

# The dipole picture and the non-relativistic expansion

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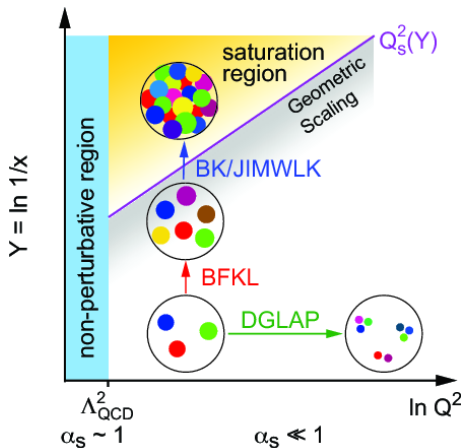
Xacobeo 2021

Phys.Rev.D 101 (2020) 3, 034030. Done in collaboration with Tuomas Lappi (Jyväskylä U).

# Plan

- 1 Introduction
- 2 Quarkonium light cone wave function in the non-relativistic limit
- 3 Cross-checks
- 4 Exclusive quarkonium production in the  $Q \gg m$  limit
- 5 Conclusions

# Why is exclusive quarkonium production interesting?



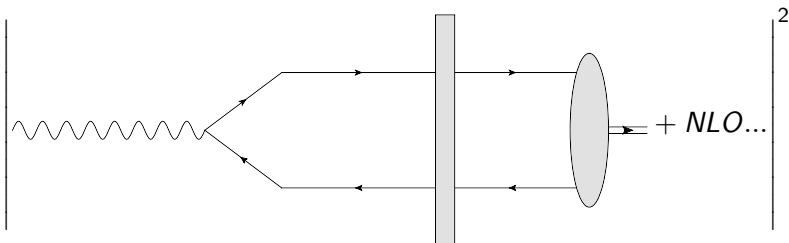
Picture taken from Marquet (2013)

Exclusive quarkonium production is an ideal way to study the gluon distribution at low  $x$  in DIS and UPCs

- Exclusive processes depend on the gluon density quadratically.
- Non-perturbative contributions are suppressed with respect to other exclusive processes.

# The dipole picture

Application of light cone perturbation theory to exclusive processes



At LO

$$\frac{d\sigma_{T,L}^{\gamma^*+N \rightarrow HQ+N}}{dt} = \frac{1}{16\pi} \left| \int d^2 r_{\perp} \int_0^1 \frac{dz}{4\pi} (\Psi_{HQ}^* \Psi_{\gamma^*})_{T,L} \sigma_{q\bar{q}} \right|^2$$

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- At higher orders the wave functions have to take into account the presence of gluons inside the photon and quarkonium. We also need to take into account  $\sigma_{q\bar{q}g}$ ,  $\sigma_{q\bar{q}gg}$  and so on.
- Our aim is to determine the properties of the nucleus. But in order to do this we need an accurate description of quarkonium wave function.

# Quarkonium's light cone wave function

- Phenomenological approaches, boosted gaussian, gaussLC... See Kowalski, Motyka and Watt (2006) for a review.
- Obtained by solving the Schrödinger equation using a phenomenological potential. Cepila, Nemchik, Krelina and Pasechnik (2019).
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## Non-relativistic assumption

- Widely used in the literature in other contexts, generally combined with EFT approaches (NRQCD, pNRQCD). Inclusive production, spectroscopy, decays.
- Well-defined limit of QCD. Theoretically interesting.
- It has already been used in the dipole model in its simplest form. Ryskin (1993) and Brodsky, Frankfurt, Gunion, Mueller and Strikman (1994).

# Energy scales in a non-relativistic quarkonium

- The mass of the heavy quark  $m$  is by definition bigger than  $\Lambda_{QCD}$ . Perturbative physics.
- The typical velocity of the heavy quarks around the center of mass  $v$  is small. Hence  $p \sim \frac{1}{r} \sim mv$ . It is estimated that for charmonium  $v^2 \sim 0.3$  while for bottomonium  $v^2 \sim 0.1$ .
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## The non-relativistic limit simplifies the treatment because...

- It allows to separate the computation of scale  $m$  effects, which are perturbative.
- From the point of view of the scales smaller than  $m$  the production of heavy quarks is a local process.

