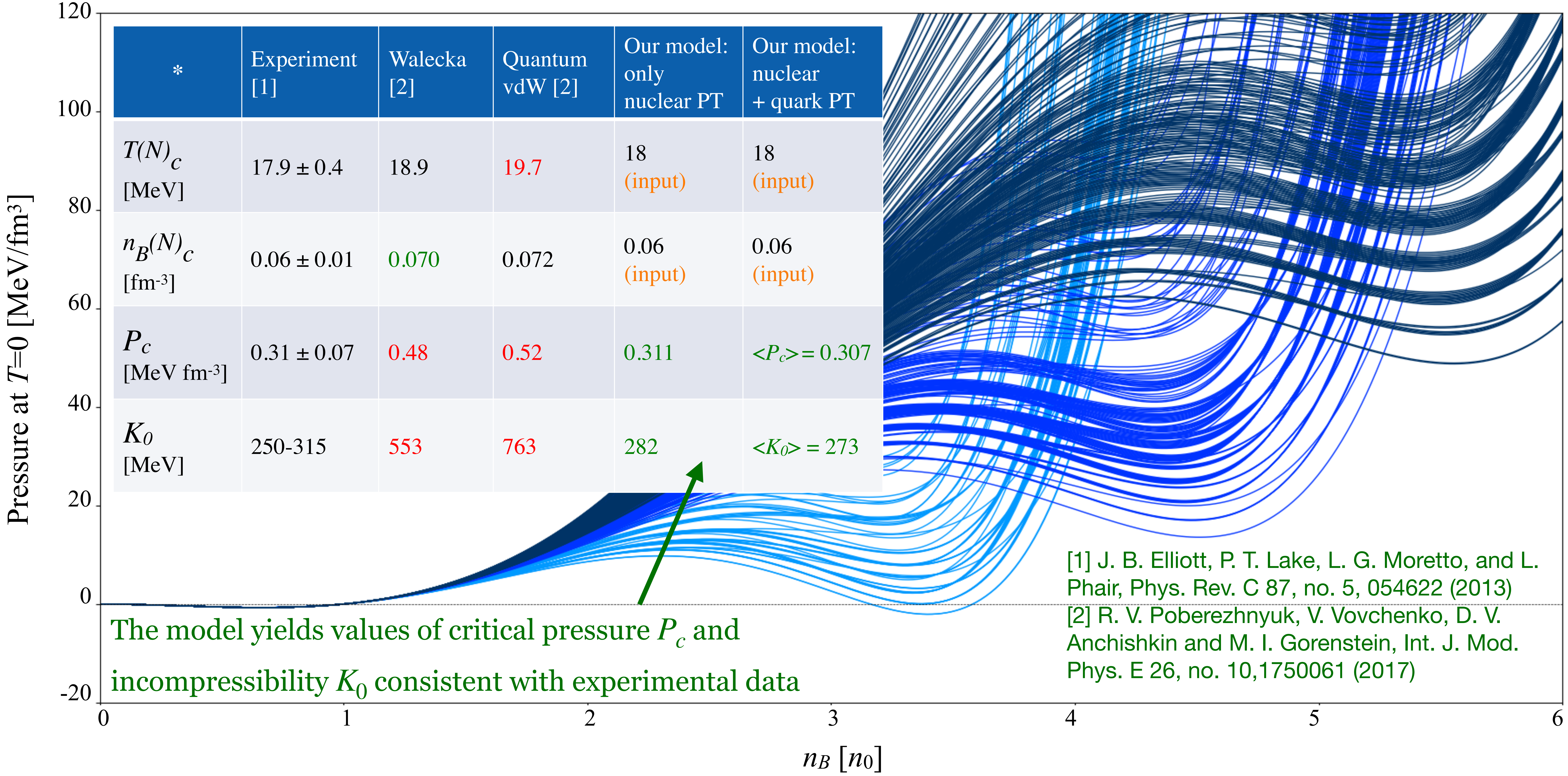
we check that EOMs + Boltzmann equation reproduce energy-momentum tensor

