# Prompt hadroproduction of C-even quarkonia in the light-front $k_T$-factorization approach

Wolfgang Schäfer [1]

[1] Institute of Nuclear Physics, PAN, Kraków

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# Outline

**Introduction**

$\gamma^*\gamma^* \to \eta_c(1S, 2S), \chi_{c0}$ **transition form factors: light front description**

$k_T$**-factorization formulation: off-shell matrix element**

$\eta_c$**: Results for LHCb kinematics - "forward production"**

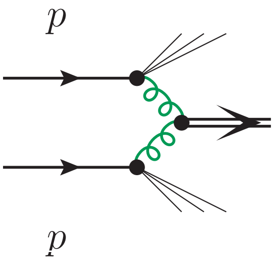
$\eta_c$**: Results for ATLAS/CMS kinematics - "central production"**

$\chi_{c0}$**: Results for transition form factors and hadroproduction**

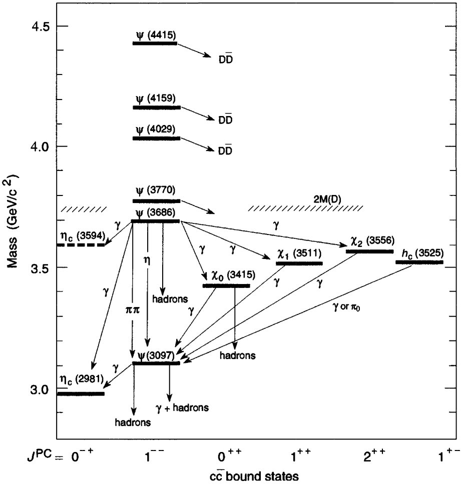
**Conclusions**


- I. Babiarz, R. Pasechnik, W. S. and A. Szczurek, "Hadroproduction of scalar $P$-wave quarkonia in the light-front $k_T$-factorization approach," JHEP **06** (2020), 101
- I. Babiarz, R. Pasechnik, W. S. and A. Szczurek, "Prompt hadroproduction of $\eta_c(1S, 2S)$ in the $k_T$-factorization approach," JHEP **2002**, 037 (2020)
- I. Babiarz, V. P. Goncalves, R. Pasechnik, W. S. and A. Szczurek, "$\gamma^*\gamma^* \to \eta_c(1S, 2S)$ transition form factors for spacelike photons," Phys. Rev. D **100**, no. 5, 054018 (2019).

# Introduction



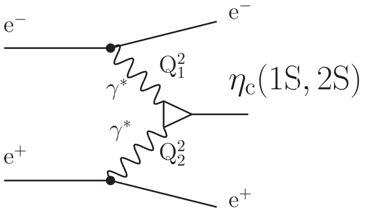

$\eta_{c,b}, \chi_{c,b}$

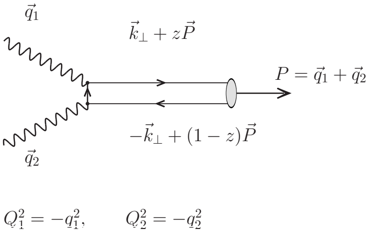



- Even $C$-parity quarkonia ($c\bar{c}$ or $b\bar{b}$ states) : prompt production via gluon-gluon fusion.
- a good probe for transverse momentum dependent gluon densities. In $k_T$ factorization: need off shell matrix element for the fusion of two spacelike virtual gluons.

# Description of the mechanism $\gamma^*\gamma^* \to \eta_c(1S, 2S)$

Production of $\eta_c$ in double-tagged $e^+e^-$ collisions measures the $\gamma^*\gamma^*\eta_c$ transition form factor.




