QCD evolution of twist-2 and twist-4 fragmentation function for heavy quarkonium production

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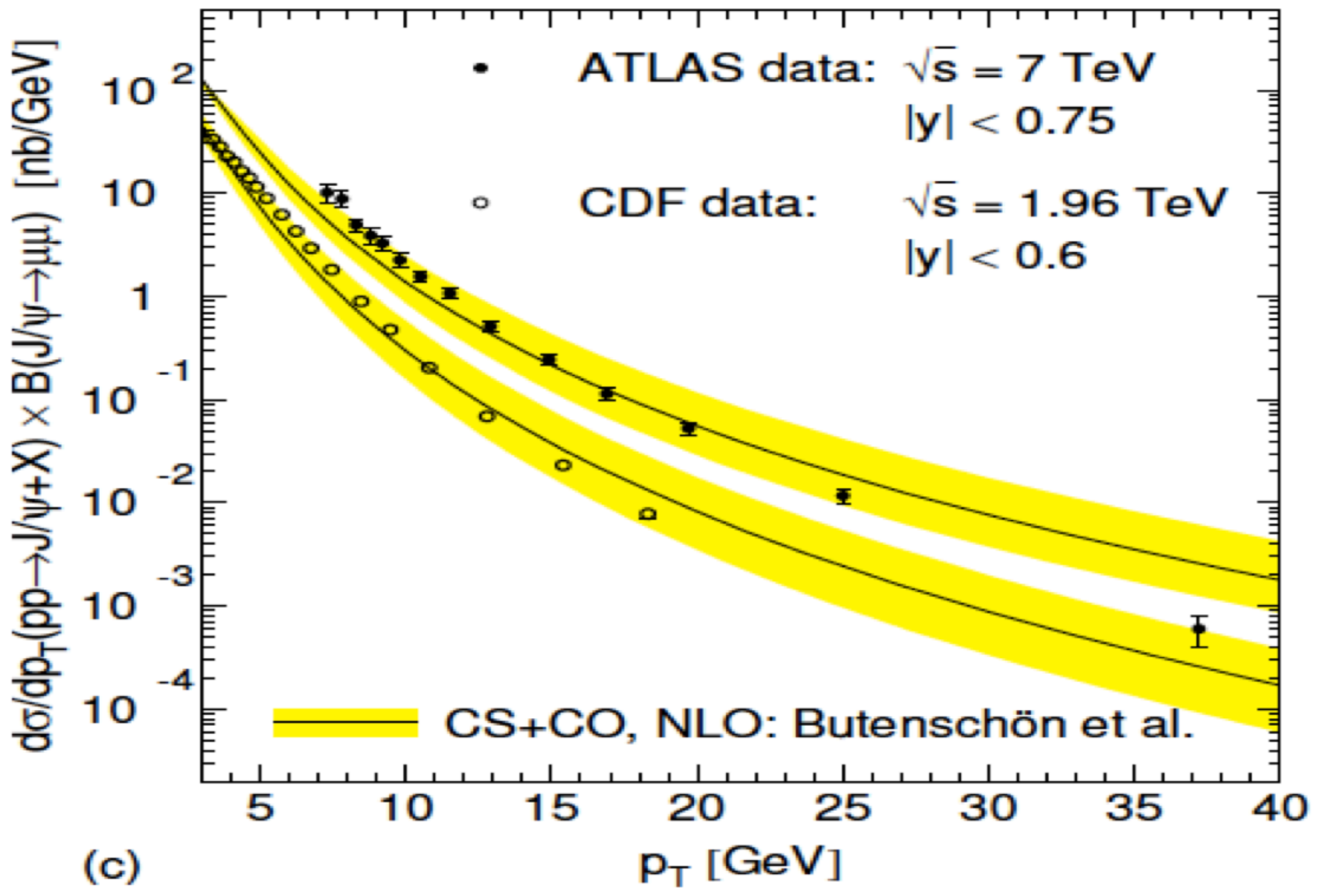
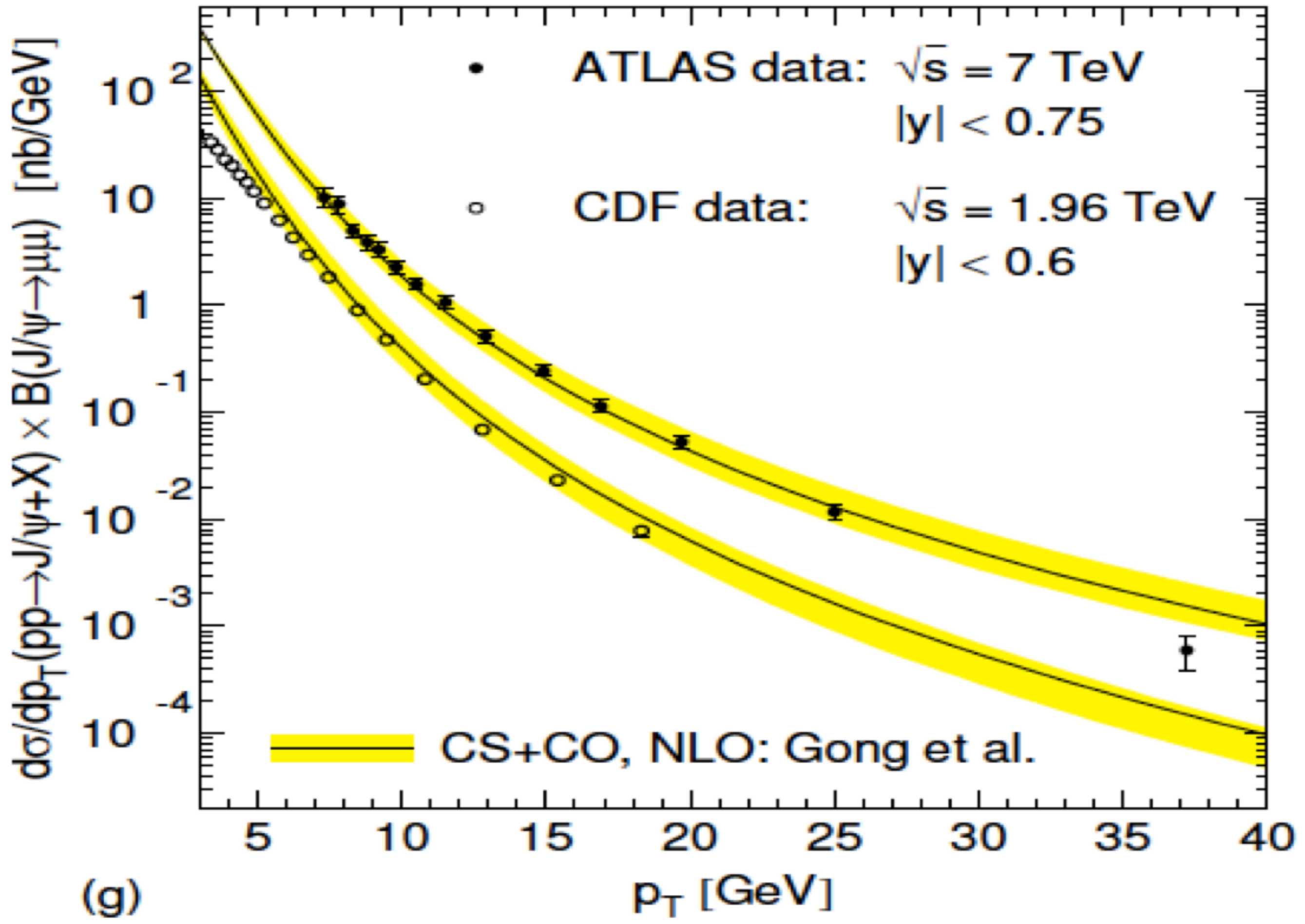
Subatech

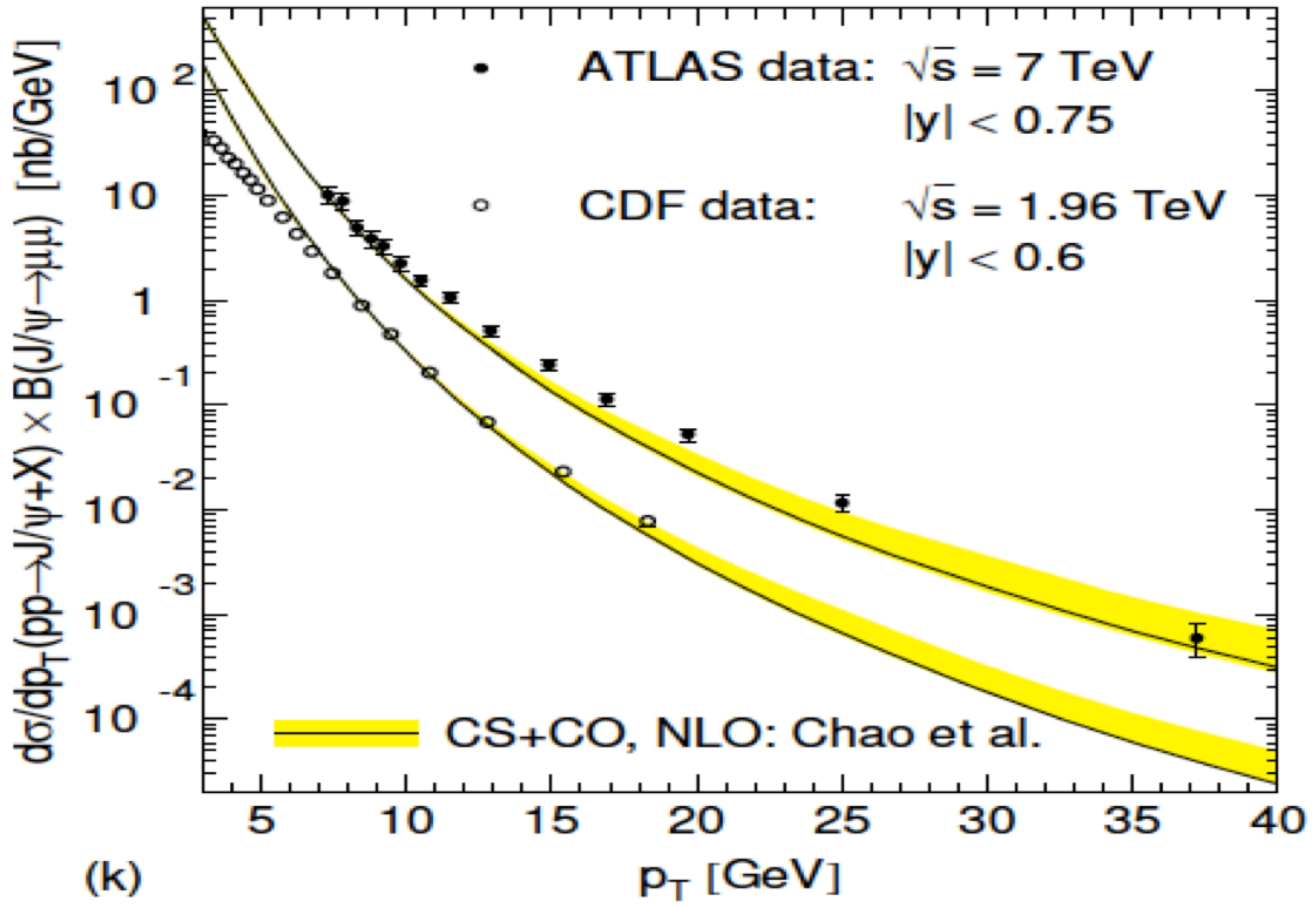
with Kyle Lee, Jianwei Qiu, George Sterman, in preparation

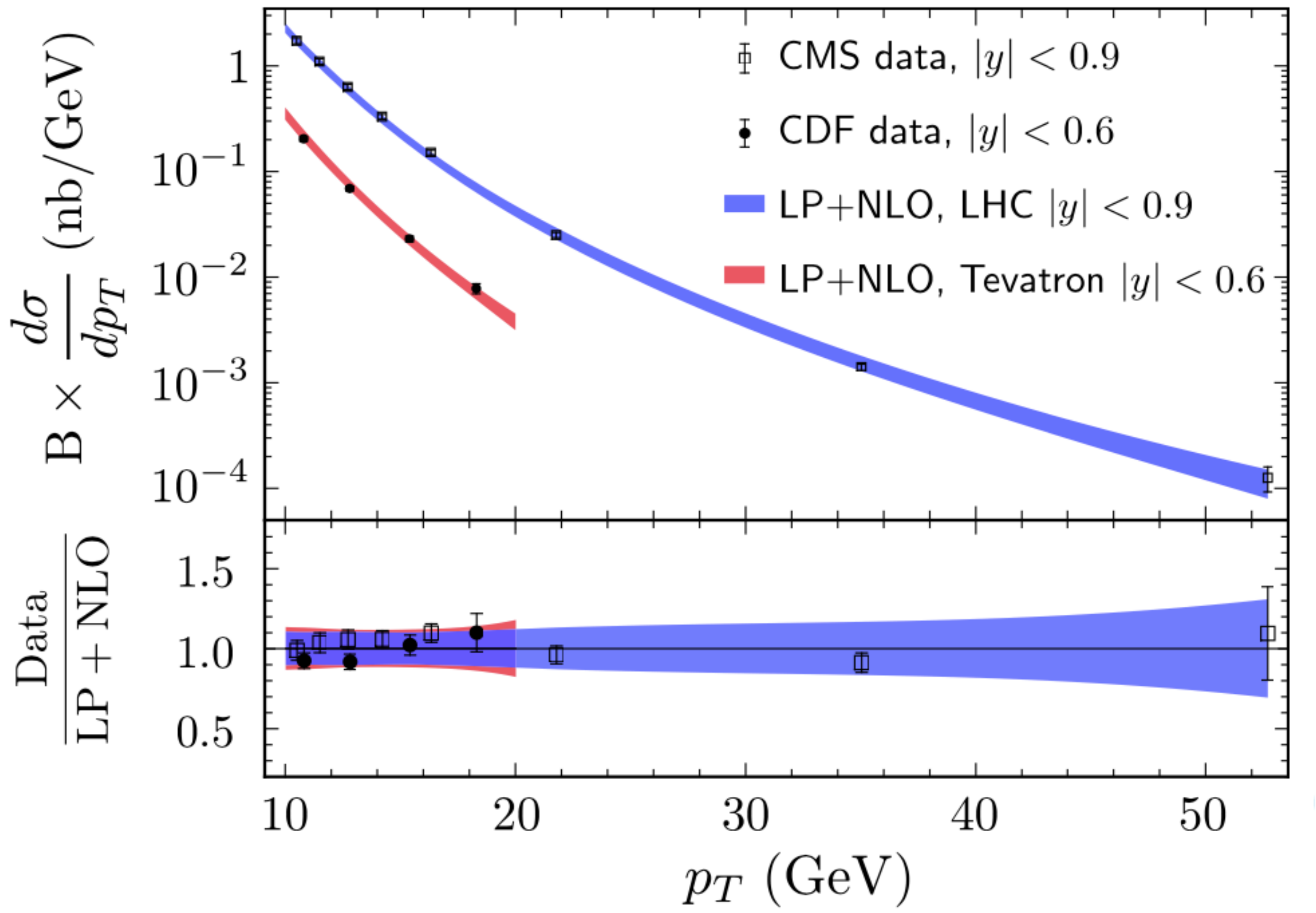
The XXVIII DIS workshop@SBU, 04/13/2021

NRQCD vs. data









Butenschoen, Kniehl, PRD84, 051501 (2011).
Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012).
Gong, Wan, Wang, Zhang, PRL110, 042002 (2013).
Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014).

