RHIC/AGS Annual Users' Meeting 2020

Relativistic Hydrodynamics at Large Baryon Densities

Modeling the transport of coupled charges

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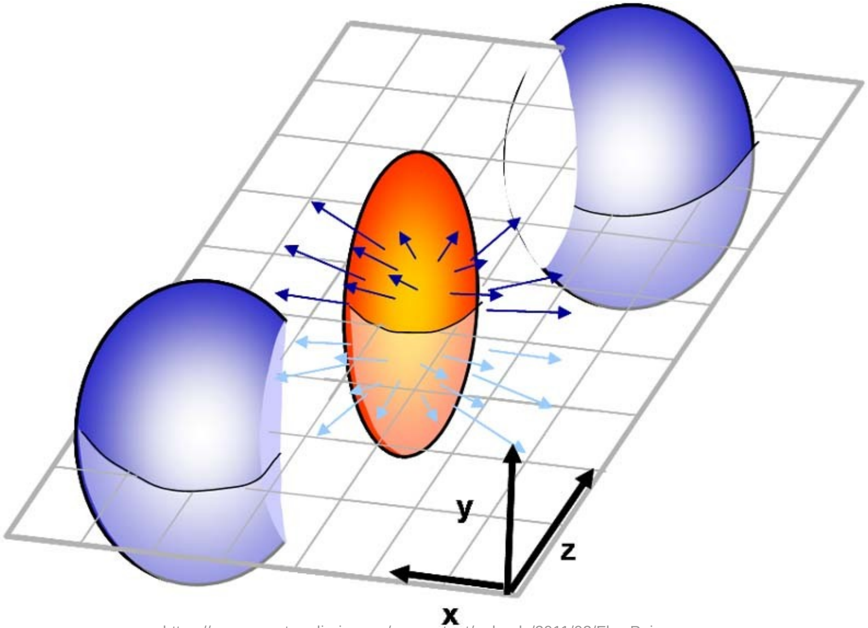
Fluid dynamics with multiple conserved charges

Traditionally:

Viewed as 'blob' of one type of matter (single component) with one velocity field

- usually 'blob' of energy without charge



https://www.quantumdiaries.org/wp-content/uploads/2011/02/FlowPr.jpg

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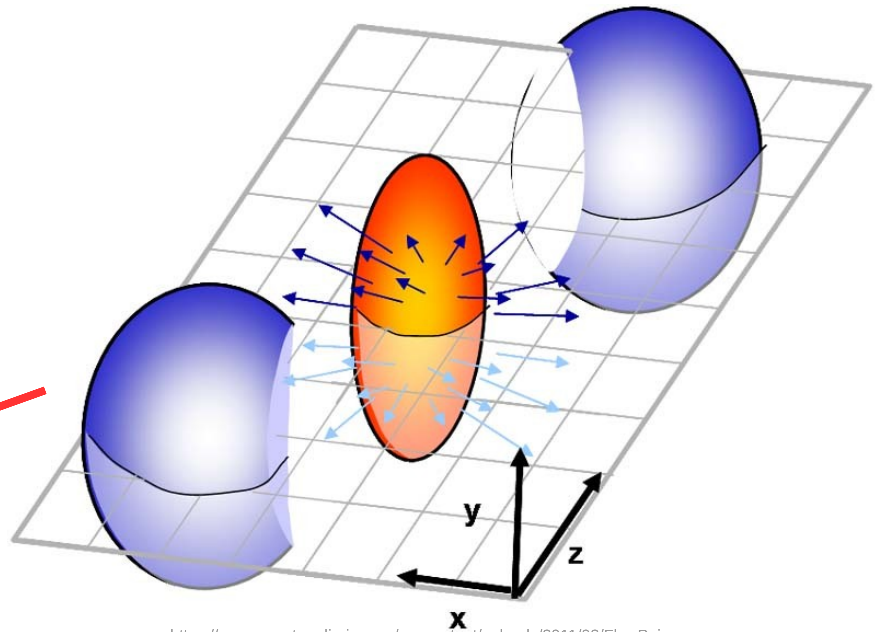

