Hadronic corrections to the anomalous magnetic moment of the muon

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Lepton dipole moments: experimental status

- Dipole moments: definition

$$\mathcal{H} = -\boldsymbol{\mu}_\ell \cdot \boldsymbol{B} - \boldsymbol{d}_\ell \cdot \boldsymbol{E}$$

$$\boldsymbol{\mu}_\ell = -g_\ell \frac{e}{2m_\ell}\boldsymbol{S} \qquad \boldsymbol{d}_\ell = -\eta_\ell \frac{e}{2m_\ell}\boldsymbol{S} \qquad a_\ell = \frac{g_\ell - 2}{2}$$

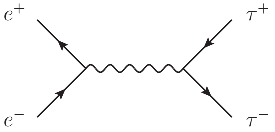
- Anomalous magnetic moments Hanneke et al. 2008, Bennett et al. 2006

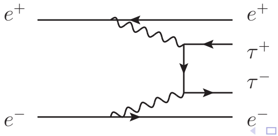
$$a_e^{\text{exp}} = 1{,}159{,}652{,}180.73(28) \times 10^{-12} \qquad a_\mu^{\text{exp}} = 116{,}592{,}089(63) \times 10^{-11}$$

- Electric dipole moments Andreev et al. 2018, Bennett et al. 2009

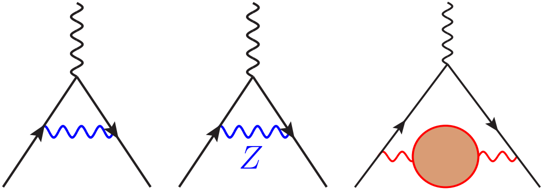
$$|d_e| < 1.1 \times 10^{-29}\, e\,\text{cm} \qquad |d_\mu| < 1.5 \times 10^{-19}\, e\,\text{cm} \qquad 90\%\ \text{C.L.}$$

- Not much known about τ dipole moments, some limits from





Anomalous magnetic moment of the electron



- **SM prediction for $(g-2)_\ell$**

$$a_\ell^{\text{SM}} = a_\ell^{\text{QED}} + a_\ell^{\text{EW}} + a_\ell^{\text{had}}$$

- For electron: electroweak and hadronic contributions under control
- For a precision calculation need:
  - Independent input for α
  - Higher-order QED contributions

Anomalous magnetic moment of the electron: QED

- QED expansion

$$a_e^{\mathrm{QED}} = A_1 + A_2\left(\frac{m_e}{m_\mu}\right) + A_2\left(\frac{m_e}{m_\tau}\right) + A_3\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right)$$

$$A_i = \left(\frac{\alpha}{\pi}\right) A_i^{(2)} + \left(\frac{\alpha}{\pi}\right)^2 A_i^{(4)} + \left(\frac{\alpha}{\pi}\right)^3 A_i^{(6)} + \cdots$$

- Numerical calculation up to five loops Aoyama, Kinoshita, Nio
- Recent developments
  - Analytic cross check of $A_{2,3}$ at 4 loops Kurz et al. 2014
  - Semi-analytic calculation of A_1 at 4 loops Laporta 2017
  - Independent calculation of 5-loop coefficient Volkov 2019

$$A_1^{(10)}\Big|_{\text{no lepton loops, AKN}} = 7.668(159) \qquad A_1^{(10)}\Big|_{\text{no lepton loops, Volkov}} = 6.793(90)$$

$\hookrightarrow$ 4.8σ difference

- Five-loop coefficient not an issue right now, but will become important in the future

Anomalous magnetic moment of the electron: fine-structure constant

- Input from **atom interferometry**

$$\alpha^2 = \frac{4\pi R_\infty}{c} \times \frac{m_{\text{atom}}}{m_e} \times \frac{\hbar}{m_{\text{atom}}}$$

- With **Rb measurement** LKB 2011

$$a_e^{\text{exp}} = 1{,}159{,}652{,}180.73(28) \times 10^{-12}$$
$$a_e^{\text{SM}} = 1{,}159{,}652{,}182.03(1)_{\text{5-loop}}(1)_{\text{had}}(72)_{\alpha(\text{Rb})} \times 10^{-12}$$
$$a_e^{\text{exp}} - a_e^{\text{SM}} = -1.30(77) \times 10^{-12}[1.7\sigma]$$

$\hookrightarrow$ α limiting factor, but more than an order of magnitude to go in theory

