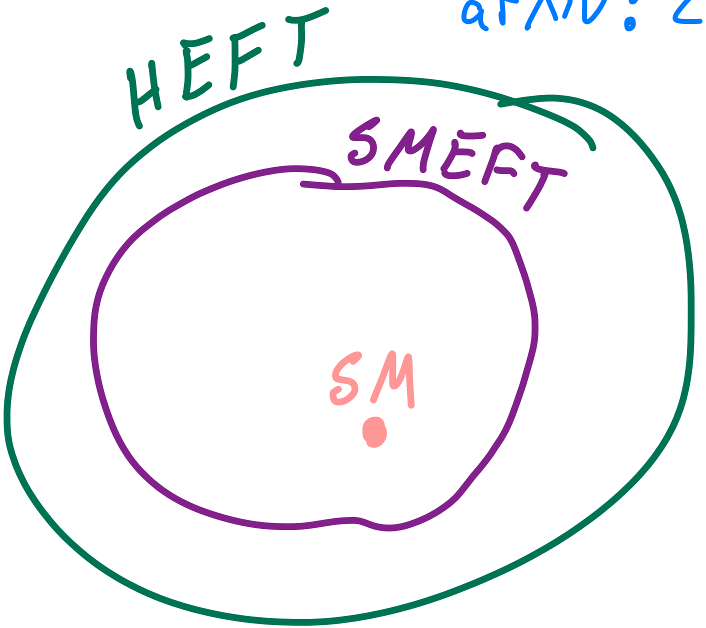
Is SMEFT Enough?

arXiv: 2008.08597



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BNL HET Seminar, Dec 16

Where's the new physics?

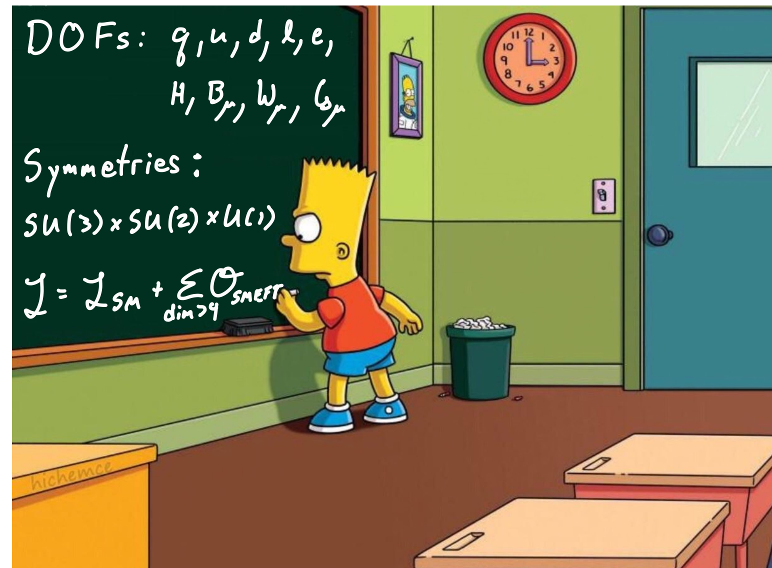
How best to search?

Can we be systematic?

How to organize BSM predictions?

Simplified Models

Effective Field Theory



The Standard Model as EFT

"Heavy physics decouples"

- Only SM dofs
- Symmetries: Lorentz + $SU(3) \times SU(2) \times U(1)$

Realize electroweak symmetry

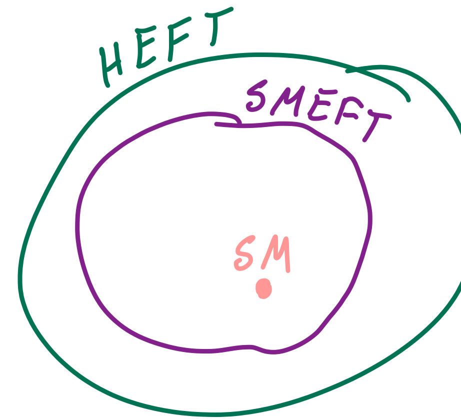
linearly or non-linearly

SMEFT HEFT

To SMEFT or to HEFT?