	$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3P_0^{[8]})\rangle$ 10^{-2} GeV^5
Set I (Butenschoen *et al.*)	1.32	3.04	0.16	−0.91
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LDMEs should be universality, however:

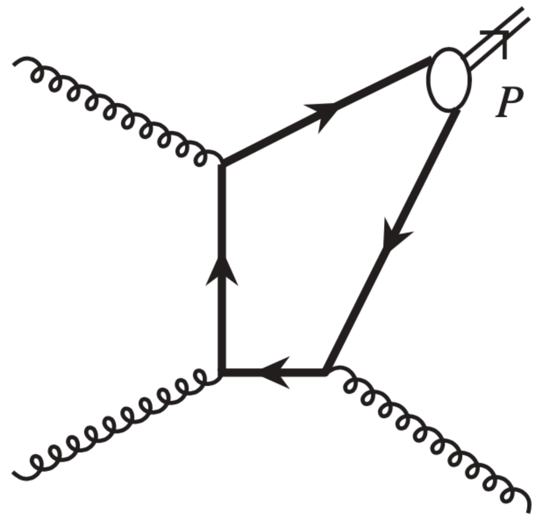
- Numbers are not the same.
- Not even the sign.

More work is needed!

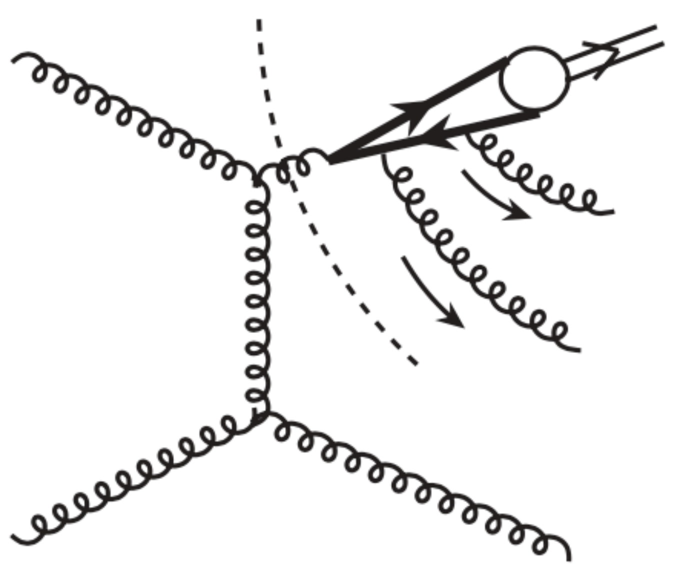
Heavy quarkonium production of high p_T

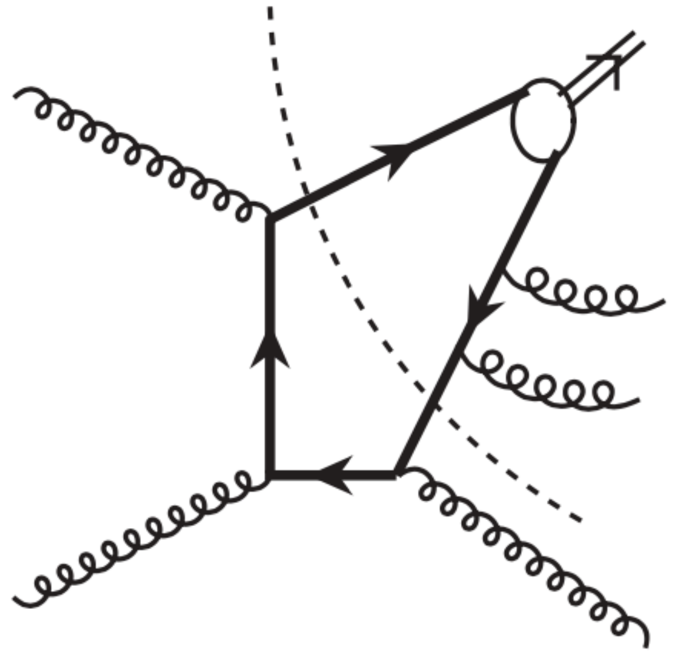
- High p_T quarkonium production ($p_T^2 \gg (2m)^2 \gg \Lambda_{\rm QCD}^2$) allows us to separate the short distance part for a quark pair production and the long distance part for the bound state formation.

- Modern approaches for tackling the bound state formation;



  1. Color Evaporation Model: useful for phenomenology but the long distance part is blinded.
  2. Color Singlet Model (direct production at the early stage): $d\sigma(Q\bar{Q}[^3S_1^{[1]}]) \propto \alpha_s^3 m^4/p_T^8$ at LO.
  3. Color Octet Model or NRQCD factorization approach: Long-Distance Matrix Elements (LDMEs) are organized by the power of the quark velocity $v^2 \sim q_T^2/m^2 < 1$.





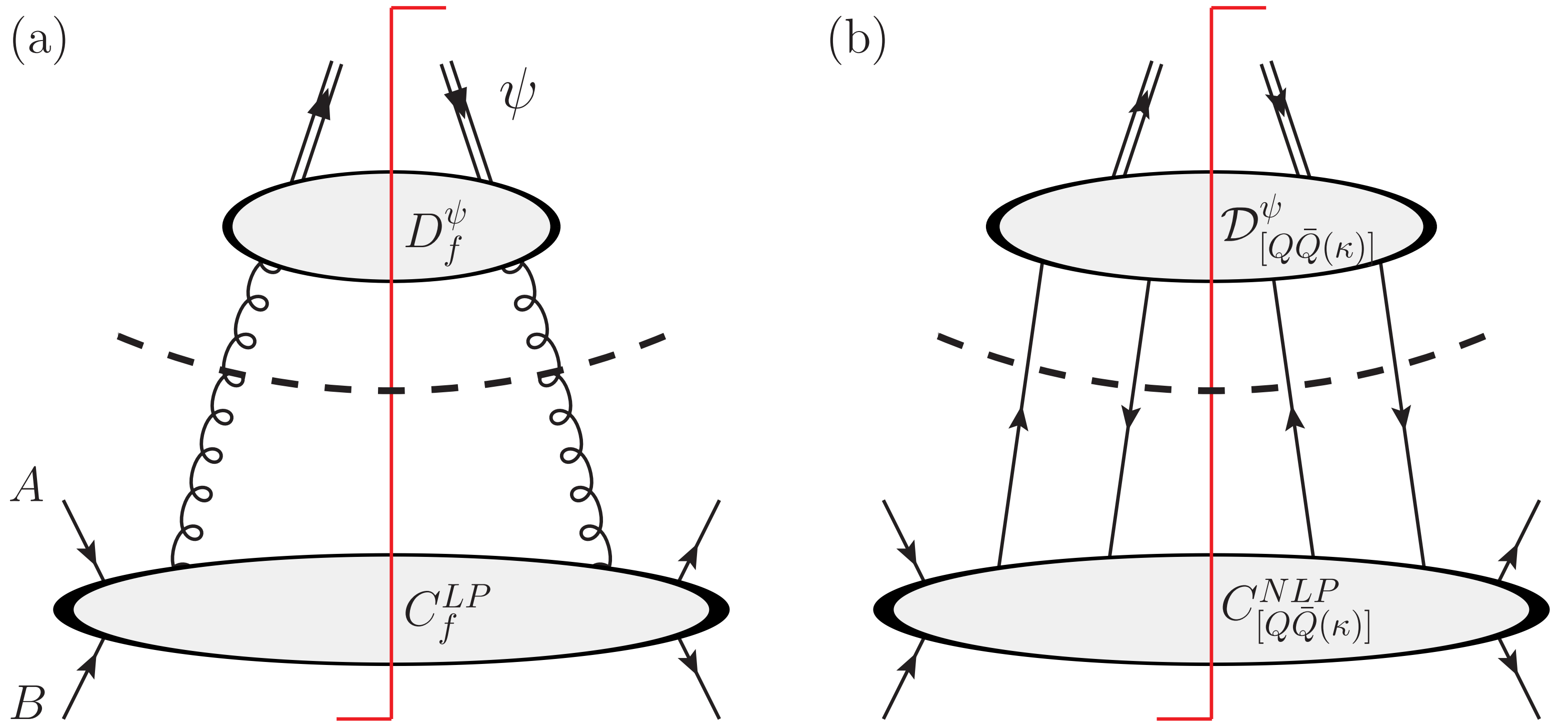
- At high p_T higher order corrections must be essential:
$d\sigma(Q\bar{Q}[^3S_1^{[1]}]) \propto \alpha_s^3 m^4/p_T^8 \times \alpha_s p_T^2/m^2 = \alpha_s^4 m^2/p_T^6$

- More importantly, the gluon jet fragmentation gives $d\sigma \propto \alpha_s^5/p_T^4$ as well as $d\sigma \propto \alpha_s^2/p_\perp^4 \times \alpha_s^3 \ln(p_T^2/m^2)$. The later is enhanced even if $\alpha_s \ll 1$; we may not obtain reliable predictions by considering only diagrams in the naive α_s expansion as well as v expansion.

- We shall study high p_T quarkonium production in the QCD factorization by employing fragmentation functions.

QCD factorization approach

Leading power (LP)



Nayak, Qiu, Sterman, PRD72 (2005) 114012
Kang, Qiu, Sterman, PRL108 (2012) 102002
Kang, Ma, Qiu, Sterman, PRD90 (2014) 3, 034006, PRD91 (2015) 1, 014030

Subleading power (NLP): critical at moderate $p_T \sim \mathcal{O}(2m)$

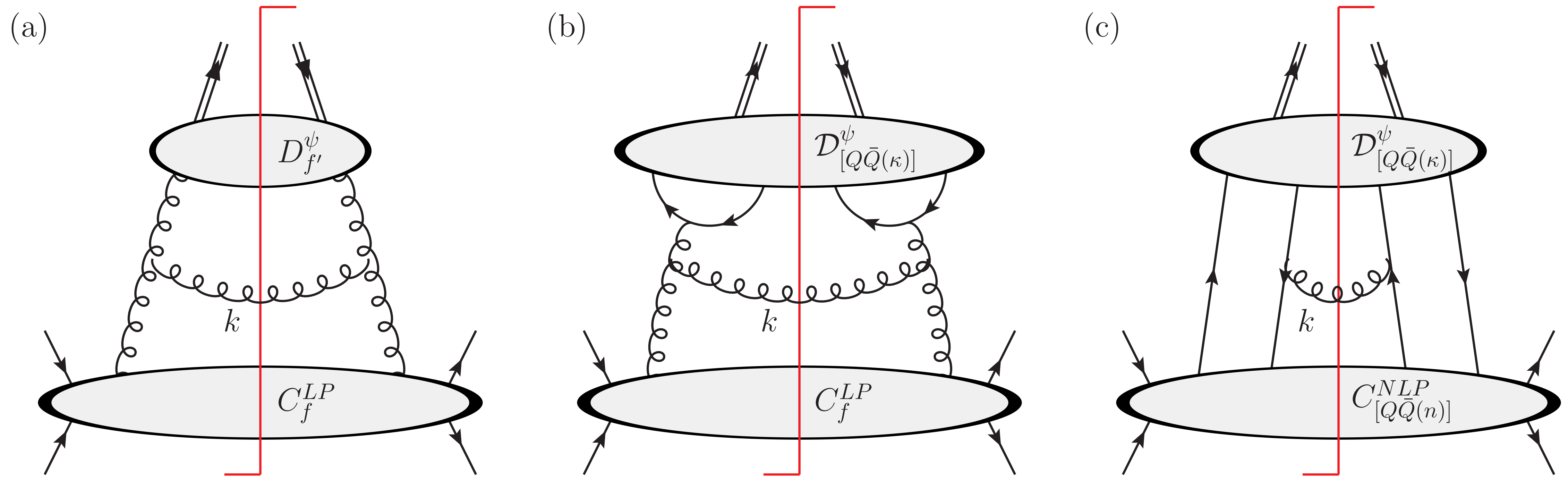
$$d\sigma^{\mathrm{QCD-Res}}_{A+B\to[f,Q\bar{Q}]\to\psi+X}(\mu) = \sum_{f=q,\bar{q},g} C^{\mathrm{LP}}_{A+B\to[f]+X}(\mu) \otimes D_{[f]\to\psi}(\mu) + \frac{1}{p_\perp^2}\left[\sum_n C^{\mathrm{NLP}}_{A+B\to[Q\bar{Q}(n)]+X}(\mu) \otimes \mathscr{D}_{[Q\bar{Q}(n)]\to\psi}(\mu)\right]$$

- In hadronic collisions, short distance coefficients (SDCs) C^{LP} and C^{NLP} are available up to NLO and LO in α_s expansion, respectively. C^{NLP} at NLO has been calculated recently in e^+e^- collisions. Lee, Sterman, JHEP 09 (2020) 046
- Single (twist-2) and double parton (twist-4) fragmentation functions D_f and $\mathscr{D}_{Q\bar{Q}}$ are employed.
- Matching condition:

$$d\sigma_{A+B\to\psi+X}(m\neq 0) = d\sigma^{\mathrm{QCD-Evol}}_{A+B\to\psi+X}(m=0) - d\sigma^{\mathrm{QCD-(n)}}_{A+B\to\psi+X}(m=0) + d\sigma^{\mathrm{NRQCD-(n)}}_{A+B\to\psi+X}(m\neq 0)$$

$$\Rightarrow \begin{cases} d\sigma^{\mathrm{QCD-Evol}}_{A+B\to\psi+X} & \text{when} \quad p_\perp \gg m;\ d\sigma^{\mathrm{NRQCD-(n)}} \approx d\sigma^{\mathrm{QCD-(n)}} \\ d\sigma^{\mathrm{NRQCD-(n)}}_{A+B\to\psi+X} & \text{when} \quad p_\perp \to m;\ d\sigma^{\mathrm{QCD-Evol}} \approx d\sigma^{\mathrm{QCD-(n)}} \end{cases}$$

Nonlinear QCD evolution



- Twist-2 evolution equation: DGLAP + nonlinear term

Kang, Ma, Qiu, Sterman, PRD 90 (2014) 3, 034006

$$\frac{dD_{[f]\to\psi}}{d\ln\mu^2} = \gamma_{[f]\to[f']} \otimes D_{[f']\to\psi} + \frac{1}{\mu^2}\gamma_{[f]\to[Q\bar{Q}(\kappa)]} \otimes \boxed{\mathscr{D}_{[Q\bar{Q}(\kappa)]\to\psi}}$$

- Twist-4 "DGLAP like" evolution equation:

$$\boxed{\frac{d\mathscr{D}_{[Q\bar{Q}(n)]\to\psi}}{d\ln\mu^2}} = \Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]} \otimes \mathscr{D}_{[Q\bar{Q}(\kappa)]\to\psi}$$

Ma, Qiu, Zhang, PRD89 (2015) 094029, 094030

- Input fragmentation functions:

Long Distance Matrix Elements (LDMEs)

$$D_{f\to\psi}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \pi\alpha_s \left\{ \hat{d}^{(1)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(2)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathscr{O}(\alpha_s^2) \right\} \frac{\boxed{\langle \mathscr{O}^{\psi}_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}}{m^{2L+3}}$$

$$\mathscr{D}_{[Q\bar{Q}(\kappa)]\to\psi}(z,u,v;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,u,v;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,u,v;m,\mu_0,\mu_\Lambda) + \mathscr{O}(\alpha_s^2) \right\} \frac{\langle \mathscr{O}^{\psi}_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+1}}$$

$\mu_0 = \mathscr{O}(2m)$: pQCD factorization scale, $\mu_\Lambda = \mathscr{O}(m)$: NRQCD factorization scale

Preliminaries

i) Fixed order calculations in the NRQCD factorization at NLO

- Polarizations and p_T spectra of quarkonia can be reproduced in the NRQCD at NLO, however, it depends on LDMEs.

