Saturation corrections to dilute-dense particle production

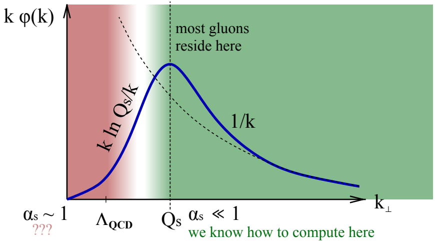
Vladi Skokov

based on: S. Schlichting, V. S.: Phys. Lett. B 806 (2020) 135511

RBRC
RIKEN BNL Research Center
BROOKHAVEN NATIONAL LABORATORY
NC STATE UNIVERSITY

Saturation regime/CGC

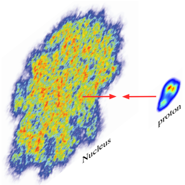
- High energy $\rightsquigarrow$ high gluon density $\rightsquigarrow$ formation of semi-hard scale, Q_s



- Particle production is dominated by $k_\perp \sim Q_s$
- Weak coupling methods can be applied $\alpha_s(Q_s) \ll 1$
- Still non-perturbative, as fields are strong, $A \sim \frac{1}{g}$ $\rightsquigarrow$ non-linearity is important
- Actual analytical calculations can be very hard

What do we know analytically?

Asymmetric collisions, when Q_s of projectile $\neq Q_s$ of target, is the easiest case.



Single inclusive production

- In general

$$\frac{dN}{d^3k} = \frac{1}{\alpha_s} f\left(\frac{Q_{sp}^2}{k_\perp^2}, \frac{Q_{sA}^2}{k_\perp^2}\right)$$

$f\left(\frac{Q_{sp}^2}{k_\perp^2}, \frac{Q_{sA}^2}{k_\perp^2}\right)$ is known only numerically; for large $k_1 \gg Q_{sA}^2$: $\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2}\frac{Q_{sA}^2}{k_\perp^2} f^{(1,1)}$

A. Krasnitz, R. Venugopalan, arXiv:9809433 *E. Kuraev, L. Lipatov, V. Fadin, '77*

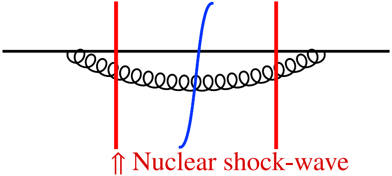
- If $k_\perp > Q_{sp}$,

$$\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2} f^{(1)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \frac{1}{\alpha_s}\left(\frac{Q_{sp}^2}{k_\perp^2}\right)^2 f^{(2)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \cdots$$

Functions $f^{(n)}$ are calculable!

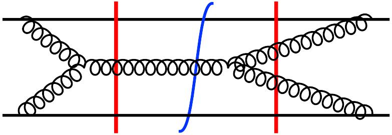
$$\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2} f^{(1)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \frac{1}{\alpha_s}\left(\frac{Q_{sp}^2}{k_\perp^2}\right)^2 f^{(2)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \cdots$$

- $f^{(1)}$ is known since '98



Y. V. Kovchegov and A. H. Mueller, arXiv:hep-ph/9802440
A. Dumitru and L. D. McLerran, arXiv:hep-ph/0105268
J.-P. Blaizot, F. Gelis, R. Venugopalan, arXiv:0402256

- $f^{(2)}$: no complete result yet



I. Balitsky, arXiv:hep-ph/0409314
G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106

Double inclusive production

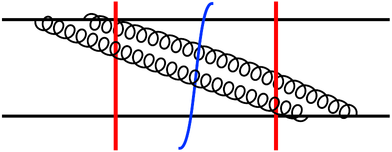
$$\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 \, h^{(1)}(Q_{sA}) + \frac{1}{\alpha_s^2} Q_{sp}^6 \, h^{(2)}(Q_{sA}) + \cdots$$

Momentum dependence is omitted to simplify notation

- Dilute-dilute "Glasma" graph: $\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 Q_{sA}^4 \, h^{(1,1)}$

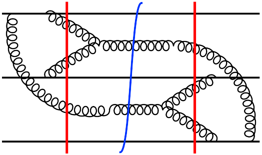
A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, arXiv:0804.3858