Anomalous magnetic moment of the electron: fine-structure constant

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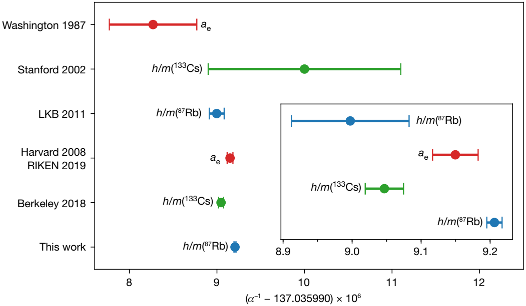
- With **Cs measurement** Berkeley 2018, Science 360 (2018) 191

$$a_e^{\text{SM}} = 1{,}159{,}652{,}181.61(1)_{\text{5-loop}}(1)_{\text{had}}(23)_{\alpha(\text{Cs})} \times 10^{-12}$$

$$a_e^{\text{exp}} - a_e^{\text{SM}} = -0.88(36) \times 10^{-12}[2.5\sigma]$$

$\hookrightarrow$ for the first time a_e^{exp} limiting factor

Anomalous magnetic moment of the electron: fine-structure constant



LKB 2020

- Tensions
  - Berkeley 2018 vs. LKB 2020: 5.4σ
  - LKB 2011 vs. LKB 2020: 2.4σ

- With new **Rb measurement** LKB 2020, Nature 588 (2020) 61

$$a_e^{\text{SM}} = 1{,}159{,}652{,}180.25(1)_{\text{5-loop}}(1)_{\text{had}}(9)_{\alpha(\text{Rb})} \times 10^{-12}$$

$$a_e^{\text{exp}} - a_e^{\text{SM}} = 0.48(30) \times 10^{-12}[1.6\sigma]$$

$\hookrightarrow$ on the opposite side of a_e^{exp}!

What does this mean for BSM?

- There seems to be an experimental issue in the determination of α
- Expectations from a_μ, depending on **mass scaling**:
  - m_ℓ^2: $a_e^{\text{BSM}} \sim 0.065(18) \times 10^{-12}$
  - m_ℓ: $a_e^{\text{BSM}} \sim 13.5(3.7) \times 10^{-12}$
- Compare to
  - LKB 2020 sensitivity: 0.095×10^{-12}
  - LKB 2020 vs. Berkeley 2018: $1.36(25) \times 10^{-12}$
  - LKB 2020 vs. a_e^{exp}: $0.48(30) \times 10^{-12}$

$\hookrightarrow$ LKB 2020 close to quadratic regime, but the tensions start much earlier

- Situation unclear, improved a_e^{exp} all the more important Gabrielse

The Standard Model prediction for $(g-2)_\mu$: QED

- **5-loop QED** result Aoyama, Kinoshita, Nio 2018:

$$a_\mu^{\text{QED}} = 116\,584\,719.0(1) \times 10^{-11}$$

$\hookrightarrow$ insensitive to input for α (at this level)

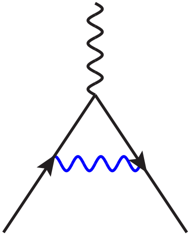
- QED coefficients enhanced by $\log m_\mu / m_e$
- Enhancement from naive RG expectation for 6-loop QED

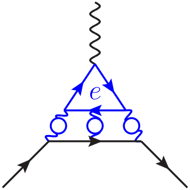
$$10 \times \frac{2}{3}\pi^2 \log\frac{m_\mu}{m_e} \times \left(\frac{2}{3}\log\frac{m_\mu}{m_e}\right)^3 \sim 1.6 \times 10^4$$

$\hookrightarrow$ would imply $a_\mu^{\text{6-loop}} \sim 0.2 \times 10^{-11}$

- Refined RG estimate Aoyama, Hayakawa, Kinoshita, Nio 2012

$$a_\mu^{\text{6-loop}} \sim 0.1 \times 10^{-11}$$





The Standard Model prediction for $(g-2)_\mu$: electroweak

- Electroweak contribution Gnendiger et al. 2013

$$a_\mu^{\text{EW}} = (194.8 - 41.2) \times 10^{-11} = 153.6(1.0) \times 10^{-11}$$

- Remaining uncertainty dominated by $q = u, d, s$ loops

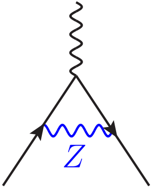
$\hookrightarrow$ nonperturbative effects Czarnecki, Marciano, Vainshtein 2003

