The deconvolution problem of deeply virtual Compton scattering

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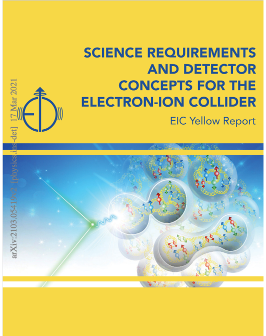
Exciting experimental promises



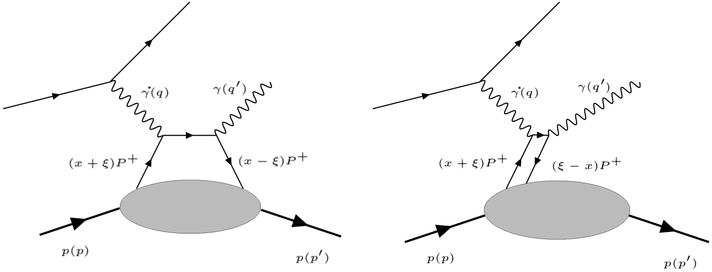
- With the **EIC yellow report** and **Chinese EIcC white paper**, deeply virtual Compton scattering (DVCS) will enter an era of more precise data over a much larger kinematic range.
- It is considered as a golden channel of extraction of generalised parton distributions (GPDs) and already provides many observables for fits. It is therefore necessary to re-examine the problem of unbiased extraction of GPDs from DVCS data.

Overview

Deeply virtual Compton scattering and the structure of hadrons

DVCS is the scattering of a lepton on a hadron via a photon of large virtuality, producing a real photon in the final state.

- x is the average light-front plus-momentum (longitudinal momentum in a fast moving hadron) fraction of the struck parton
- ξ describes the light-front plus-momentum transfer, linked to Björken's variable x_B
- $t = \Delta^2$ is the total four-momentum transfer squared



Tree-level depiction of DVCS for $x > |\xi|$ (left) and $\xi > |x|$ (right)

Deeply virtual Compton scattering and the structure of hadrons

Similarly to the introduction of **parton distribution functions** (PDFs) in the study of DIS,

- For a large photon virtuality $Q^2 = -q^2$, finite x_B and small total four-momentum transfer squared t, **factorisation theorems** describe DVCS in terms of a hard scattering part computable thanks to perturbative QCD, and a soft non-perturbative part described by **generalised parton distributions** (GPDs).
- The amplitude of DVCS is parametrised by **Compton form factors** (CFFs) $\mathcal{F}$, which write as convolutions of perturbative **coefficient functions** T_F^a and the **GPDs** F^a:

CFF convolution (leading twist)

$$\mathcal{F}(\xi, t, Q^2) = \sum_{\text{parton type } a} \int_{-1}^{1} \frac{\mathrm{d}x}{\xi}\, T_F^a\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) F^a(x, \xi, t, \mu^2) \tag{1}$$

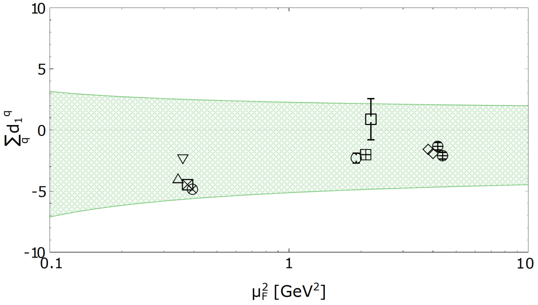
$F^a(x, \xi, t, \mu^2) \to F^g(x, \xi, t, \mu^2)/x$ for the usual definition of gluon GPD

μ is the factorisation / renormalisation scale, α_s the strong coupling.

Deeply virtual Compton scattering and the structure of hadrons

see talk by P. Sznajder on Thursday, 8:18 EDT (Spin physics session)

GPDs allow access to gravitational form factors of the **energy-momentum tensor** (EMT) $\rightarrow$ first data-driven extractions of **mechanical properties** of hadronic matter (*e.g.* pressure distribution) [Burkert, *et al.*, 2018], [Kumericki, 2019], [Dutrieux, *et al.*, 2021]



In green, 68% confidence interval found for $\sum_q d_1^q(t = 0, \mu^2)$, a critical parameter to evaluate pressure profiles and results obtained by other studies (black markers). The parameter is compatible with 0 with current experimental data. [Dutrieux, *et al.*, 2021]