- $h^{(1)}$ is known since '12 ; invariant under $(k_\perp \to -k_\perp)$



A. Kovner and M. Lublinsky, arXiv:1211.1928
Y. V. Kovchegov and D. E. Wertepny, arXiv:1212.1195

- $h^{(2)}$: no complete result yet



L. McLerran and V. S., arXiv:1611.09870;
Yu. Kovchegov and V. S., arXiv:1802.08166

Inspiration from Single Transverse Spin Asymmetry

- Consider single gluon production

$$\frac{d\sigma}{d^2k} \sim |M(\underline{k})|^2 = \int d^2x\, d^2y\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\, M(\underline{x})\, M^*(\underline{y})$$

- Amplitude may have two contributions

$$M(\underline{x}) = M_1(\underline{x}) + M_3(\underline{x}) + \ldots$$

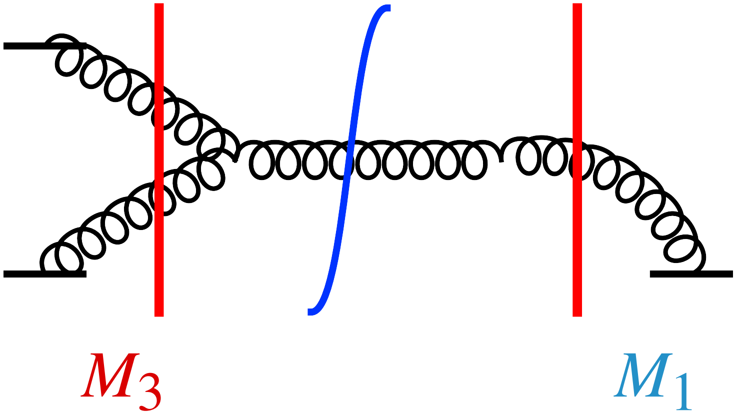
- Asymmetry under $\underline{k} \to -\underline{k}$ would mean that

$$M_1(\underline{x})\, M_3{}^*(\underline{y}) + M_3(\underline{x})\, M_1{}^*(\underline{y}) = -M_1(\underline{y})\, M_3{}^*(\underline{x}) - M_3(\underline{y})\, M_1{}^*(\underline{x})$$

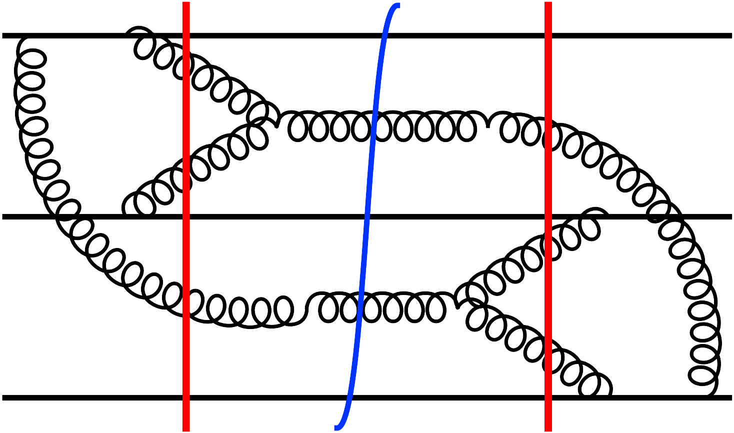
$\rightsquigarrow$ $M_1(\underline{x})\, M_3{}^*(\underline{y})$ is imaginary

$\rightsquigarrow$ Phase difference between M_1 and M_3 in coordinate space

In coordinate space, but not dissimilar from STSA
S. Brodsky, D. S. Hwang, Y. Kovchegov, I. Schmidt, M. Sievert, arXiv:1304.5237

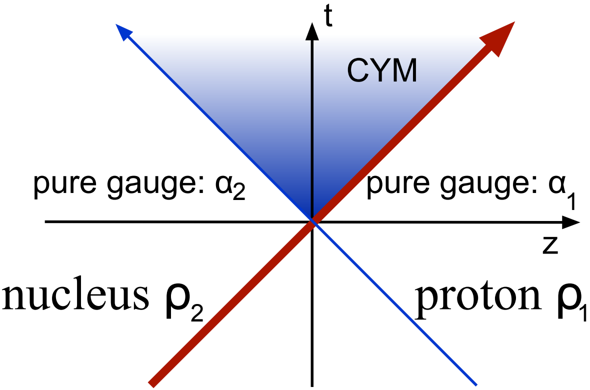


- Vanishes for single-inclusive production after performing average with respect to projectile configurations...