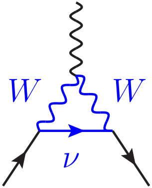
- Two-loop calculation recently revisited without asymptotic expansion Ishikawa, Nakazawa, Yasui 2019

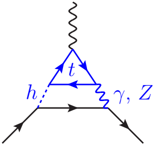
$$a_\mu^{\text{EW}} = 152.9(1.0) \times 10^{-11}$$

- 3-loop corrections?
  - 3-loop RG estimate accidentally cancels in scheme chosen by Gnendiger et al. 2013, with an error of 0.2×10^{-11}
  - α_s corrections to t-loop should scale as

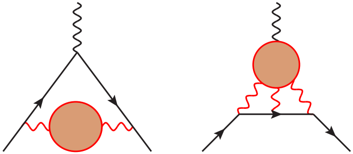
$$a_\mu^{t\text{-loop}}\big|_{\text{2-loop}} \times \frac{\alpha_s}{\pi} \lesssim 0.3 \times 10^{-11}$$







The Standard Model prediction for $(g-2)_\mu$: hadronic effects



- **Hadronic vacuum polarization**: need hadronic two-point function

$$\Pi_{\mu\nu} = \langle 0|T\{j_\mu j_\nu\}|0\rangle$$

- **Hadronic light-by-light scattering**: need hadronic four-point function

$$\Pi_{\mu\nu\lambda\sigma} = \langle 0|T\{j_\mu j_\nu j_\lambda j_\sigma\}|0\rangle$$

- Rest of the talk: how to evaluate the hadronic contributions

The Muon $g-2$ Theory Initiative

- Formed in 2017, series of workshops since (last plenary one at the INT in Sep 2019) `https://indico.fnal.gov/event/21626/`
- Map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental results
- White paper 2006.04822: **The anomalous magnetic moment of the muon in the Standard Model** `https://muon-gm2-theory.illinois.edu/`

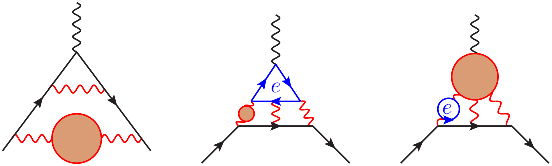
The anomalous magnetic moment of the muon in the Standard Model

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

Table 1: Summary of the contributions to a_μ^{SM}. After the experimental number from E821, the first block gives the main results for the hadronic contributions from Secs. 2 to 5 as well as the combined result for HLbL scattering from phenomenology and lattice QCD constructed in Sec. 8. The second block summarizes the quantities entering our recommended SM value, in particular, the total HVP contribution, evaluated from e^+e^- data, and the total HLbL number. The construction of the total HVP and HLbL contributions takes into account correlations among the terms at different orders, and the final rounding includes subleading digits at intermediate stages. The HVP evaluation is mainly based on the experimental Refs. [37–89]. In addition, the HLbL evaluation uses experimental input from Refs. [90–109]. The lattice QCD calculation of the HLbL contribution builds on crucial methodological advances from Refs. [110–116]. Finally, the QED value uses the fine-structure constant obtained from atom-interferometry measurements of the Cs atom [117].

Now waiting for E989!

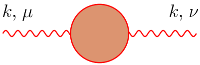
The Standard Model prediction for $(g-2)_\mu$: higher-order hadronic effects



- Once $\Pi_{\mu\nu}$ and $\Pi_{\mu\nu\lambda\sigma}$ known, higher-order iterations determined
- Standard for NLO HVP Calmet et al. 1976
- NNLO HVP found to be relevant recently Kurz et al. 2014
- NLO HLbL already further suppressed Colangelo et al. 2014

Hadronic vacuum polarization

- General principles yield **direct connection with experiment**
  - **Gauge invariance**



$$= -i(k^2 g^{\mu\nu} - k^\mu k^\nu)\Pi(k^2)$$

  - **Analyticity**

$$\Pi_{\text{ren}} = \Pi(k^2) - \Pi(0) = \frac{k^2}{\pi} \int_{4M_\pi^2}^{\infty} ds \frac{\text{Im}\,\Pi(s)}{s(s-k^2)}$$