# Plan

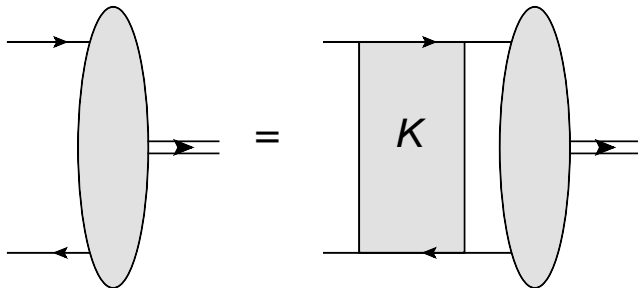
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# The non-relativistic limit

## Basic assumptions

- The leading order light cone wave function can be computed taking into account only non-relativistic quarks and their interaction.
- Relativistic degrees of freedom can appear, but they are a perturbation  $\rightarrow$  can appear during small times.
- Non-relativistic quarks (in light-cone perturbation theory) are defined by having a  $p_{\perp}$  much smaller than  $m$  and a momentum fraction very close to  $\frac{1}{2}$ .

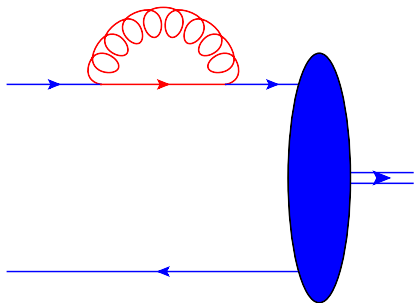
# The leading order wave function



- Contains only non-relativistic components.
- Fulfills a Bethe-Salpeter equation which can be expressed as a Schrödinger equation.
- Relation between this and the wave function in potential models/NRQCD studied in Bodwin, Kang and Lee (2006).



# Relativistic corrections to the non-relativistic wave function



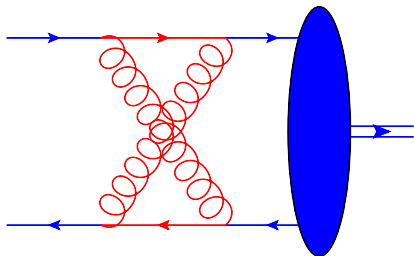
Type 1. Contribution of the relativistic degrees of freedom to the wave function renormalization of the non-relativistic quark.

Computed in Mustaki, Pinsky, Shigemitsu and Wilson (1991).

## Color code

- Relativistic particles and gluons with virtuality of order  $m^2$ .
- Non-relativistic particles and softer gluons.

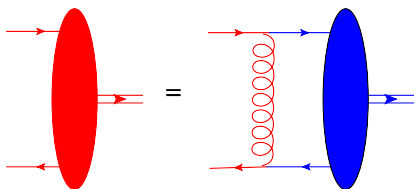
# Relativistic corrections to the non-relativistic wave function



Type 2. Contributions that can be encoded as a redefinition of the potential between non-relativistic quarks.

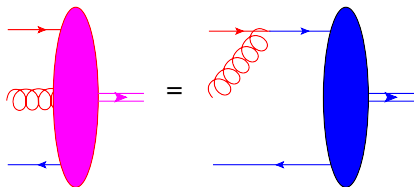
In our case, it will not appear explicitly in our equations. However, it modifies the value of the non-relativistic wave-function at the origin.

# Relativistic quark-antiquark pair component



- Can be computed in perturbation theory.  $\rightarrow$  Proportional to  $\alpha_s(m)$ .
- Point like interaction from the point of view of non-relativistic quarks.  $\rightarrow$  Proportional to the leading order non-relativistic wave function at the origin.

# Relativistic quark, hard gluon and non-relativistic antiquark



- Proportional to  $g(m)$ .
- Cross-check. One can recover the wave-function renormalization by computing the square of this contribution.

$$\int dz f(z) \Psi_{HQ}^n(z, x_\perp) = \sum_{m,k} \int dz f(z) C_{n \leftarrow m}^k(z, x_\perp) \left( \frac{\nabla}{m} \right)^k \int \frac{d\lambda}{4\pi} \phi^m(\lambda, 0)$$