- Non-zero!

Classical Yang-Mills



- Just after collision, $\tau \to 0+$, initial conditions are known (Fock-Schwinger gauge $A_\tau = 0$)

  A. Kovner, L. McLerran, H. Weigert, arXiv:9506320

- In forward light-cone $[D_\mu, F^{\mu\nu}] = 0$
- Solve equations perturbatively in ρ_1; use LSZ

Gluon production

- Leading order and the first saturation correction

$$\frac{dN^{\rm even}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_t\right] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\underline{k})\left[\Omega^a_{lm}(\underline{k})\right]^\star$$

$$\frac{dN^{\rm odd}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_T\right] = \frac{2}{(2\pi)^3}{\rm Im}\left\{\frac{g}{\underline{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{{\rm Sign}(\underline{k}\times\underline{l})}{l^2|\underline{k}-\underline{l}|^2}f^{abc}\Omega^a_{ij}(\underline{l})\Omega^b_{mn}(\underline{k}-\underline{l})\left[\Omega^c_{rp}(\underline{k})\right]^\star\times\right.$$

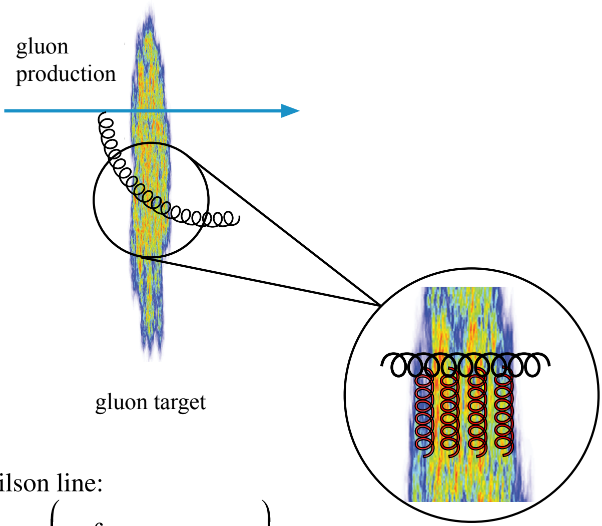
$$\left.\left[\left(\underline{k}^2\epsilon^{ij}\epsilon^{mn}-\underline{l}\cdot(\underline{k}-\underline{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\underline{k}\cdot(\underline{k}-\underline{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\right\}$$

Here $\delta_{ij}\Omega_{ij}=\Omega_{xx}+\Omega_{yy}$ and $\epsilon_{ij}\Omega_{ij}=\Omega_{xy}-\Omega_{yx}$ and

$$\underbrace{\Omega^a_{ij}(\mathbf{x}_\perp) = g\left[\frac{\partial_i}{\partial^2}\overbrace{\rho^b(\mathbf{x}_\perp)}^{\text{val. sour.}}\right]\partial_j\overbrace{U^{ab}(\mathbf{x}_\perp)}^{\text{target W line}}}_{\text{valence sources rotated by the target}}$$

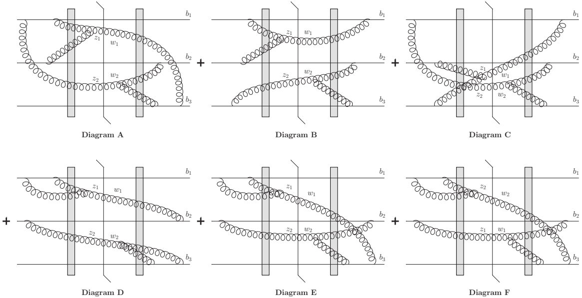
$\frac{dN^{\rm odd}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_T\right]$ is suppressed by extra $\alpha_s\rho_p$