  - **Unitarity**

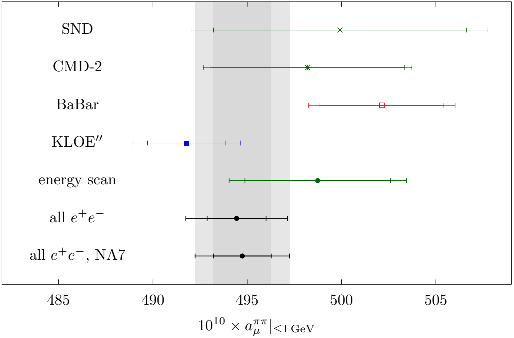
$$\text{Im}\,\Pi(s) = \frac{s}{4\pi\alpha}\sigma_{\text{tot}}(e^+e^- \to \text{hadrons}) = \frac{\alpha}{3}R(s)$$

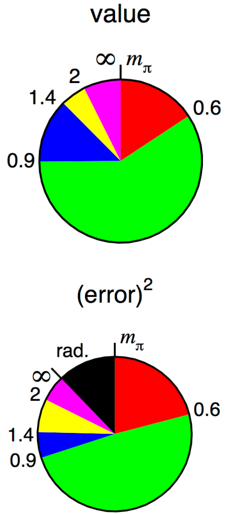
- 1 Lorentz structure, 1 kinematic variable, no free parameters
- **Dedicated e^+e^- program** under way, hopefully new results from SND (under review), CMD3, BaBar, Belle II, BESIII soon

Hadronic vacuum polarization: two-pion channel

- HVP accuracy goal: 0.6% (present) $\rightarrow$ 0.2% (experiment)
- Main contender: $\pi\pi$ channel
- Current status Colangelo, MH, Stoffer 2019

  $\hookrightarrow$ tension between BaBar and KLOE data sets





Keshavarzi et al. 2018

Hadronic vacuum polarization: global constraints

- Direct integration: **local error inflation** wherever tensions between data sets arise
- **Analyticity**, **unitarity**, and **crossing symmetry** imply strong constraints on hadronic cross sections

  $\hookrightarrow$ defines **global fit function**, very few parameters

  - Can one describe the data with an acceptable χ^2 in this way?
  - How do the results compare to direct integration?

  $\hookrightarrow$ **internal consistency**, combination of data sets

- Implemented for 2π and 3π (80% of total HVP)
  - Problems for some data sets discovered, but BaBar/KLOE tension unaffected
  - Interpolation issue in 3π discovered
  - Uncertainty estimates from direct integration corroborated

  $\hookrightarrow$ how to deal with the BaBar/KLOE tension?

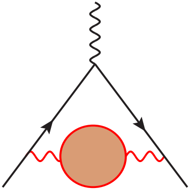
Hadronic vacuum polarization: white paper

- **BaBar/KLOE tension** drives differences between compilations Davier et al. 2019, Keshavarzi et al. 2019
- In the Muon $g-2$ Theory Initiative, we developed a prescription to account for the respective systematic effect

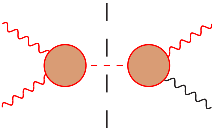
$$a_\mu^{\text{HVP}} = 6\,931(28)_{\text{exp}}(28)_{\text{sys}}(7)_{\text{DV+QCD}} \times 10^{-11} = 6\,931(40) \times 10^{-11}$$

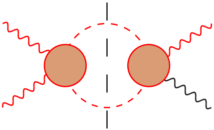
- Merits of this value:
  - Conservative but realistic
  - Merges different methodologies, including global constraints
  - Accounts for BaBar/KLOE tension beyond a (local) χ^2 inflation
  - New high-statistics data sets will help remove the added systematic effect

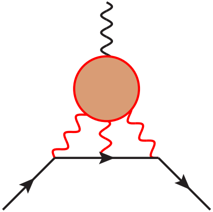
Hadronic light-by-light scattering

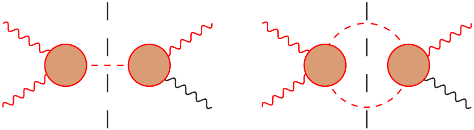


- So far: hadronic models, inspired by various QCD limits, but error estimates difficult
- Our suggestion: use again **analyticity**, **unitarity**, **crossing**, and **gauge invariance** for data-driven approach Colangelo, MH, Procura, Stoffer 2014, 2015
- For simplest intermediate states: relation to $\pi^0 \to \gamma^*\gamma^*$ **transition form factor** and $\gamma^*\gamma^* \to \pi\pi$ **partial waves**