HEFT >> SMEFT

Goal: What (perturbative) BSM models require HEFT??



Liturature

- Alonso, Jenkins, Manohar (arXiv: 1605.03602 + ...)
- Falkowski, Rattazzi (arXiv: 1902.05936)
- Chang, Luty (arXiv: 1902.05556 + ...)
- Brivio, Trott (arXiv: 1706.08945)

Outline

I. Define SMEFT & HEFT

II. Light BSM & Analyticity

III. EFT Convergence (Why HEFT?)

IV. Curvature Criterion (When HEFT?)

V. Unitarity (Where HEFT?)

VI. Summary and Outlook

I. Define SMEFT & HEFT

Define SMEFT + HEFT

Scope (AJM)

Scalar sector only

Operators up to two derivatives

Custodial symmetry

$SU(2) \times U(1) \to SO(4)$

Field redefinitions do not include ∂ interactions

SMEFT $(v=0)$

Let $\vec{\varphi}$ be an $O(4)$ vector

$$\vec{\varphi} \to O\,\vec{\varphi}$$

Identify $H = \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_4 + i\varphi_3 \end{pmatrix}$

s.t. $\langle H \rangle \neq 0 \Leftrightarrow \langle \varphi_4 \rangle \neq 0$

SMEFT $(v=0)$

$$\mathcal{L}_{SMEFT} = A(|H|^2)\,|\partial H|^2 + \frac{1}{2}B(|H|^2)\left[\partial(|H|^2)\right]^2 - \tilde{V}(|H|^2) + \mathcal{O}(\partial^4)$$

$A, B, \tilde{V}$ are real analytic @ origin $|H| = 0$

$\Rightarrow$ have convergent Taylor expansion @ $|H| = 0$

($A(0) = 1 \Rightarrow$ canonical norm)

Think of φ_i as Cartisian coords on manifold

$\vec{\varphi} = 0$ is invariant point under $O(4)$ transformations

HEFT ($v \neq 0$)

Non-linearly realized sym breaking

$O(4)/O(3)$ Calan, Coleman, Wess, Zumino (1969)

Polar coordinates: h (physical Higgs)
$\vec{n}$ (Goldstone bosons)

$$\vec{\varphi} = (v_0 + h)\,\vec{n} \qquad w/ \qquad \vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_i^2} \end{pmatrix}$$

$\vec{n} \in S^3 \qquad \vec{n} \cdot \vec{n} = 1$

HEFT $(v \neq 0)$

$O(4)$ transformation: $h \to h$, $\vec{n} \to O\vec{n}$

$\Rightarrow$ $\vec{n}$ in non-linear rep

$$\mathcal{L}_{HEFT} = \frac{1}{2}\left[\mathbb{K}(h)\right]^2 (\partial h)^2 + \frac{1}{2}\left[v F(h)\right]^2 (\partial \vec{n})^2$$

$$- V(h) + \mathcal{O}(\partial^4)$$

$\langle h \rangle = 0$

$(\mathbb{K}(0) = 1$ is canonical norm$)$

HEFT ($v \neq 0$)

Does HEFT know that $\langle H \rangle = v$?

AJM $\Rightarrow$ There might be special place on manifold $h_* = -v$ where $O(4)$ symmetry is manifest

Determined by $F(h_*) = 0$

If $h = h_*$ exists $\Rightarrow$ HEFT $\rightarrow$ SMEFT possible

HEFT $\longrightarrow$ SMEFT ?