- In this formula it is assumed that  $x_\perp \sim \frac{1}{m}$  or smaller. The momentum fraction of a non-relativistic quark is  $\lambda + \frac{1}{2}$  where  $\lambda \ll 1$ .
- $\phi$  represents the non-relativistic part.  $n$  and  $m$  label de components of the Fock space. For example,  $C_{q\bar{q} \leftarrow q\bar{q}}^k(z, x_\perp)$  means how the  $q\bar{q}$  component of the full wave function depends on the same component of the non-relativistic wave function.
- The terms in the rhs scale as  $v^k$ . Note that if  $mv^2 \gg \Lambda_{QCD}$  then  $v \sim \alpha_s(mv)$ .
- $C_{n \leftarrow m}^k(z, x_\perp)$  can be computed as and expansion in  $\alpha_s(m)$ .

# The cross-section in the non-relativistic expansion

$$16\pi \frac{d\sigma_{T,L}^{\gamma^*+N \rightarrow HQ+N}}{dt} = \left| \sum_{n,m,k} \left( \left( \frac{\nabla}{M} \right)^k \int \frac{d\lambda}{4\pi} \phi^m(\lambda, 0) \right) \int d^2 r_\perp \int_0^1 \frac{dz}{4\pi} \left( (C_{n \leftarrow m}^k(z, r_\perp))^* \Psi_{\gamma^*} \right)_{T,L}^n \sigma_n \right|^2$$

## Power counting

- The first correction from terms with  $k \neq 0$  will enter at NNLO in  $\alpha_s$ . Studied by Lappi, Mäntysaari and Penttala (Phys.Rev.D 102 (2020) 5, 054020).
- At NLO we only need to take into account  $C_{q\bar{q} \leftarrow q\bar{q}}^0$  and  $C_{q\bar{q}g \leftarrow q\bar{q}}^0$ .
- In this power counting we did not consider the difference between  $\alpha_s(m)$  and  $\alpha_s(mv)$ .

## Results: $q\bar{q}$ longitudinal polarization

$$\begin{aligned} \int d\theta_r C_{q\bar{q} \leftarrow q\bar{q}}^0(z, r_\perp; \lambda_1, \lambda_2; \lambda'_1, \lambda'_2)_{long} = & 8\pi^2 \delta(z - \frac{1}{2})(1 + \delta Z) \delta_{\lambda_1 \lambda'_1} \delta_{\lambda_2 \lambda'_2} \\ & + \frac{4g^2 C_F z(1-z)}{(z - \frac{1}{2})^2} \delta_{\lambda_1, \lambda'_1} \delta_{\lambda_2, \lambda'_2} \left\{ K_0(\tau) \right. \\ & + \left( \theta(z - \frac{1}{2})(1-z) + \theta(\frac{1}{2} - z)z \right) \left[ \frac{(z - \frac{1}{2})^2 - \frac{1}{2}}{z(1-z)} (K_0(\tau) \right. \\ & \left. \left. - \frac{\tau}{2} K_1(\tau) \right) - \frac{(z - \frac{1}{2})^2}{2z(1-z)} \tau K_1(\tau) \right] \left. \right\} \end{aligned}$$

Longitudinal polarization. We do the transverse angle integration to get a more compact expression.  $\tau = 2m(z - \frac{1}{2})r_\perp$ .

## Results: $q\bar{q}$ transverse polarization

$$\begin{aligned} \int d\theta_r C_{q\bar{q} \leftarrow q\bar{q}}^0(z, r_\perp; \lambda_1, \lambda_2; \lambda'_1, \lambda'_2)_{trans} = & 8\pi^2 \delta(z - \frac{1}{2})(1 + \delta Z) \delta_{\lambda_1 \lambda'_1} \delta_{\lambda_2 \lambda'_2} \\ & + \frac{4g^2 C_F z(1-z) \delta_{\lambda_1, \lambda'_1} \delta_{\lambda_2, \lambda'_2}}{(z - \frac{1}{2})^2} [K_0(\tau) \\ & - \frac{(\theta(z - \frac{1}{2})(1-z) + \theta(\frac{1}{2} - z)z)}{2z(1-z)} \left( K_0(\tau) - \frac{\tau}{2} K_1(\tau) \right)] \end{aligned}$$

Transverse polarization. We do the transverse angle integration to get a more compact expression.  $\tau = 2m \left( z - \frac{1}{2} \right) r_\perp$ .



