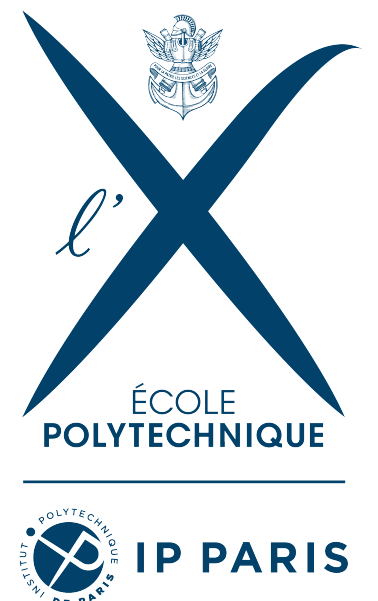


Quarkonium production and gluon TMDs at the EIC

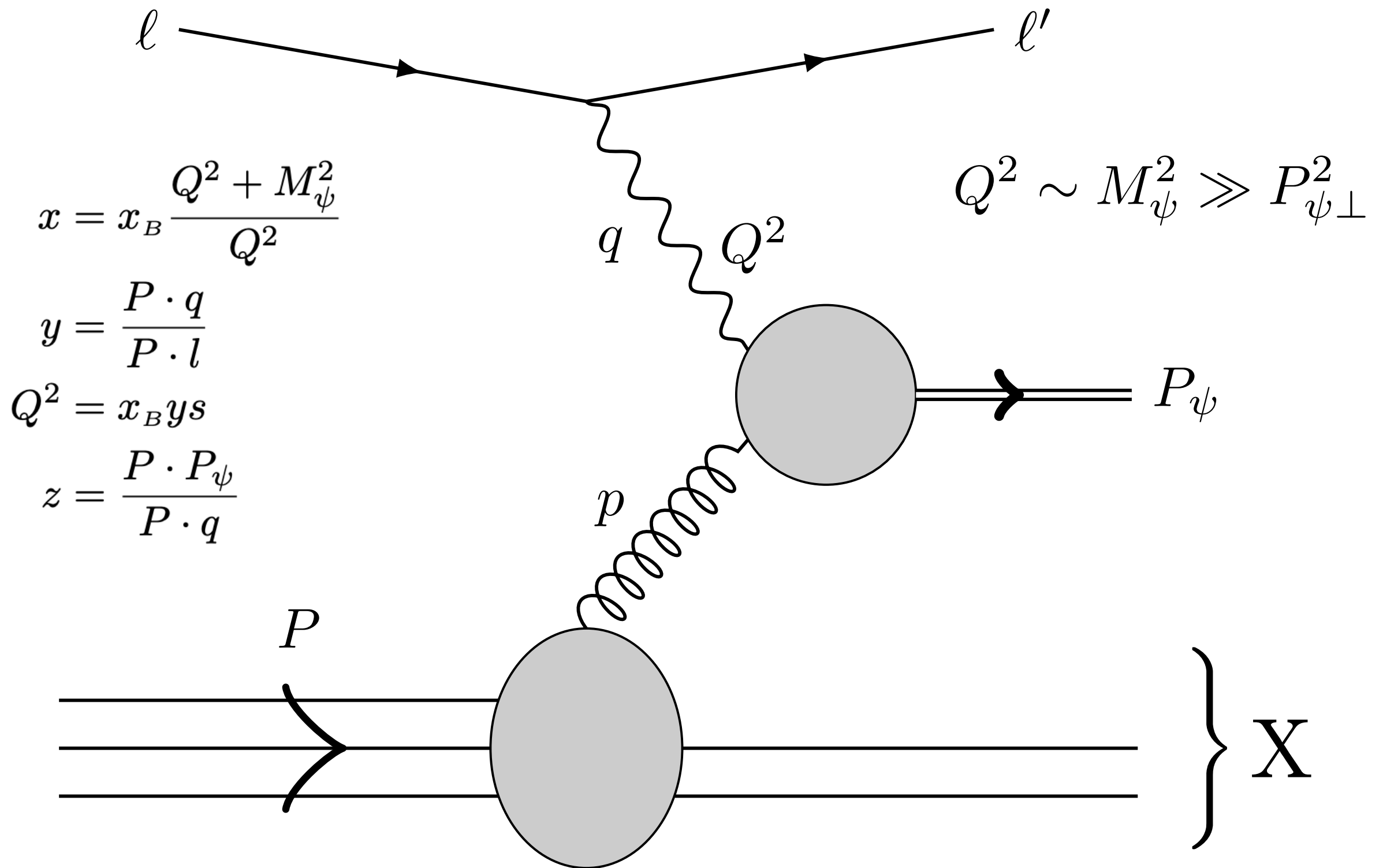


Pieter Tael
CPHT, École polytechnique, France



Semi-inclusive J/ψ production

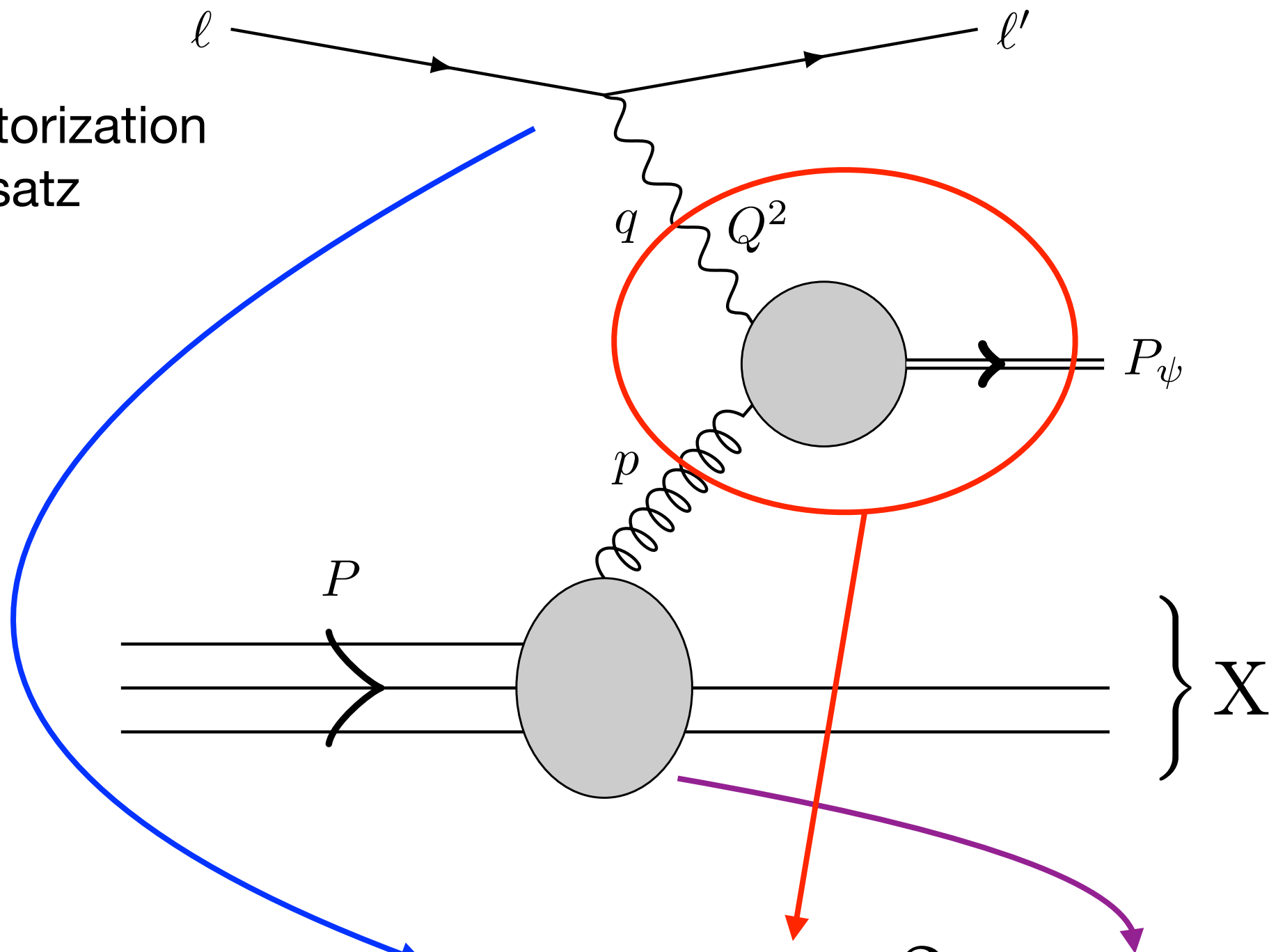
$$\ell + p \rightarrow \ell + J/\psi + X$$