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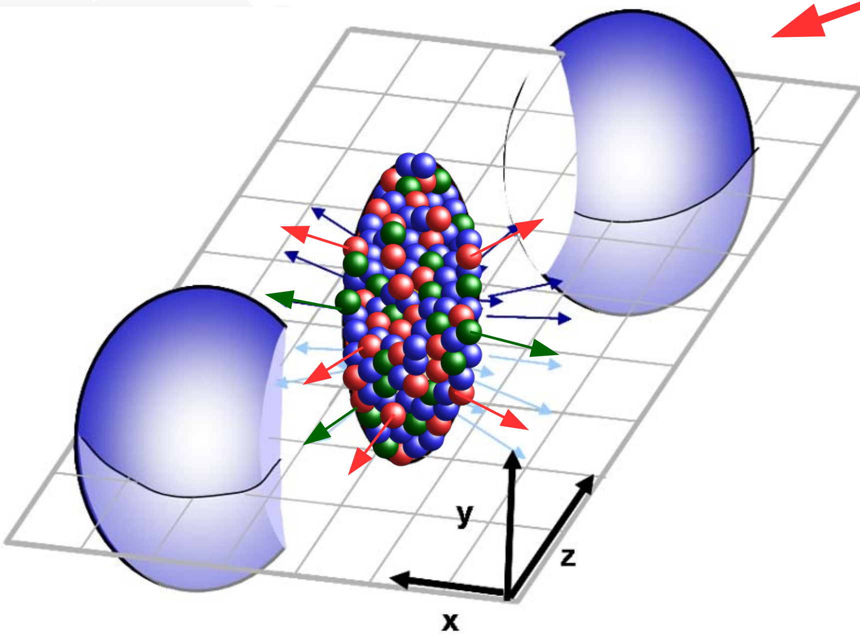

In general:

Consists of <u>multiple components</u> with <u>various properties</u> with <u>multiple velocity fields</u>

- with **multiple conserved quantities** (e.g. energy, electric charge, baryon number, strangeness, …)
- mixed chemistry → <u>**coupled charge currents!**</u>

Fluid dynamics with multiple conserved charges

<u>Hydrodynamics:</u> macroscopic effective field theory of thermal matter close to local equilibrium

Conservation of Energy and Momentum:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = 0$$

Here: single-fluid approximation $u_i^\mu \approx u^\mu$

Conservation of charge:

$$N_q^\mu = n_q u^\mu + V_q^\mu$$

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N_{ch} equations

$$10 + 4N_{\mathrm{ch}} \text{ degrees of freedom}, \quad 4 + N_{\mathrm{ch}} \text{ equations} \quad \rightarrow \quad 6 + 3N_{\mathrm{ch}} \text{ unknowns}$$

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- Equation of state
- Equations of motion for dissipative fields & <u>transport coefficients</u>
- Initial state
- Freeze-out and δf-correction

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Fluid dynamics with conserved baryon number:
Denicol et al., PRC 98, 034916 (2018)
Du et al., Comp. Phys. Comm. 251, 107090 (2020)
Li et al., PRC 98, 064908 (2018)

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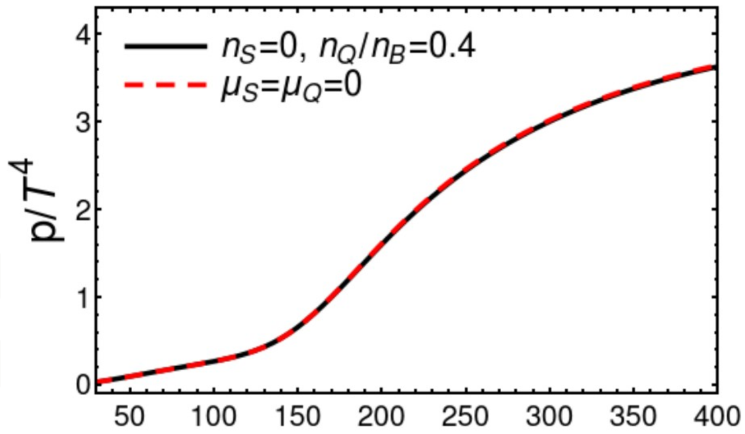
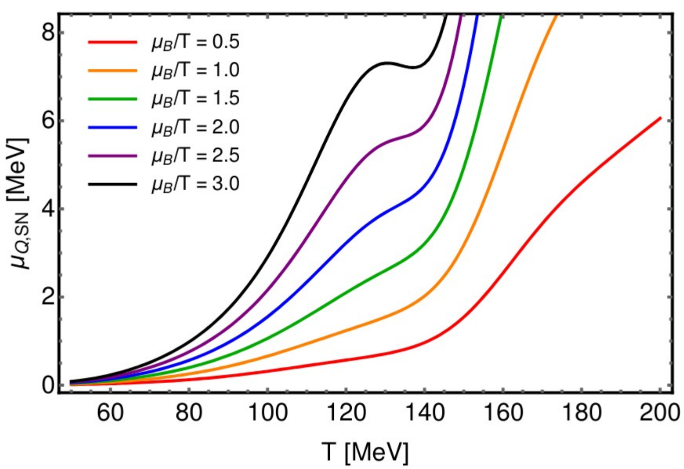
Equation of state with multiple conserved charges