Properties of generalised parton distributions

- Several types of GPDs: H, E, $\widetilde{H}$, $\widetilde{E}$, ... depending on helicity considerations.
- A GPD is a function of (x, ξ, t, μ^2), ξ-even, with physical region $(x, \xi) \in [-1, 1]^2$ and the dependence on μ^2 is given by renormalisation group equations.
- The **forward limit** gives back the PDF:

$$H^q(x, \xi = 0, t = 0, \mu^2) = f_q(x, \mu^2) \tag{2}$$

- **Polynomiality property**: due to Lorentz covariance,

$$\int_{-1}^{1} \mathrm{d}x \, x^n H^q(x, \xi, t, \mu^2) = \sum_{k=0 \text{ even}}^{n+1} H^q_{n,k}(t, \mu^2)\xi^k \tag{3}$$

This property implies that the GPD is the Radon transform of a **double distribution** F (DD) with an added *D*-term on the support $\Omega = \{(\beta, \alpha) \mid |\beta| + |\alpha| < 1\}$:

Double distribution formalism

$$H^q(x, \xi, t, \mu^2) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, \delta(x - \beta - \alpha\xi) \left[F(\beta, \alpha, t, \mu^2) + \xi\delta(\beta)D(\alpha, t, \mu^2)\right] \tag{4}$$

Deconvoluting a Compton form factor

Position of the problem

Assuming a CFF has been extracted from experimental data with excellent precision*, we are left with the convolution:

$$\int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^q \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) H^q(x, \xi, t, \mu^2) = T^q(Q^2, \mu^2) \otimes H^q(\mu^2) \tag{5}$$

where T^q is a coefficient function computed in pQCD. **Can we then "de-convolute" eq. (5) to recover $H^q(x, \xi, t, \mu^2)$ from $T^q(Q^2, \mu^2) \otimes H^q(\mu^2)$?**

Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in [Freund, 1999], but the question remains essentially open.
- We show that GPDs exist which bring contributions to the LO and NLO CFF of only subleading order even under evolution. We call them **LO and NLO shadow GPDs**.

Definition of a LO shadow GPD

For a given scale μ_0^2,

$$\forall \xi, \forall t, \; T^q_{LO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \tag{6}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2, \; T^q_{LO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s(\mu^2)) \tag{7}$$

- Let H^q be a LO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same LO CFF up to a numerically small and theoretically subleading contribution.

Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in [Freund, 1999], but the question remains essentially open.
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Definition of an NLO shadow GPD

For a given scale μ_0^2,

$$\forall \xi, \forall t,\ T^q_{NLO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \tag{6}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2,\ T^q_{NLO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s{}^2(\mu^2)) \tag{7}$$

- Let H^q be an NLO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same NLO CFF up to a numerically small and theoretically subleading contribution.

Shadow GPDs at leading order

- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\mathrm{Im}\ T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We also omit t since it is untouched by the convolution.
- **Leading order** It is well-known that the LO CFF only probes the GPD on the $x = \xi$ line and the D-term, so a LO shadow GPD is simply given by:

$$\mathrm{Im}\ T^q_{LO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) \propto H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \quad (8)$$

$$H^q(x, \xi = 0, \mu_0^2) = 0 \quad (9)$$

where $H^{q(+)}$ denotes the singlet GPD (x-odd part of the GPD).

Shadow GPDs at leading order

- We search our DD as a polynomial of order N in (β, α), characterised by $\sim N^2$ coefficients c_{mn}:

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn}\, \alpha^m \beta^n \tag{10}$$

- The associated GPD is obtained by the linear Radon transform, given by the matrix R for $x > |\xi|$ *(not diverging for $|\xi| \to 1$ thanks to the cancellation of poles when $x \to 1$)*:

$$H^{q(+)}(x, \xi, \mu_0^2) = \sum_{u=1}^{N+1} \frac{1}{(1+\xi)^u} + \frac{1}{(1-\xi)^u} \sum_{v=0}^{N+1} q_{uv}\, x^v \quad \text{where} \quad q_{uv} = \sum_{m,n} R_{uv}^{mn} c_{mn} \tag{11}$$

$$R_{uv}^{mn} = \sum_{j=0}^{n} \frac{(-1)^{u+v+j}}{m+j+1} \binom{n}{j} \binom{j}{m-u+j+1} \binom{m+j+1}{v-n+j} \tag{12}$$

Shadow GPDs at leading order

- For our LO shadow GPD, we first want $H^{q(+)}(\xi,\xi,\mu_0^2)=0$, so we notice that

$$H^{q(+)}(\xi,\xi,\mu_0^2)=\sum_{w=1}^{N+1}\frac{k_w}{(1+\xi)^w}\quad\text{where}\quad k_w=\sum_{u,v}C_w^{uv}\,q_{uv}\,,\quad C_w^{uv}=(-1)^{u+v+w}\binom{v}{u-w}$$