Rajesh, Kishore, Mukherjee (2018)
 Bacchetta, Boer, Pisano, PT (2018)

$$\ell + p \rightarrow \ell + J/\psi + X$$

TMD factorization
Ansatz



$$d\sigma = L \otimes |M|^2 \otimes \Gamma$$

Lepton tensor

Hard part

Hadron tensor

Hadron tensor

$$\Gamma_U^{\mu\nu}(x, \mathbf{q}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu} f_1^g(x, \mathbf{q}_T) + \left(\frac{q_T^\mu q_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\mathbf{q}_T^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{q}_T) \right\}$$

unpolarized gluons

linearly polarized gluons

Sivers function

circularly polarized gluons

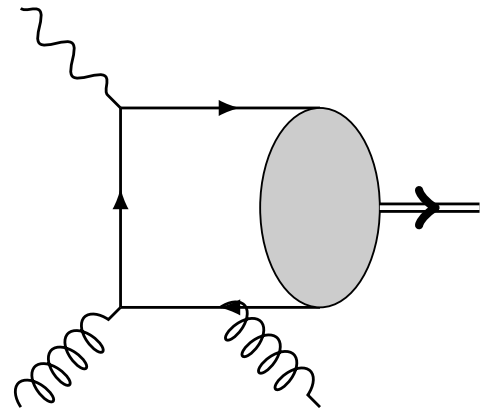
$$\Gamma_T^{\mu\nu}(x, \mathbf{q}_T) = \frac{x}{2} \left\{ g_T^{\mu\nu} \frac{\epsilon_T^{\rho\sigma} q_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x, \mathbf{q}_T) + i\epsilon_T^{\mu\nu} \frac{q_T \cdot S_T}{M_h} g_{1T}^g(x, \mathbf{q}_T) \right. \\ \left. + \frac{q_{T\rho} \epsilon_T^{\rho\{\mu} q_T^{\nu\}}}{2M_p^2} \frac{q_T \cdot S_T}{M_p} h_{1T}^{\perp g}(x, \mathbf{q}_T) - \frac{q_{T\rho} \epsilon_T^{\rho\{\mu} S_T^{\nu\}} + S_{T\rho} \epsilon_T^{\rho\{\mu} q_T^{\nu\}}}{4M_p} h_{1T}^g(x, \mathbf{q}_T) \right\}$$

linearly polarized gluons

Mulders & Rodrigues (2001); Meissner, Metz & Goeke (2007)

Hard part: non-relativistic QCD (NRQCD)

CS mechanism: colorless bound state in quantum numbers of J/ψ

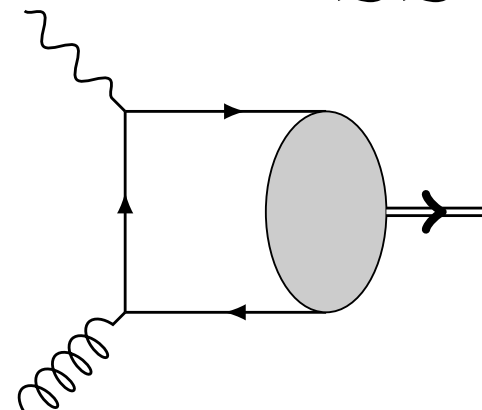


$$\langle \mathcal{O}_1^{J/\psi} (^3S_1) \rangle \sim v^0$$

$$v_c^2 \approx 0.3$$

$$v_b^2 \approx 0.1$$

CO mechanism: $Q\bar{Q}$ pair in some colored excited state, hadronizes later



$$\langle \mathcal{O}_8^{J/\psi} (^1S_0) \rangle \sim v^3$$

$$\langle \mathcal{O}_8^{J/\psi} (^3P_J) \rangle \sim v^4$$

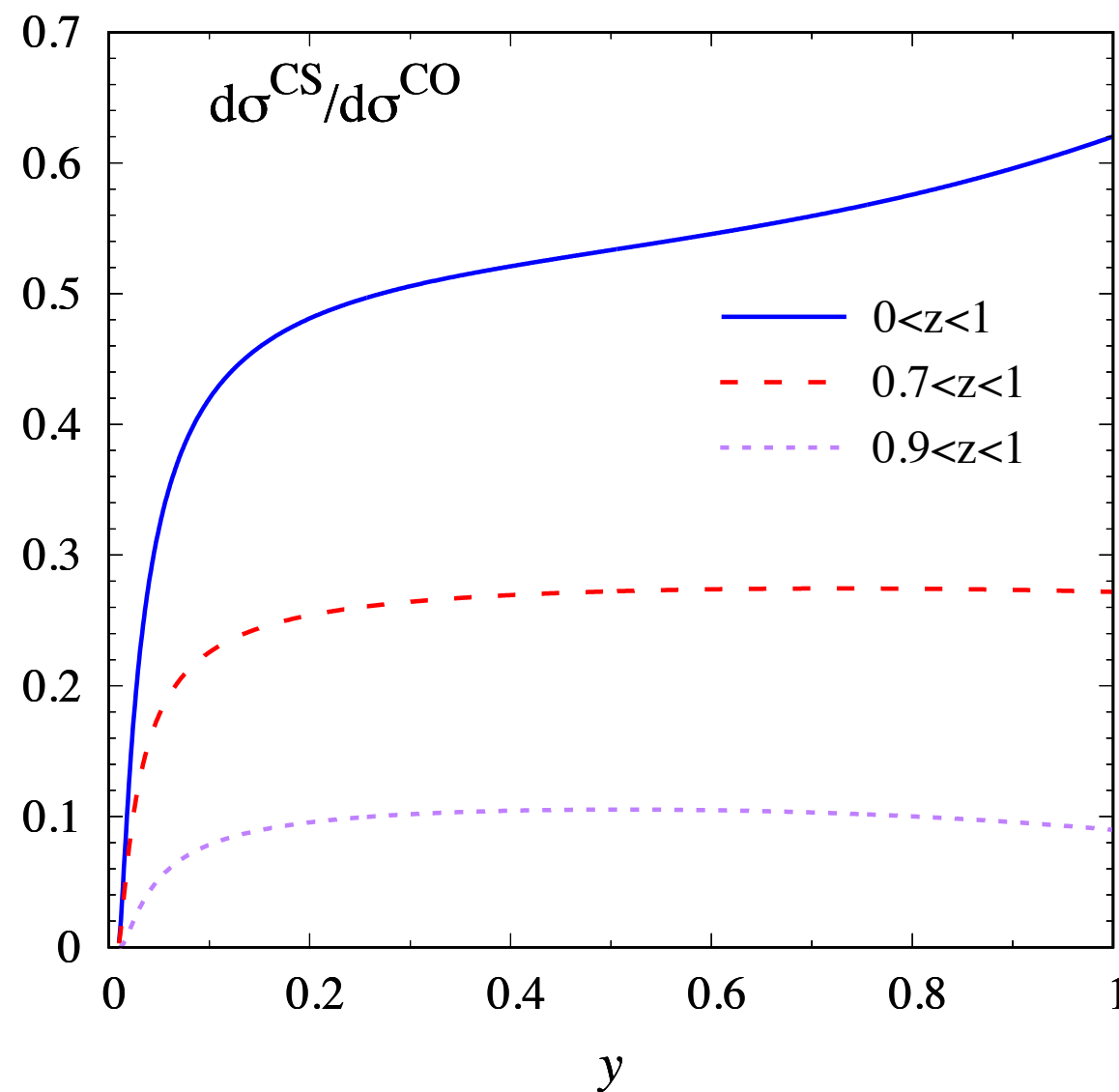
Indeed, available extractions: $\frac{\langle \mathcal{O}_1^{J/\psi} (^3S_1) \rangle}{\langle \mathcal{O}_8^{J/\psi} (^3S_0) \rangle} \sim O(10^{1-2})$