- Transverse components of SDCs in the NRQCD for producing $Q\bar{Q}$ in P-state can be negative: cancellations between different intermediate states happen.

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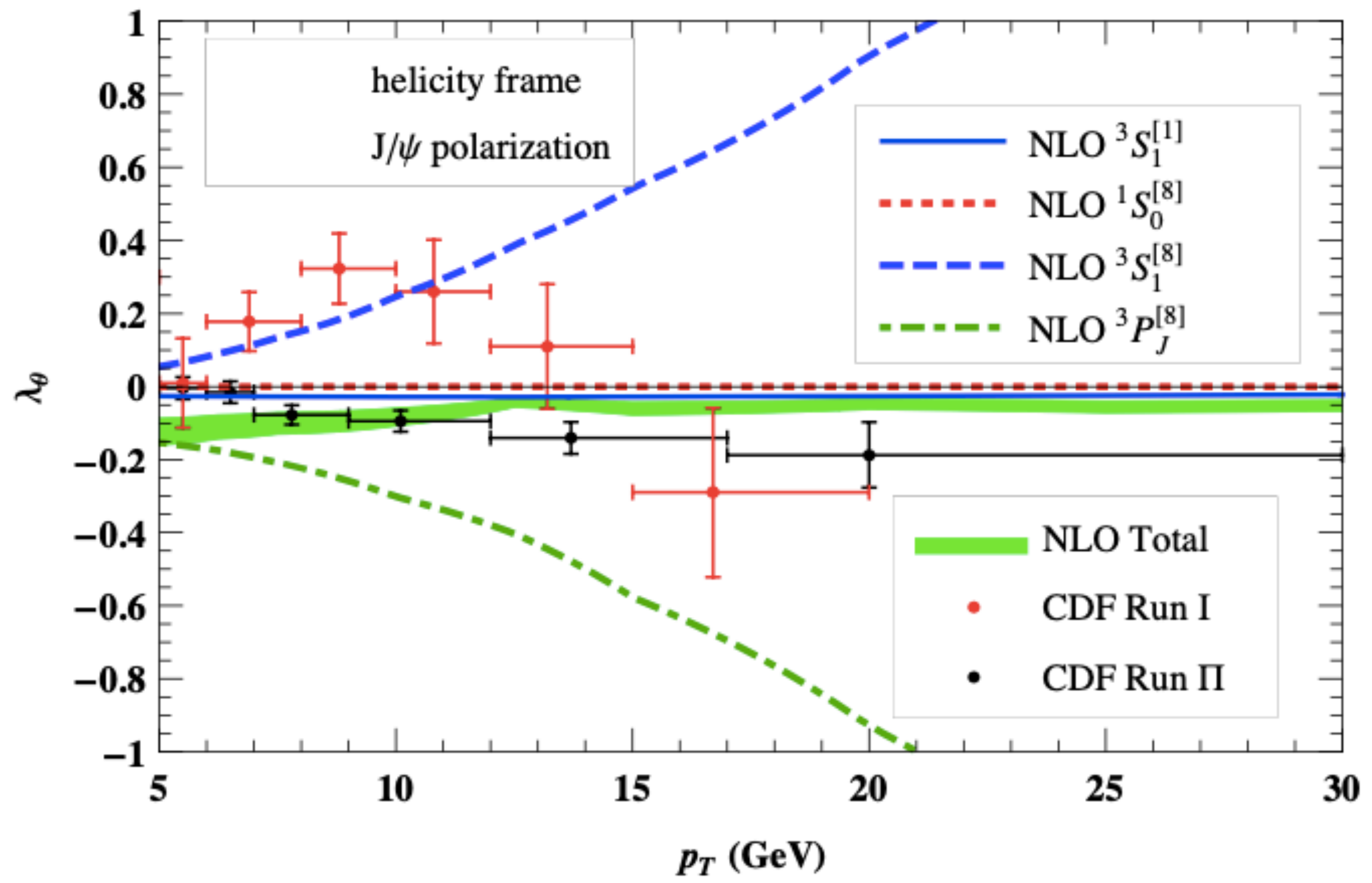
Chao, Ma, Shao, Wang, Zhang, PRL108 (2012) 242004



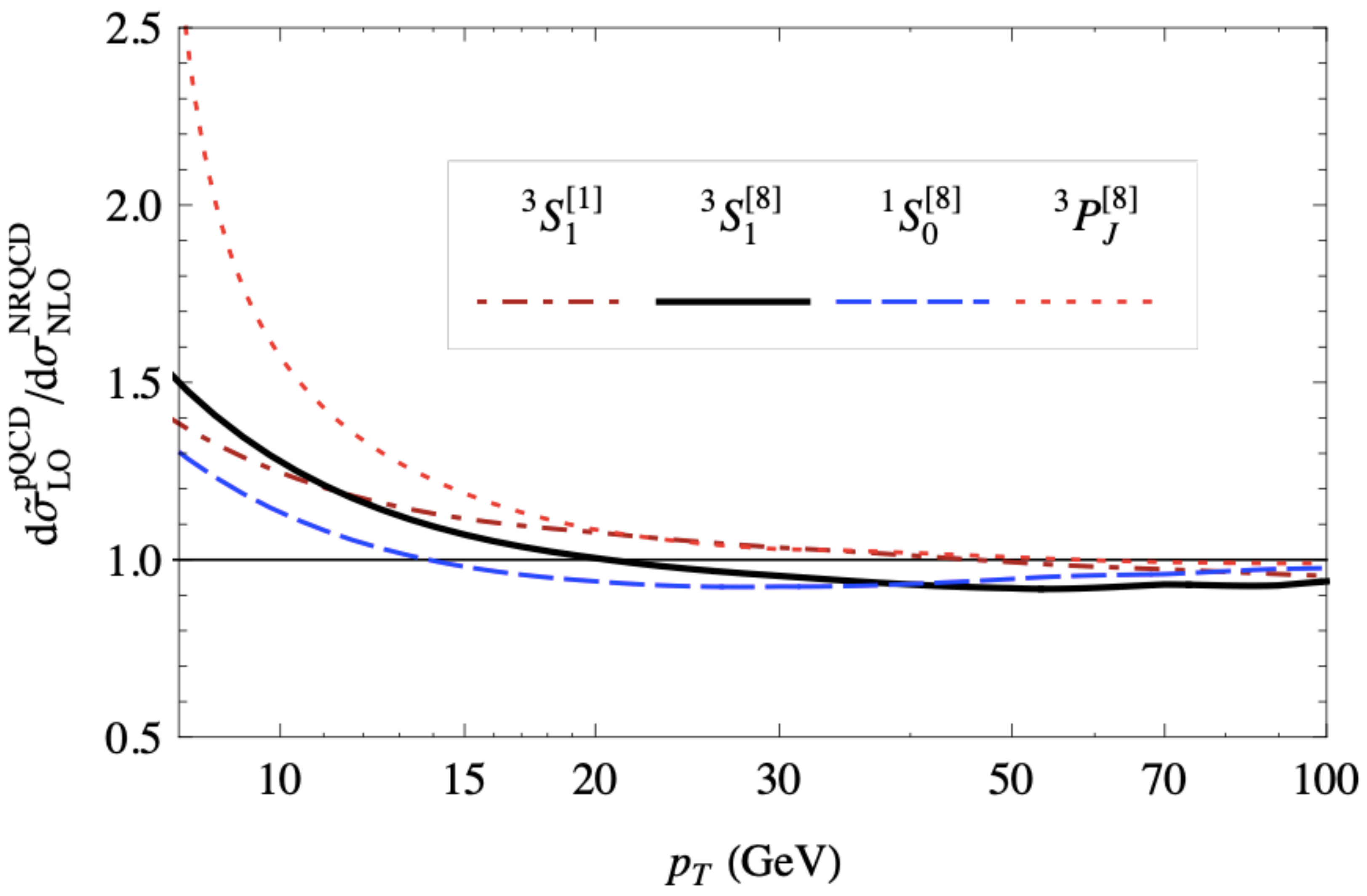
ii) QCD factorization at LO with input FFs

- LP (NLP) SDCs at LO (α_s^2 (α_s^3)) combined with LP (NLP) input FFs at NLO (α_s^2 (α_s)) have the same powers of α_s as the NLO NRQCD ($\alpha_s^4 v^4$).

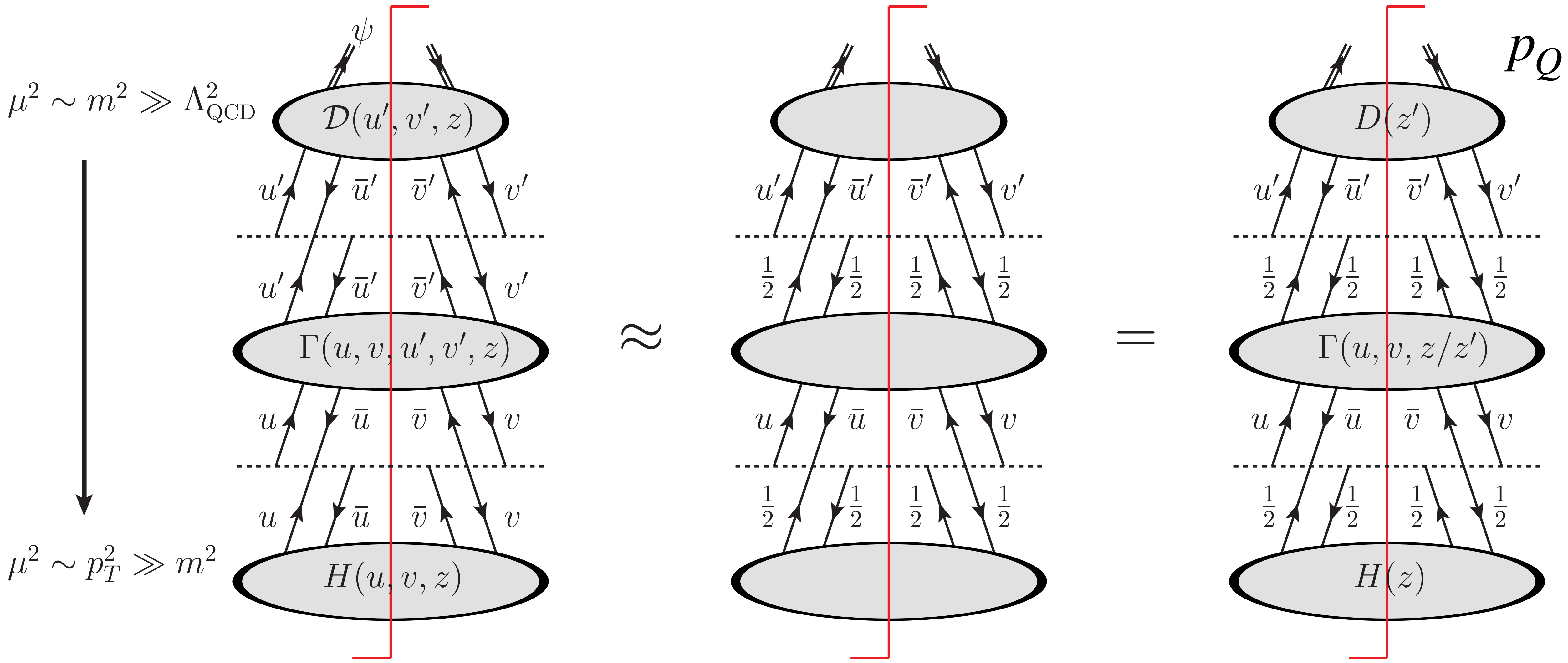
- LO QCD factorization results with input FFs reproduce complicated NRQCD results at NLO channel-by-channel.

- Need to include the QCD evolution effects.