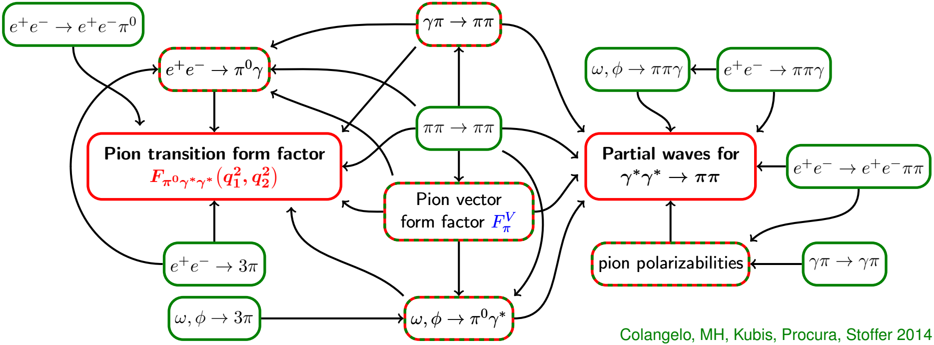


- List of challenges
  - Find a suitable Lorentz basis Bardeen, Tung 1968, Tarrach 1975

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{138} \Pi_i T_i^{\mu\nu\lambda\sigma}$$

$\hookrightarrow$ 41 gauge-invariant structures

  - There is no minimal Lorentz basis free of kinematic singularities Tarrach 1975
  - General kinematics: $s, t, u, q_1^2, q_2^2, q_3^2, q_4^2$, but $q_4 \to 0$ in the end
  - Identify Π_i relevant for $(g-2)_\mu$, in the correct kinematic configuration
  - Combine all that with partial-wave expansion



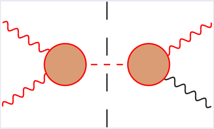
Colangelo, MH, Kubis, Procura, Stoffer 2014

- Reconstruction of $\gamma^*\gamma^* \to \pi\pi, \pi^0$: combine experiment and theory constraints
- Implementation
  - π^0 pole done MH et al. 2018
  - First results for $\pi\pi$ Colangelo, MH, Procura, Stoffer 2017

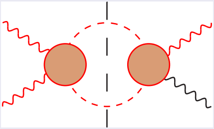
Impact on HLbL contribution

Numbers

$$a_\mu^{\pi^0\text{-pole}} = 62.6^{+3.0}_{-2.5} \times 10^{-11}$$



$$a_\mu^{\pi\text{-box}} + a_{\mu,J=0}^{\pi\pi,\pi\text{-pole LHC}} = -24(1) \times 10^{-11}$$



- Well-defined contributions with controlled error estimates
- Plan towards a full evaluation of HLbL
  - η, η' poles with dispersion relations
  - $K\bar{K}$ and $\pi\pi$ beyond π-pole LHC and S-waves
  - Asymptotics of HLbL tensor, matching to pQCD Bijnens et al. 2019, Colangelo et al. 2019
  - Resonance estimates for higher intermediate states

On the implementation of short-distance constraints

- **Short-distance constraints** on HLbL scattering important to constrain high- and mixed-energy regions in $g-2$ integral
  - All photon virtualities q_i^2 large Bijnens et al. 2019

    $\hookrightarrow$ pQCD quark loop first term in systematic OPE
  - One virtuality remains small $q_3^2 \ll q_1^2, q_2^2$ Melnikov, Vainshtein 2004

    $\hookrightarrow$ exact relation **in the chiral limit**
- Implementation in Melnikov, Vainshtein 2004 in terms of pseudoscalar poles
  - Take $F_{\pi^0\gamma^*\gamma^*}(q_3^2, 0) \to F_{\pi^0\gamma^*\gamma^*}(0, 0)$ (without changing anything else)
  - This increases the π^0, η, η' contributions by 38×10^{-11} (40% increase!)
  - Severe distortion of low-energy properties of HLbL that cannot be justified
- Implementation in terms of **excited pseudoscalars** Colangelo et al. 2019
  - Resum series of pseudoscalar poles to get the asymptotics right
  - Works for physical quark masses, but not in the chiral limit
  - Model dependence reduced significantly by matching to the pQCD quark loop
  - Find for dominant longitudinal contribution $13(6) \times 10^{-11}$