Cancelling the LO CFF

$$H^{q(+)}(\xi,\xi,\mu_0^2)=0\implies(c_{mn})_{m,n}\in\ker(C.R)\tag{13}$$

Shadow GPDs at leading order

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- We then want $H^{q(+)}(x,\xi=0,\mu_0^2)=0$, so we notice that

$$H^{q(+)}(x,0,\mu_0^2)=\sum_{w=0}^{N+1} q_w x^w \quad \text{where} \quad q_w=\sum_{u,v} Q_w^{uv}\, q_{uv}\,, \quad Q_w^{uv}=2\delta_w^v$$

Cancelling the forward limit

$$H^{q(+)}(x,\xi=0,\mu_0^2)=0 \implies (c_{mn})_{m,n}\in \ker(Q.R) \tag{14}$$

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{13}$$

Cancelling the forward limit

$$H^{q(+)}(x, \xi = 0, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{14}$$

- Both linear systems $C.R$ and $Q.R$ are systems of $\sim N$ equations for $\sim N^2$ variables, so the number of solutions grows quadratically with N, order of the polynomial DD.

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi,\xi,\mu_0^2)=0 \implies (c_{mn})_{m,n}\in\ker(C.R) \tag{13}$$

Cancelling the forward limit

$$H^{q(+)}(x,\xi=0,\mu_0^2)=0 \implies (c_{mn})_{m,n}\in\ker(Q.R) \tag{14}$$

LO shadow GPDs

Here is an example of an infinite family of LO shadow DDs, each being of degree $N\geq 9$ odd

$$F_N(\beta,\alpha,\mu_0^2)=\beta^{N-8}\Big[\alpha^8-\frac{28}{9}\alpha^6\left(\frac{N^2-3N+20}{(N+1)N}+\beta^2\right)+\frac{10}{3}\alpha^4\left(\frac{N^2-7N+40}{(N+1)N}+\frac{2(N^2-3N+44)}{3(N+1)N}\beta^2+\beta^4\right)$$
$$-\frac{4}{3}\alpha^2\left(\frac{N^2-11N+60}{(N+1)N}-\frac{N-8}{N}\beta^2-\frac{N^2-3N-28}{(N+1)N}\beta^4+\beta^6\right)+\frac{1}{9}(1-\beta^2)^2\left(\frac{N^2-15N+80}{(N+1)N}-\frac{2(N-8)}{N}\beta^2+\beta^4\right)\Big] \tag{15}$$

Shadow GPDs at next-to-leading order

- **First sudy beyond leading order:** The NLO CFF is composed of a collinear part (compensating LO evolution applied to the tree-level LO CFF) and a genuine 1-loop NLO part. An explicit calculation of each term for our polynomial double distribution gives that

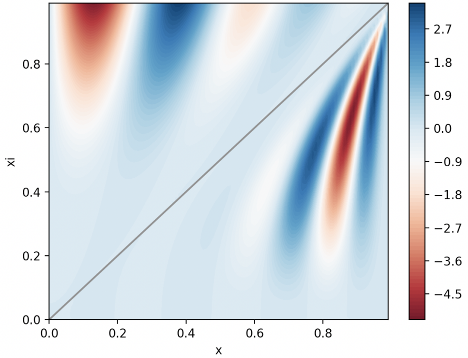
$$\mathrm{Im}\ T^q_{coll}(Q^2,\mu^2)\otimes H^q(\mu^2) \propto$$
$$\alpha_s(\mu^2)\log\left(\frac{\mu^2}{Q^2}\right)\left[\left(\frac{3}{2}+\log\left(\frac{1-\xi}{2\xi}\right)\right)\mathrm{Im}\ T^q_{LO}\otimes H^q(\mu^2)+\sum_{w=1}^{N+1}\frac{k_w^{(coll)}}{(1+\xi)^w}\right] \tag{16}$$