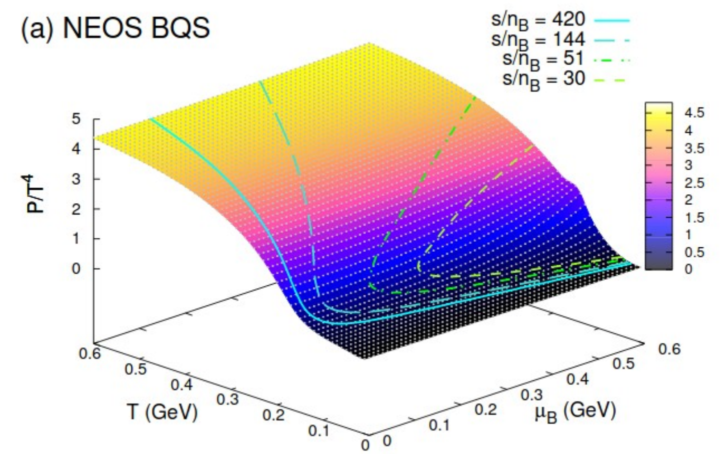
$$P_0(T) \quad \rightarrow \quad P_0(T, \mu_{\mathrm{B}}, \mu_{\mathrm{Q}}, \mu_{\mathrm{S}})$$

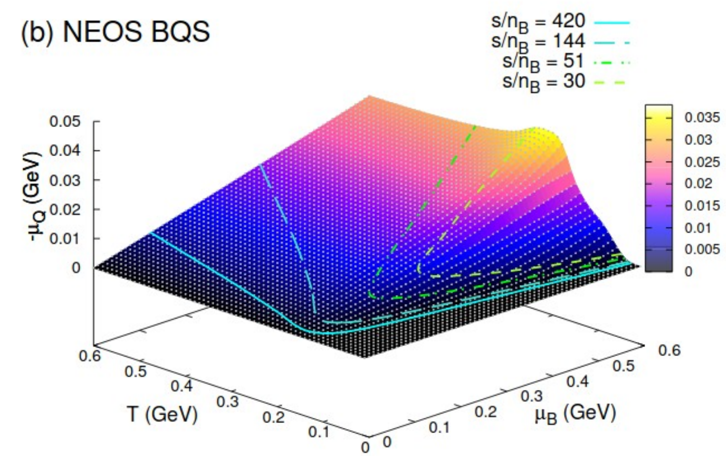
Noronha-Hostler et al., PRC 100, 064910 (2019)