CO vs CS in DIS

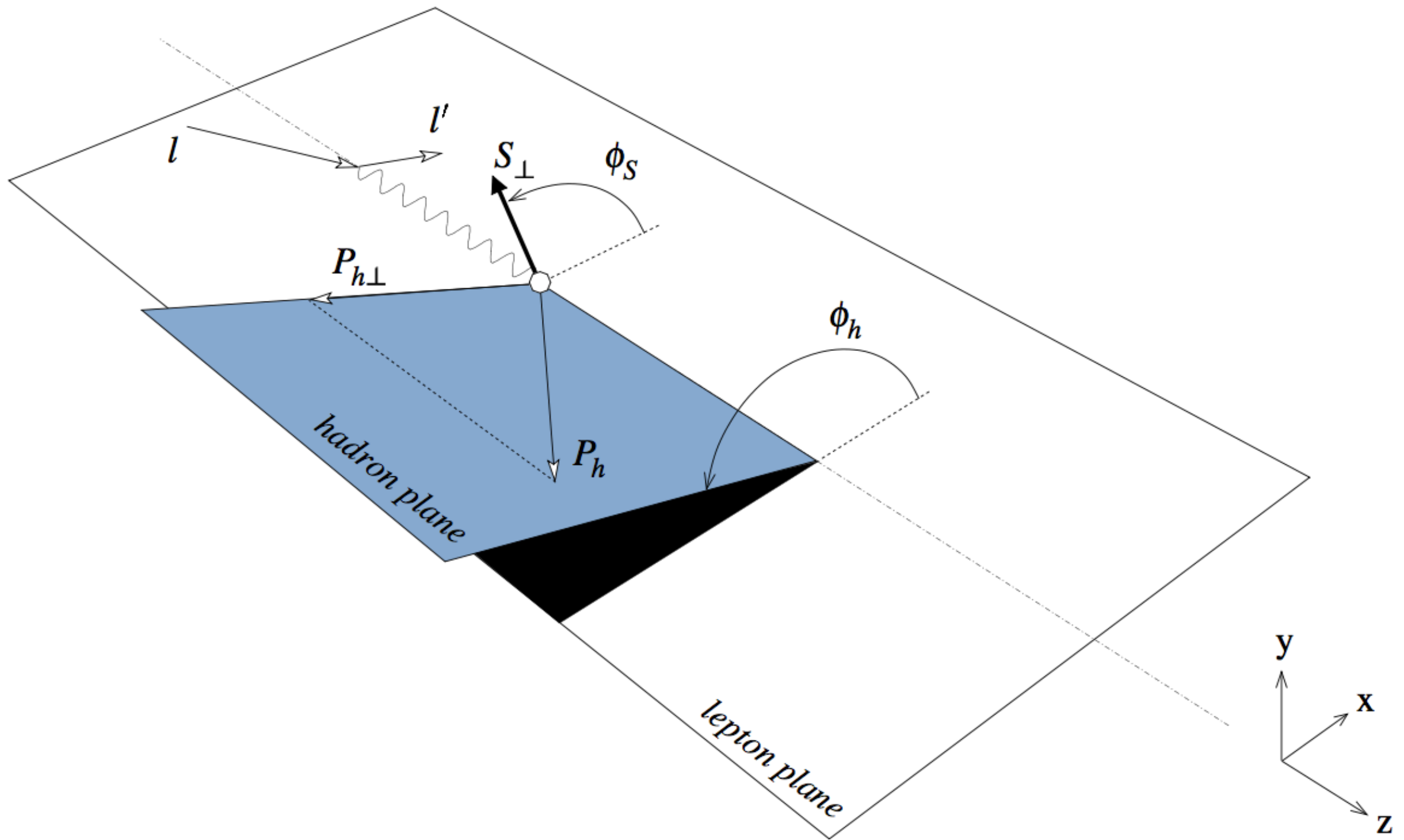
In DIS for large z and $Q^2 \geq M_\psi^2$, CO mechanism is dominant

Parametrically: $\frac{\alpha_s}{\pi v^3} \sim \frac{1}{2}$, numerically much stronger suppression

$$Q^2 = 10 \text{ GeV}^2$$
$$P_{\psi\perp} = 1 \text{ GeV}$$



Reference frame



Bacchetta, D'Alesio, Diehl & Miller (2004)

Cross section

$$\frac{d\sigma}{dy dx_B d^2\mathbf{q}_T} \equiv d\sigma^U(\phi_T) + d\sigma^T(\phi_T, \phi_S)$$

$$d\sigma^U = \mathcal{N} \left[A^U f_1^g(x, \mathbf{q}_T^2) + \frac{\mathbf{q}_T^2}{M_p^2} B^U h_1^{\perp g}(x, \mathbf{q}_T^2) \cos 2\phi_T \right]$$

unpolarized

linearly polarized

T even

Sivers function

$$d\sigma^T = \mathcal{N} |\mathbf{S}_T| \frac{|\mathbf{q}_T|}{M_p} \left\{ A^T f_{1T}^{\perp g}(x, \mathbf{q}_T^2) \sin(\phi_S - \phi_T) \right. \\ \left. + B^T \left[h_1^g(x, \mathbf{q}_T^2) \sin(\phi_S + \phi_T) - \frac{\mathbf{q}_T^2}{2M_p^2} h_{1T}^{\perp g}(x, \mathbf{q}_T^2) \sin(\phi_S - 3\phi_T) \right] \right\}$$

T odd → only CO!

linearly polarized

Azimuthal asymmetries

probe *ratios* of gluon TMDs

$$A^{W(\phi_S, \phi_T)} \equiv 2 \frac{\int d\phi_S d\phi_T W(\phi_S, \phi_T) d\sigma(\phi_S, \phi_T)}{\int d\phi_S d\phi_T d\sigma(\phi_S, \phi_T)}$$

...we have:

$$\int d\phi_S d\phi_T d\sigma(\phi_S, \phi_T) = (2\pi)^2 \mathcal{N} A^U f_1^g(x, \mathbf{q}_T^2)$$

