Energy loss beyond multiple soft or single hard approximations

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in collaboration with
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Thursday, Oct 22nd
Jets in medium

- Several medium-induced effects will change a “pp jet” into a “PbPb jet”
Jets in medium

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- Medium-induced energy loss

- Collisional energy loss

- Re-scattering

- Fast evolving medium
Jets in medium

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Jet Energy Loss

ATLAS [1805.05635]

Particle Energy Loss

PHENIX [2002.11156]
Jets in medium

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Jets in medium

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Medium-induced energy loss

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Re-scattering

Fast evolving medium

Jet Substructure

Need accurate theoretical description to withdraw QGP characteristics!
Medium-induced energy loss

- Understand the stopping power of matter for colour-charged particles

Fast evolving medium
Medium-induced energy loss

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- From a pQCD view:
  - QGP is a collection of static scattering centres
  - Multiple interactions enhance gluon radiation
Medium-induced energy loss

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- From a pQCD view:
  - QGP is a collection of static scattering centres
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    - Number of interactions is not fixed
Medium-induced energy loss

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- From a pQCD view:
  - QGP is a collection of static scattering centres
  - Multiple interactions enhance gluon radiation
    - Number of interactions is not fixed

  ⇒ Need resumation up to all orders
  or
  ⇒ Opacity expansion (finite interactions with the medium)
Medium-induced gluon radiation

- Accumulation of momenta enhances gluon radiation:

\[
\omega \frac{dI}{d\omega d^2k} = \frac{2\alpha_s C_R}{(2\pi)^2 \omega^2} \text{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{pq} p \cdot q \tilde{\mathcal{K}}(t', q; t, p) \mathcal{P}(\infty, \mathbf{k}; t', q)
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Momentum Broadening:

\[
\mathcal{P}(t'', k; t', q) \equiv \int d^2z e^{-i(k-q) \cdot z} \exp \left\{ -\frac{1}{2} \int_{t'}^{t''} ds n(s) \sigma(z) \right\}
\]

Density of scattering centres:

\[
n(x_+) = \int dx_i \delta(x_+ - x_i).
\]

Dipole cross-section:

\[
\sigma(r) = \int_q V(q) (1 - e^{iqr})
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\]

Collision rate
(parton-medium interaction)

Momentum Broadening:

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Emission Kernel:

\[ \tilde{K} (t', z; t, y) \equiv \int_{pq} e^{i(q \cdot z - p \cdot y)} \tilde{K} (t', q; t, p) \]

\[ = \int_{r(t')=z}^{r(t)=y} \mathcal{D}r \exp \left[ \int_t^{t'} ds \left( \frac{i\omega}{2} r^2 - \frac{1}{2} n(s) \sigma(r) \right) \right] \]

Solution to the path integral (for an arbitrary potential) poses significant technical challenges...
H. Oscillator

- Analytical solution to medium-induced gluon radiation for finite size medium

- 2 free parameters: $\hat{q}$ and $L$

- Resums scatterings over medium length

Useful to gain qualitative insight into experimental observations

[Baier, Dokshitzer, Mueller, Peigné, Schiff (97-00), Zakharov (96)]
[Wiedemann (00), Arnold, Moore, Yaffe (01)]
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Target from several theoretical developments: finite energy corrections, interplay between energy loss and transverse momentum broadening, interferences between successive emitters, …
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- Only valid when medium is dense:
  
  $n(s)\sigma(r) \approx \frac{1}{2} \hat{q}(s)r^2 + \mathcal{O}(r^2 \ln r^2)$,  
  
  $\hat{q} = \frac{<k^2_{\perp}>}{\lambda_{mfp}}$

- Ignores perturbative tails at high transverse momentum.