# Results: $q\bar{q}g$

$$\begin{aligned}
 & \int \frac{d\lambda}{4\pi} \left\{ \Psi_{HQ}^{q\bar{q}g} \right\}_{\lambda_{RQ}, \lambda_G, \lambda_{\bar{Q}}}^i (x, l_{\perp}; \lambda, r_{\perp} = 0) = \\
 & - \frac{ig \left(1 - \frac{x}{2}\right) x l_{\perp}^i \epsilon_{\perp}^{*A,j}(\lambda_G)}{2\pi \sqrt{2x(1-x)} l_{\perp}} \left( \frac{mx}{2\pi l_{\perp} \mu^2} \right)^{\frac{D-4}{2}} K_{\frac{D-2}{2}}(mx l_{\perp}) \sum_{\lambda_Q} \bar{u}(\hat{p}_{RQ}, \lambda_{RQ}) T^A \\
 & \times \left[ \delta^{ij} - \frac{x}{4 \left(1 - \frac{x}{2}\right)} [\gamma_{\perp}^i, \gamma_{\perp}^j] \right] \not{n} u(mv, \lambda_Q) \int \frac{d\lambda}{4\pi} \phi_{q\bar{q}}^i(\lambda, 0) \\
 & + \frac{g \sqrt{x(1-x)} x}{8\sqrt{2}\pi(1-x)} \left( \frac{mx}{2\pi l_{\perp} \mu^2} \right)^{\frac{D-4}{2}} K_{\frac{D-4}{2}}(mx l_{\perp}) \\
 & \times \sum_{\lambda_Q} \bar{u}(\hat{p}_{RQ}, \lambda_{RQ}) \not{\epsilon}_{\perp}^*(\lambda_G) \not{n} u(mv, \lambda_Q) \int \frac{d\lambda}{4\pi} \phi_{q\bar{q}}^i(\lambda, 0)
 \end{aligned}$$

$l_{\perp}$  is the distance between the gluon and the relativistic quark.  $x$  is the momentum fraction of the gluon with respect to the non-relativistic quark.


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# Light-cone distribution amplitude

- The light-cone distribution amplitude is a function that is defined in collinear factorization.
- In light-cone gauge, it is equal to the light-cone wave function at  $r_{\perp} = 0$ .
- It must fulfil the Efremov-Radyushkin-Brodsky-Lepage (ERBL)<sup>1</sup> equation.

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<sup>1</sup>Lepage and Brodsky (1980), Chernyak and Zhitnitsky (1984) 

# Light-cone distribution amplitude

The limit  $r_\perp \rightarrow 0$  introduces ultraviolet divergences, they can be regulated in dimensional regularization.

$$\lim_{r_\perp \rightarrow 0} f(z, r_\perp) = \int \frac{d^{D-2} p_\perp}{(2\pi)^{D-2}} f(z, p_\perp)$$

For the longitudinal polarization

$$\begin{aligned} D^3(z) = & 4\pi(1 + \delta Z) \delta\left(z - \frac{1}{2}\right) \int \frac{d\lambda}{4\pi} \phi_{q\bar{q}}^3(\lambda, 0) \\ & - \frac{2g^2 C_F z(1-z)}{\pi \left(z - \frac{1}{2}\right)^2} \left[ \left( \frac{1}{D-4} + \frac{1}{2} \log\left(\frac{m^2 \left(z - \frac{1}{2}\right)^2}{\pi \mu^2}\right) + \frac{\gamma_E}{2} \right) \right. \\ & \times \left( 1 + \left(\theta\left(z - \frac{1}{2}\right)(1-z) + \theta\left(\frac{1}{2} - z\right)z\right) \frac{(D-2) \left(z - \frac{1}{2}\right)^2 - 1}{2z(1-z)} \right) \\ & \left. + \frac{(\theta\left(z - \frac{1}{2}\right)(1-z) + \theta\left(\frac{1}{2} - z\right)z)}{4z(1-z)} \left( 2(D-2) \left(z - \frac{1}{2}\right)^2 - 1 \right) \right] \int \frac{d\lambda}{4\pi} \phi_{q\bar{q}}^3(\lambda, 0) \end{aligned}$$