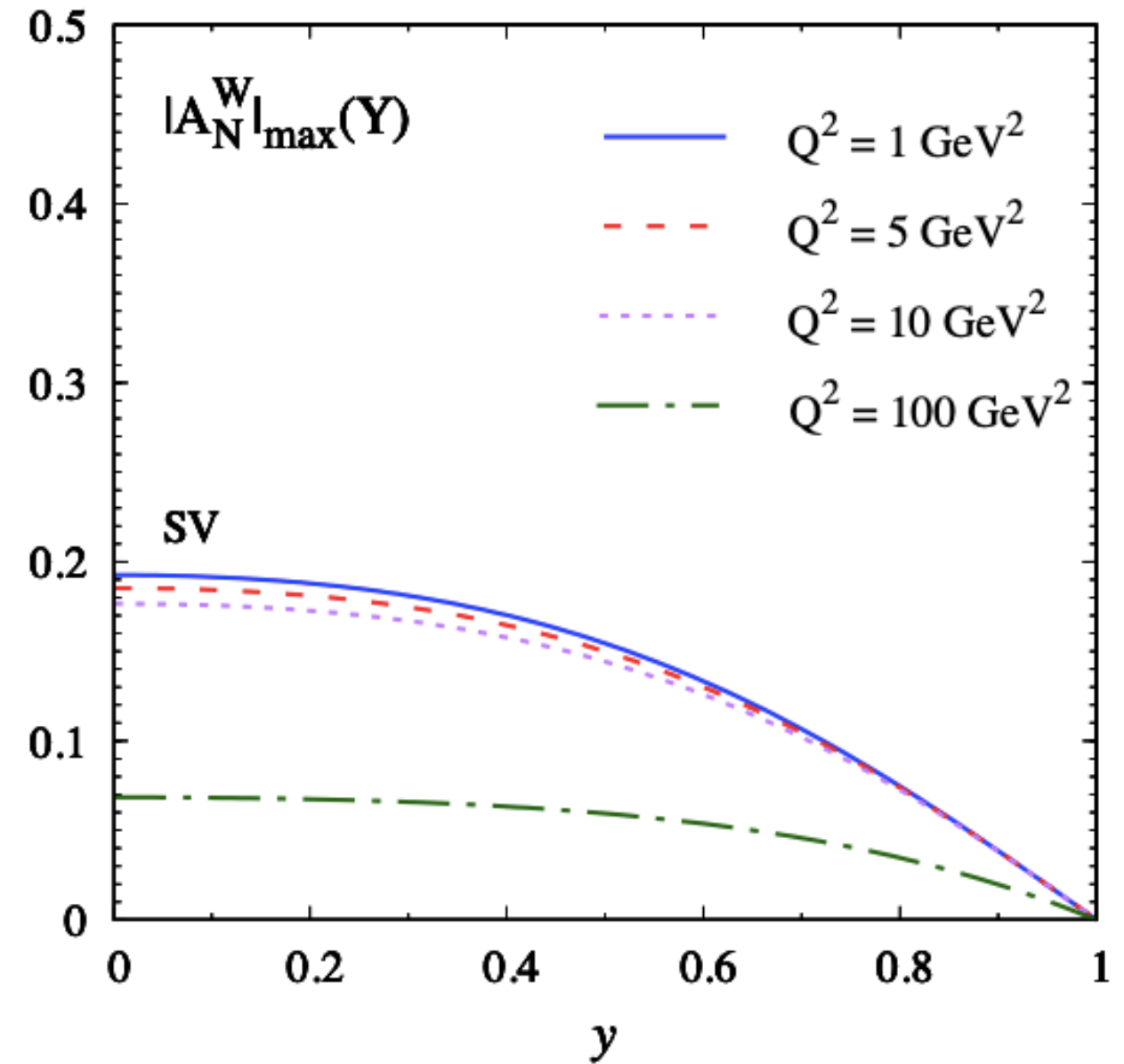
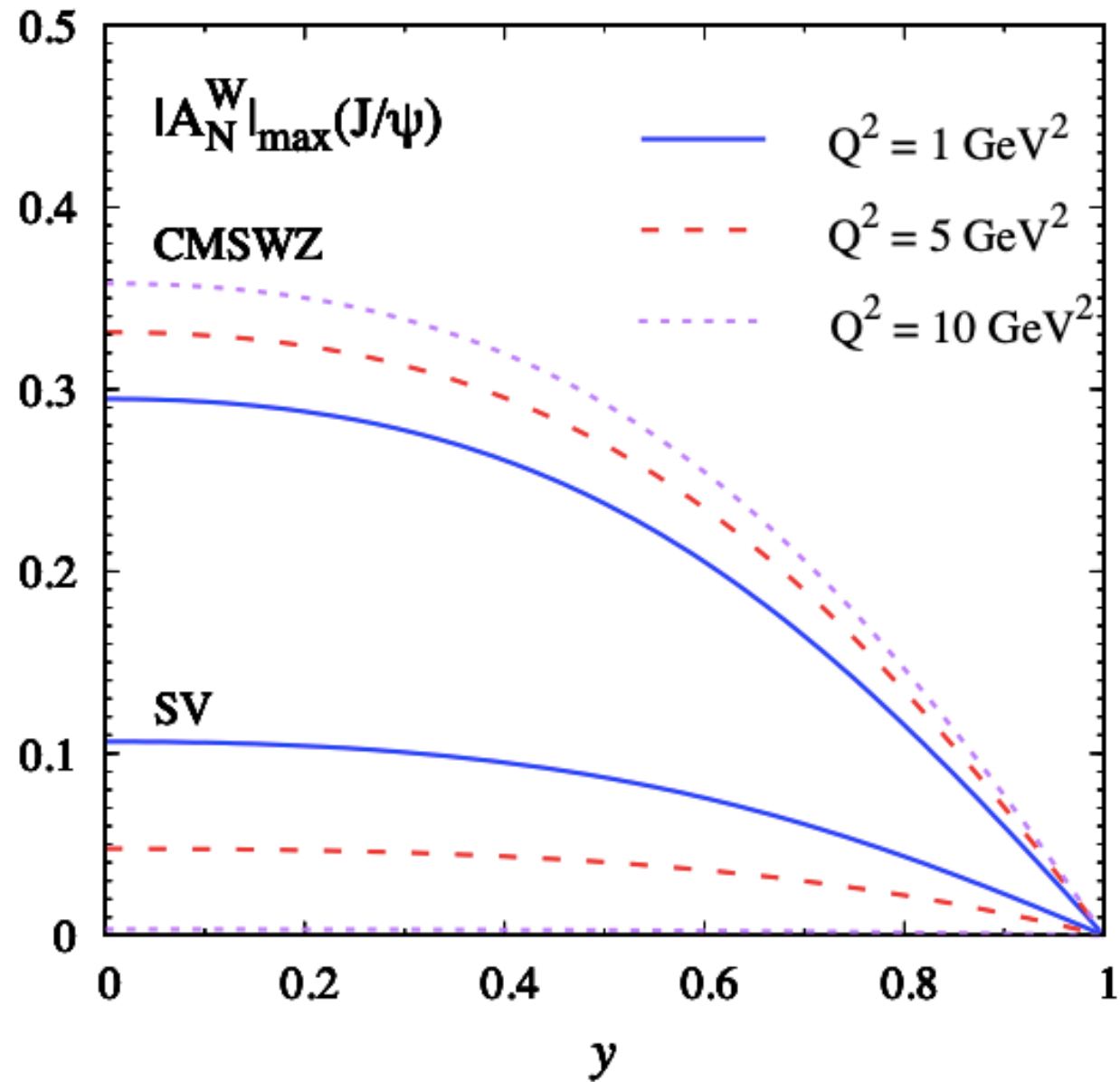
$$A^{\cos 2\phi_T} = H(y, M_\psi, Q) \frac{\mathbf{q}_T^2}{M_p^2} \frac{h_1^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)} \leq H(y, M_\psi, Q)$$

$$A^{\sin(\phi_S - \phi_T)} = \frac{|\mathbf{q}_T|}{M_p} \frac{f_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)} \leq 1$$

$$A^{\sin(\phi_S + \phi_T)} = H(y, M_\psi, Q) \frac{|\mathbf{q}_T|}{M_p} \frac{h_1^g(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)} \leq H(y, M_\psi, Q)$$

$$|A^{\sin(\phi_S - 3\phi_T)}| = H(y, M_\psi, Q) \frac{|\mathbf{q}_T|^3}{M_p^3} \frac{h_{1T}^{\perp g}(x, \mathbf{q}_T^2)}{f_1^g(x, \mathbf{q}_T^2)} \leq H(y, M_\psi, Q)$$