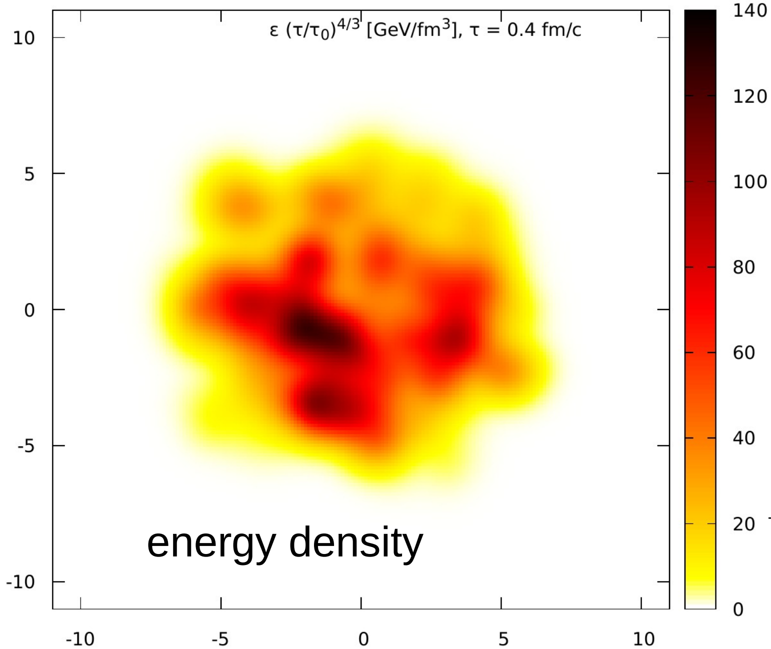


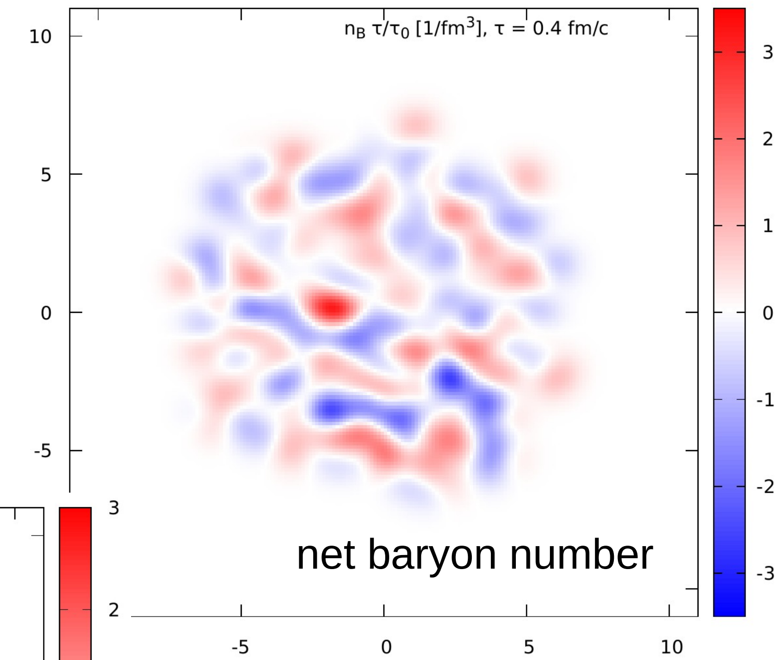
Monnai et al., PRC 100, 024907 (2019)

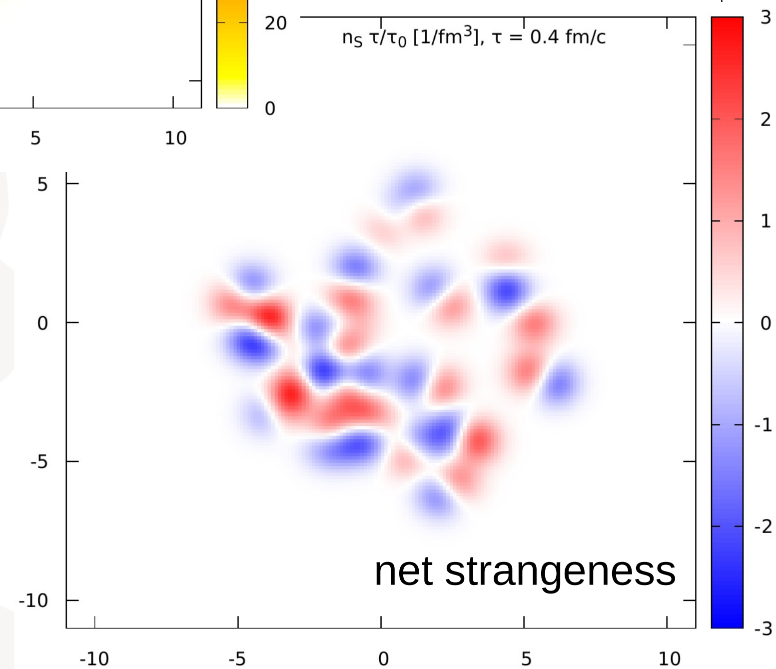




Initial state with multiple conserved charges







Smoothed initial condition for Pb Pb @ 5 TeV from ICCING

Martinez et al., arXiv:1911.10272, arXiv:1911.12454

Plots not published!