HLbL scattering: white paper

- Reference points:

$$a_\mu^{\text{HLbL}}\Big|_{\text{“Glasgow consensus” 2009}} = 105(26)\times 10^{-11}$$

$$a_\mu^{\text{HLbL}}\Big|_{\text{Jegerlehner, Nyffeler 2009}} = 116(39)\times 10^{-11}$$

- Strategy in the white paper
    - Take well-controlled results for the low-energy contributions
    - Combine errors in quadrature
    - Take best guesses for medium-range and short-distance matching
    - Add these errors linearly, since errors hard to disentangle at the moment
- Estimate from **phenomenology** (including charm loop)

$$a_\mu^{\text{HLbL}} = 92(19)\times 10^{-11}$$

- Compare to **lattice QCD**: first complete calculation RBC/UKQCD 2019

$$a_\mu^{\text{HLbL}}[uds] = 79(35)\times 10^{-11}$$

- Final recommendation

$$a_\mu^{\text{HLbL}} = 90(17)\times 10^{-11}$$

HVP from lattice QCD

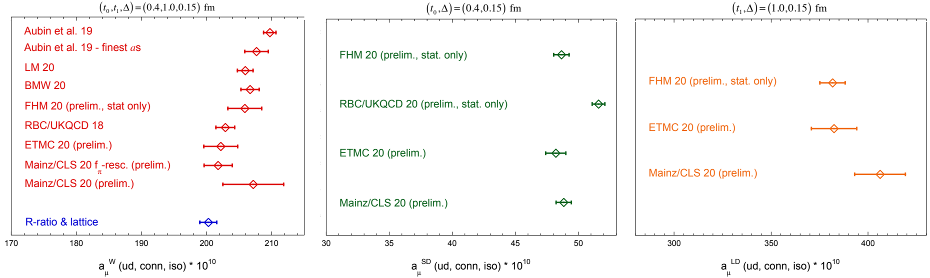
	e^+e^- from WP	lattice average from WP	BMWc v2
$a_\mu^{\text{HVP, LO}} \times 10^{11}$	$6\,931(40)$	$7\,116(184)$	$7\,087(53)$
difference to e^+e^-		1.0σ	2.3σ
tension with BNL	3.7σ	0.5σ	1.5σ

- Calculation from BMWc in tension with e^+e^- data
- How can we test this result?
  - Independent lattice calculations at same level of accuracy
  - Hadronic running of α
  - Correlations with low-energy hadron phenomenology

Crosschecks

“Window” quantities

(Plots from Davide Giusti)



- Straightforward reference quantities
- Can be applied to individual contributions (light, strange, charm, disconnected,...)
- Comparison with e^+e^-/R-ratio may require tuning of the window

Summary talk by H. Wittig at Muon $g-2$ Theory Initiative virtual workshop
“The hadronic vacuum polarization from lattice QCD at high precision”

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)\times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

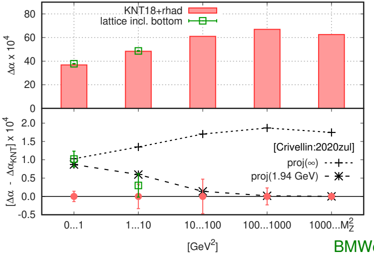
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)=\frac{\alpha M_Z^2}{3\pi}P\int_{s_{\text{thr}}}^{\infty}\text{d}s\frac{R_{\text{had}}(s)}{s(M_Z^2-s)}$$

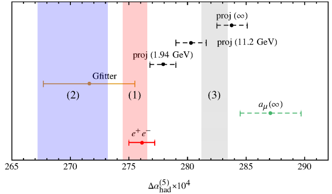
- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)=\frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2)+\frac{\alpha}{\pi}\big(\hat{\Pi}(M_Z^2)-\hat{\Pi}(-M_Z^2)\big)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020



Crivellin, MH, Manzari, Montull 2020

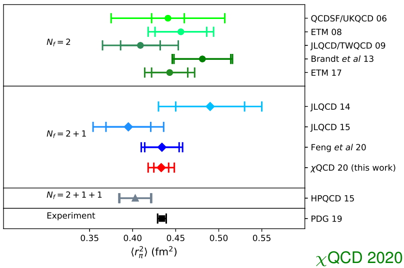
- BMWc results for $\Delta\alpha_{\text{had}}$ suggest that the change needs to come from low energies
  $\hookrightarrow \pi\pi$ channel
- $a_\mu^{\text{HVP, LO}}$ and $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ expressed in terms of **pion vector form factor**

$$R_{\text{had}}(s) = \frac{1}{4}\bigg(1-\frac{4M_\pi^2}{s}\bigg)^{3/2}\big|F_\pi^V(s)\big|^2$$