# Light-cone distribution amplitude

For the transverse polarization

$$\begin{aligned} D^i(z) = & 4\pi(1 + \delta Z)\delta\left(z - \frac{1}{2}\right) \int \frac{d\lambda}{4\pi} \phi_{q\bar{q}}^3(\lambda, 0) \\ & - \frac{2g^2 C_F z(1-z)}{\pi \left(z - \frac{1}{2}\right)^2} \left[ \left( \frac{1}{D-4} + \frac{1}{2} \log\left(\frac{m^2 \left(z - \frac{1}{2}\right)^2}{\pi\mu^2}\right) + \frac{\gamma_E}{2} \right) \right. \\ & \quad \left( 1 - \frac{(\theta(z - \frac{1}{2})(1-z) + \theta(\frac{1}{2} - z)z)}{2z(1-z)} \right) \\ & \quad \left. - \frac{(\theta(z - \frac{1}{2})(1-z) + \theta(\frac{1}{2} - z)z)}{4z(1-z)} \right] \int \frac{d\lambda}{4\pi} \phi_{q\bar{q}}^i(\lambda, 0) \end{aligned}$$

# The ERBL equation

We have checked that it is fulfilled that

$$\frac{\partial D^i(z)}{\partial \log \mu^2} = \frac{\alpha_s C_F}{2\pi} \int_0^1 dz' K_{L,T}(z, z') D^i(z')$$

where

$$K_L(z, z') = \theta(z - z') \frac{1 - z}{1 - z'} \left( 1 + \left[ \frac{1}{z - z'} \right]_+ \right) \\ + \theta(z' - z) \frac{z}{z'} \left( 1 + \left[ \frac{1}{z' - z} \right]_+ \right) + \frac{3}{2} \delta(z - z')$$

and

$$K_T(z, z') = \theta(z - z') \frac{1 - z}{1 - z'} \left[ \frac{1}{z - z'} \right]_+ + \theta(z' - z) \frac{z}{z'} \left[ \frac{1}{z' - z} \right]_+ + \frac{3}{2} \delta(z - z')$$

# Quarkonium decay into leptons

This is a quantity that can be computed knowing the light-cone wave function. It has also been computed at one loop using NRQCD and related approaches<sup>2</sup> (in dimensional regularization). Therefore, we know

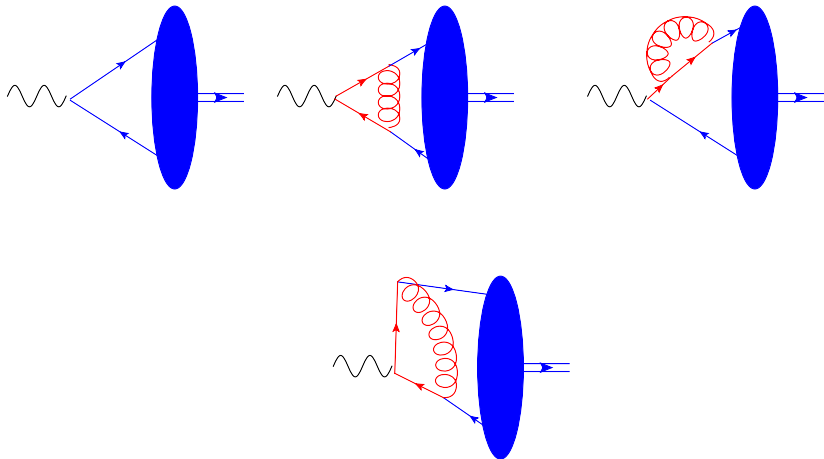
$$\int_0^1 dz \sum_n \Psi_{HQ}^n(z, 0_\perp) = \left(1 - \frac{2\alpha_s C_F}{\pi}\right) \int d\lambda \phi(\lambda, 0)$$

However, we are not using dimensional regularization.

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<sup>2</sup>Barbieri, Gatto, Kogerler and Kunszt (1975)

# Diagrams



Remark: The last diagram vanishes in the longitudinal polarization case.