Ma, Qiu, Sterman, Zhang, PRL113 (2014) 14, 142002



Evolution equations in a simplified situation



$p_Q = u' p_c,\ p_{\bar{Q}} = (1-u')p_c,\ z p_c^+ = p^+$

- The cross section and FFs involve multi-integral variables; numerical calculations get complicated.
- Since the produced heavy quark pair is dominated by its on-shell state, we may expand the SDCs and evolution kernels on lower virtuality sides at each evolution step around $u = v = 1/2$.

$$\frac{d\sigma^{\psi}_{\mathrm{NLP}}}{dy d^2 p_T} = \int dz du dv\, H_{[Q\bar{Q}]}(p_Q, p_{\bar{Q}}, \mu)\, \mathscr{D}_{[Q\bar{Q}]\to\psi}(u, v, z, \mu) \approx \int dz\, H_{[Q\bar{Q}]}(\hat{p}_Q^+ = \frac{1}{2} p_c^+, \hat{p}_{\bar{Q}}^+ = \frac{1}{2} p_c^+, \mu) \underbrace{\int du dv\, \mathscr{D}_{[Q\bar{Q}]\to\psi}(u, v, z, \mu)}_{\equiv D_{[Q\bar{Q}]\to\psi}(z,\mu)}$$

$$\frac{\partial}{\partial \ln \mu^2} D_{[Q\bar{Q}(\kappa)]\to\psi}(z, \mu) \approx \sum_n \int_z^1 \frac{dz'}{z'} \int_0^1 du \int_0^1 dv\, \Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\left(u, v, u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to\psi}\left(z', \mu\right)$$

$$\frac{\partial}{\partial \ln \mu^2} D_{f\to\psi}(z, \mu) \approx \frac{\alpha_s}{2\pi} \sum_{f'} \int_z^1 \frac{dz'}{z'} P_{f\to f'}(z/z') D_{f'\to\psi}(z') + \frac{\alpha_s^2(\mu)}{\mu^2} \sum_{[Q\bar{Q}(\kappa)]} \int_z^1 \frac{dz'}{z'} P_{f\to[Q\bar{Q}(\kappa)]}\left(u' = \frac{1}{2}, v' = \frac{1}{2}, \frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to\psi}\left(z', \mu\right)$$

We solve these evolution equations in moment space numerically. Inverse Mellin transformation gets solutions in moment space back to those in position space.

Input FFs

Perturbative SDCs of input FFs in α_s and v expansion in the NRQCD are reliable only when SDCs $\ll \mathcal{O}(1)$. Indeed, the NRQCD factorization is not reliable as $z \to 1$ where SDCs $\hat{d}(z)$ include the following terms:

1. $\delta(1-z)$ at LO in α_s expansion
2. $f(z)\ln(1-z)$ with $f(z)$ being a regular function
3. $\dfrac{f(z)}{[1-z]_+}, f(z)\left[\dfrac{\ln(1-z)}{1-z}\right]_+$ due to the perturbative cancelation of IR divergences

- For example, the transition probability $P(g \to \psi)$ should vanish at $z = 1$, but not $\delta(1-z)$. A color octet gluon needs some phase to radiate soft particles to help neutralize the color of the quark pair, so we mimic $\delta(1-z)$ to obtain $P(z \to 1) = 0$. This is the same for $Q\bar{Q} \to \psi$ channels, which have similar terms. We shall use an area under curve (first moment) to give normalization:

$$D^{[\delta]}(z) = C(\alpha_s)\delta(1-z) \to C(\alpha_s)\frac{z^\alpha(1-z)^\beta}{B[1+\alpha, 1+\beta]} \qquad (\alpha \gg 1,\ 1 > \beta > 0)$$

- $\ln(1-z)$-type unphysical divergence should be resummed. We mitigate the divergence as follows:

$$f(z)\ln(1-z) \to f(z)\ln(1-z)\, e^{-c\ln^2(1-z)} \qquad (c \sim \mathcal{O}(\alpha_s))$$

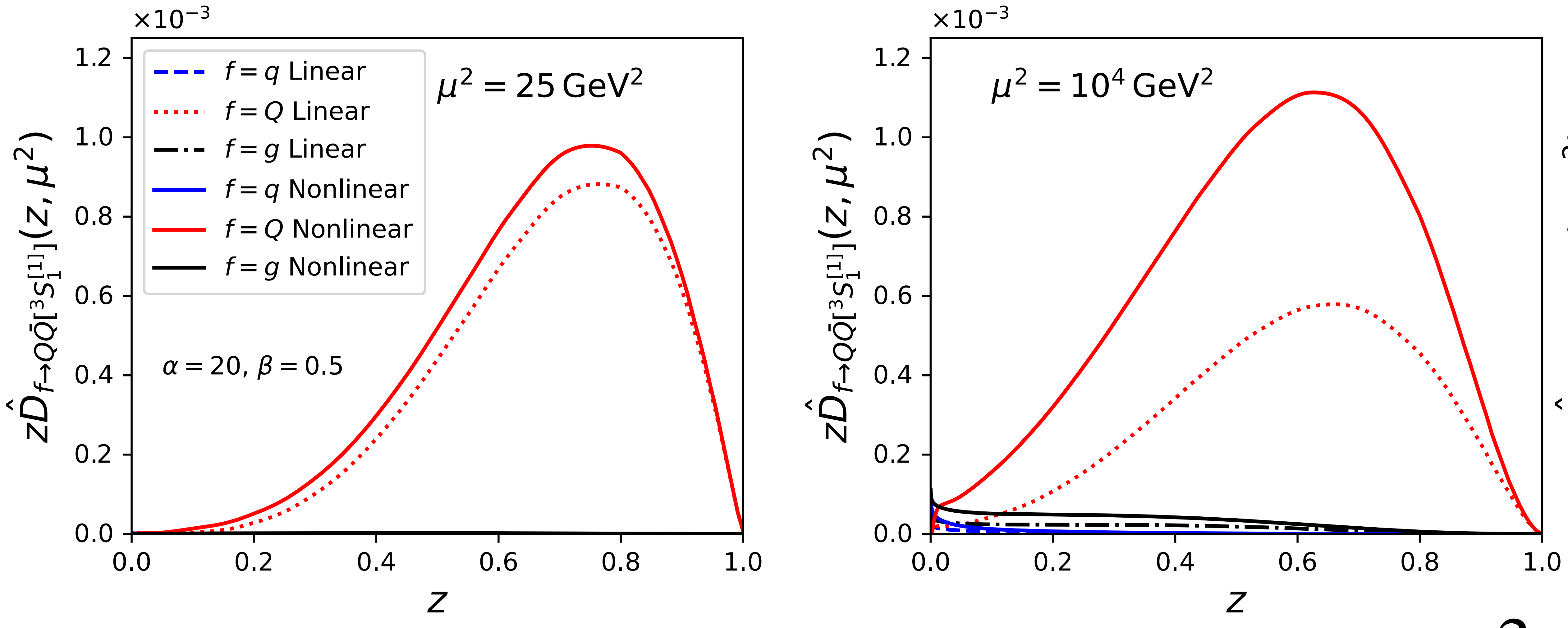
- Plus-functions are defined only under integration over z, so we add their first moments to input FFs as we do for the $\delta(1-z)$ term.

- We apply the above treatments to both single and double parton input FFs for any channels.

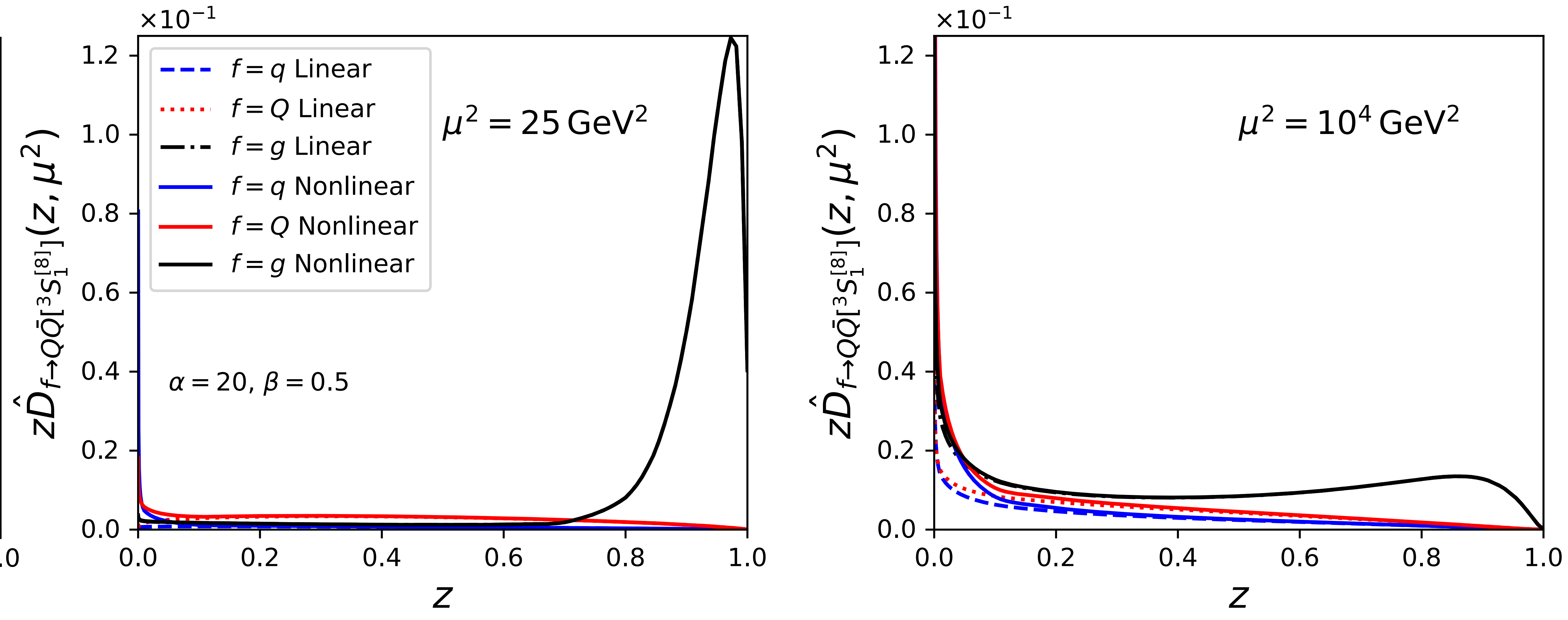
The scale evolution of single parton FFs