Positivity bounds



Huge dependence on LDME sets, all extracted at large k_T

SV = Sharma & Vitev (2013)

CMSWZ = Chao, Ma, Shao, Wang & Zhang (2012)

TMD LDMEs

$$\ell + p \rightarrow \ell + J/\psi + X:$$

matching of collinear and TMD calculation

Notion of LDMEs needs to be extended to small transverse momenta (cfr. fragmentation functions).

Collinear factorization: $\mu \sim P_{\psi\perp} \gg M_p$

TMD factorization: $\mu \gg P_{\psi\perp} \gtrsim M_p$

Overlapping region: $\mu \gg P_{\psi\perp} \gg M_p$

Strategy: *assume* TMD factorization $\rightarrow$ implies matching
 $\rightarrow$ which conditions will follow?

matching described in:

Collins, Soper & Sterman (1985, 1989)

Bacchetta, Boer, Diehl & Mulders (2008)

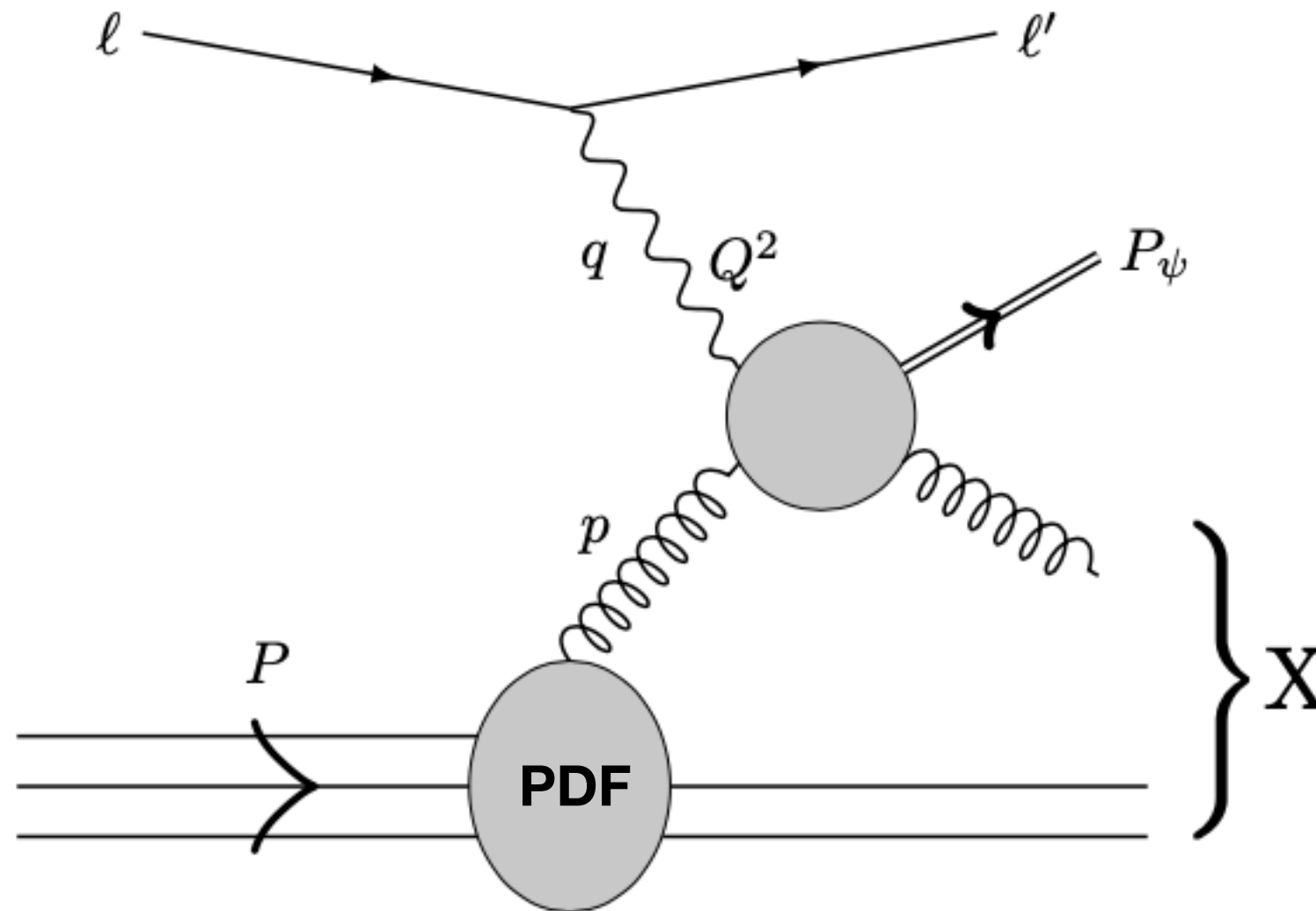
Collins (2011)

Bacchetta, Bozzi, Echevarria, Pisano, Prokudin & Radici (2019)

$\ell + p \rightarrow \ell + J/\psi + X$:
 matching of **collinear** and TMD calculation