Map:

$$|H|^2 = \frac{1}{2}\vec{\varphi}\cdot\vec{\varphi} = \frac{1}{2}(v+h)^2$$

$$|\partial H|^2 = \frac{1}{2}(\partial\vec{\varphi})^2 = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(v+h)^2(\partial\vec{n})^2$$

$$(\partial|H|^2)^2 = (\vec{\varphi}\cdot\partial\vec{\varphi})^2 = (v+h)^2(\partial h)^2$$

Naively:

$$\mathcal{L}_{HEFT} = \frac{v^2 F}{2|H|^2}|\partial H|^2 + \frac{1}{2}(\partial|H|^2)^2\frac{1}{2|H|^2}\left(K^2 - \frac{v^2F^2}{2|H|^2}\right)$$

$$+\tilde{V}(|H|^2) + \mathcal{O}(\partial^4)$$

Analytic @ $|H|=0$?

II. Light BSM and Analyticity

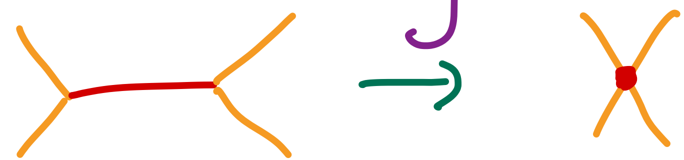
Light BSM and Analyticity

Given $\mathcal{L}_{UV}$ integrate out heavy State $\Rightarrow \mathcal{L}_{1LPI}$. Expand and truncate $\Rightarrow \mathcal{L}_{EFT}$

We will use functional methods

2 examples:

1) tree exchange



2) loop exchange

* see also arXiv: 2011.02484 by TC, Lu, Zhang

Tree Exchange

Extend SM with singlet scalar S

$$\mathcal{L}_{BSM} = \frac{1}{2} S\left(-\partial^2 - m^2 - \kappa|H|^2\right)S - a|H|^2 S + b S^3$$

Integrate out S using EOM (neglect ∂):

$$S[H] = \frac{1}{b}\left(-m^2 + \kappa|H|^2 + \sqrt{(m^2+\kappa|H|^2) - 2ab|H|^2}\right)$$

$$\Rightarrow V_{ILPI} = V_{SM} - \frac{1}{3b^2}\left(m^2+\kappa|H|^2\right)\left[\left(m^2+\kappa|H|^2\right)^2 - 3ab|H|^2\right]$$
$$+ \frac{1}{3b^2}\left[\left(m^2+\kappa|H|^2\right) - 2ab|H|^2\right]^{3/2}$$

Tree Exchange $\Rightarrow$ SMEFT

Expand

$$V_{ILPI} = V_{SM} - \frac{1}{3b^2}\left(m^2 + \lambda|H|^2\right)\left[\left(m^2 + \lambda|H|^2\right)^2 - 3ab|H|^2\right]$$
$$+ \frac{1}{3b^2}\left[\left(m^2 + \lambda|H|^2\right) - 2ab|H|^2\right]^{3/2}$$

in $|H|^2/m^2 \Rightarrow$ Local SMEFT expansion

Tree Exchange $\Rightarrow$ HEFT

<u>But</u> if $m^2 = 0$ ...

$$V_{ILPI} = V_{SM} + \frac{1}{3b^2}\left[3ab\lambda|H|^4 - \lambda^3|H|^6 + \left(-2ab|H|^2 + \lambda^2|H|^4\right)^{3/2}\right]$$

is non-analytic about $|H| = 0$

$\Rightarrow$ HEFT (will revisit)

Similar story for corrections to $|\partial H|^2$ + $(\partial |H|^2)^2$

Loop Exchange

Singlet extension w/ $a = b = 0$ (eg 2HDM)

$$\mathcal{L}_{BSM} = \frac{1}{2} S' \left(-\partial^2 - m^2 - \lambda |H|^2 \right) S'$$

Leading BSM correction at one loop

Use functional matching $\left(\begin{array}{c} \text{Coleman - Weinberg} \\ \text{for } V_{ILPI} \end{array} \right)$

$$m_S^2 = m^2 + \frac{\lambda}{2} v^2 \Rightarrow \int d^4x \, \mathcal{L}_{ILPI}^{1\text{-loop}} = i \log \det \left(\partial^2 + m^2 + \lambda |H|^2 \right)$$