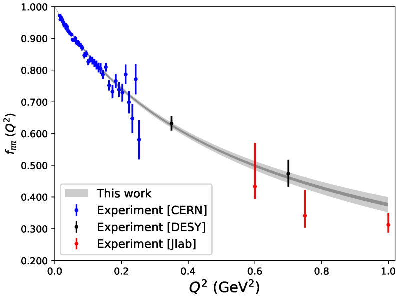
- **Pion charge radius**

$$\langle r_\pi^2\rangle = \frac{6}{\pi}\int\limits_{4M_\pi^2}^{\infty} ds \frac{\text{Im}\, F_\pi^V(s)}{s^2}$$

$\hookrightarrow$ can also be calculated on the lattice

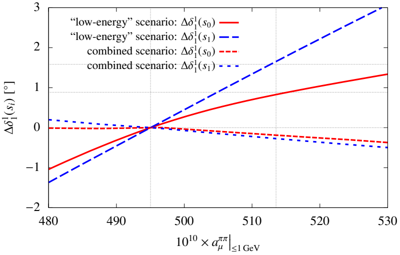


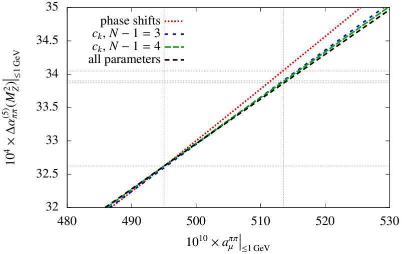
χQCD 2020



Changing HVP at low energies

- Use dispersive representation to study changes that would be allowed by analyticity and unitarity Colangelo, MH, Stoffer 2020
  - "Low-energy" physics: **$\pi\pi$ phase shifts**
  - "High-energy" physics: **inelastic effects**
  - **All parameters together**
- Consider correlations with phase shifts, hadronic running of α, pion charge radius, and space-like form factor





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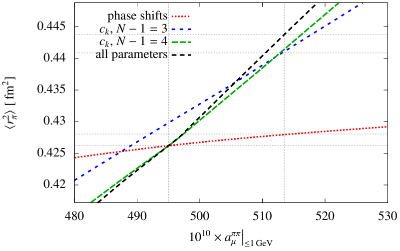

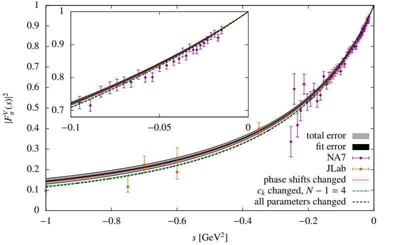

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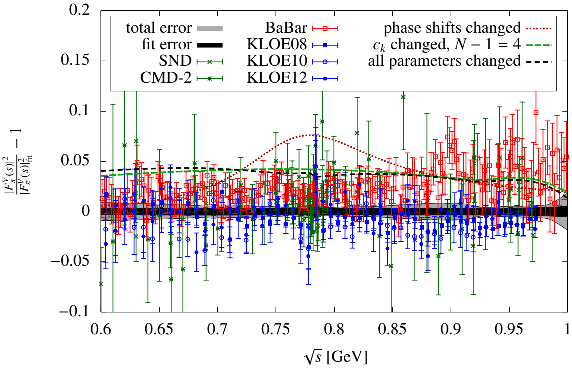

- **"Low-energy" scenario**: changes of 8% near the ρ
- **"All parameters**: uniform shift around 4%, far outside the experimental errors

  $\hookrightarrow$ could be excluded/confirmed by a precision calculation of $\langle r_\pi^2\rangle$

Conclusions

- **Electron $g-2$**
  - Quo vadis α?
- **Hadronic vacuum polarization**
  - Presently largest systematic uncertainty in $\pi\pi$ channel
  - Dispersive analysis to consolidate error estimate
  - Ultimately new data required: SND, CMD-3, BaBar
  - What will happen on the lattice?
- **Hadronic light-by-light scattering**
  - Use dispersion relations to remove model dependence as far as possible
  - Implemented for π^0 and leading $\pi\pi$ intermediate states
  - Subleading terms including asymptotic matching in progress
  - Good agreement between phenomenology and lattice