Preliminary results

$f \to Q\bar{Q}[^3S_1^{[1]}]$

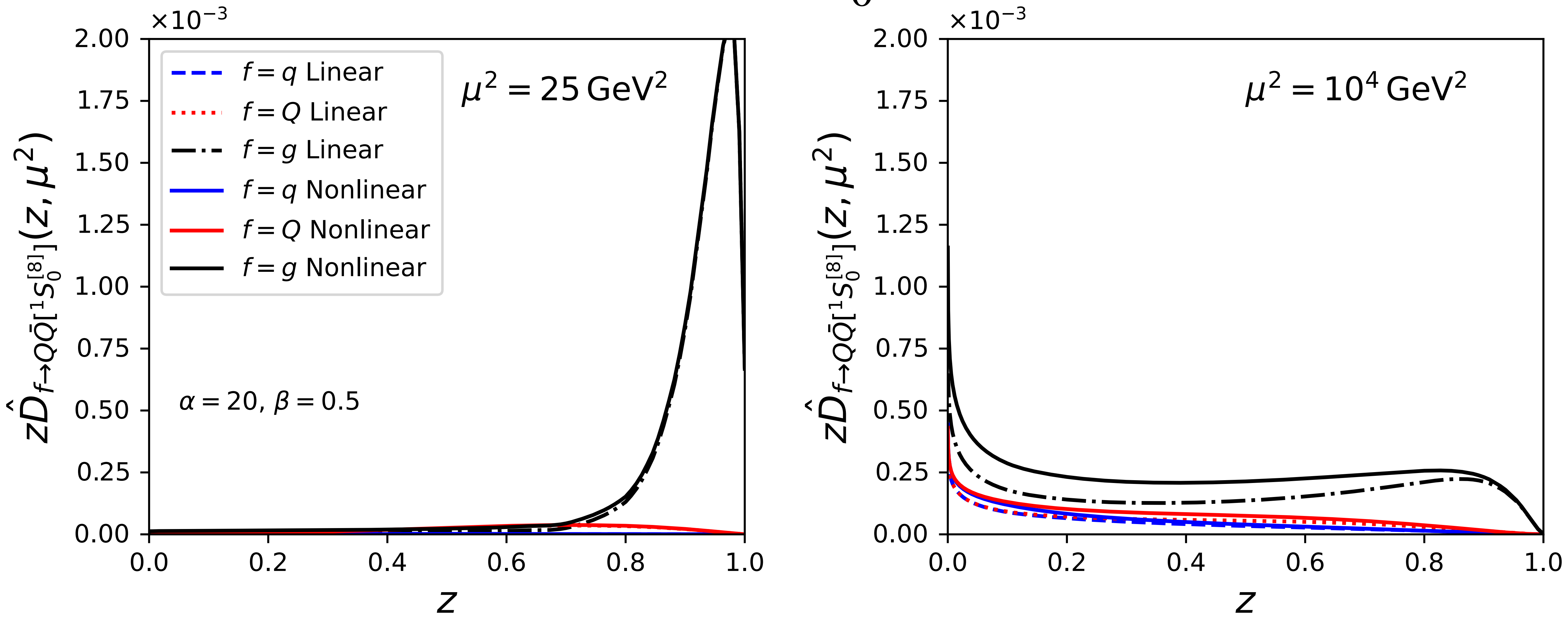


$f \to Q\bar{Q}[^3S_1^{[8]}]$

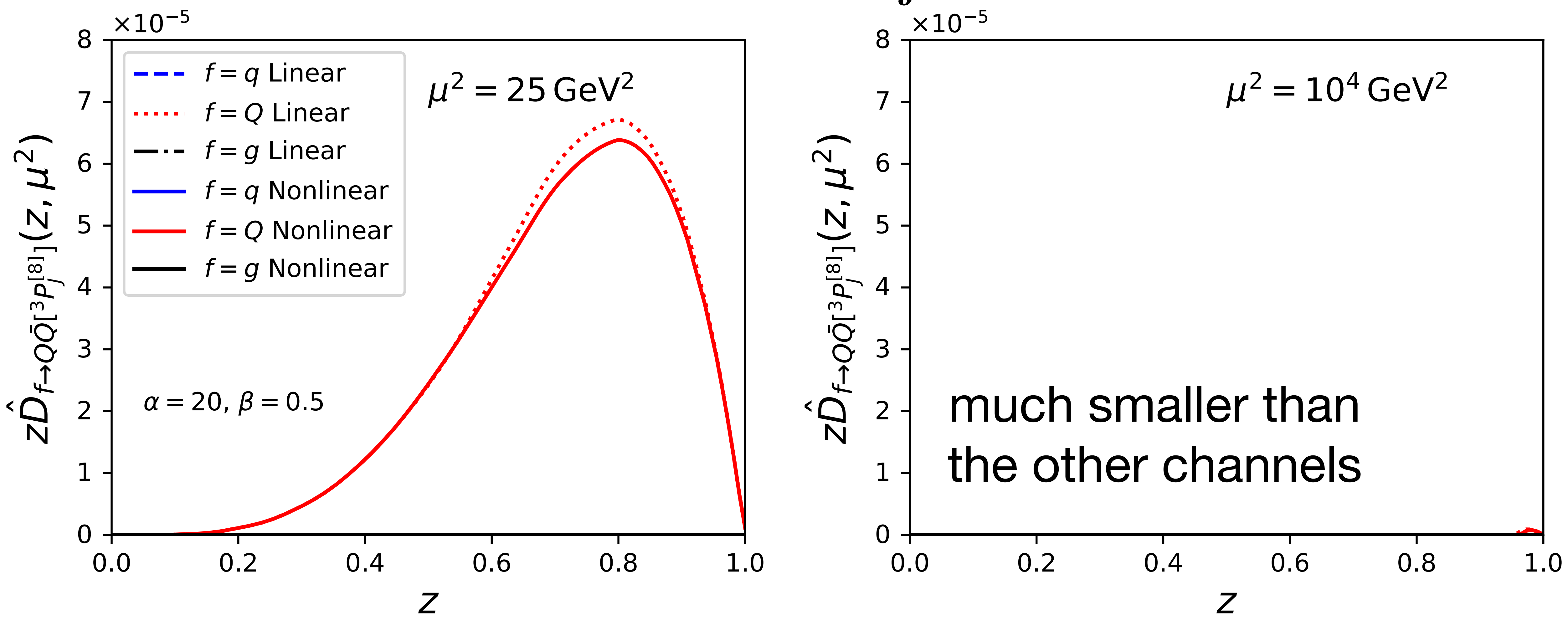


$f \to Q\bar{Q}[^1S_0^{[8]}]$

$$\mu_0 = 3m,\ \mu_\Lambda = m$$



$f \to Q\bar{Q}[^3P_J^{[8]}]$

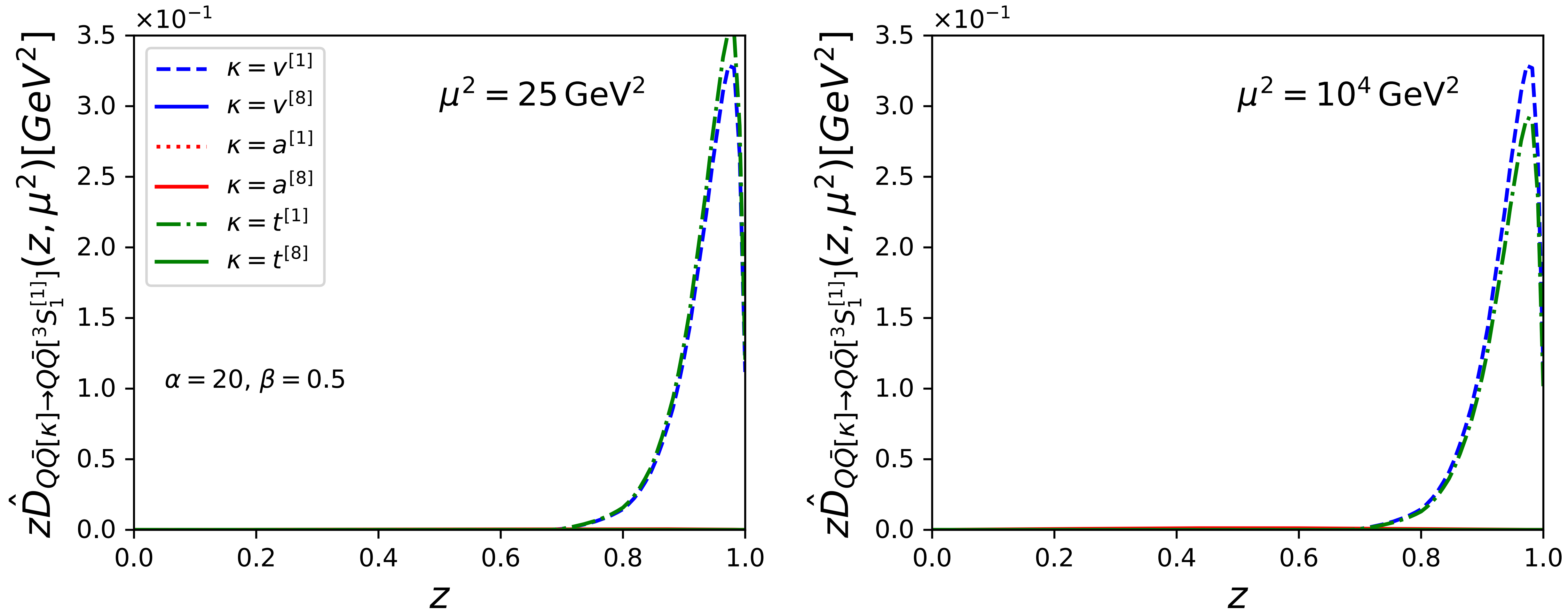


Nonlinear evolution effects are profound in $f \to Q\bar{Q}[^1S_0^{[8]}]$ and $f \to Q\bar{Q}[^3S_1^{[1]}]$.

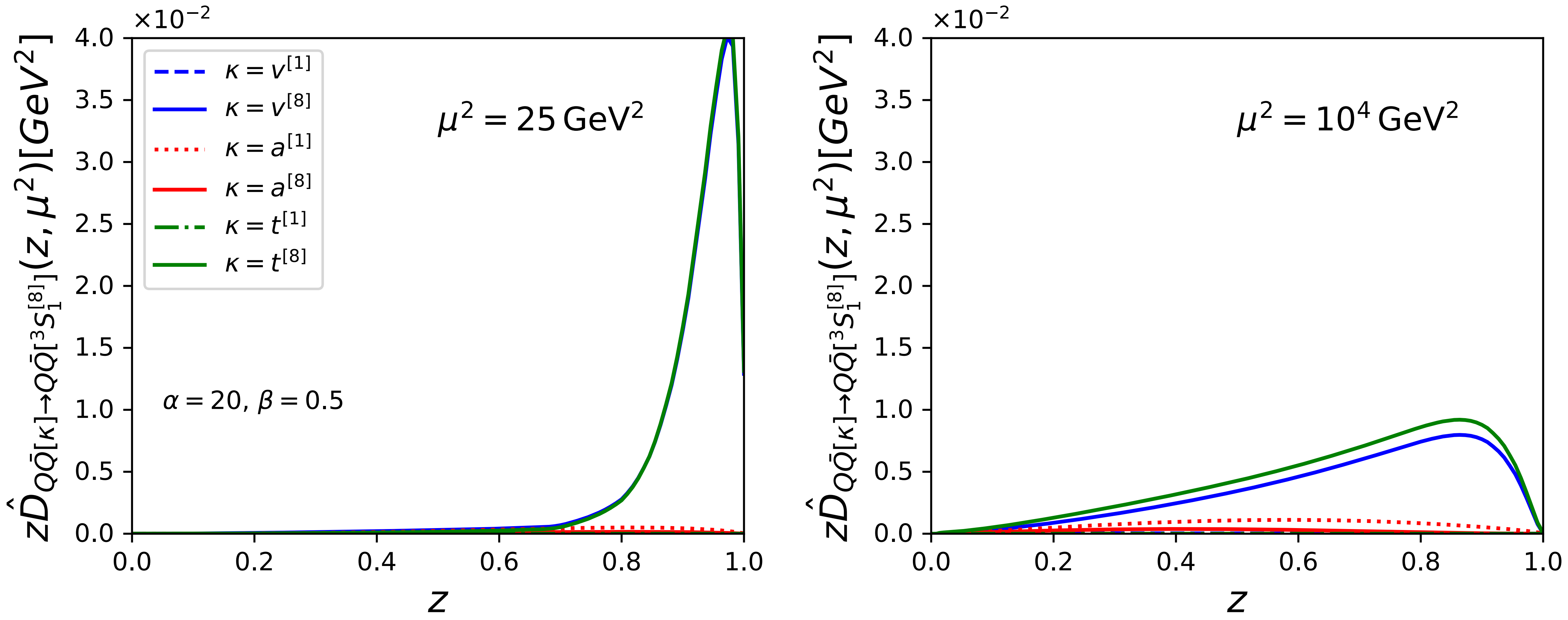
The scale evolution of double parton FFs

Preliminary results

$Q\bar{Q}[\kappa] \to Q\bar{Q}[{}^3S_1^{[1]}]$



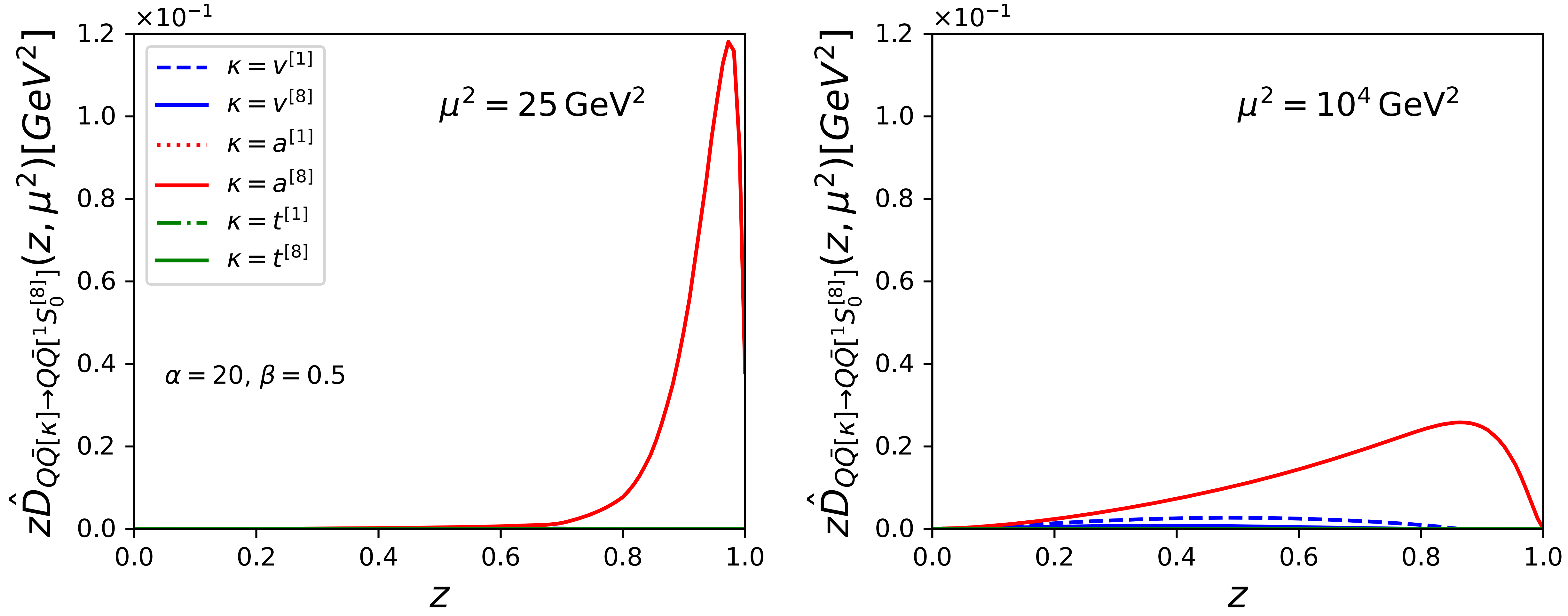
$Q\bar{Q}[\kappa] \to Q\bar{Q}[{}^3S_1^{[8]}]$