Loop Exchange

$$\mathcal{L}_{1LPI} = |\partial H|^2 - \mu_h^2 |H|^2 + \frac{1}{2}\lambda_h |H|^4$$

$$+ \frac{1}{16\pi^2}\left(m^2 + \lambda |H|^2\right)^2 \left(\ln \frac{\mu_h^2}{m^2 + \lambda |H|^2} + \frac{3}{2}\right)$$

$$+ \frac{1}{16\pi^2}\,\frac{1}{6}\,\frac{\lambda^2}{m^2 + \lambda^2 |H|^2}\left(\partial |H|^2\right)^2$$

When $m^2 \neq 0 \Rightarrow$ expand to derive local SMEFT

but $m^2 = 0 \Rightarrow \log\left(\mu_h^2 / \lambda |H|^2\right)$ non-analytic!

$\Rightarrow$ need HEFT (will revisit)

Is Analyticity Enough?

Does requiring A, B, $\tilde{V}$ be analytic ensure SMEFT?

Field redefinitions do not change physics (must be analytic and include linear term)

Cannot remove non-analyticity w/ H redef

What about using redefs of h?

Field Redefinitions of h

Let

$$\mathcal{L} = \frac{1}{2}\left(1+\frac{h}{2v}\right)^2(\partial h)^2 + \frac{1}{2}(v+h)^2\left(\frac{3}{4}+\frac{h}{4v}\right)^2(\partial\vec{n})^2 - V$$

$$= \frac{1}{4}\left(1+\frac{\sqrt{2|H|^2}}{v}+\frac{|H|^2}{2v^2}\right)|\partial H|^2$$

$$+\frac{1}{4v^2}\left(\frac{v}{\sqrt{2|H|^2}}+\frac{3}{4}\right)\frac{1}{2}\left(\partial|H|^2\right)^2 - \tilde{V}$$

$$\boxed{\begin{array}{l} \text{w/ } V = V(h) \\ V'(0)=0 \\ V \text{ analytic} \end{array}}$$

Looks like no SMEFT expansion...

Field Redefinitions of h

But let $h_1 = h + \frac{1}{4v} h^2$ (no shift in min of V)

$\Rightarrow \partial_\mu h_1 = \left(1 + \frac{h}{2v}\right) \partial_\mu h$

and $(v_1 + h_1)^2 = (v+h)^2 \left(\frac{3}{4} + \frac{h}{4v}\right)$ $v_1 = \frac{3}{4} v$

$\Rightarrow \mathcal{L} = \frac{1}{2} (\partial_\mu h_1)^2 + \frac{1}{2} (v_1 + h_1)^2 (\partial \vec{n})^2 + V$

$= |\partial H_1|^2 + \tilde{V} \Rightarrow$ SMEFT!

Field Redefinitions of h

We learned that analytic field redefs of h can obscure analyticity in terms of H.

Field redefs within HEFT can obscure SMEFT

When should/must one use HEFT?

III. EFT Convergence

(Why HEFT?)

Practical Criterion

One should match onto HEFT when integrating out a state whose mass is near (or below) the electroweak scale

Check radius of convergence for EFT expansion

Practical Criterion

Radius of convergence:

$$\mathcal{L} \supset \sum_m \frac{c_m}{\Lambda^{2m-4}} |H|^{2m} \supset \lambda_{h^3} h^3$$

$$= \sum_m \frac{2^{1-m}}{3} m(m-1)(2m-1) c_m \left(\frac{v}{\Lambda}\right)^{2m-4} v\, h^3$$

$\Rightarrow$ if $\Lambda \sim v$ and $c_m \sim 1$

$\Rightarrow$ SMEFT does not converge

Applying the Practical Criterion

Revisit singlet loop example

$$V^{1\text{-loop}} = \frac{1}{16\pi^2}\left(m^2 + \lambda_{\alpha}|H|^2\right)^2\left(\ln\frac{\Lambda^2}{m^2+\lambda_{\alpha}|H|^2} + \frac{3}{2}\right)$$