Deriving fluid dynamics from kinetic theory

Denicol et al., PRD 85, 114047 (2012)

On basis of DNMR theory: derivation from the Boltzmann equation
→ **upcoming publication!** (Fotakis, Molnár, Niemi, Denicol, Rischke, Greiner)

Also refer to: Monnai, Hirano, Nucl. Phys. A847:283-314 (2010) or Kikuchi et al. PRC 92, 064909 (2015)

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \sum_j \mathcal{C}_{ij}[f_i, f_j]$$

→

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

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1st order terms (Navier-Stokes): mixed chemistry already couples diffusion currents!
2nd order terms: couples all currents to each other; depend on all gradients!
→ 3 conserved charges: 70+ transport coefficients (!!) with $(T, \mu_\mathrm{B}, \mu_\mathrm{Q}, \mu_\mathrm{S})$-dependence

A simplistic case study

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)

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- Investigate longitudinal evolution in Milne coordinates (transversally homogeneous)

- Conserved baryon number and strangeness, ***neglect viscosity***, neglect 2nd order terms

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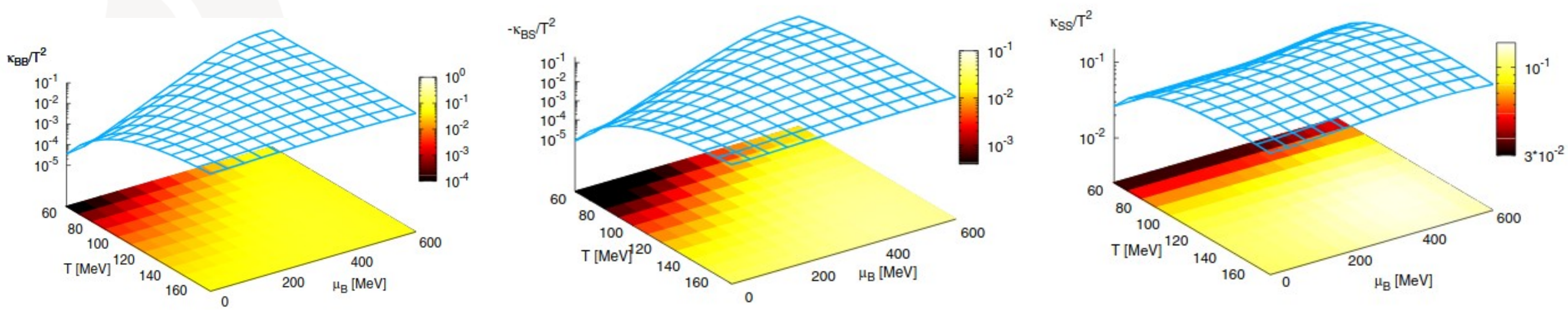

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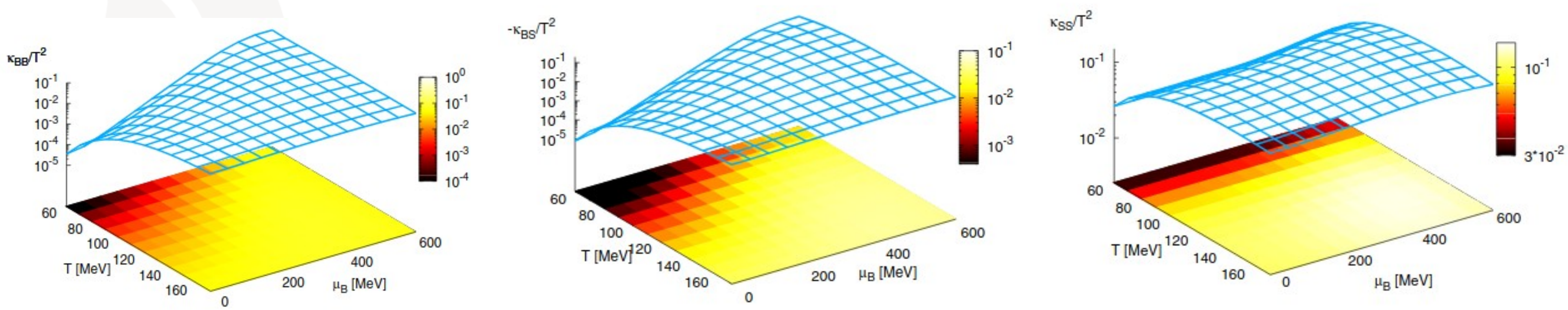