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[Wiedemann (00), Arnold, Moore, Yaffe (01)]

[Mehtar-Tani, Salgado, Tywoniuk (2010-2011)]

[LA, Armesto, Salgado (12), Blaizot, Dominguez, Iancu, Mehtar-Tani (13-14)]

[Blaizot, Iancu, Mehtar-Tani (13), Blaizot, Mehtar-Tani, Torres (14)]

[LA, Armesto, Milhano, Salgado (15)]

[...]
H. Oscillator

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    - Ignores perturbative tails at high transverse momentum.

\[ ^{\dagger}q = \frac{q_{N/T^3_{\text{DIS}}}}{\text{Au+Au at RHIC}} \]

Transport coefficient: RHIC > LHC at the same temperature

Center-of-mass energy dependent
Opacity expansion (GLV limit)

- Radiation pattern = Incoherent superposition of just a few single hard scattering processes.

\[ \mathcal{P}(t'', k; t', q) \equiv \int d^2 z \ e^{-i(k-q)\cdot z} \ \exp \left\{ -\frac{1}{2} \int_{t'}^{t''} ds \ n(s) \ \sigma(z) \right\} \]

- Expansion in terms of: \( (n(s)\sigma(r))^N \)

- Exact form of potential: \( V(q) = \frac{8\pi\mu^2}{(q^2 + \mu^2)^2} \)

- 3 parameters: n0, L, \( \mu \)

An opacity expansion of the BDMPS-ASW reproduces the GLV approach

\[ \sigma(r) = \int_q V(q) \ (1 - e^{iqr}) \]
Opacity expansion

- Exact limit when medium is dilute;

- For dense medium (large number of scattering centers):
  - Needs resuming the contributions from all orders (analytically and computationally demanding)

Recent works:
- [Vitev, Ovanesyan (2013)]
- [Arnold, Iqbal (2015)]
- [Sievert, Vitev (2018)]
Towards resummation

- Analytical expansion around the HO:
  \[ n(s)\sigma(r) \approx \frac{1}{2} \hat{q}(s) r^2 + \mathcal{O}(r^2 \ln r^2) \]

Fast convergence of the improved opacity expansion

Energy Spectrum

Momentum broadening

\[ \lambda = \ln^{-1}(Q^2/p^2) = 0.1 \]
Towards resummation

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Fast convergence of the improved opacity expansion

(still limited by an order-by-order calculation)
Towards resummation

- Full resummation of all scatterings within a MC approach:

- Result with the full resummation of all scatterings (in the soft limit) without apparent inconsistencies in temperature
Towards resummation

- Full resummation of all scatterings within a MC approach:

![Graph showing resummation of scatterings](chart)

*Result with the full resummation of all scatterings (in the soft limit) without apparent inconsistencies in temperature*

Uses involving Monte Carlo methods
(difficult to generally apply for phenomenological studies)
Towards resummation

- Solve the spectrum by using Schwinger-Dyson type equations (in momentum space):

- Evolution equations for emission kernel and broadening

\[
\partial_\tau \mathcal{P}(\tau, \mathbf{k}; s, l) = -\frac{1}{2} n(\tau) \int_{\mathbf{k}'} \sigma(\mathbf{k} - \mathbf{k}') \mathcal{P}(\tau, \mathbf{k}'; s, l)
\]

\[
\partial_t \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) = \frac{i\mathbf{p}^2}{2\omega} \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) + \frac{1}{2} n(t) \int_{\mathbf{k}'} \sigma(\mathbf{k}' - \mathbf{p}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{k}')
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\]

Set of integro-partial differential equations that can be numerically solved to any (realistic) potential

Contains the resummation of all scattering scatterings, in the soft limit, without further approximations!
Equations to solve numerically

- Set of integro-differential equations of that can be solve numerically:

  - Start with broadening and dipole cross-section equation:
    \[ \partial_{\tau} \phi(\tau, k; s, q) = -\frac{1}{2} n(\tau) \int_{k'} \sigma(k - k') \phi(\tau, k'; s, q) \]
    \[ \phi(s, k; s, q) = n(s) \left( \frac{k}{k^2} - \frac{q}{q^2} \right) \sigma(k - q) \]