$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)$$

Light-front representation of the transition form factor ($Q_i^2 = -\boldsymbol{q}_i^2$):

$$F(Q_1^2, Q_2^2) = e_c^2\sqrt{N_c}4m_c \cdot \int \frac{dzd^2\mathbf{k}}{z(1-z)16\pi^3}\psi(z,\mathbf{k})$$
$$\left\{\frac{1-z}{(\mathbf{k}-(1-z)\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2} + \frac{z}{(\mathbf{k}+z\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2}\right\}.$$

## $\gamma^*\gamma^* \to M,\ M = \eta_c(1S, 2S), \chi_{c0}$

$$\mathcal{M}_{\mu\nu} = \int \frac{dzd^2\boldsymbol{k}}{z(1-z)16\pi^3} \sum_{\lambda,\bar{\lambda}} \Psi^*_{\lambda\bar{\lambda}}(z,\boldsymbol{k}) \mathcal{M}^{\lambda\bar{\lambda}}_{\mu\nu}(\gamma^*\gamma^* \to Q_\lambda(zP_+, \boldsymbol{p}_Q)\bar{Q}_{\bar{\lambda}}((1-z)P_+, \boldsymbol{p}_{\bar{Q}})\,.$$

General form of the amplitude – invariant form factors:

- pseudoscalar:

$$\frac{1}{4\pi\alpha_{\rm em}} \mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = (-i)\varepsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)$$

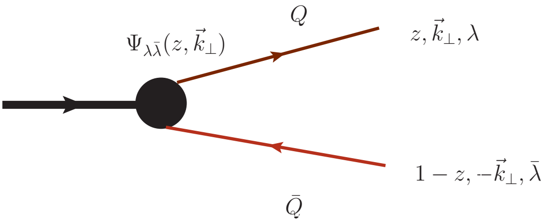
- scalar:

$$\frac{1}{4\pi\alpha_{\rm em}} \mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \chi_{c0}) = -\underbrace{\delta^\perp_{\mu\nu}(q_1, q_2)\, F_{TT}(Q_1^2, Q_2^2)}_{\text{transverse}} + \underbrace{e^L_\mu(q_1) e^L_\nu(q_2)\, F_{LL}(Q_1^2, Q_2^2)}_{\text{longitudinal}}\,.$$

- we can extract the invariant form factors by calculating the projection on light-front directions

$$n^{+\mu} n^{-\nu} \mathcal{M}_{\mu\nu} \text{ in the frame } q_{1\mu} = q_1^+ n^+_\mu + q^\perp_{1\mu},\, q_{2\mu} = q_2^- n^-_\mu + q^\perp_{2\mu},$$

# Light-front wave functions




Frame-independent $Q\bar{Q}$ component from LF-Fock-state expansion:

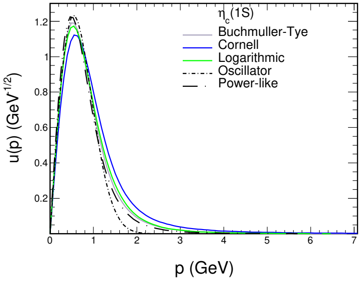
$$|\mathrm{Meson}; P_+, \mathbf{P}\rangle = \sum_{i,j,\lambda,\bar\lambda} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} \Psi_{\lambda\bar\lambda}(z,\mathbf{k}) |Q_{i\lambda}(zP_+, \mathbf{p}_Q) \bar{Q}^j_{\bar\lambda}((1-z)P_+, \mathbf{p}_{\bar Q})\rangle + \dots$$

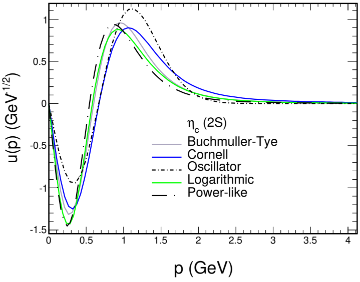
- $z$=fraction of meson's plus-momentum carried by quark, $\boldsymbol{k} = (1-z)\boldsymbol{p}_Q - z\boldsymbol{p}_{\bar Q}$
- dominance of 2-body Fock state:

$$\sum_{\lambda,\bar\lambda} \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} |\Psi_{\lambda\bar\lambda}(z,\mathbf{k})|^2 = N_{Q\bar Q} \sim 1.$$

- We ideally would like to use LFWF's from a direct formulation of the bound-state eqn's on the LF. For the time being we use a prescription based on the rest-frame WF.

# Nonrelativistic quarkonium wave functions




Radial momentum space wave function for different potentials.