- ***Simple initial state:*** T = 160 MeV, no initial net strangeness, longitudinal double-gaussian profile in net baryon number, no inititial dissipative currents

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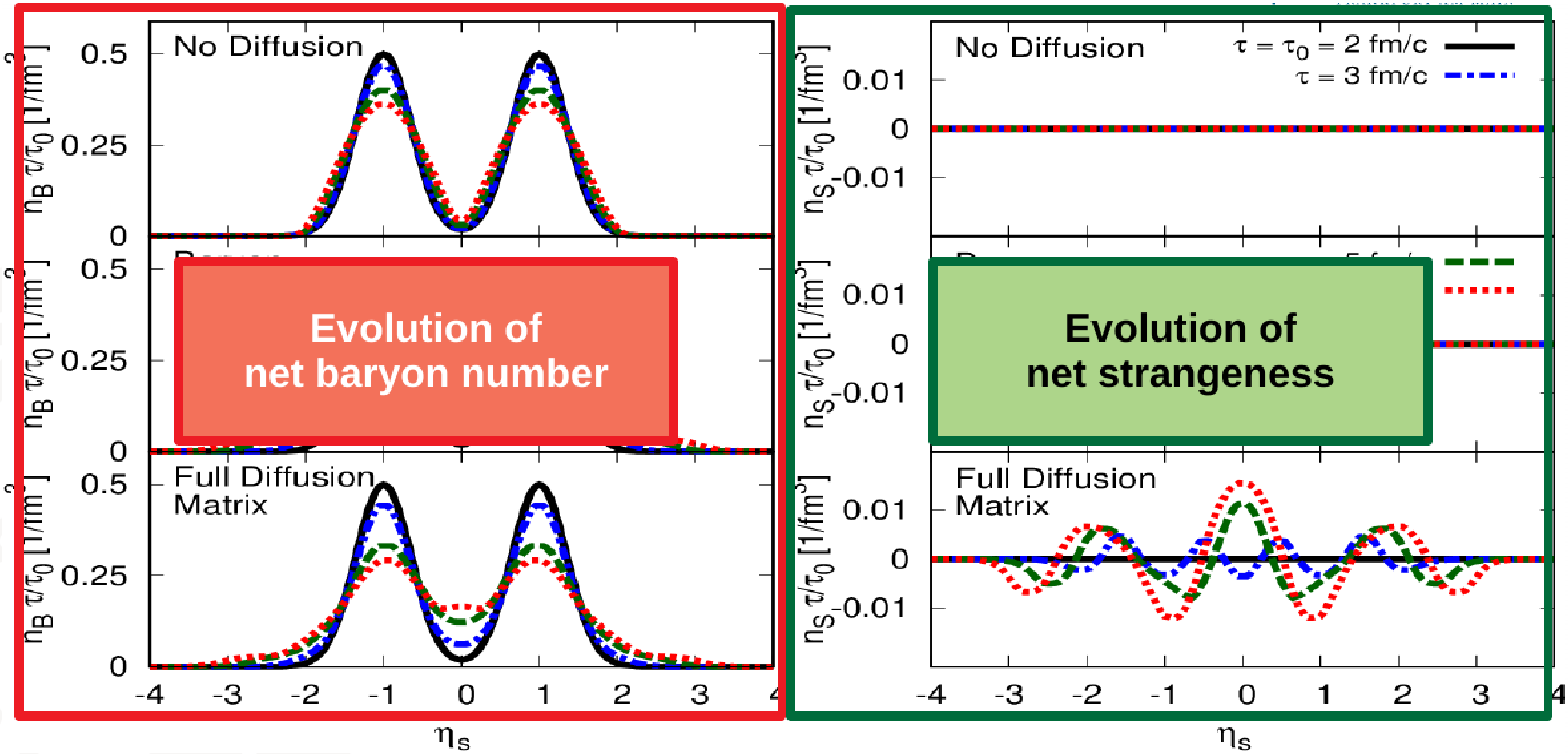