L. McLerran and V. S., arXiv:1611.09870



Adjoint Wilson line:

$$U(\mathbf{x}_\perp) = \mathcal{P}\exp\left(ig\int dx^+ \underbrace{A^-}_{\text{adj.}}(x^+, \mathbf{x}_\perp)\right) \quad \Leftarrow \quad \text{multiple rescattering}$$



- Reproduces result obtained in Fock-Schwinger gauge!
- In Golec-Biernat–Wusthoff model & Large N_c & at high momentum:

$$\frac{d\sigma_{odd}}{d^2k_1\, dy_1\, d^2k_2\, dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b \left[T_1(\underline{B}-\underline{b})\right]^3 g^8\, Q_{s0}^6(b)\, \frac{1}{\underline{k}_1^6\, \underline{k}_2^6} \left\{ \underbrace{\left[\frac{(\underline{k}_1^2+\underline{k}_2^2+\underline{k}_1\cdot\underline{k}_2)^2}{(\underline{k}_1+\underline{k}_2)^6} - \frac{(\underline{k}_1^2+\underline{k}_2^2-\underline{k}_1\cdot\underline{k}_2)^2}{(\underline{k}_1-\underline{k}_2)^6}\right]}_{A} + \underbrace{\frac{10\, c^2}{(2\pi)^2}\, \frac{1}{\Lambda^2}\, \frac{\underline{k}_1\cdot\underline{k}_2}{k_1\, k_2}}_{B} \right.$$

$$\left. + \underbrace{\frac{1}{4\pi}\, \frac{k_1^4}{\Lambda^4} \left[\delta^2(\underline{k}_1-\underline{k}_2) - \delta^2(\underline{k}_1+\underline{k}_2)\right]}_{C} \right\}$$

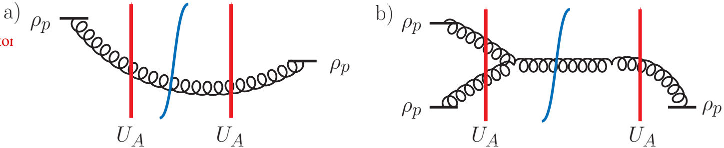
Yu. Kovchegov and V. S., arXiv:1802.08166

- Leading order and the first saturation correction

$$\text{a)}\ \frac{dN^{\text{even}}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_t\right] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\underline{k})\left[\Omega^a_{lm}(\underline{k})\right]^\star$$

$$\text{b)}\ \frac{dN^{\text{odd}}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_T\right] = \frac{2}{(2\pi)^3}\text{Im}\left\{\frac{g}{\underline{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{\text{Sign}(\underline{k}\times\underline{l})}{\underline{l}^2|\underline{k}-\underline{l}|^2}f^{abc}\Omega^a_{ij}(\underline{l})\Omega^b_{mn}(\underline{k}-\underline{l})\left[\Omega^c_{rp}(\underline{k})\right]^\star\times\right.$$
$$\left.\left[\left(\underline{k}^2\epsilon^{ij}\epsilon^{mn}-\underline{l}\cdot(\underline{k}-\underline{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\underline{k}\cdot(\underline{k}-\underline{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\right\}$$

Recall that $\Omega \propto \rho_{\text{proton}}$



Multiplicity dependence: scaling argument

- Two-particle anisotropy coefficients can be simply expressed as

$$v_n^2\{2\}(N_{\rm ch}) = \int \mathcal{D}\rho_p \mathcal{D}\rho_t \, W[\rho_p]\, W[\rho_t]\, |Q_n\left[\rho_p,\rho_t\right]|^2 \,\delta\left(\frac{dN}{dy}\left[\rho_p,\rho_t\right] - N_{\rm ch}\right)$$

with

$$Q_{2n}\left[\rho_p,\rho_t\right] = \frac{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i2n\phi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}, \quad Q_{2n+1}\left[\rho_p,\rho_t\right] = \frac{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i(2n+1)\phi} \frac{dN^{\rm odd}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}$$

- Assuming that multiplicity is due to fluctuations in ρ_p
- To study multiplicity dependence, rescale $\rho_p \to c\,\rho_p$
- Under this rescaling:

$$\frac{dN}{dy} \to c^2 \frac{dN}{dy}; \qquad v_{2n}^2\{2\} \to v_{2n}^2\{2\}; \qquad v_{2n+1}^2\{2\} \to c^2\, v_{2n+1}^2\{2\}$$