Radial wave function are obtained from the Schrödinger equation

J. Cepila, J. Nemchik, M. Krelina and R. Pasechnik, Eur. Phys. J. C **79**, no. 6, 495 (2019).

$$\frac{\partial^2 u(r)}{\partial r^2} = (V_{\rm eff}(r) - \epsilon)u(r)\,, \quad u(r) = \sqrt{4\pi}\, r\psi(r)\,, \quad u(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty r dr j_0(pr) u(r)$$

$$\Psi_{\sigma\bar{\sigma}}(\vec{p}) = \underbrace{\frac{1}{\sqrt{2}}\chi_\sigma^\dagger i\sigma_2 \chi_{\bar{\sigma}}}_{\rm spin-singlet} \underbrace{\frac{u(p)}{p}\frac{1}{\sqrt{4\pi}}}_{\rm s-wave}\,.$$

# Light-front wave functions

Frame-independent $q\bar{q}$ component from LF-Fock-state expansion:

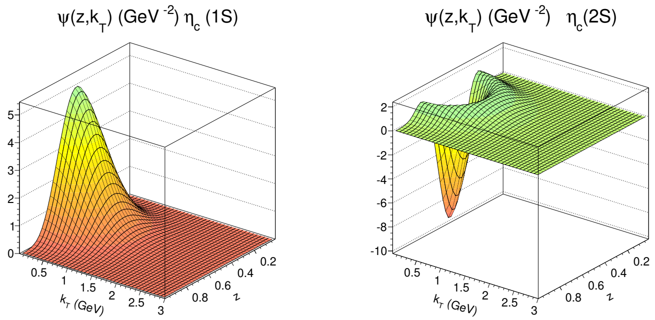
$$|\eta_c; P_+, \mathbf{P}\rangle = \sum_{i,j,\lambda,\bar{\lambda}} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} \Psi_{\lambda\bar{\lambda}}(z,\mathbf{k}) |c_{i\lambda}(zP_+, \mathbf{p}_c) \bar{c}^j_{\bar{\lambda}}((1-z)P_+, \mathbf{p}_{\bar{c}})\rangle + \ldots$$

$$\begin{aligned} \Psi_{\lambda\bar{\lambda}}(z,\mathbf{k}) &= \bar{u}_\lambda(zP_+, \mathbf{k}) \gamma_5 v_{\bar{\lambda}}((1-z)P_+, -\mathbf{k})\, \psi(z,\mathbf{k}) \\ \psi(z,\mathbf{k}) &= \frac{\pi}{\sqrt{2M_{c\bar{c}}}} \frac{u(p)}{p}\,. \end{aligned}$$

Terentev prescription valid for weakly bound system

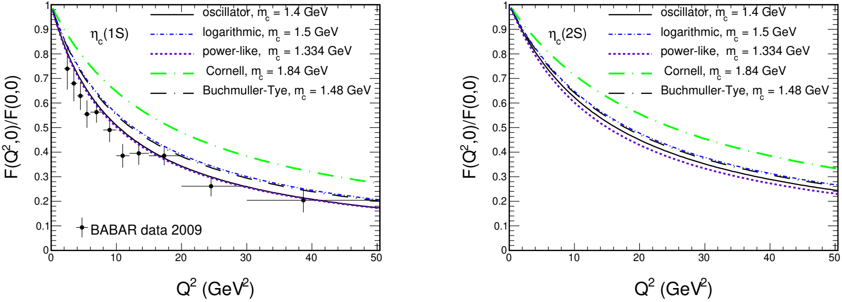
$\mathbf{p} = \mathbf{k}, \quad p_z = (z - \frac{1}{2}) M_{c\bar{c}}, \; M^2_{c\bar{c}} = \frac{\mathbf{k}^2 + m_c^2}{z(1-z)}$.

Helicity dependence from Melosh-Trf. of RF-WF




Radial light-front wave function for Buchmüller-Tye potential.

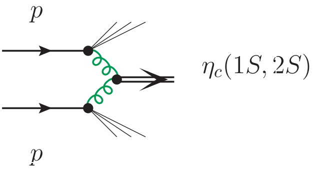
# Normalized transition form factor $\tilde{F}(Q^2, 0)$




**Normalized** transition form factor $\tilde{F}(Q^2, 0)$ as a function of photon virtuality $Q^2$. The BaBar data are shown for comparison.