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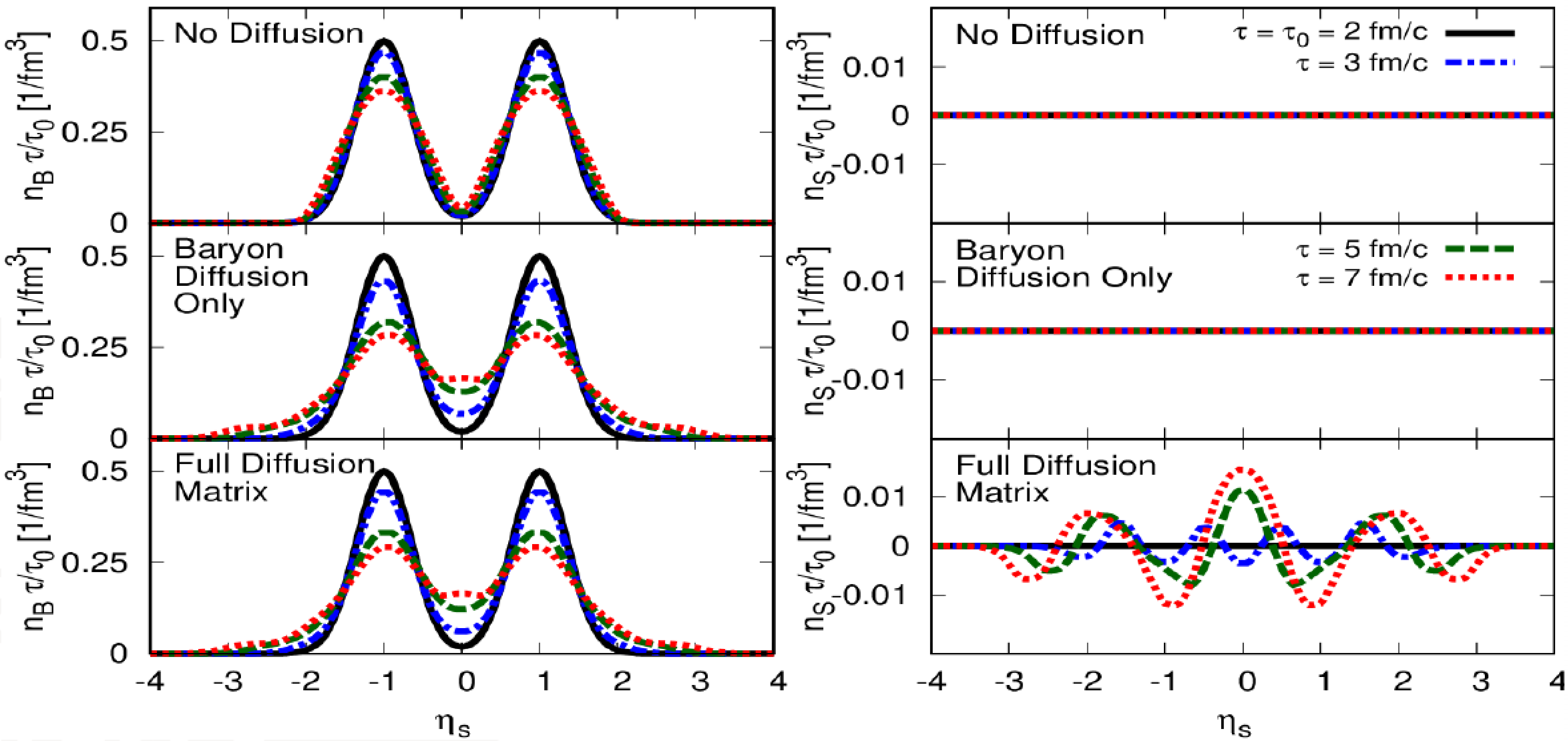

Mixed chemistry couples diffusion currents → Generation of domains of non-vanishing local net charge (here net strangeness)!

Outlook

- Implement derived fluid dynamic theory in existing (3+1)D-hydro code
- Evaluate 2nd order transport coefficients from linearized Boltzmann equation
- Use more realistic initial state and equation of state (see above)
- Apply freeze-out routines, take δf-correction from derived theory
- Find observables sensitive to charge-coupling $\rightarrow$ investigate impact