- Therefore in first approximation: $v_{2n}\{2\}$ is independent of multiplicity

$$v_{2n+1}\{2\} \propto \sqrt{\frac{dN}{dy}}$$

Beyond leading order

- Only leading order contributions to $\langle N \rangle$ and even/odd harmonics in dilute-dense expansion are known
- Therefore there no robust way to estimate truncation error within approach itself

Some recent progress: Ming Li et al – to be reported elsewhere

- Compare with numerics of dense-dense CYM
  - This is can be done in a uniform way, as initial conditions in forward light cone are the same for both approaches
  - Difference: CYM has to be solved numerically in forward light cone $\tau > 0$; dilute dense approximation allows to find the solution analytically
  - Change Q_s^{P} and Q_s^{A} to define region of applicability of dilute-dense approximation

Set up

- We consider the color charge distribution in the dilute projectile as

$$\langle \rho_a^{(p)}(\underline{x}) \rho_b^{(p)}(\underline{x}') \rangle = \left(\frac{g^2 \mu}{Q_s} \right)^2 \; Q_s^{(P)2} \left(\frac{\underline{x} + \underline{x}'}{2} \right) \delta_{ab} \, \delta^{(2)}(\underline{x} - \underline{x}') \, ,$$

  where the local saturation scale $Q_s^{(P)2}\left(\underline{x}\right)$ is determined by

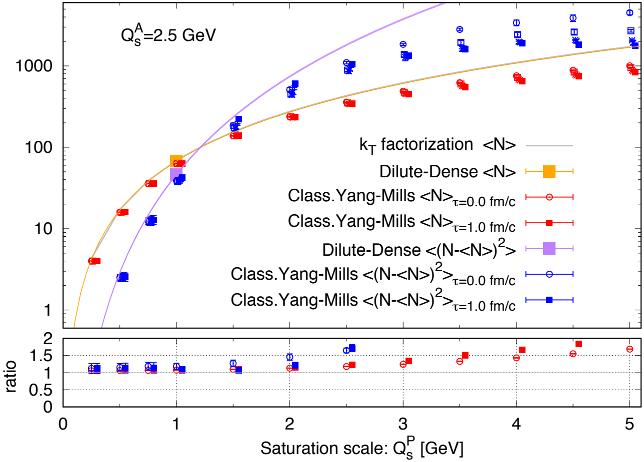
$$Q_s^{(P)2}\left(\underline{x}\right) = 2\pi R_p^2 \; T(\underline{x}) \; (Q_{s,0}^{(p)})^2 = (Q_{s,0}^{(p)})^2 \; \exp\left(- \frac{\underline{x}^2}{2R_p^2} \right)$$

  This can be considered as a minimal saturation model for the proton.
- Similarly, a spatially homogeneous color charge distribution of the dense target

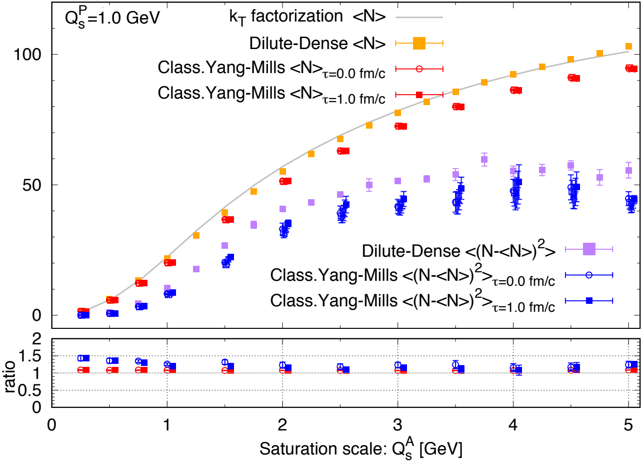
$$\langle \rho_a^{(t)}(\underline{x}) \rho_b^{(t)}(\underline{x}') \rangle = \left(\frac{g^2 \mu}{Q_s} \right)^2 \; (Q_{s,0}^{(A)})^2 \, \delta_{ab} \, \delta^{(2)}(\underline{x} - \underline{x}')$$