# Result

$$\int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, 0_\perp) = \left( 1 + \frac{\alpha_s C_F}{\pi} \left( \frac{1}{x_0} - 2 \right) \right) \int d\lambda \phi(\lambda, 0)$$

$$\int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, 0_\perp) = \left( 1 + \frac{\alpha_s C_F}{\pi} \left( \frac{1}{x_0} - 2 \right) \right) \int d\lambda \phi(\lambda, 0)$$

- We are using a cut-off  $x_0$  to regulate the integration in  $z$  and DR to regulate the transverse component.
- Power like divergences do not appear in dimensional regularization (DM) but they can appear in our case.
- The divergence comes from the region in which  $p_\perp \sim m x_0 \ll m$  and  $|z - \frac{1}{2}| \sim \frac{x_0}{2} \ll 1$ . Coulomb singularity.

# Result

$$\int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, 0_\perp) = \left( 1 + \frac{\alpha_s C_F}{\pi} \left( \frac{1}{x_0} - 2 \right) \right) \int d\lambda \phi(\lambda, 0)$$

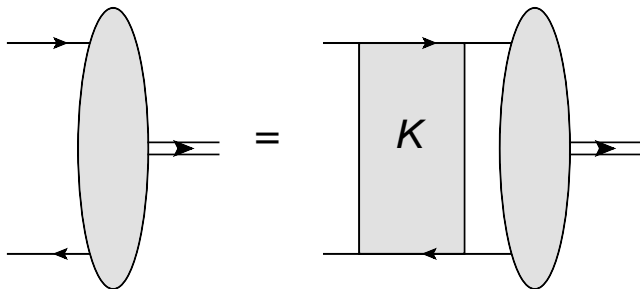
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We need that...

$$\frac{d}{dx_0} \int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, 0_\perp) = -\frac{\alpha_s C_F}{\pi x_0^2} \int d\lambda \phi(\lambda, 0) + \frac{d}{dx_0} \int d\lambda \phi(\lambda, 0) = 0$$

# Coulomb singularity

We want to check if the condition is fulfilled.



- We look at the ultraviolet behaviour, we can substitute the kernel by a Coulomb exchange.
- We focus on the case in which  $|z - \frac{1}{2}| \sim \frac{x_0}{2}$  and  $m \gg p_{\perp} \sim mx_0 \gg mv$ .
- Indeed, the dependence of  $\int d\lambda \phi$  on  $x_0$  makes decay width finite.

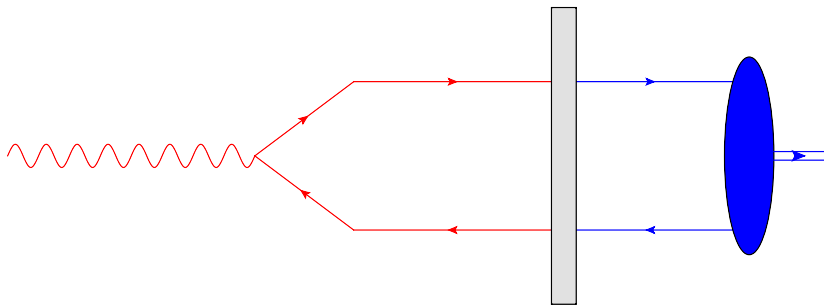
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# Exclusive quarkonium production in the $Q \gg m$ limit at NLO

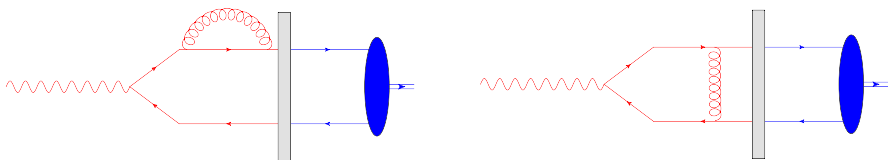
- The general case  $Q \sim m$  has been very recently studied using this quarkonium wave function. ArXiv:2104.02349, Mäntysaari and Penttala. See Penttala's talk on Thursday.
- For this we need the NLO photon wave function with massive quarks. This result was recently published, ArXiv:2103.14549, Beuf, Lappi and Paatelainen.
- Here, we check that all divergences cancel in the  $Q \gg m$  limit. We focus on the simpler, longitudinal polarization case.
- We get consistent results compatible with B-JIMWLK evolution.

# Tree level



Dependence with  $x_0$  hidden in two terms.  $\sigma_{q\bar{q}}$ , which fulfils B-JIMWLK evolution, and the non-relativistic wave function, which depends on  $x_0$  in the way we described when discussing the decay into leptons.