J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **81**, 052010 (2010) [arXiv:1002.3000 [hep-ex]].

# Hadroproduction of $\eta_c(1S, 2S)$ via gluon-gluon fusion




$$\frac{d\sigma}{dyd^2\boldsymbol{p}} = \int \frac{d^2\boldsymbol{q}_1}{\pi \boldsymbol{q}_1^2}\mathcal{F}(x_1, \boldsymbol{q}_1^2) \int \frac{d^2\boldsymbol{q}_2}{\pi \boldsymbol{q}_2^2}\mathcal{F}(x_2, \boldsymbol{q}_2^2) \times \delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{p}) \frac{\pi}{(x_1 x_2 s)^2} \overline{|\mathcal{M}|^2},$$

where the momentum fractions of gluons are fixed as $x_{1,2} = m_T \exp(\pm y)/\sqrt{s}$. The off-shell matrix element is written in terms of the Feynman amplitude as (we restore the color-indices):

Catani, Ciafaloni & Hautmann; Gribov, Levin & Ryskin, Collins & Ellis

$$\mathcal{M}^{ab} = \frac{q_{1\perp}^{\mu} q_{2\perp}^{\nu}}{|\boldsymbol{q}_1||\boldsymbol{q}_2|}\mathcal{M}^{ab}_{\mu\nu} = \frac{q_{1+}q_{2-}}{|\boldsymbol{q}_1||\boldsymbol{q}_2|} n^+_\mu n^-_\nu \mathcal{M}^{ab}_{\mu\nu} = \frac{x_1 x_2 s}{2|\boldsymbol{q}_1||\boldsymbol{q}_2|} n^+_\mu n^-_\nu \mathcal{M}^{ab}_{\mu\nu}\,.$$

In covariant form, the matrix element reads:

$$\mathcal{M}^{ab}_{\mu\nu} = (-i)4\pi\alpha_s\, \varepsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \frac{Tr[t^a t^b]}{\sqrt{N_c}}\, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)\,.$$

To the lowest order, it is proportional to the matrix element for the $\gamma^*\gamma^*\eta_c$ vertex. In particular, the form factor $I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)$ is related to the $\gamma^*\gamma^*\eta_c$ transition form factor $F(Q_1^2, Q_2^2)$, $Q_i^2 = \boldsymbol{q}_i^2$ as

$$F(Q_1^2, Q_2^2) = e_c^2 \sqrt{N_c}\, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)\,,$$

# $k_T$-factorization

$$n^+_\mu n^-_\mu \mathcal{M}^{ab}_{\mu\nu} \quad = \quad 4\pi\alpha_s(-i)[\boldsymbol{q}_1, \boldsymbol{q}_2] \frac{Tr[t^a t^b]}{\sqrt{N_c}} \, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) \, = 4\pi\alpha_s(-i) \frac{1}{2} \delta^{ab} \frac{1}{\sqrt{N_c}} [\boldsymbol{q}_1, \boldsymbol{q}_2] \, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) \, ,$$

and averaging over colors, we obtain our final result:

$$\frac{d\sigma}{dy d^2\boldsymbol{p}} = \int \frac{d^2\boldsymbol{q}_1}{\pi \boldsymbol{q}_1^4} \mathcal{F}(x_1, \boldsymbol{q}_1^2) \int \frac{d^2\boldsymbol{q}_2}{\pi \boldsymbol{q}_2^4} \mathcal{F}(x_2, \boldsymbol{q}_2^2) \, \delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{p}) \, \frac{\pi^3 \alpha_s^2}{N_c(N_c^2 - 1)} |[\boldsymbol{q}_1, \boldsymbol{q}_2] \, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)|^2 .$$

Parametrizing the transverse momenta as $\boldsymbol{q}_i = (q_i^x, q_i^y) = |\boldsymbol{q}_i|(\cos\phi_i, \sin\phi_i)$, we can write the vector product $[\boldsymbol{q}_1, \boldsymbol{q}_2]$ as

$$[\boldsymbol{q}_1, \boldsymbol{q}_2] = q_1^x q_2^y - q_1^y q_2^x = |\boldsymbol{q}_1||\boldsymbol{q}_2| \sin(\phi_1 - \phi_2) \, .$$

Note: gluons are *linearly polarized* (cf. gluon distributions $f(x, \boldsymbol{q}), h_1(x, \boldsymbol{q})$ in TMD factorization).