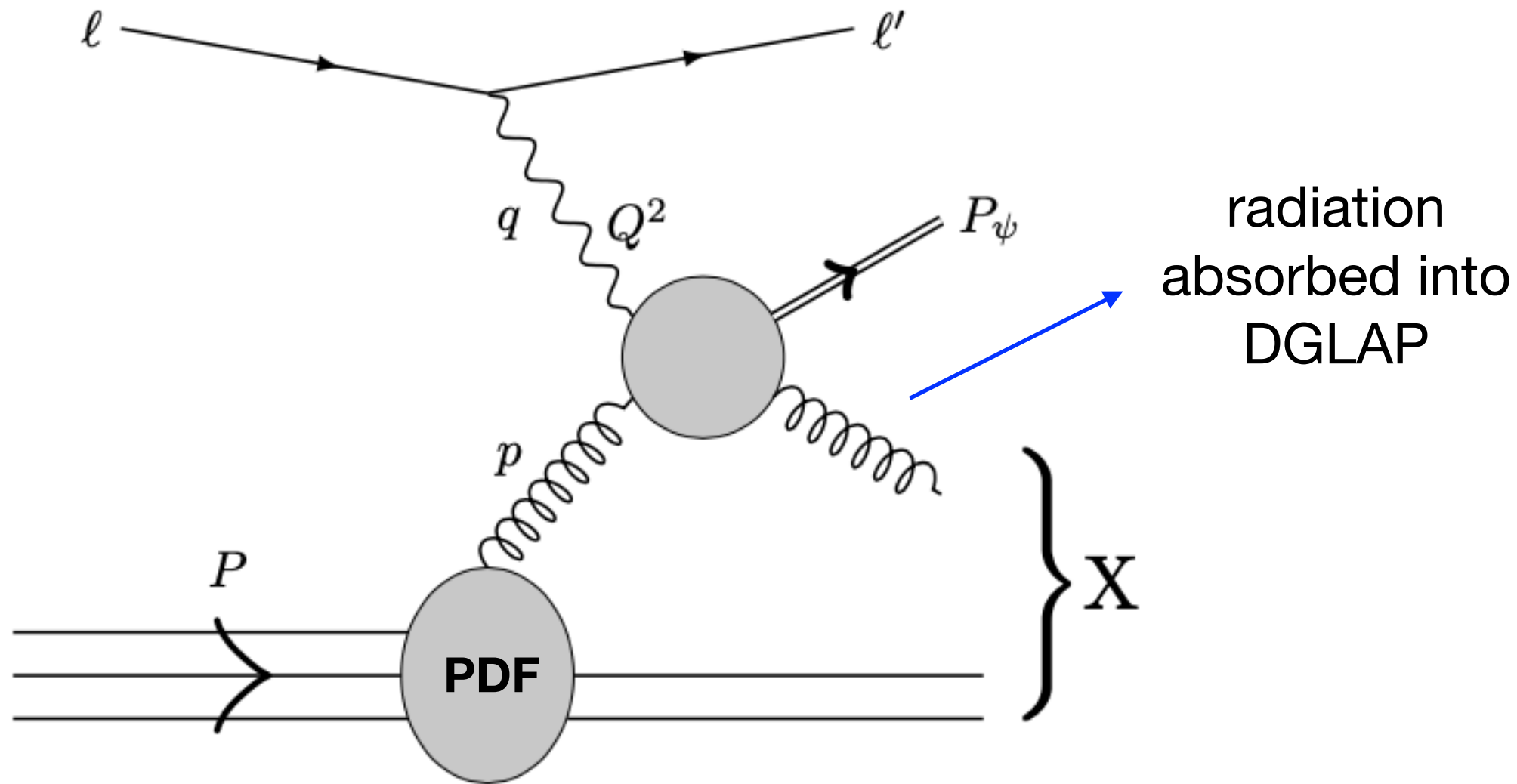
In the collinear regime: $\mu \sim P_{\psi\perp} \gg M_p$ with $\mu = \sqrt{Q^2 + M_\psi^2}$

$P_{\psi\perp}$ is generated by recoil off hard parton



$\ell + p \rightarrow \ell + J/\psi + X$:
 matching of **collinear** and TMD calculation

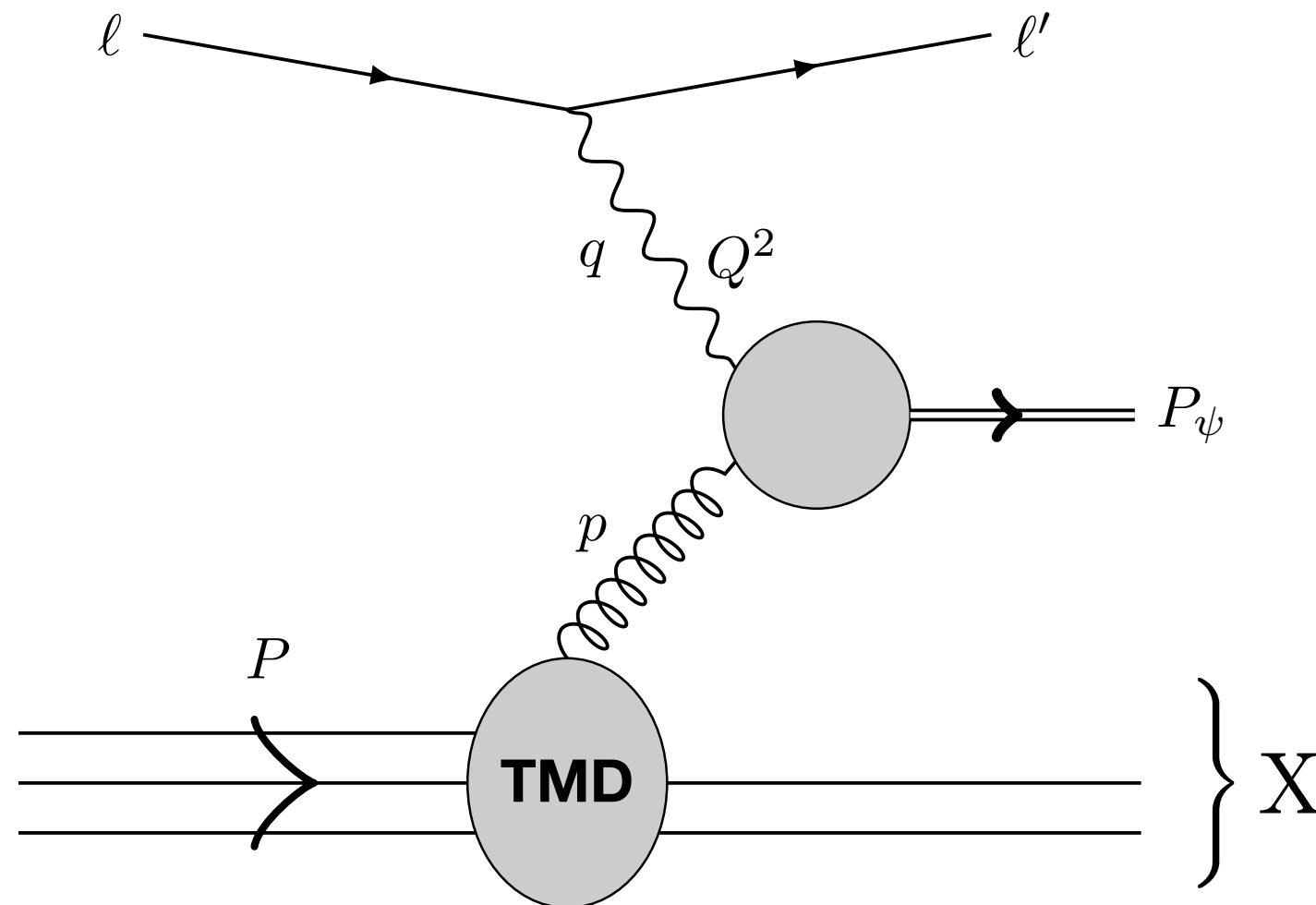
$\mu \gg P_{\psi\perp}$ limit:



$\ell + p \rightarrow \ell + J/\psi + X$:
 matching of collinear and **TMD** calculation