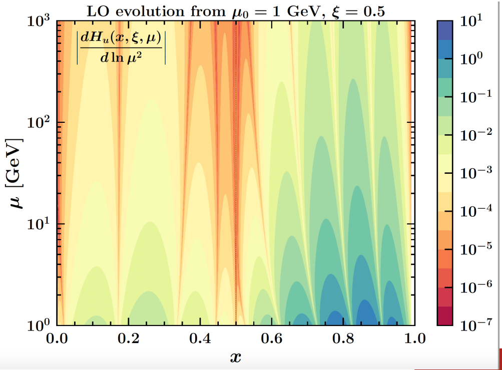
and assuming $\mathrm{Im}\ T^q_{LO}\otimes H^q(\mu^2)=0$,

$$\mathrm{Im}\ T^q_1(Q^2,\mu^2)\otimes H^q(\mu^2)\propto\alpha_s(\mu^2)\left[\log\left(\frac{1-\xi}{2\xi}\right)\mathrm{Im}\ T^q_{coll}\otimes H^q(\mu^2)+\sum_{w=1}^{N-1}\frac{k_w^{(1)}}{(1+\xi)^w}\right] \tag{17}$$

Shadow GPDs at next-to-leading order

- Cancelling both terms gives rise to two additional systems with a linear number of equations. The first NLO shadow GPD is found for $N = 21$, and adding the condition that the DD vanishes at the edges of its support gives a first solution for $N = 25$ (see below).

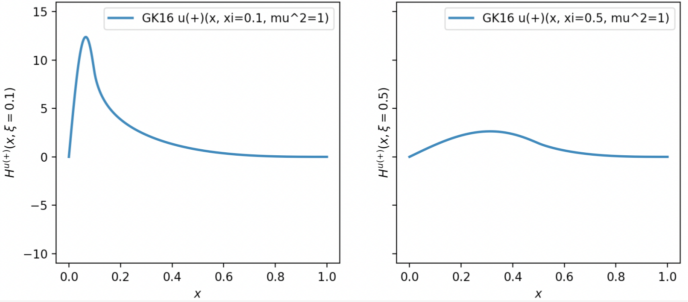




Color plot of an NLO shadow GPD at initial scale 1 GeV^2, and its evolution for $\xi = 0.5$ up to 10^6 GeV^2 via APFEL++ and PARTONS [Bertone].

Shadow GPDs at next-to-leading order

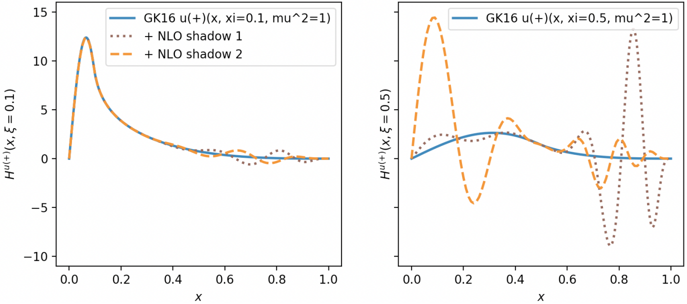
- **On a lever-arm in Q^2 of $[1, 100]$ GeV^2** (typical collider kinematics), the NLO CFF generated by the NLO shadow GPD varies as $\mathcal{O}(\alpha_s^2(Q^2))$ and **is** 10^5 **times smaller than the typical size of the shadow GPD** at a typical $\xi = 0.1$.
- Consider this Goloskov-Kroll GPD model (via PARTONS) at scale 1 GeV^2



$\xi = 0.1$ (left) and $\xi = 0.5$ (right)

Shadow GPDs at next-to-leading order

- The orange and brown models are **GK + NLO shadow GPDs**. For ξ close to 0 and x close to ξ, by design, they are very close, but vastly different otherwise. They give rise to NLO CFFs which are exactly identical at this scale, and different by a negligible amount for expected Q^2 lever arm.



$\xi = 0.1$ (left) and $\xi = 0.5$ (right)

Conclusion

- Explicit demonstration of LO and NLO shadow GPDs of considerable size with a very small and subleading contribution to CFFs. **Such shadow GPDs will be hidden in typical statistical and systematic uncertainties of DVCS**. TCS or LO DVMP face similar issues. We foresee that our discussion can be extended to higher order DVCS. Other exclusive processes will help discriminate the DVCS shadow GPDs. Especially DDVCS or Lattice QCD for instance should escape the dimensionality of data problem.
- Potential impact on **hadron tomography** due to the $\xi \to 0$ extrapolation, determination of **OAM** and mechanical properties to study.
- An extraction of GPDs with lesser systematic uncertainty requires a **multi-channel analysis**, and the development of integrated analysis tools, like **PARTONS**
- More precise data over a much larger Q^2 range promised by future colliders will be very welcomed here and for the extraction of mechanical properties as well.
- More theoretical constraints, like **positivity** could play a significant role in reducing the uncertainty.