In our numerical calculations presented below, we set the factorization scale to $\mu_F^2 = m_T^2$, and the renormalization scale is taken in the form:

$$\alpha_s^2 \to \alpha_s(\max\{m_T^2, \boldsymbol{q}_1^2\})\alpha_s(\max\{m_T^2, \boldsymbol{q}_2^2\}) \, .$$

Application to exclusive diffraction: talk by Izabela Babiarz on Thursday

# Normalization

$$\Gamma(\eta_c \to \gamma\gamma) = \Gamma_{\rm LO}(\eta_c \to \gamma\gamma)\left(1 - \frac{20-\pi^2}{3}\frac{\alpha_s}{\pi}\right),$$
$$\Gamma(\eta_c \to gg) = \Gamma_{\rm LO}(\eta_c \to gg)\left(1 + 4.8\,\frac{\alpha_s}{\pi}\right).$$

**Table:** Total decay widths as well as $|F(0,0)|$ obtained from $\Gamma_{tot}$ using the next-to-leading order approximation (see Eq. (1)).

| | Experimental values $\Gamma_{tot}$ (MeV) | Derived from NLO $\lvert F(0,0)\rvert_{gg}[GeV^{-1}]$ |
|---|---|---|
| $\eta_c(1S)$ | 31.9±0.7 | 0.119±0.001 |
| $\eta_c(2S)$ | 11.3±3.2±2.9 | 0.053±0.010 |

**Table:** Radiative decay widths as well as $|F(0,0)|$ obtained from $\Gamma_{\gamma\gamma}$ using leading order and next-to-leading order approximation.

| | Experimental values $\Gamma_{\gamma\gamma}$(keV) | Derived from LO $\lvert F(0,0)\rvert[GeV^{-1}]$ | Derived from NLO $\lvert F(0,0)\rvert_{\gamma\gamma}[GeV^{-1}]$ |
|---|---|---|---|
| $\eta_c(1S)$ | 5.0 ±0.4 | 0.067±0.003 | 0.079±0.003 |
| $\eta_c(2S)$ | $1.9 \pm 1.3 \cdot 10^{-4} \cdot \Gamma_{\eta_c(2S)}$ | 0.033±0.012 | 0.038±0.014 |

M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, no. 3, 030001 (2018).

# Results in LHCb kinematics

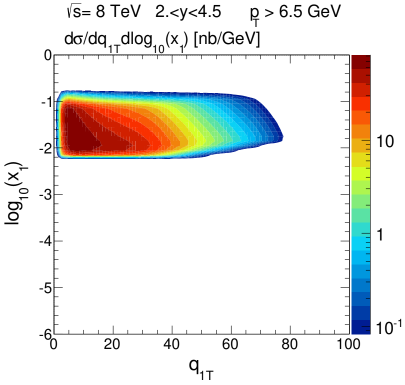



**Figure:** Two-dimensional distributions in $(x_1, q_{1T})$ (left panel) and in $(x_2, q_{2T})$ (right panel) for $\eta_c(1S)$ production for $\sqrt{s} = 8$ TeV. In this calculation the KMR UGD was used for illustration.

# Results in LHCb kinematics

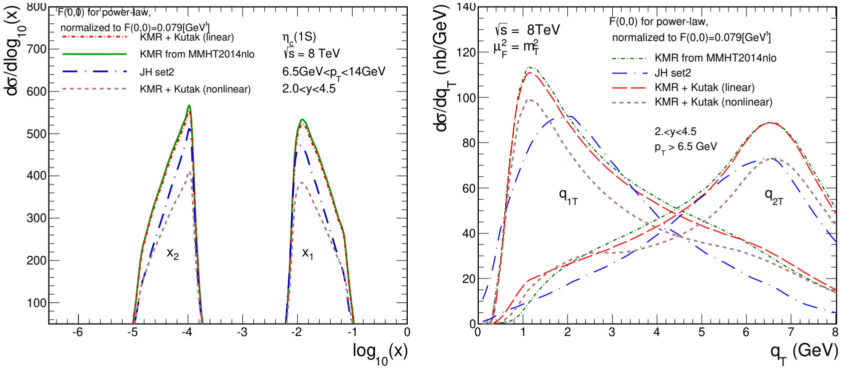



**Figure:** Distributions in $\log_{10}(x_1)$ or $\log_{10}(x_2)$ (left panel) and distributions in $q_{1T}$ or $q_{2T}$ (right panel) for the LHCb kinematics. Here the different UGDs were used in our calculations. Here we show an example, where $\sqrt{s} =$ 7 TeV.