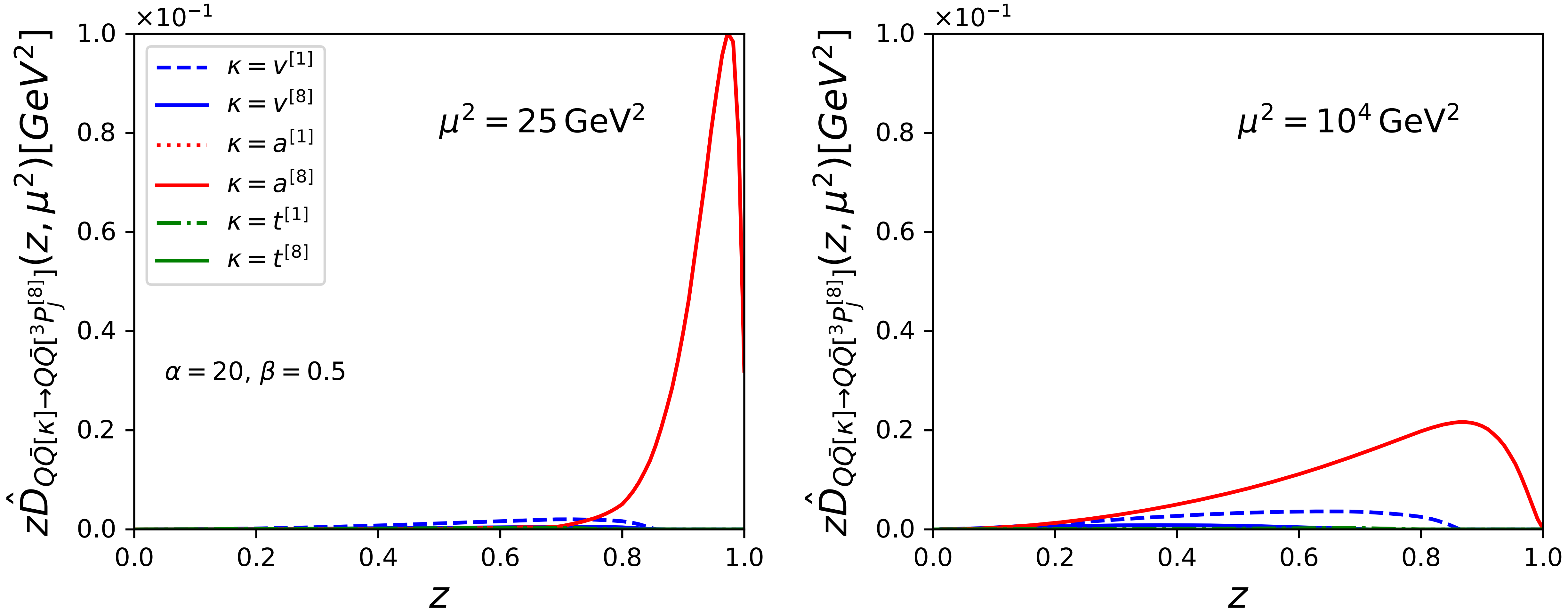


$Q\bar{Q}[\kappa] \to Q\bar{Q}[{}^1S_0^{[8]}]$

$$\mu_0 = 3m,\ \mu_\Lambda = m$$

$Q\bar{Q}[\kappa] \to Q\bar{Q}[{}^3P_J^{[8]}]$

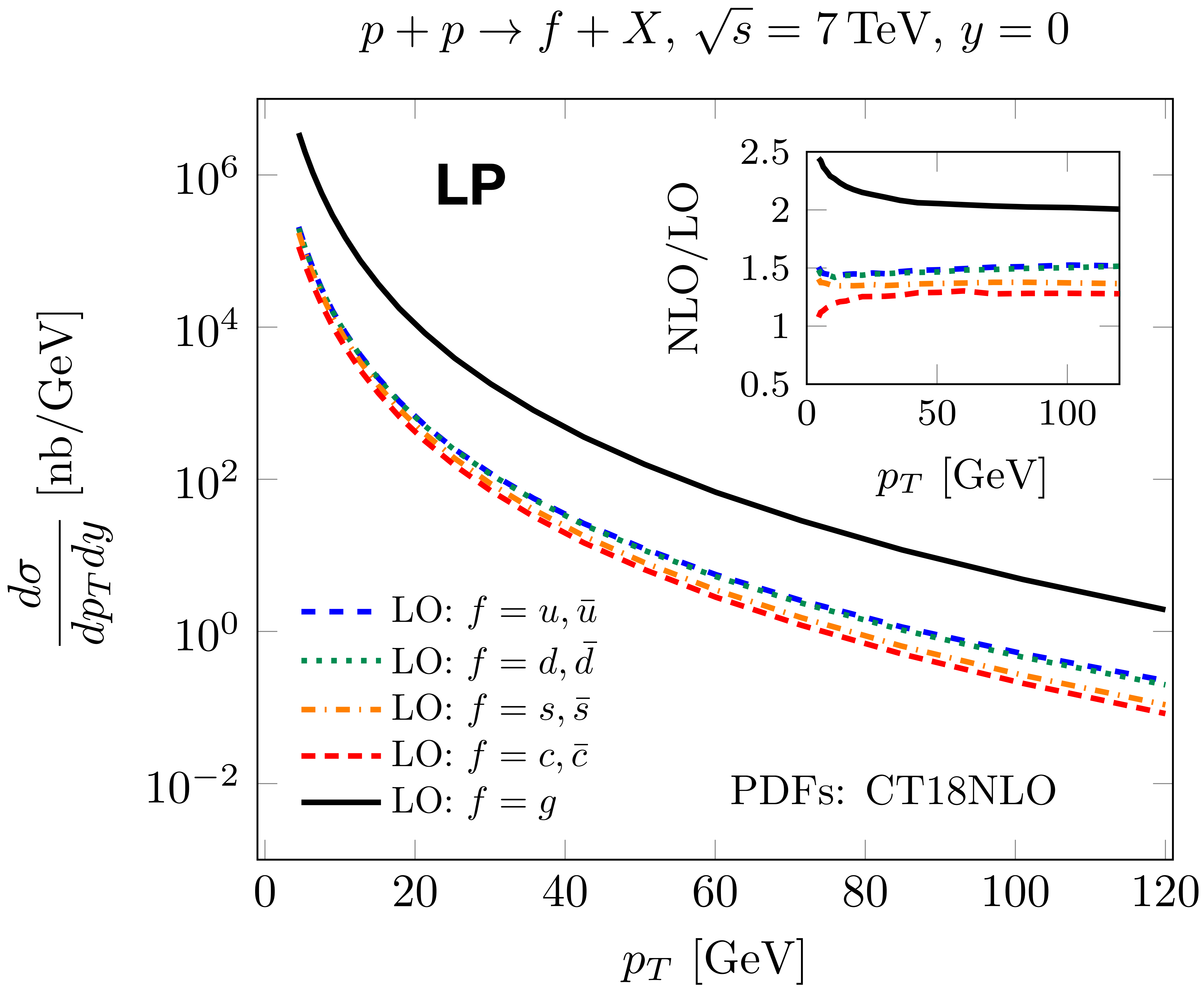




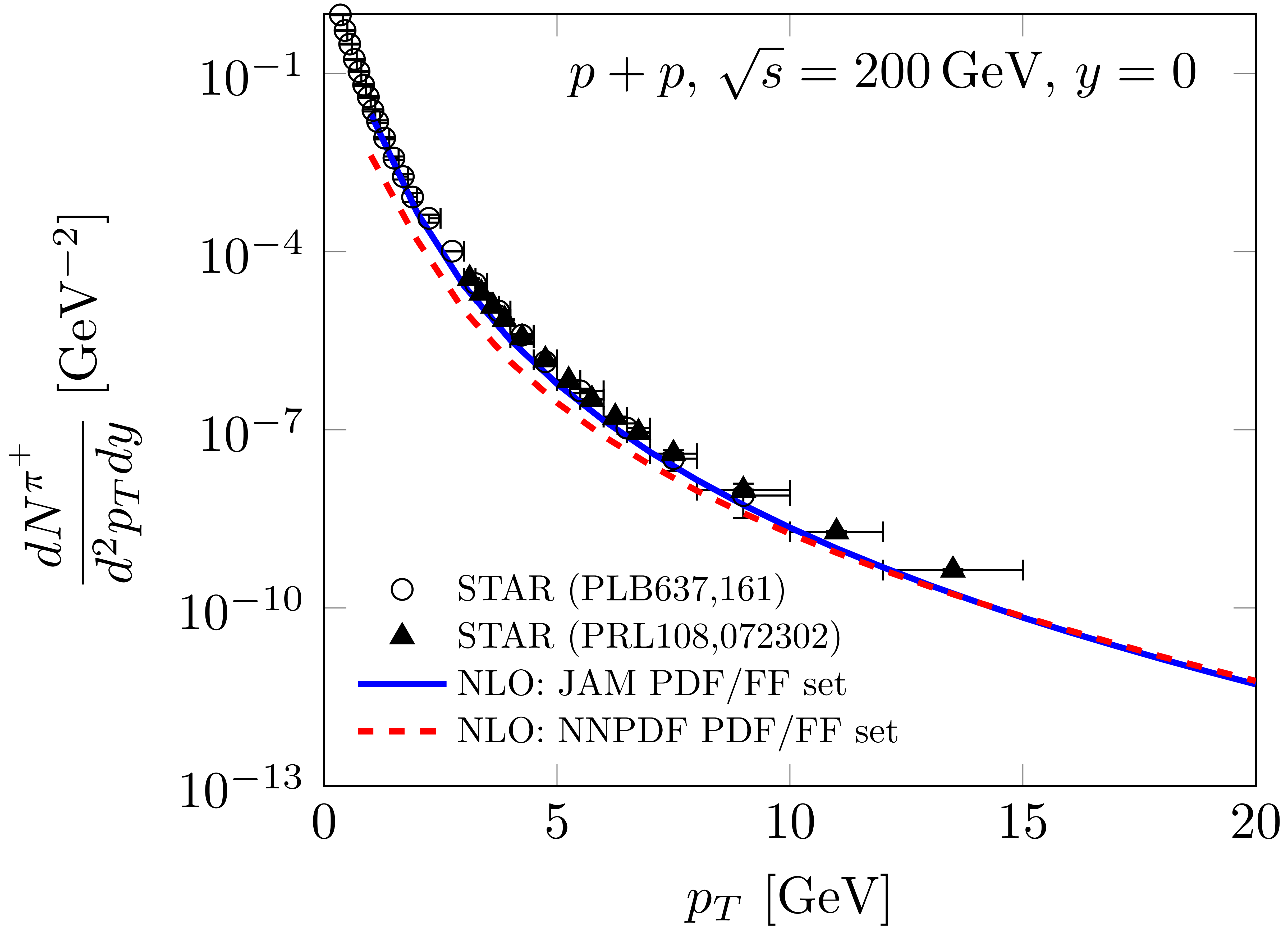
Note that tensor channels $t^{[1]}$ and $t^{[8]}$ are irrelevant in the current simplified setup.

Single parton production cross section

Aversa, Chiappetta, Greco, Guillet, NPB327 (1989) 105



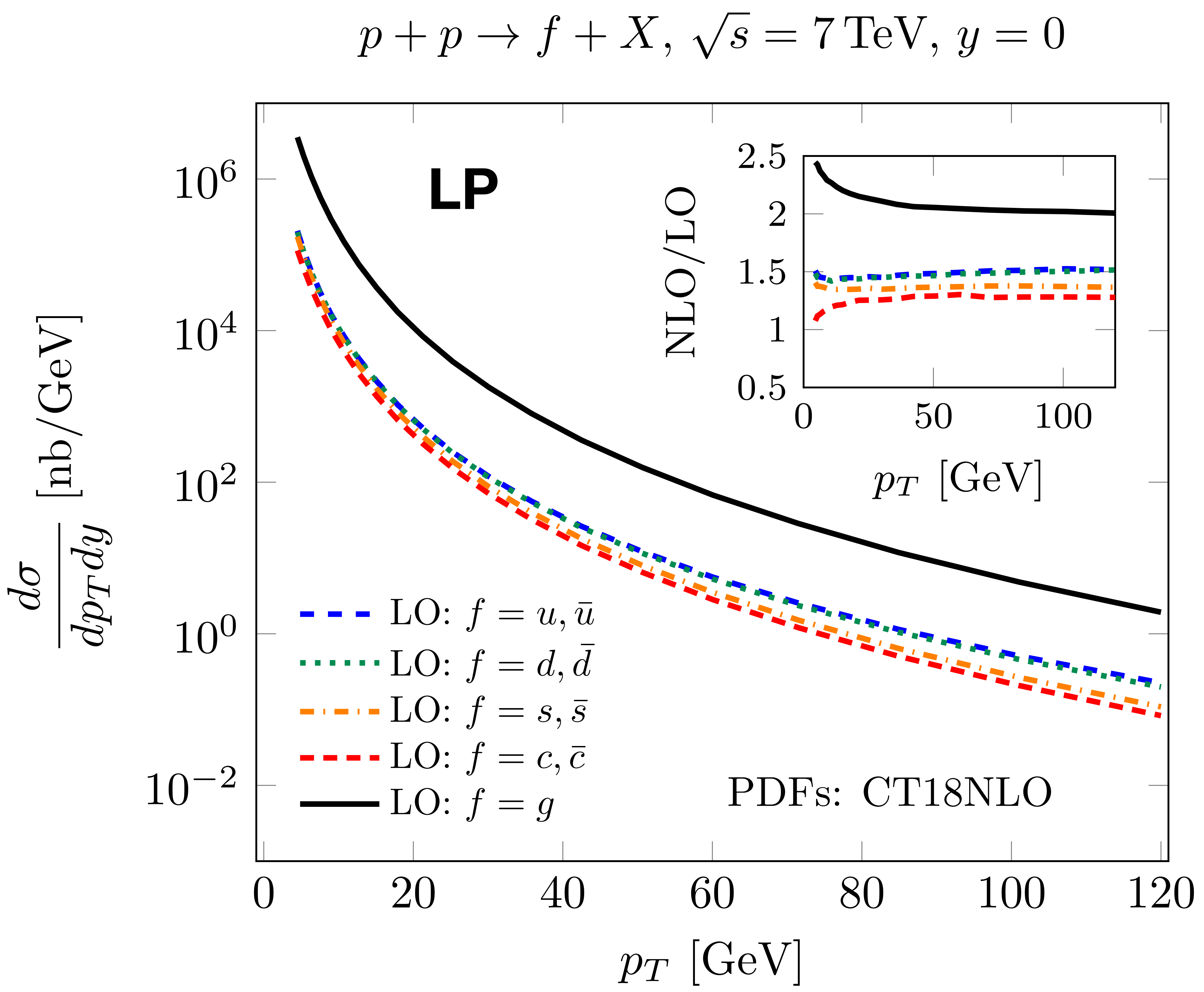
Preliminary results

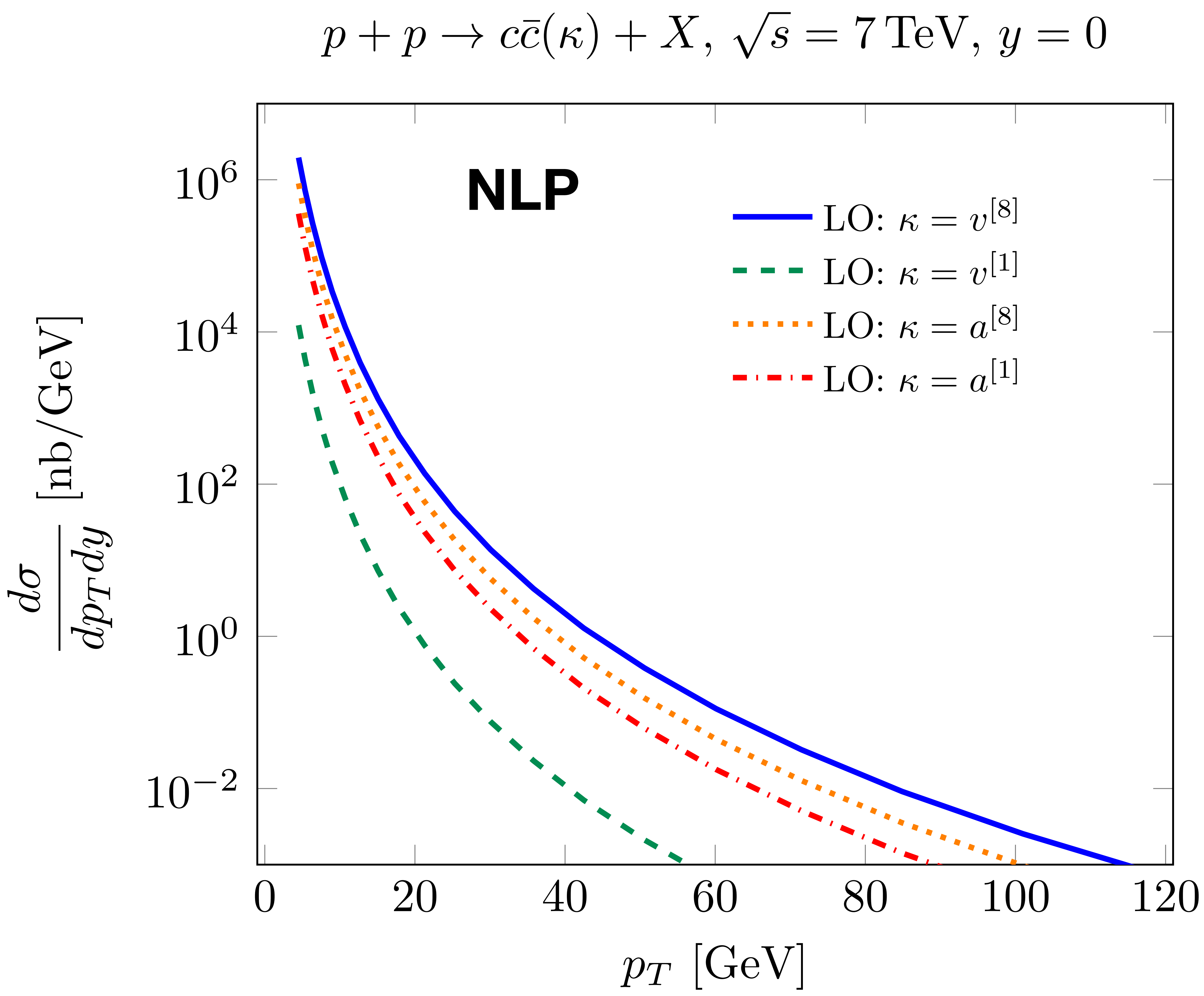


- For the single parton production, gluons are more produced than light and heavy quarks. $K = 1.5 \sim 2$.
- We can reproduce π^+ production yield in pp collisions at RHIC using the LP single parton cross section.