  - Use \( \psi \) as initial condition for:
    \[ \psi_I(s, k; s, p) = \phi(L, k; s, p) \]
    \[ \partial_{t} \psi_I(s, k; t, p) = \frac{1}{2} n(t) \int_{k'} e^{i \frac{p^2}{2\sigma}(s-t)} \sigma(k' - p) e^{-i \frac{p^2}{2\sigma}(s-t)} \psi_I(s, k; t, k') \]

  - Finally, calculate:
    \[ \omega \frac{dI}{d\omega d^2k} = \frac{2\alpha_s C_R}{(2\pi)^2 \omega} \text{Re} \int_0^L ds \int_t^s dt \int_p i e^{-i \frac{p^2}{2\sigma}(s-t)} p \cdot \psi_I(s, k; t, p) \]
GLV vs Full solution

- Specifying the interaction potential: \( \sigma(r) = \int q V(q) (1 - e^{iqr}) \)

- Yukawa-type interaction:
  \[ V(q) = \frac{8\pi\mu^2}{(q^2 + \mu^2)^2} \]

- Parameters: \( n_0, L, \mu \)
  \[ \kappa^2 = \frac{k^2}{\mu^2} \]
  \[ x^{-1} = \frac{\mu^2 L}{2\omega} \]

\( n_0 L = 1 \) ("dilute")
GLV vs Full solution

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  \[ \kappa^2 = \frac{\kappa^2}{\mu^2} \]
  \[ x^{-1} = \frac{\mu^2 L}{2\omega} \]

\[ n_0 L = 5 \text{ (“dense”)} \]
GLV vs Full solution

- Specifying the interaction potential: \[ \sigma(r) = \int \frac{V(q)}{q} \left(1 - e^{iqr}\right) \]

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GLV vs Full solution

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- Parameters: $n_0$, $L$, $\mu$

---

Solid: Full Solution (our approach)
Dashed: GLV ($N = 1$)

\[ \mu = 0.6 \text{ GeV} \]
\[ n_0 = 1 \text{ fm}^{-1} \]

Energy spectrum

$L = 5 \text{ fm}$

$L = 3 \text{ fm}$
HO vs Full solution

- Specifying the interaction potential:
  \[ \sigma(r) = \int_q V(q) \left(1 - e^{iqr}\right) \]

- Yukawa-type interaction:
  \[ V(q) = \frac{8\pi\mu^2}{(q^2 + \mu^2)^2} \]

- Parameters (our): \( n_0, L, \mu \)

- Parameters (HO): \( \hat{q}, L \)

Only qualitative comparison:

\[ \hat{q}L \approx (n_0L)\mu^2 \ln \frac{q_{\text{max}}}{\mu} \rightarrow 1.3(n_0L)\mu^2 \]

\[ x^{-1} = \frac{\mu^2 L}{2\omega} \quad \kappa^2 = \frac{k^2}{\mu^2} \]
Comparing QGP potential models

• Comparing two potentials:
  
  • Yukawa:
  \[
  V(q) = \frac{8\pi\mu^2}{(q^2 + \mu^2)^2}
  \]

  • Hard Thermal Loop (HTL):
  \[
  \frac{1}{2} n V(q) = \frac{g_s^2 N_c m_D^2 T}{q^2(q^2 + m_D^2)}
  \]

Matching small distance behaviour:

\[
 n_0\mu^2 = \alpha_s N_c T m_D^2, \quad m_D^2 = \epsilon \mu^2
\]
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Non-universal, model dependent, contributions seem to be negligible
Summary

- Novel analytical approach: resummation of all multiple scatterings

- Comparison with GLV limit and HO approximation:
  - GLV valid for single hard scattering; overestimate true contribution from soft and low momentum gluons
  - HO more suitable than GLV to describe low energy gluons; underestimate true contribution from hard gluons (single soft scattering)
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  \( \text{Phase space to pin down QGP main characteristics} \)
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Phase space to pin down QGP main characteristics

Thank you!