SMEFT: expand in $X_{SMEFT} = \frac{\lambda_{\alpha}|H|^2}{m^2}$

HEFT: expand in $X_{HEFT} = \frac{\lambda_{\alpha}\left(|H|^2 - \frac{1}{2}v^2\right)}{m^2 + \frac{1}{2}\lambda_{\alpha} v^2}$

so that $m^2(1 + X_{SMEFT}) = m_{phys}^2(1 + X_{HEFT})$

Applying the Practical Criterion

$$V^{1\text{-loop}} = \frac{1}{16\pi^2}\left(m^2+\lambda|H|^2\right)^2\left(\ln\frac{\Lambda_n^2}{m^2+\lambda|H|^2}+\frac{3}{2}\right)$$

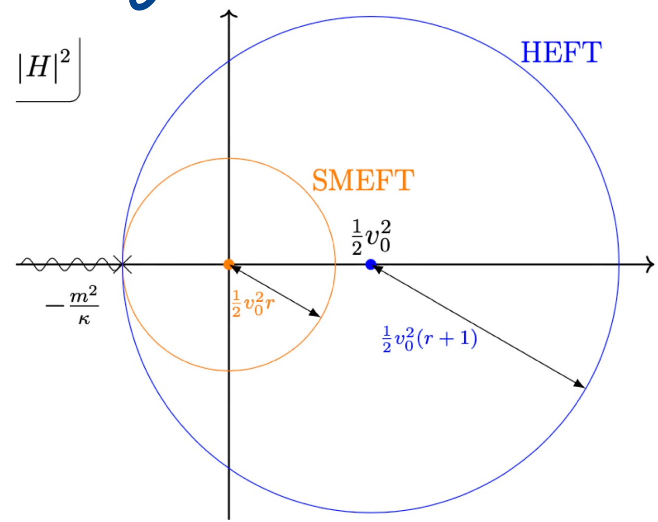
$$V_{SMEFT} \supset \dots + \sum_{k=3}^{k_{max}} \frac{2(-1)^k}{k(k-1)(k-2)} \mathcal{X}_{SMEFT}^k$$

$$V_{HEFT} \supset \dots + \sum_{k=3}^{k_{max}} \frac{2(-1)^k}{k(k-1)(k-2)} \mathcal{X}_{HEFT}^k$$