Power corrections at twist-4

Preliminary results

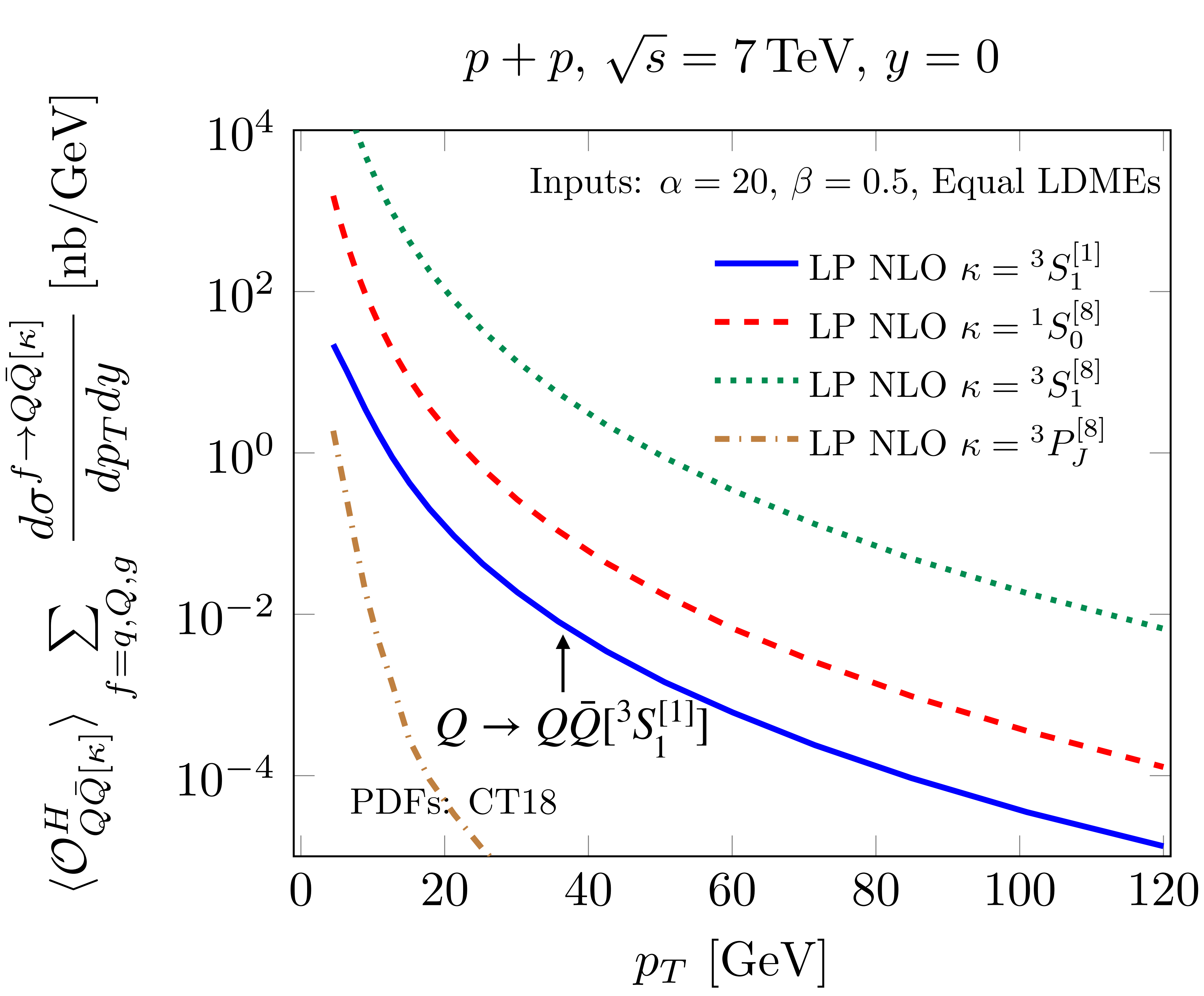


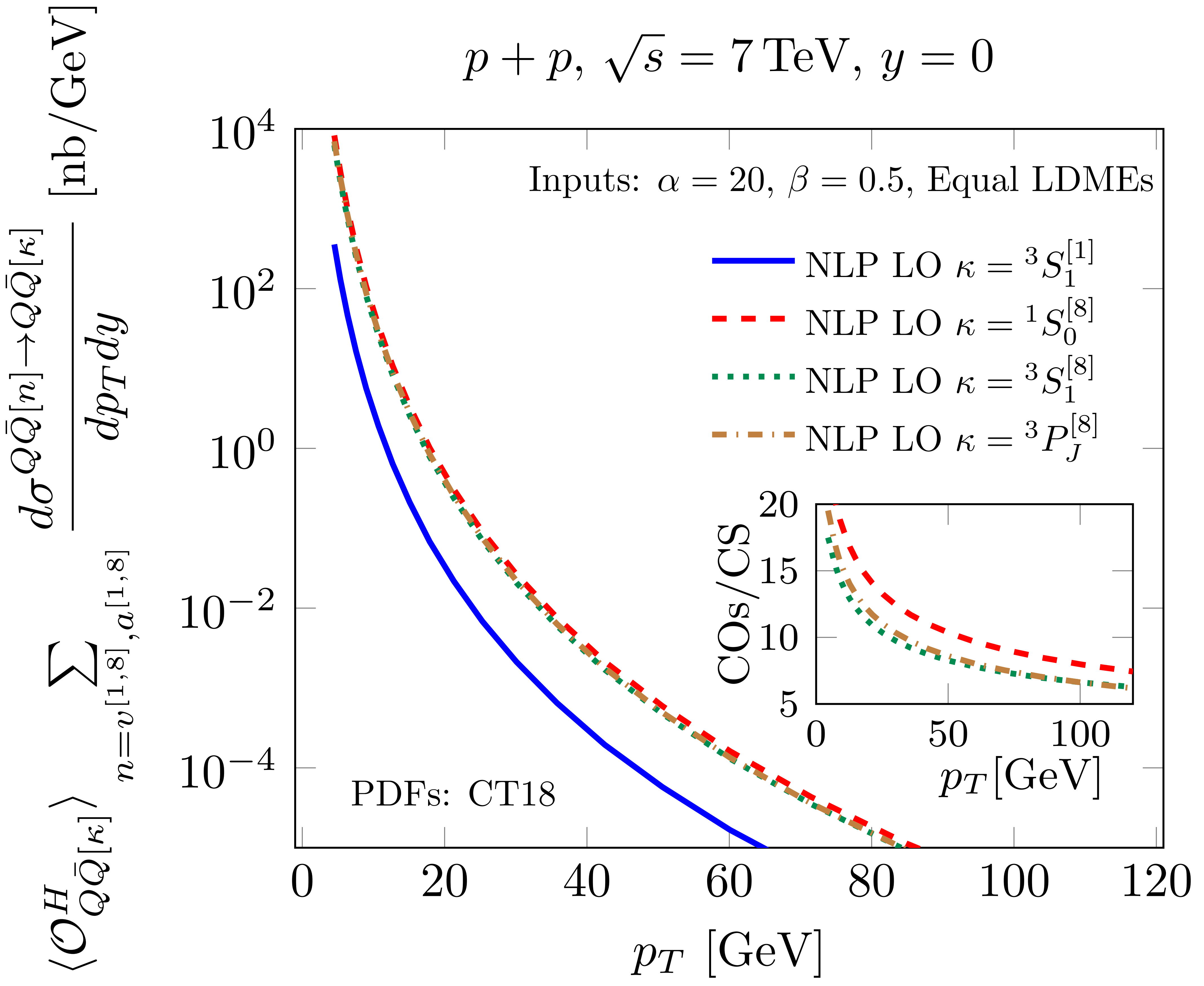


- Power corrections are suppressed by $1/p_T^2$ but sizable at moderate p_T ($p_T \sim \mathcal{O}(2m)$).
- For the double parton production, $v^{[8]}, a^{[8]}, a^{[1]}$ are important. The SDCs for $t^{[8]}$ and $t^{[1]}$ vanish.

Convolution with FFs w/o LDMEs

Preliminary results





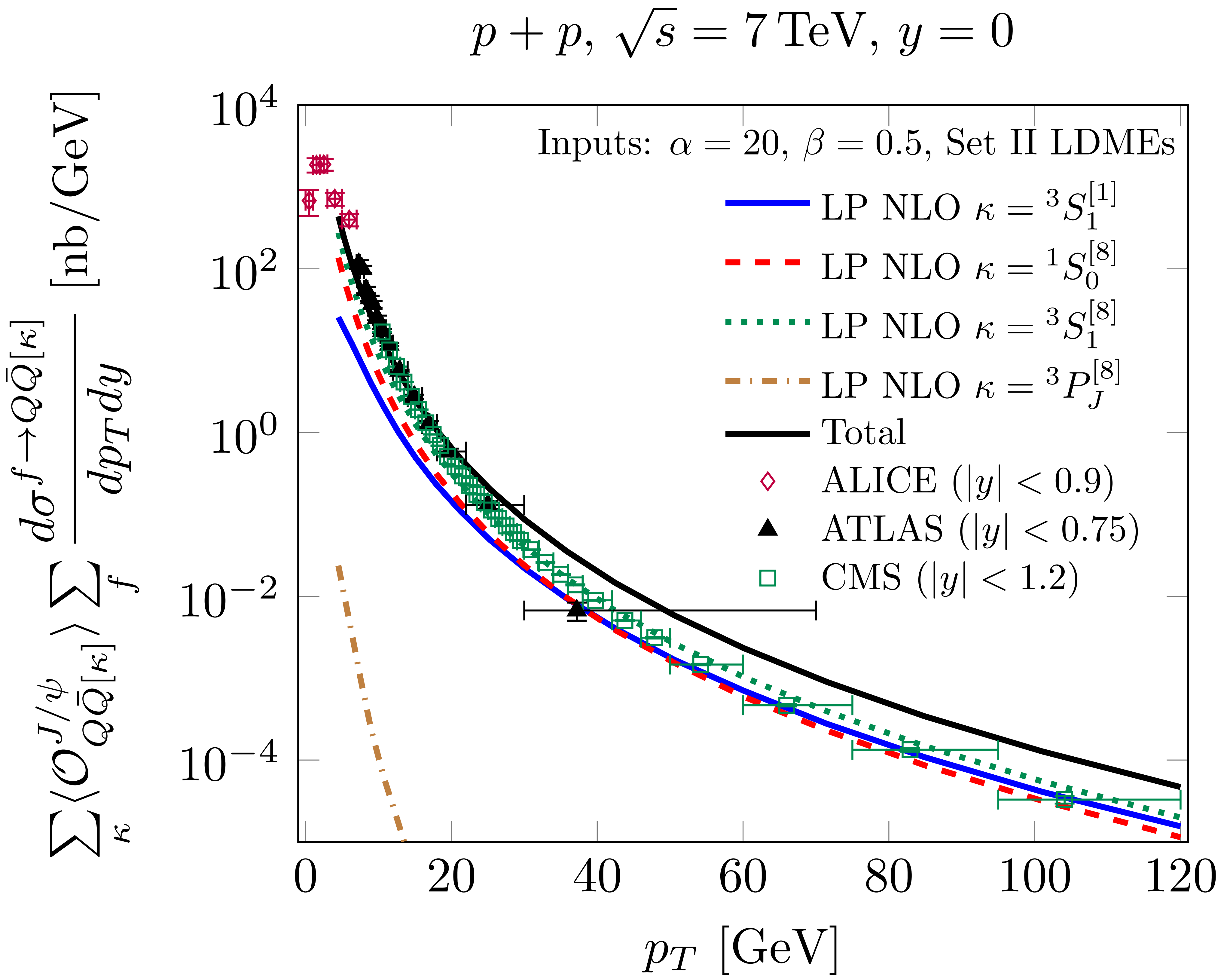
'Equal' weights $\quad \langle\mathcal{O}({}^3S_1^{[1]})\rangle/\mathrm{GeV}^3 = \langle\mathcal{O}({}^1S_0^{[8]})\rangle/\mathrm{GeV}^3 = \langle\mathcal{O}({}^3S_1^{[8]})\rangle/\mathrm{GeV}^3 = \langle\mathcal{O}({}^3P_0^{[8]})\rangle/\mathrm{GeV}^5 = 1$