# Unintegrated gluon distributions

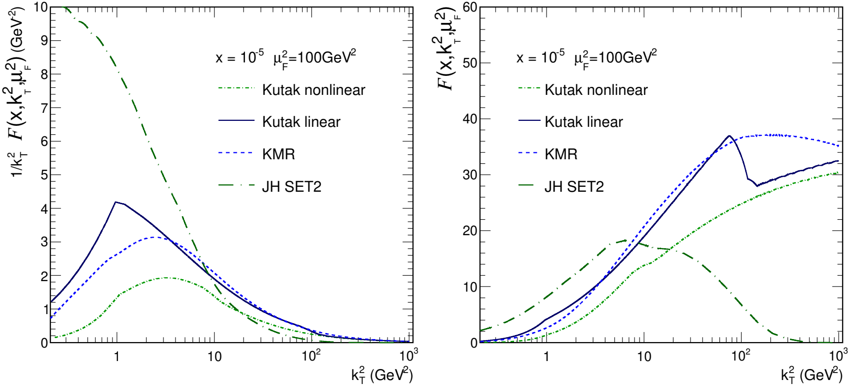



**Figure:** Unintegrated gluon densities for typical scale $\mu^2$= 100 GeV$^2$ for $\eta_c(1S)$ production in proton-proton scattering at LHCb kinematics.

$$xg(x,\mu_F^2) = \int^{\mu_F^2} \frac{d\boldsymbol{k}^2}{\boldsymbol{k}^2} \mathcal{F}(x,\boldsymbol{k}^2,\mu_F^2)$$

# $p_T$ distributions in LHCb kinematics

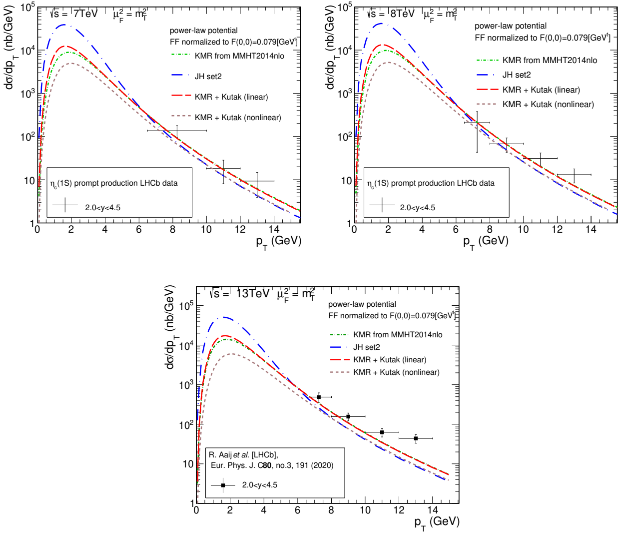



**Figure:** Differential cross section as a function of transverse momentum for prompt $\eta_c(1S)$ production compared with the LHCb data for $\sqrt{s} = 7, 8, 13\,\mathrm{TeV}$. Different UGDs were used. Here we used the $g^*g^* \to \eta_c(1S)$ form factor calculated from the power-law potential.