Radius of Convergence

Define $r = \dfrac{m^2}{\kappa v^2/2}$

s.t. $r \to \infty$ as $m^2 \to \infty$



Then

$$X_{\text{SMEFT}} \sim \frac{1}{r}$$

$$X_{\text{HEFT}} \sim \frac{1}{r+1}$$

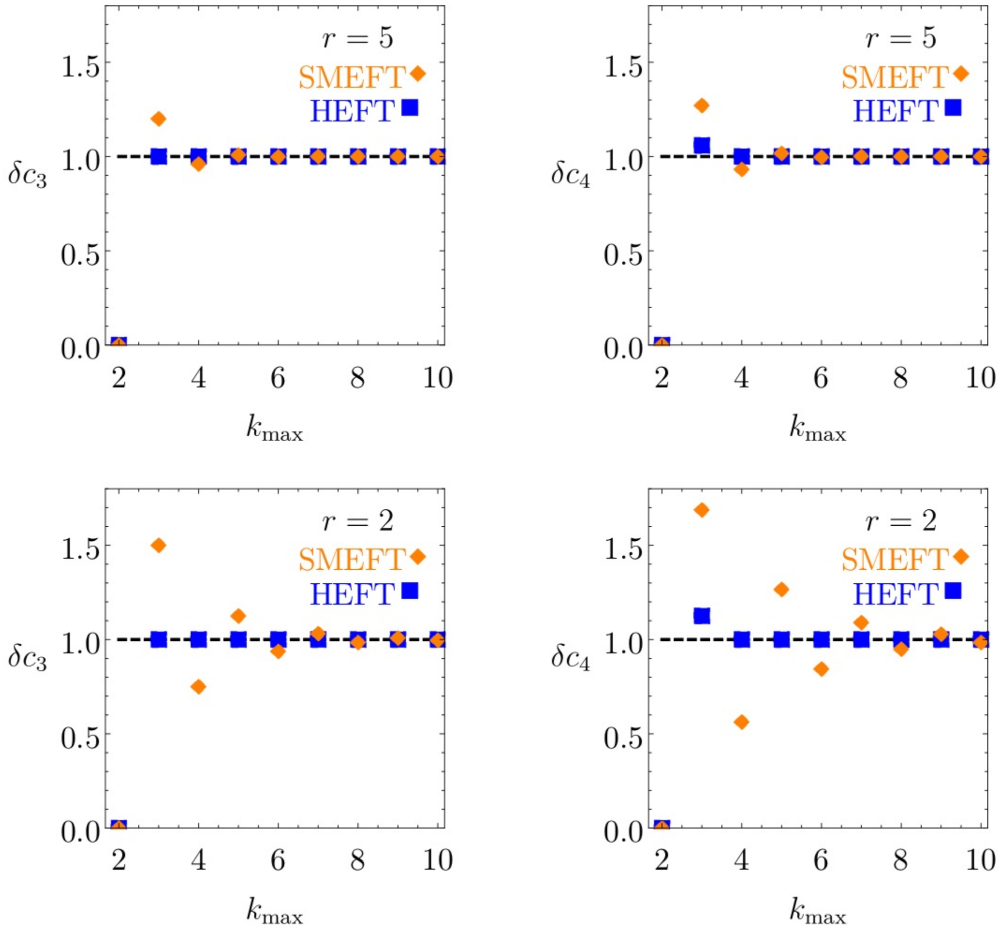
$r > 1$: both converge

$0 < r \leq 1$: SMEFT does not converge

$r = 0$: SMEFT does not exist

Rate of Convergence

Correction to h^3



Correction to h^4

IV. Curvature Criterion

(Why HEFT?)

Curvature Invariants

We saw examples where particle gets all of its mass from Higgs

$\Rightarrow$ SMEFT does not exist

But could be obscured by field redef

Want basis independent check

Goal classify UV theories that require HEFT

Curvature Invariants

(AJM)

Analog w/ GR: define metric on moduli space

Note $(\partial\vec{n})^2 = \left(\delta_{ij} + \frac{n_i n_j}{1-n^2}\right)(\partial^\mu n_i)(\partial_\mu n_j)$

$$\Rightarrow \quad \mathcal{L}_{HEFT} \supset \frac{1}{2}\left[\underline{K}(h)\right]^2(\partial h)^2 + \frac{1}{2}\left[v\,F(h)\right]^2(\partial\vec{n})^2$$

$\Rightarrow$ metric

$$g_{hh} = \underline{K}^2$$

$$g_{ij} = vF^2\left(\delta_{ij} + \frac{n_i n_j}{1-n^2}\right)$$

Curvature Invariants

metric

$$g_{hh} = \mathcal{K}^2$$

$$g_{ij} = v F^2 \left(\delta_{ij} + \frac{n_i n_j}{1-n^2} \right)$$

Ricci scalar

$$R = -\frac{2N_\varphi}{\mathcal{K}^2 F}\left[(\partial_h^2 F) - (\partial_h \mathcal{K})\left(\frac{1}{\mathcal{K}} \partial_h F \right) \right]$$

$$+ \frac{N_\varphi (N_\varphi - 1)}{v^2 F^2}\left[1 - \left(\frac{v}{\mathcal{K}} \partial_h F \right)^2 \right]$$

Curvature Criterion

See paper for proof

A HEFT can be expressed as SMEFT iff

1) $F(h=h_*)=0$: Candidate $O(4)$ invariant point

2) The metric is analytic @ h_*
   - F + $\overline{K}$ have convergent Taylor exp @ h_*
   - Curvature invariants $(\nabla^{2n})R$ are finite @ h_*

3) The potential is analytic @ h_*
   - V has convergent Taylor exp @ h_*
   - $(\nabla^{2n})V$ are finite @ h_*

HEFT is a Black Hole

Conjecture: Checking finiteness of R & V is sufficient.