- Common prefactor $\left(\frac{g^2 \mu}{Q_s} \right) = 1.423$ can always be absorbed into a redefinition of the saturation momenta $Q_{s,0}^{(p/A)}$

DILUTE-DENSE VS DENSE-DENSE: Q_s^{P} DEPENDENCE

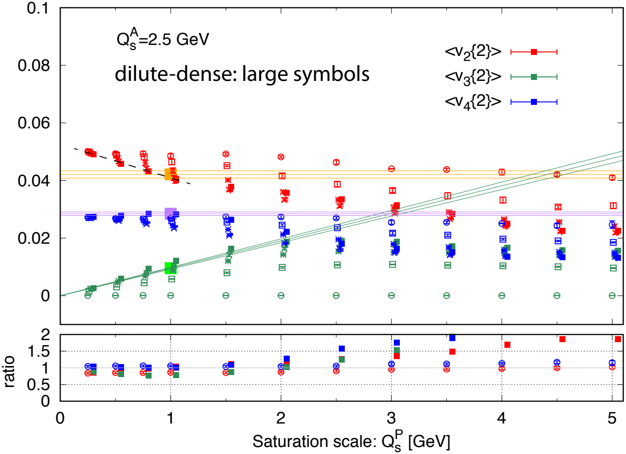


- Orange and purple curves: analytical result for dilute-dense ($Q_s^{\mathrm{P}\,2}$ and $Q_s^{\mathrm{P}\,4}$) with proportionality coefficient fixed by dilute-dense numerics
- $k_\perp$ factorization result overlaps with the dilute dense line



- Systematic deviation between dilute-dense and dense-dense results might be due to relative difference in the dependence on $f^{(1)}$ and $f^{(2)}$ on Q_{sA}

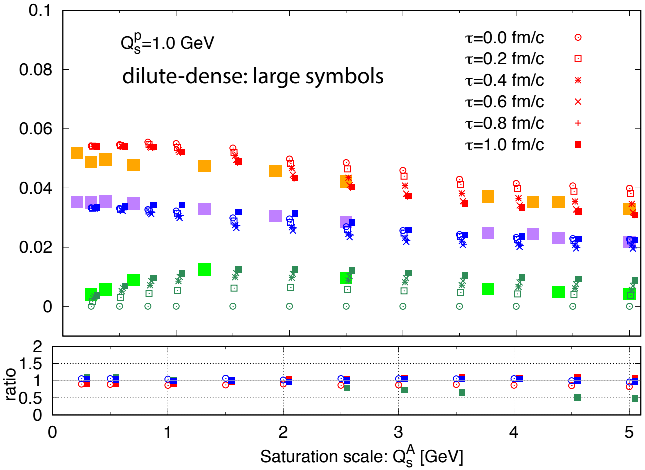
$$\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2} f^{(1)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \frac{1}{\alpha_s}\left(\frac{Q_{sp}^2}{k_\perp^2}\right)^2 f^{(2)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \cdots$$



- Dense-dense results obtained for different times $\tau = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$ fm$/c$
- Even harmonics: dense-dense numerics → negative corrections to the leading order:

$$\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 \, h^{(1)}(Q_{sA}) + \frac{1}{\alpha_s^2} Q_{sp}^6 \, h^{(2)}(Q_{sA}) + \cdots$$

Dilute-dense vs dense-dense: Q_s^A dependence of harmonics



- Overall a good agreement

Conclusions

- Dilute-dense expansion: Current analytical results allow us to evaluate only leading order contribution to $\langle N \rangle$, even and odd harmonics.
- In absence of analytical results for higher orders, to assess corrections we compared dilute-dense to dense-dense numerics for a minimal model of p-A collisions
- Results shows, that dilute-dense results start to deviate from full dense-dense calculations when $Q_s^{(P)} > (0.6 - 0.8) \times Q_s^{(A)}$. (Naively, one would expect breaking down of leading order dilute-dense result at smaller values of $Q_s^{(P)}$.)