# One loop corrections to photon wave function



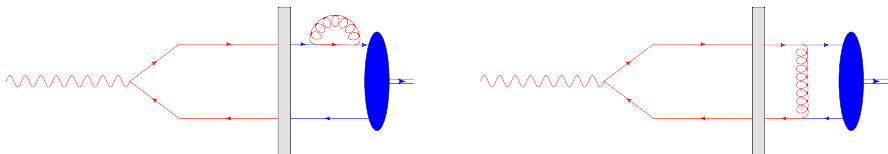
$$\Psi_{\gamma}(z, r_{\perp})|_{NLO} = \Psi_{\gamma}(z, r_{\perp})|_{LO} (1 + \delta Z_{\gamma}(z, r_{\perp}))$$

Recently computed in Beuf (2017). In our case we need the value at  $z = \frac{1}{2}$ .

$$\delta Z_{\gamma}\left(\frac{1}{2}, r_{\perp}\right) = -\frac{2C_F\alpha_s}{\pi} \left( \left( \log x_0 + \frac{3}{4} \right) \left( \frac{1}{D-4} - \frac{\gamma_E}{2} - \frac{1}{2} \log(\pi\mu^2 r_{\perp}^2) \right) + \frac{\pi^2}{24} - \frac{3}{4} \right)$$



# One loop corrections to quarkonium wave function



## Dependence with $\mu$

Comes only from the wave function renormalization and fulfils that

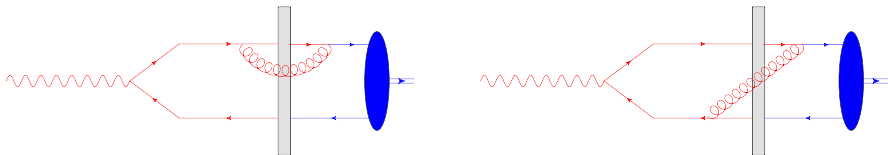
$$\frac{d\delta Z}{d\mu} = \frac{d\delta Z_\gamma\left(\frac{1}{2}, r_\perp\right)}{d\mu}$$

## Dependence with $x_0$

Can be divided into two pieces:

- One which cancels the  $x_0$  dependence of the non-relativistic wave function. Coulomb singularity.
- One whose derivative is proportional to  $\frac{d\delta Z_\gamma\left(\frac{1}{2}, r_\perp\right)}{dx_0}$ .

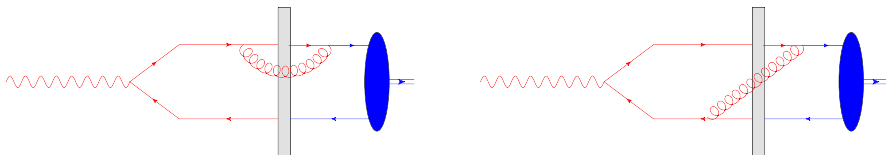
# Contribution of the $q\bar{q}g$ Fock state



## Dependence with $\mu$

Note that in the ultraviolet  $\sigma_{q\bar{q}g} \rightarrow \sigma_{q\bar{q}}$ . It has a divergence that cancels that of the wave functions of the photon and quarkonium.

# Contribution of the $q\bar{q}g$ Fock state



## Dependence with $x_0$

Can be divided in two terms:

- One proportional to  $(\sigma_{q\bar{q}g} - \sigma_{q\bar{q}})$  which cancels the B-JIMWLK evolution of the target.
- One proportional to  $\sigma_{q\bar{q}}$  which cancels the divergences of the wave functions of the photon and quarkonium, except for the piece related with the Coulomb singularity

# Plan

- 1 Introduction
- 2 Quarkonium light cone wave function in the non-relativistic limit
- 3 Cross-checks
- 4 Exclusive quarkonium production in the  $Q \gg m$  limit
- 5 Conclusions

- We have computed the NLO corrections to the quarkonium wave function in the non-relativistic limit.
- We have checked that the light-cone distribution amplitude obtained in this framework fulfils ERBL equation.
- We recovered known results for the decay of quarkonium into leptons. To our knowledge, first computation in light-cone gauge.
- We have checked that when the wave function is applied to compute exclusive quarkonium production all divergences will cancel.
- See Penttala's talk on Thursday for applications of these results.