In the TMD regime: $\mu \gg P_{\psi\perp} \gtrsim M_p$

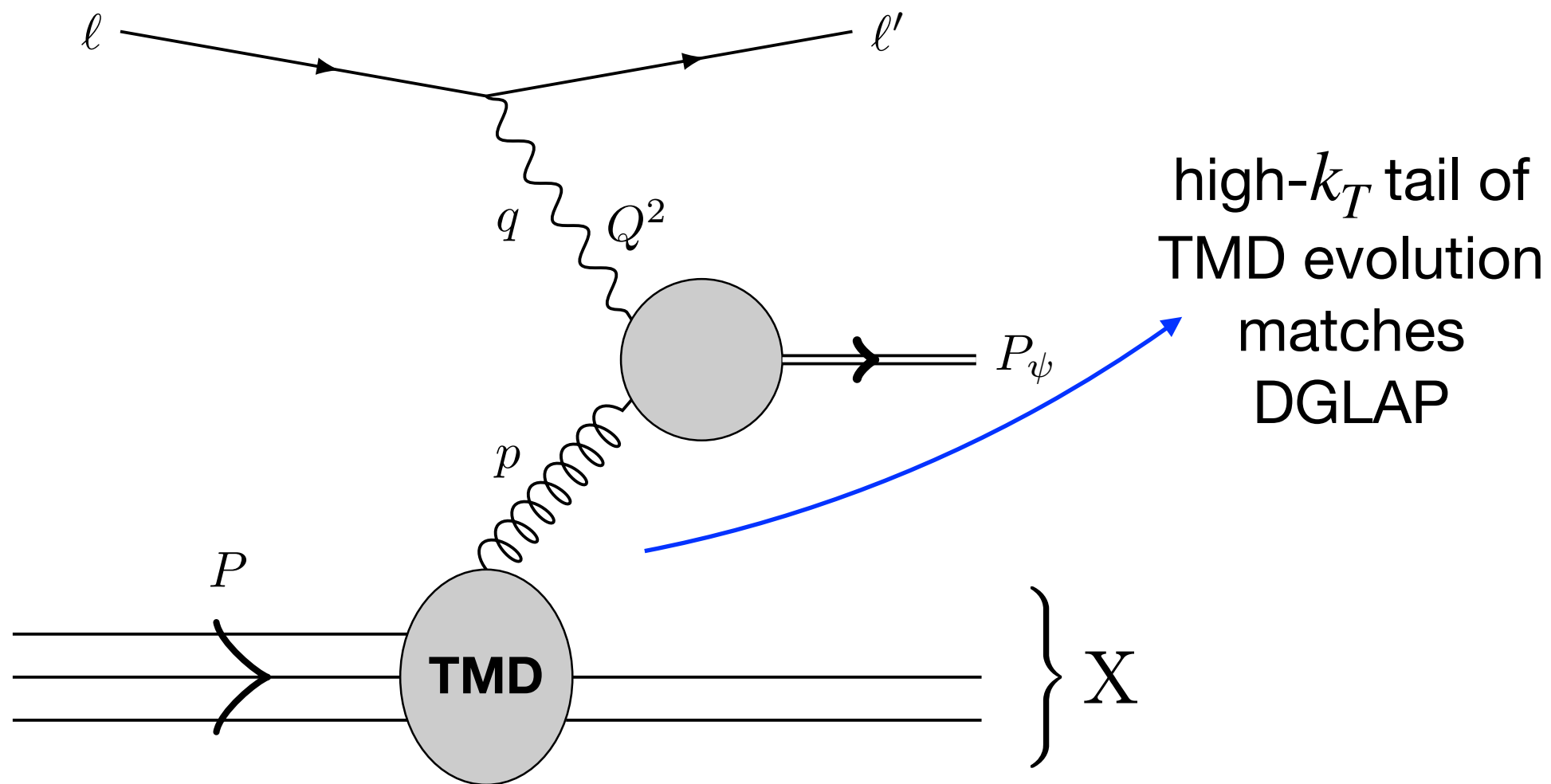
$P_{\psi\perp}$ stems from intrinsic transverse momentum in target, or from soft emissions



Boer, D'Alesio, Murgia, Pisano, PT (2020)

$\ell + p \rightarrow \ell + J/\psi + X$:
 matching of collinear and **TMD** calculation

$P_{\psi\perp} \gg M_p$ limit:



$\ell + p \rightarrow \ell + J/\psi + X$: matching of collinear and TMD calculation

Collinear calculation at small q_T :

$$\frac{d\sigma}{dy dx_B dz dq_T^2 d\phi_\psi} = \frac{\alpha^2}{y} \left[\underbrace{\frac{1+(1-y)^2}{Q^2} F_{UU,T}}_{\text{transverse } \gamma^*} + 4(1-y) \underbrace{F_{UU,L}}_{\text{longitudinal } \gamma^*} + (1-y) \cos 2\phi_\psi \underbrace{F_{UU}^{\cos 2\phi_\psi}}_{\text{lin. pol. } \gamma^*} \right]$$

TMD calculation at high q_T :

$$\frac{d\sigma}{dy dx_B dz d\mathbf{q}_T^2 d\phi_\psi} = \frac{\alpha^2}{y} \left[\frac{1+(1-y)^2}{Q^2} \mathcal{F}_{UU,T} + 4(1-y) \mathcal{F}_{UU,L} + (1-y) \cos 2\phi_\psi \mathcal{F}_{UU}^{\cos 2\phi_\psi} \right] \delta(1-z)$$

In kinematic region of overlap:

$$\begin{aligned} \mathcal{F}_{UU,T} &= F_{UU,T} - \sigma_{UU,T} C_A \ln \left(\frac{Q^2 + M_\psi^2}{q_T^2} \right) \\ \mathcal{F}_{UU,L} &= F_{UU,L} - \sigma_{UU,L} C_A \ln \left(\frac{Q^2 + M_\psi^2}{q_T^2} \right) \\ \mathcal{F}_{UU}^{\cos 2\phi_\psi} &= F_{UU}^{\cos 2\phi_\psi} \end{aligned}$$

$\ell + p \rightarrow \ell + J/\psi + X$:
matching of collinear and TMD calculation

Therefore, assuming both TMD factorization and enforcing the matching:

$$\mathcal{F}_{UU,T} = \sum_n \mathcal{H}_{UU,T}^{[n]} \mathcal{C}[f_1^g \Delta^{[n]}](x, \mathbf{q}_T^2)$$

$$\mathcal{F}_{UU,L} = \sum_n \mathcal{H}_{UU,L}^{[n]} \mathcal{C}[f_1^g \Delta^{[n]}](x, \mathbf{q}_T^2)$$

$$\mathcal{F}_{UU}^{\cos 2\phi_\psi} = \sum_n \mathcal{H}_{UU,}^{[n], \cos 2\phi_\psi} \mathcal{C}[w h_1^\perp g \Delta^{[n]}](x, \mathbf{q}_T^2)$$

we have to introduce the following ‘smearing’ function:

$$\Delta^{[n]}(\mathbf{k}_T^2, \mu^2) = \frac{\alpha_s}{2\pi^2 \mathbf{k}_T^2} C_A \langle 0 | \mathcal{O}_c(n) | 0 \rangle \ln \frac{\mu^2}{\mathbf{k}_T^2}$$

Same k_T -dependence for $^1S_0^{[8]}$ and $^3P_J^{[8]}$ states

Boer, D’Alesio, Murgia, Pisano, PT (2020)

Echevarria (2019); Fleming, Makris, Mehen (2019)

Idea(s) for LDME CO extraction

Comparison with heavy-quark pair production

Unpolarized-beam cross sections have $f_1^g(x, \mathbf{q}_T)$ and $h_1^{\perp g}(x, \mathbf{q}_T)$

-dependent part

$$D^{\mathcal{Q}_U} \equiv \int d\phi_T \frac{d\sigma}{dy dx_B d^2\mathbf{q}_T}$$

$$D^{Q\bar{Q}} \equiv \int d\phi_T d\phi_{\perp} \frac{d\sigma^{Q\bar{Q}}}{dz dy dx_B d^2\mathbf{K}_{\perp} d^2\mathbf{q}_T}$$

$$N^{\mathcal{Q}_U} \equiv \int d\phi_T \cos 2\phi_T \frac{d\sigma}{dy dx_B d^2\mathbf{q}_T}$$

$$N^{Q\bar{Q}} \equiv \int d\phi_T d\phi_{\perp} \cos 2\phi_T \frac{d\sigma^{Q\bar{Q}}}{dz dy dx_B d^2\mathbf{K}_{\perp} d^2\mathbf{q}_T}$$