BACKUP

Cumulants & Phenomenological Conclusion I

- Average number of gluons

$$\kappa_1 = \frac{N_c^2 - 1}{8\pi} \underbrace{S_\perp \mu_{\rm p}^2}\, \mathfrak{D}\, \ln \frac{k_{\min}^2}{\Lambda^2}$$

- Higher order cumulants

$$\kappa_{n\geq 2} = \frac{\partial}{\partial t^n} \ln G_{\rm LO}(t)\Big|_{t=0} = (n-2)!\, \frac{(N_c^2-1)\, S_\perp \Lambda^2}{8\pi} \left(\frac{\mu_{\rm p}^2 \mathfrak{D}}{\Lambda^2}\right)^n$$

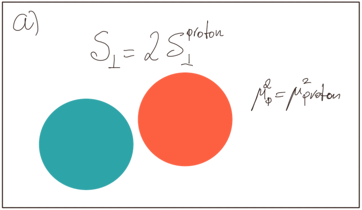
- Properties ($\Lambda^2 = S_\perp^{\rm proton}$)
  - κ_1 is a function of $\underbrace{S_\perp\, \mu_{\rm p}^2}$
  - Consider configurations a) and b):

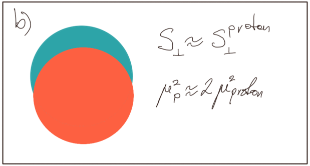
$\kappa_1[a)] = \kappa_1[b)]$

$\kappa_n[b)] \propto \mathbf{2}\, S_\perp^{\rm proton} \left(\mu_p^{\rm proton}\right)^{2n}$

$\kappa_n[b)] \propto \mathbf{2^n}\, S_\perp^{\rm proton} \left(\mu_p^{\rm proton}\right)^{2n}$

High multiplicity tail ≡
≡ configurations with
overlapping nucleons





More functionals

- BE is property of projectile
- Need for effective theory of gluons in projectile
- Constraint effective action for projectile gluon distribution

$$e^{-V_{\rm eff}[\eta(\underline{q})]} = \frac{1}{Z_{\rm p}} \int \mathcal{D}\rho_{\rm p} \underbrace{W(\rho_{\rm p})}_{\text{all possible fluct.}} \underbrace{\delta\left(\eta(\underline{q}) - \frac{g^2 {\rm tr}|A^+(\underline{q})|^2}{\langle g^2 {\rm tr}|A^+(\underline{q})|^2\rangle}\right)}_{\text{keeping only interesting stuff}}$$

$$A^+(\underline{q}) = g/q^2 \rho_{\rm p}(q), \qquad \langle g^2 {\rm tr}|A^+(\underline{q})|^2\rangle = \tfrac{1}{2}(N_c^2-1) S_\perp \frac{g^4\mu_{\rm p}^2}{q^4}$$

- Exact expression for effective potential (modulo $S_\perp^{-1}$ corrections)

$$V_{\rm eff}[\eta(\underline{q})] = \frac{1}{2}(N_c^2-1)S_\perp \int \frac{d^2q}{(2\pi)^2}\left\{\eta(\underline{q}) - 1 - \ln\eta(\underline{q})\right\} \approx \frac{1}{2}(N_c^2-1)S_\perp \int \frac{d^2q}{(2\pi)^2}\frac{1}{2}\ln^2\eta(\underline{q})$$

A. Dumitru & V. S., arXiv:1704.05917

LIOUVILLE POTENTIAL & HIGH MULTIPLICITY TAIL

- Back to generating function

$$G_{\rm LO}(t) = \left\langle \exp\left[t \int_\Lambda^{k_{\rm min}} \frac{d^2q}{(2\pi)^2} \rho^a(-\underline{q}) \frac{\mathfrak{D}}{2q^2} \rho^a(\underline{q}) \right] \right\rangle_{\rm p}$$

- In terms of effective potential

$$G_{\rm LO}(t) = \int \mathcal{D}\eta \exp\left(-V_{\rm eff}[\eta(q)] + \underbrace{\frac{1}{2}(N_c^2-1)S_\perp \int_\Lambda^{k_{\rm min}} \frac{d^2q}{(2\pi)^2}\, t\, \frac{\mu_{\rm p}^2 \mathfrak{D}}{q^2}\, \eta(\underline{q})}_{\text{reweighting! derivatives in } t \text{ probe Liouville potential}} \right)$$