# Predictions for $\eta_c(2S)$

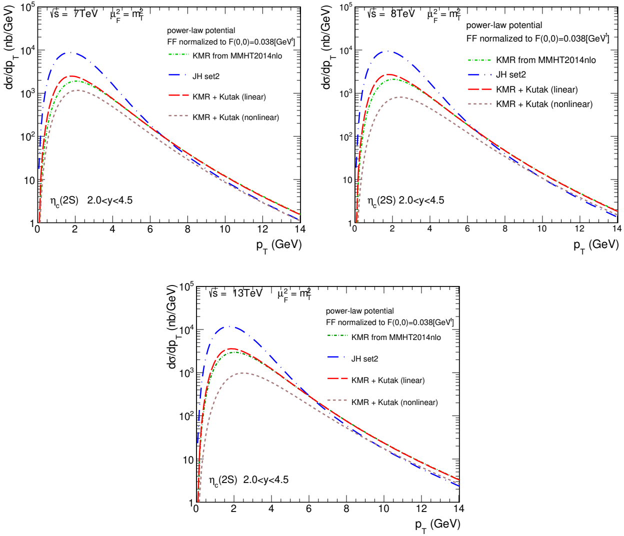



**Figure:** Differential cross section as a function of transverse momentum for prompt $\eta_c(2S)$ production for $\sqrt{s} = 7, 8, 13\,\mathrm{TeV}$.

# Integrated cross section with LHCb cuts

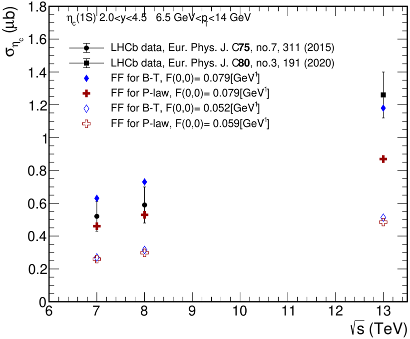



**Figure:** The integrated cross section computed within LHCb range of $p_T$ and $y$ with our transition form factors, compared to experimental values. Here red crosses represent values for Buchmüller-Tye potential (B-T) and deltoids for Power-law potential (P-law). Data are from [1],[2]


[1] R. Aaij *et al.* [LHCb Collaboration], Eur. Phys. J. C **75**, no. 7, 311 (2015).
[2] R. Aaij *et al.* [LHCb Collaboration],Eur. Phys. J. C **80**, no. 3, 191 (2020).

# Importance of the form factor

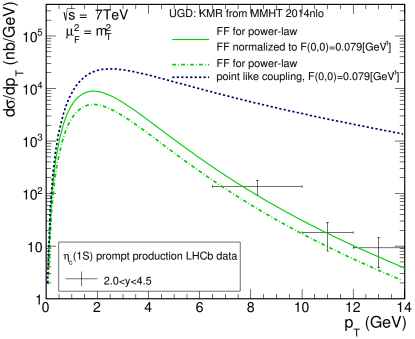



**Figure:** Comparison of results for two different transition form factor, computed with the KMR unintegrated gluon distribution. We also show result when the $(q_{1T}^2, q_{2T}^2)$ dependence of the transition form factor is neglected.

- pointlike coupling of $\eta_c$ to gluons $\propto \eta_c F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$ badly overshoots data. Off-shell form factor is essential.

## Hadroproduction of $\chi_{c0}, \chi_{b0}$ via gluon-gluon fusion

$$
\begin{aligned}
\frac{d\sigma}{dy d^2\boldsymbol{p}} &= \int \frac{d^2\boldsymbol{q}_1}{\pi \boldsymbol{q}_1^2} \mathcal{F}(x_1, \boldsymbol{q}_1^2) \int \frac{d^2\boldsymbol{q}_2}{\pi \boldsymbol{q}_2^2} \mathcal{F}(x_2, \boldsymbol{q}_2^2)\, \delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{p}) \frac{\pi^3 \alpha_S^2}{N_c(N_c^2-1)} \\
&\times \Big( G_1(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) + \cos(\phi_1 - \phi_2) G_2(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) \Big)^2 ,
\end{aligned}
$$

We parametrize the transverse momenta as $\boldsymbol{q}_i = (q_i^x, q_i^y) = |\boldsymbol{q}_i|(\cos\phi_i, \sin\phi_i)$.