Two classes of models need HEFT:

Conical singularity: BSM state gets all of its mass from H

Horizon: BSM sources of symmetry breaking

Conical Singularity

Ex: Singlet w/ $S|H|^2$ & S^3 $\Rightarrow$ tree level

$$\Rightarrow R(h=-v) = \frac{a^2}{m^4} N_\varphi (N_\varphi + 1)$$

finite w/ $m^2 \neq 0$ but diverges as $m^2 \to 0$

Ex: Singlet w/ $S^2|H|^2$ $\Rightarrow$ loop level

$$\Rightarrow R(h=-v) = \frac{1}{192\pi^2} \frac{\lambda}{3m^2} N_\varphi (N_\varphi + 1)$$

but $R|_{m^2=0} = \frac{N_\varphi (N_\varphi - 1)}{(v+h)^2} \frac{\lambda}{96\pi^2 + \lambda} \xrightarrow[h \to -v]{} \infty$

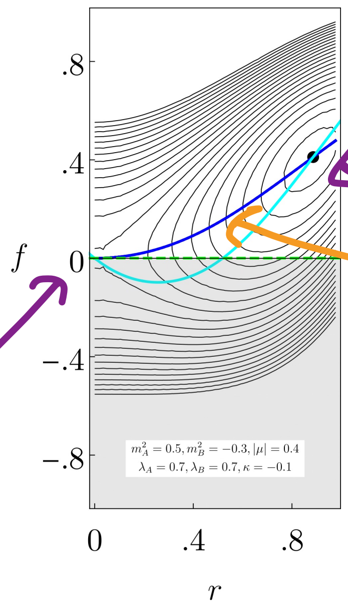
Horizon

We provide three examples in paper.

Rely on "EFT submanifold" picture

Ex: Abelian toy model w/ vevs f & r

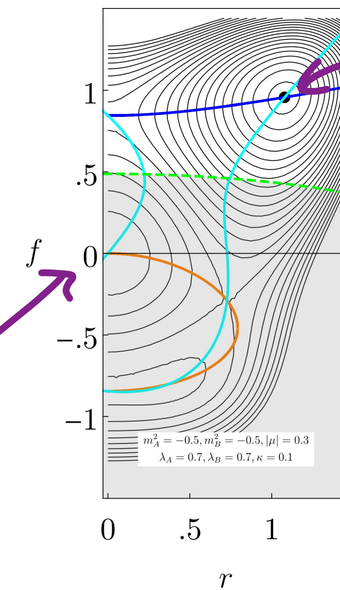
$m^2_{BSM} > 0$



physical vac

SMEFT trajectory

fixed point



physical vac

$m^2_{BSM} < 0$

fixed point

no SMEFT trajectory

⇒ HEFT

V. Unitarity

(Where HEFT?)

Unitarity

In forthcoming paper, we follow AJM to compute scattering amplitudes in terms of curvature invariants, generalizing unitarity arguments of Falkowski, Rattazzi.

- HEFT violates perturbative unitarity when $s \sim 4\pi v$

VI. Summary and Outlook

Summary

- EFT is useful for parametrizing BSM
- SMEFT: linear realized EW sym
  decoupling manifest
- HEFT: non-linear realized EW sym
  useful when new physics scale
  is near v
- HEFT required
  - BSM state gets all mass from H
  - BSM source of sym breaking
- HEFT violates unitarity @ $s \sim 4\pi v$

Outlook

- Paper on Unitarity is in progress

- Paper exploring pheno of "HEFTons" in progress (+ Ian Banta)

- Paper exploring submanifold picture for 2HDM in progress ("Unitary" vs "mass" basis)

- Extension to field redefs w/ ∂ is unknown