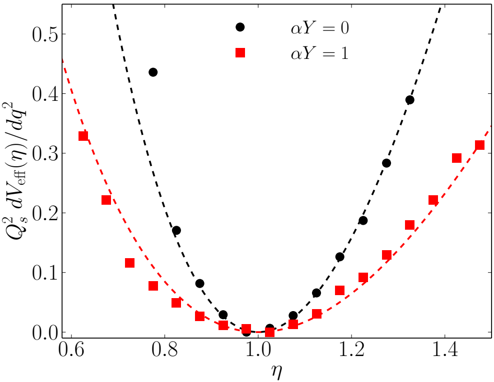
- For large $S_\perp$: saddle point approximation

$$\eta_s(\underline{q}) = \begin{cases} \left(1 - t\frac{\mu_{\rm p}^2\mathfrak{D}}{q^2}\right)^{-1}, & \text{if } \Lambda \le q \le k_{\rm min} \\ 1, & \text{otherwise} \end{cases}$$

to yield

$$\ln G_{\rm LO}(t) = \frac{1}{2}(N_c^2-1)S_\perp \int \frac{d^2q}{(2\pi)^2} \ln \eta_s(\underline{q}) = -\frac{1}{2}(N_c^2-1)S_\perp \int_\Lambda^{k_{\rm min}} \frac{d^2q}{(2\pi)^2} \ln\left(1 - t\frac{\mu_{\rm p}^2\mathfrak{D}}{q^2}\right)$$

- We recovered previously derived result. Origin of ln ≡ Liouville's ln!

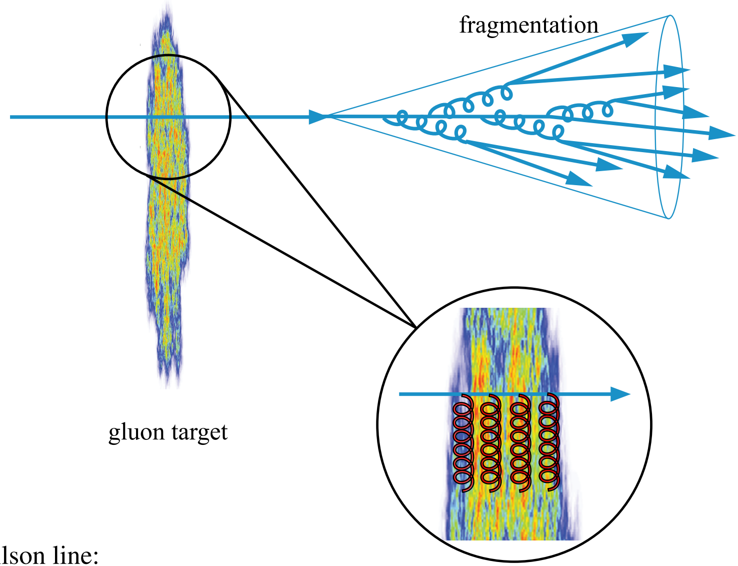


A. Dumitru & V. S., arXiv:1704.05917

- Form does not change $V_{\rm eff}[\eta(\underline{q})] \approx \frac{1}{2}(N_c^2 - 1)S_\perp \int \frac{d^2q}{(2\pi)^2}\,\frac{1}{2}\ln^2 \eta(\underline{q})$
- $S_\perp \to S_\perp^{\rm eff} \equiv \frac{S_\perp}{\sigma^2}$:

  partially responsible for phenomenological parameter σ
- C.f. $P[\rho] \propto \exp\left[-\frac{\rho^2}{2\sigma^2}\right]$ with $\rho \equiv \ln Q_s^2/\bar{Q}_s^2$

L. McLerran & P. Tribedy, arXiv:1508.03292



Fundamental Wilson line:

$$V(\mathbf{x}_\perp) = \mathcal{P} \exp\left(ig \int dx^+ A^-(x^+, \mathbf{x}_\perp)\right) \quad \Leftarrow$$

Odd contribution is buried somewhere in multiple rescattering i.e. in high order $h^{(N\gg1)}$ $\Downarrow$

$$\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4\, h^{(1)}\left(Q_{sA}\right) + \frac{1}{\alpha_s^2} Q_{sp}^6\, h^{(2)}\left(Q_{sA}\right) + \cdots$$