Putting $b_1 = 2$ reproduces vector interactions known e.g. from the Walecka model:

$$A^\lambda(b_1 = 2; x) = C_1 j^\lambda \,, \qquad \varepsilon_{\mathbf{p}} = \epsilon_{kin} + C_1 n_B \,, \qquad \mathcal{E} = \int \frac{d^3p}{(2\pi)^3}\, \epsilon_{kin}\, f_{\mathbf{p}} + \frac{C_1}{2} n_B^2$$

Relativistic DF: two 1st order phase transitions

Properties of ordinary nuclear matter are well known, but few constraints for $n_B \geq 1.5 n_0$



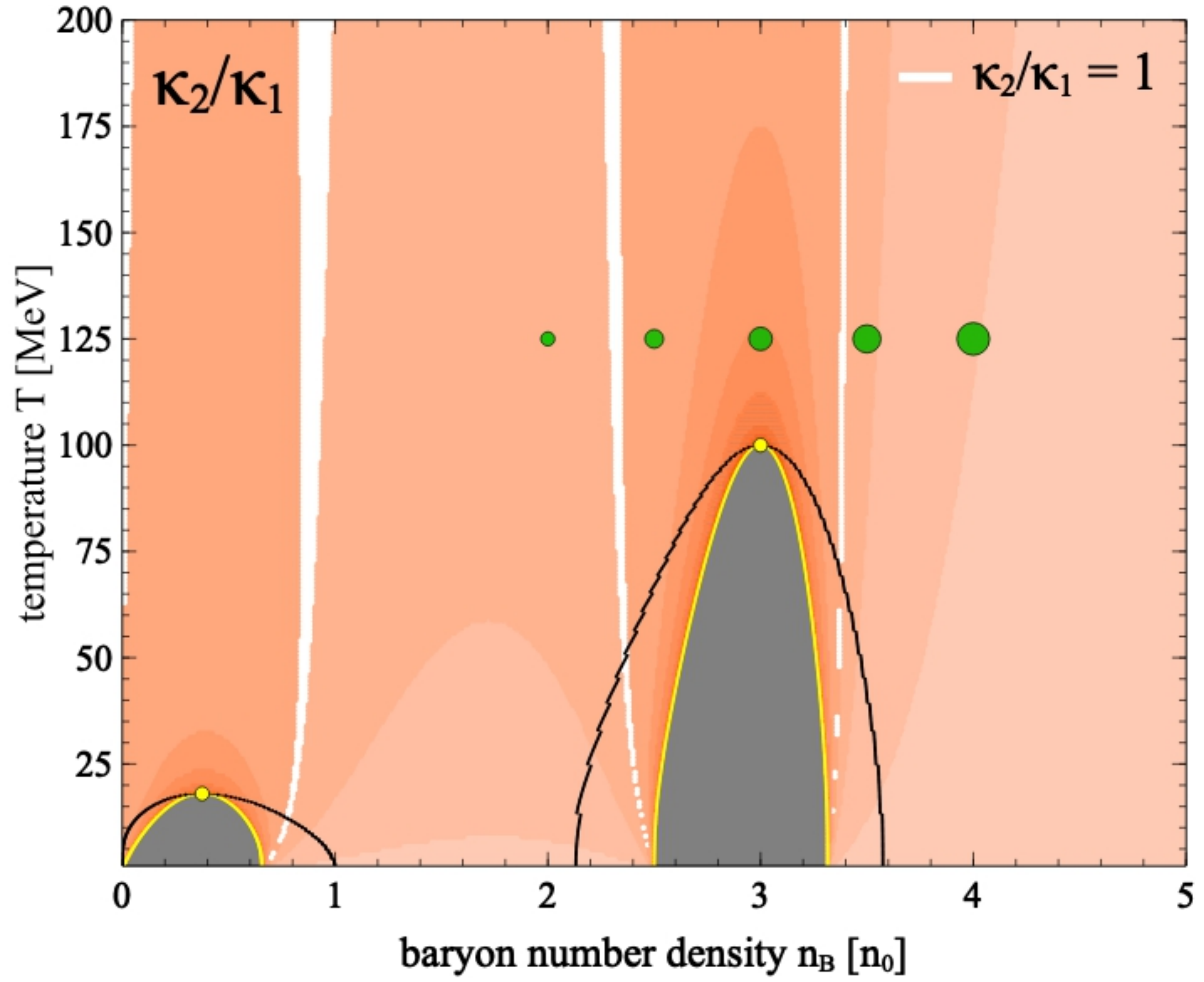
*	Experiment [1]	Walecka [2]	Quantum vdW [2]	Our model: only nuclear PT	Our model: nuclear + quark PT
$T(N)_c$ [MeV]	17.9 ± 0.4	18.9	19.7	18 (input)	18 (input)
$n_B(N)_c$ [fm^{-3}]	0.06 ± 0.01	0.070	0.072	0.06 (input)	0.06 (input)
P_c [MeV fm^{-3}]	0.31 ± 0.07	0.48	0.52	0.311	$\langle P_c \rangle$ = 0.307
K_0 [MeV]	250-315	553	763	282	$\langle K_0 \rangle$ = 273