$$
\begin{aligned}
G_1(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) &= |\boldsymbol{q}_1||\boldsymbol{q}_2| \frac{4m_Q}{\boldsymbol{q}_2^2} \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \psi(z, \boldsymbol{k})\, 2z(1-z)(2z-1) \left[ \frac{1}{\boldsymbol{l}_A^2 + \varepsilon^2} - \frac{1}{\boldsymbol{l}_B^2 + \varepsilon^2} \right] \\
G_2(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) &= 4m_Q \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \psi(z, \boldsymbol{k}) \left[ \frac{1-z}{\boldsymbol{l}_A^2 + \varepsilon^2} + \frac{z}{\boldsymbol{l}_B^2 + \varepsilon^2} \right] \\
&+ \frac{4m_Q}{\boldsymbol{q}_2^2} \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \psi(z, \boldsymbol{k}) 4z(1-z) \left[ \frac{\boldsymbol{q}_2 \cdot \boldsymbol{l}_A}{\boldsymbol{l}_A^2 + \varepsilon^2} - \frac{\boldsymbol{q}_2 \cdot \boldsymbol{l}_B}{\boldsymbol{l}_B^2 + \varepsilon^2} \right] .
\end{aligned}
$$

$$
\varepsilon^2 = z(1-z)\boldsymbol{q}_1^2 + m_Q^2 , \qquad \boldsymbol{l}_A = \boldsymbol{k} - (1-z)\boldsymbol{q}_2 , \qquad \boldsymbol{l}_B = \boldsymbol{k} + z\boldsymbol{q}_2 .
$$

- Form factors $G_1, G_2$ are linear combinations of the helicity form factors $F_{TT}, F_{LL}$.

# Gluon polarizations: transverse vs. longitudinal

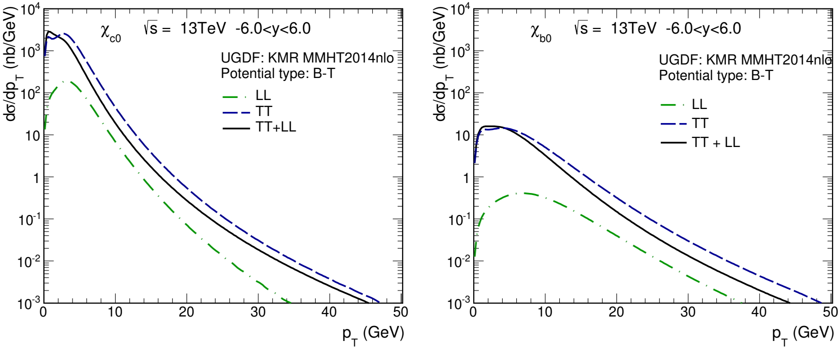



**Figure:** The comparison of meson transverse momentum distribution with only the *LL* amplitude and only *TT* amplitude as well as for the full, coherent sum of *TT* and *LL* amplitudes. Here the B-T potential was used.

- production of scalar particle probes a density matrix of transverse (linear - along the gluon transverse mom.) and longitudinal polarizations of gluons.

# Conclusions

- The $\gamma^*\gamma^*$ transition form factors for different wave functions obtained as a solution of the Schrödinger equation for the $c\bar{c}$ system for different phenomenological $c\bar{c}$ potentials from the literature, was calculated.
- We have studied the transition form factors for $\gamma^*\gamma^* \to \eta_c$ (1S,2S) for two space-like virtual photons, which can be accessed experimentally in future measurements of the cross section for the $e^+e^- \to e^+e^-\eta_c$ process in the double - tag mode.
- $k_T$-factorization approach leads to good description of LHCb data for inclusive $pp \to \eta_c$ for $\sqrt{s} = 7, 8$ TeV, and somewhat worse for $\sqrt{s} = 13$ TeV. Some room for a color octet contribution is left.
- In the LHCb kinematics very small $x$ are probed. Asymmetric kinematics where the small-$x$ gluon transfers bulk of $p_T$.
- Despite sensitivity to small $x$, no sign of gluon saturation is observed.
- Predictions for $\eta_c(2S)$. A measurement could help to pin down possible color octet.
- Uncertainty due to $g^*g^*\eta_c, \chi_c$ form factors somewhat smaller than the one from UGD.
- For the case of $\chi_{c0}, \chi_{b0}$ a gluon density matrix of transverse and longitudinal polarizations are probed.
- Predictions for $\chi_{c0}, \chi_{b0}$ at LHC. Difficult to measure?