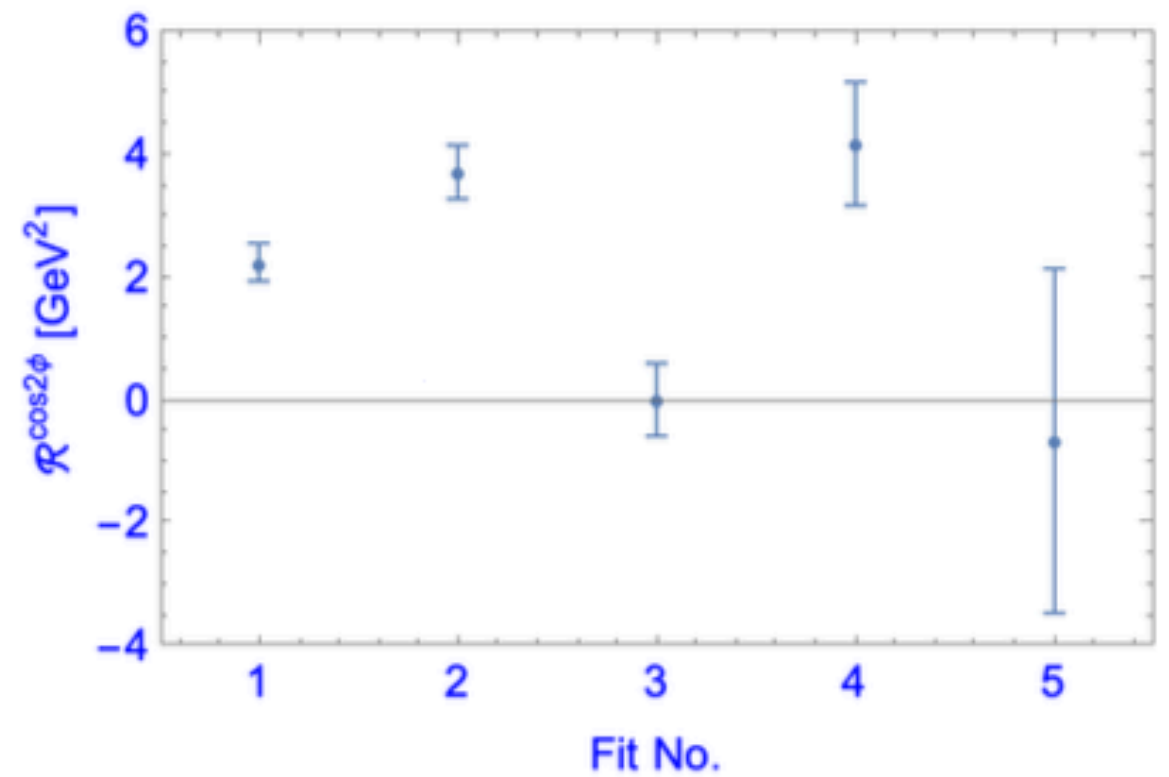
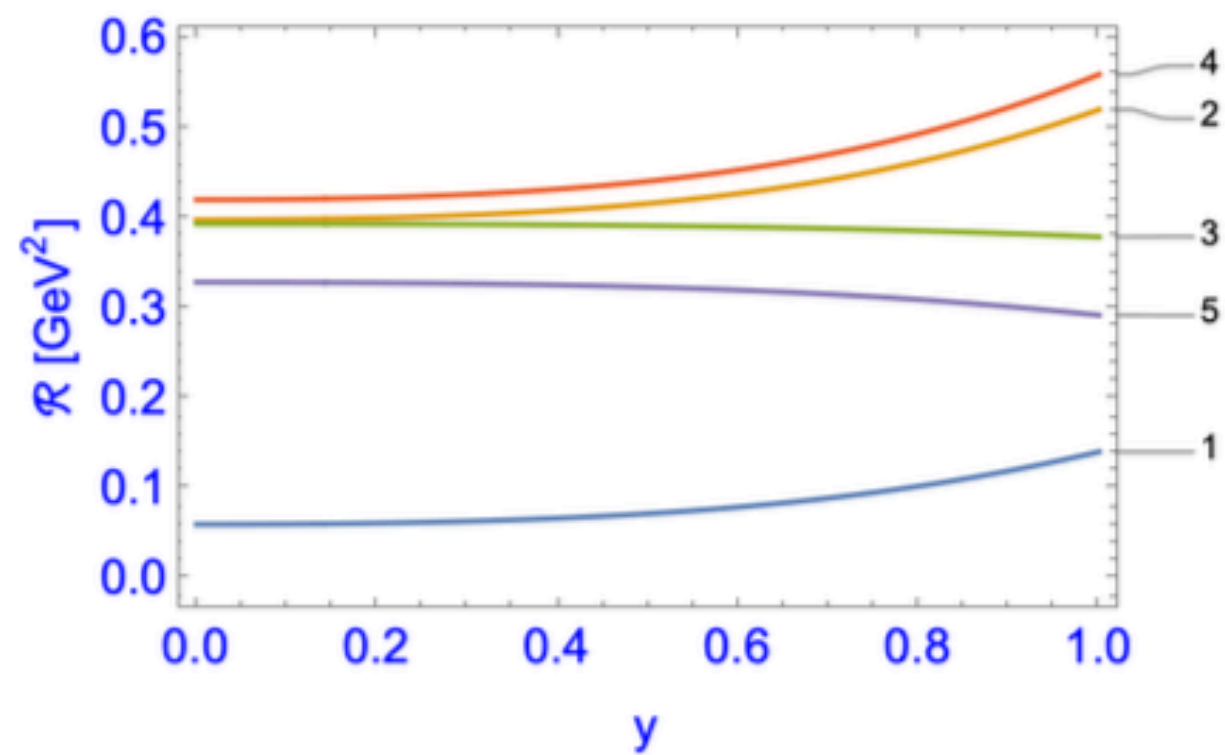
TMDs disappear from ratios of cross sections!

$$\mathcal{R} \equiv \frac{D^{\mathcal{Q}_U}}{D^{Q\bar{Q}}} = \frac{27\pi^2}{4} \frac{1}{M_Q} \frac{[1 + (1 - y)^2] \mathcal{O}_8^S + (10 - 10y + 3y^2) \mathcal{O}_8^P / M_Q^2}{26 - 26y + 9y^2}$$

$$\mathcal{R}^{\cos 2\phi_T} \equiv \frac{N^{\mathcal{Q}_U}}{N^{Q\bar{Q}}} = \frac{27\pi^2}{4} \frac{1}{M_Q} \left[\mathcal{O}_8^S - \frac{1}{M_Q^2} \mathcal{O}_8^P \right].$$

Bacchetta, Boer, Pisano, PT (2018)
Boer, Pisano, PT (2021)

Comparison with heavy-quark pair production



Bacchetta, Boer, Pisano, PT (2018)
Boer, Pisano, PT (2021)

Conclusions & outlook

Leptoproduction of J/ψ (+jet) at EIC very promising way to access gluon TMDs

Need for dedicated fit of TMD LDMEs
= reminiscent of light hadron FF

Perturbative part of TMD LDMEs can be calculated by matching

Comparison of open charm with quarkonium production can help to discriminate between fits

Thanks for your attention !