The model yields values of critical pressure P_c and incompressibility K_0 consistent with experimental data

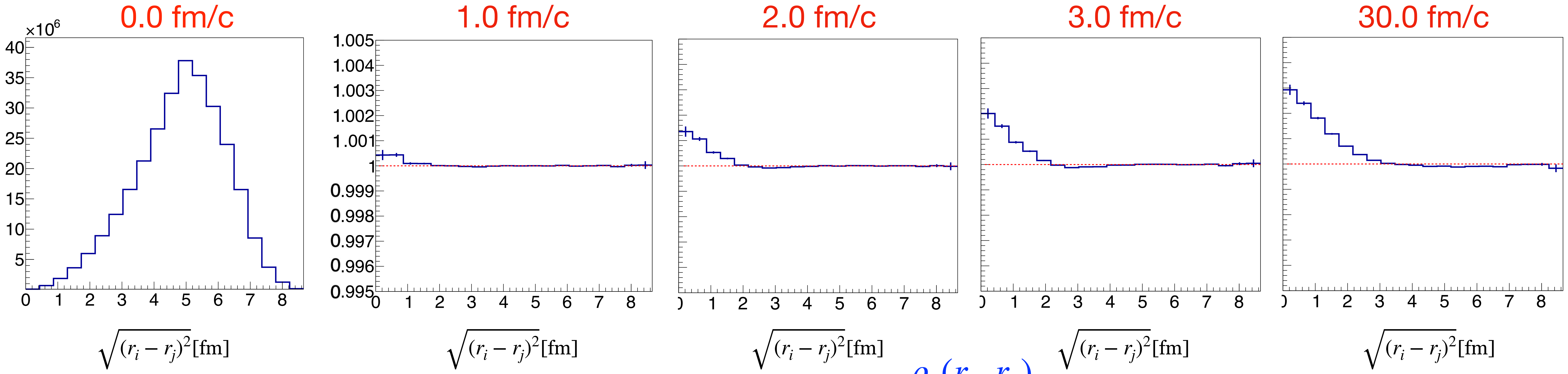
[1] J. B. Elliott, P. T. Lake, L. G. Moretto, and L. Phair, Phys. Rev. C 87, no. 5, 054622 (2013)
[2] R. V. Poberezhnyuk, V. Vovchenko, D. V. Anchishkin and M. I. Gorenstein, Int. J. Mod. Phys. E 26, no. 10,1750061 (2017)

SMASH: critical behavior above the “GQP-like” phase transition

T = 125 MeV $n_B = 3n_0$



The correlation develops within a short time



$\rho_2(r_1, r_2)$

$$\frac{\rho_2(r_1, r_2)}{\rho_1(r_1)\rho_1(r_2)}$$