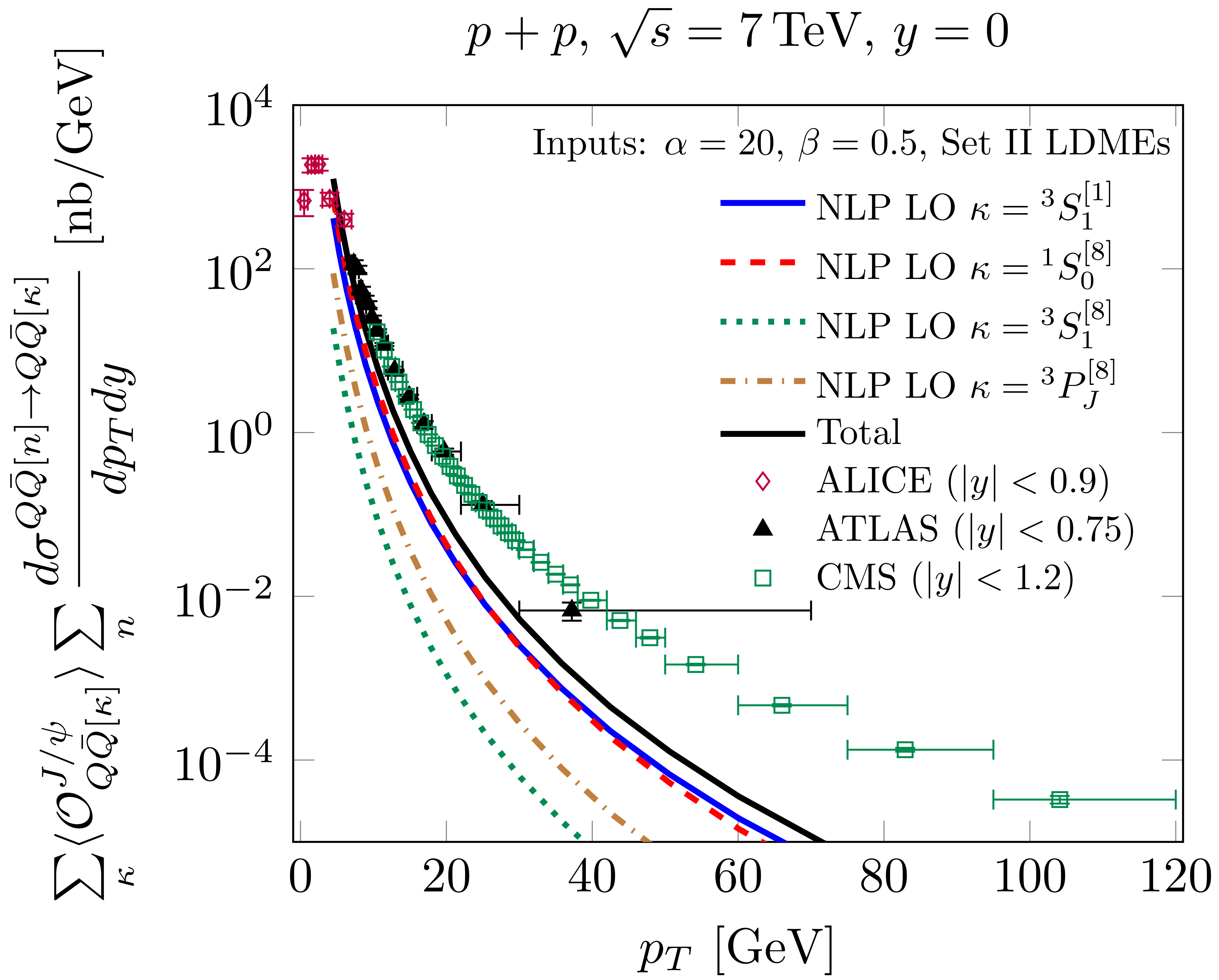
We have to sum the leading four channels with relative weights c_κ: $\langle\mathcal{O}(\kappa)\rangle/\mathrm{GeV}^{3\text{ or }5} = c_\kappa$

$$\sum_{J=0,1,2} D_{{}^3P_J^{[8]}} = \boxed{\langle\mathcal{O}({}^3P_0^{[8]})\rangle}\left[\hat{D}_{{}^3P_0^{[8]}} + 3\hat{D}_{{}^3P_1^{[8]}} + 5\hat{D}_{{}^3P_2^{[8]}}\right]$$

p_T spectra of hadronic J/ψ production

Preliminary results





	$\langle\mathcal{O}({}^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}({}^1S_0^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}({}^3S_1^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}({}^3P_0^{[8]})\rangle$ 10^{-2} GeV^5
Set I (Butenschoen *et al.*)	1.32	3.04	0.16	−0.91
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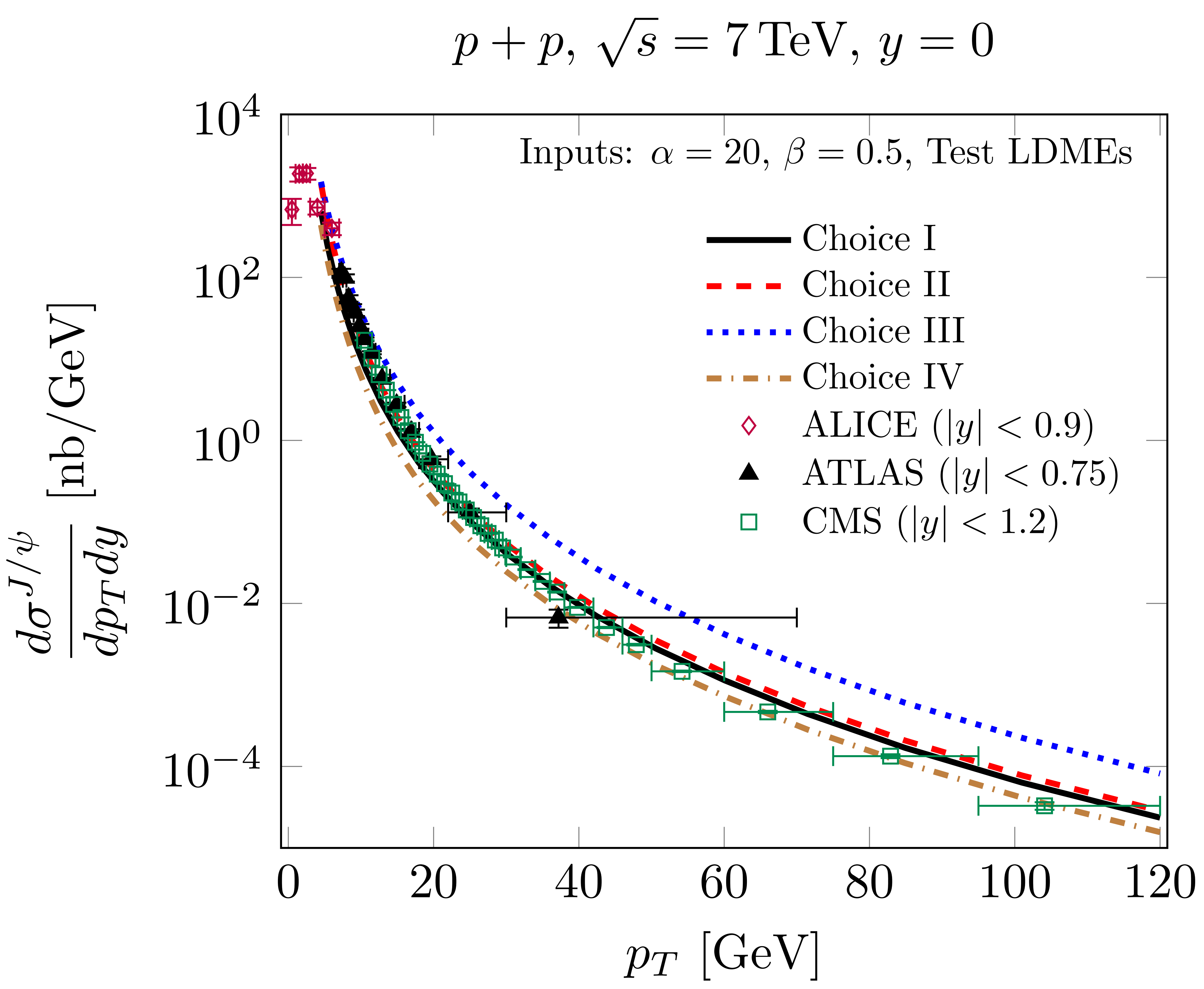
LP $\gg$ NLP at high p_T : $p_\perp \gg 10\,\text{GeV}$

NLP $\gtrsim$ LP at moderate p_T : $p_\perp \lesssim 10\,\text{GeV} = \mathcal{O}(2m)$

All SDCs are positive in the QCD factorization, while transverse components of SDCs in the NRQCD can be negative.

Tuning LDMEs

Preliminary results



We can try to use the following sets of LDMEs:

	$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3P_0^{[8]})\rangle$ 10^{-2} GeV^5
Choice I	1.16	1.0	0.10	0.10
Choice II	1.16	10.0	0	0
Choice III	1.16	0	1.0	1.0
Choice IV	1.16	0	0	0

Given smaller values of the LDMEs for $^3S_1^{[8]}$ and $^3P_0^{[8]}$, our approach describes data about the p_T spectra and polarization of J/ψ in hadronic collisions.

- The QCD factorization framework with our FFs should give a set of LDMEs by fitting data about quarkonium yield and polarization. The input FFs can be improved.

Summary

- We have solved the nonlinear QCD evolution equations for the twist-2 single parton FFs and the twist-4 double parton FFs; perturbative calculations tell reliable orders of the FFs.
- We have demonstrated numerically that the LP contributions are significant for quarkonium production at high $p_\perp$ while the NLP contributions are sizable at lower p_T but different in shape; One needs to take their superposition.
- LDMEs have not yet been refitted in this formalism. In the QCD factorization formalism, all SDCs are positive, which should make possible a new global analysis.
- Matching between the QCD factorization and fixed order NRQCD factorization should enable us to describe hadronic quarkonium production in a broader p_T region.

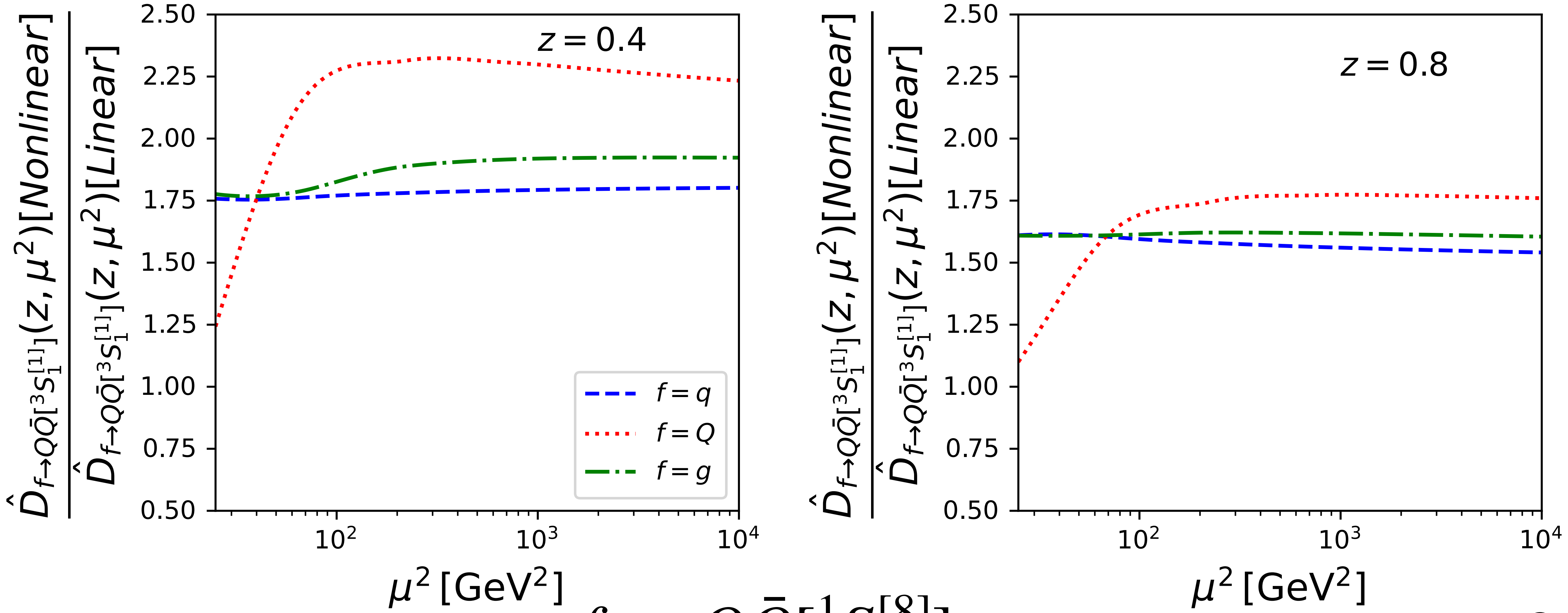
Thank you!

backup

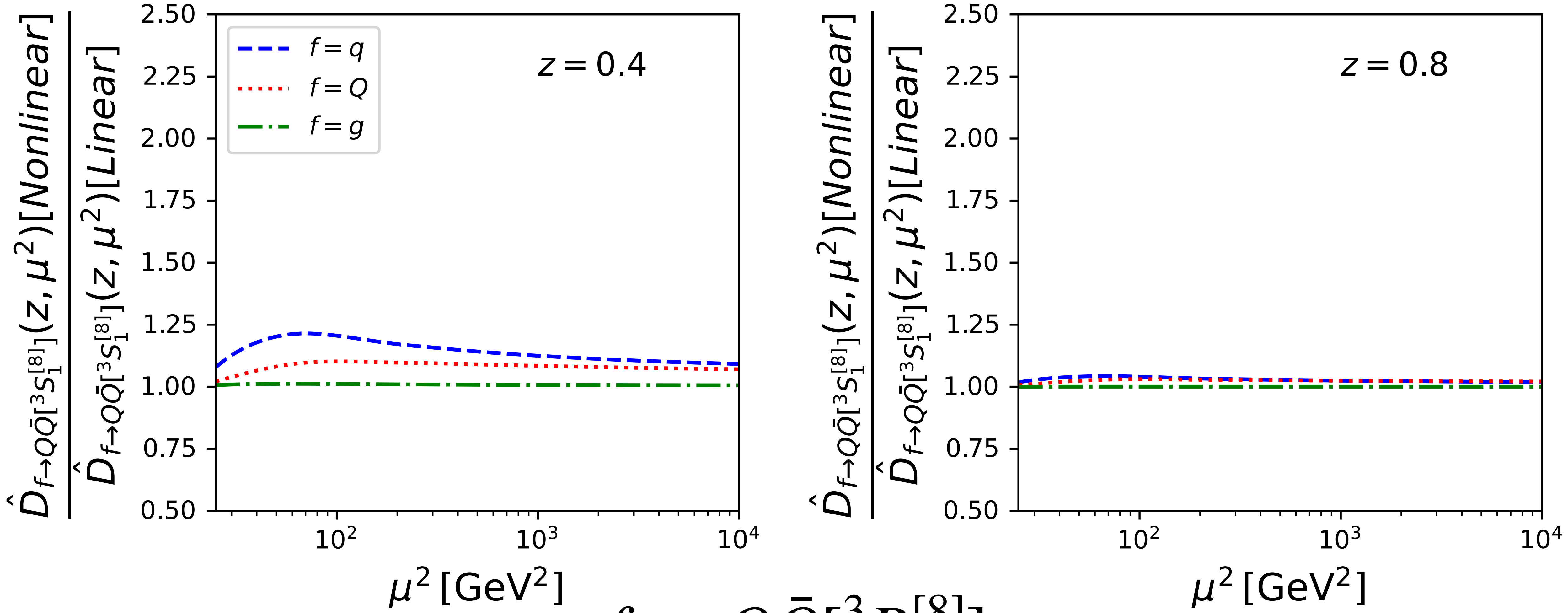
Nonlinear vs. Linear evolution effects

Preliminary results

$f \to Q\bar{Q}[^3S_1^{[1]}]$

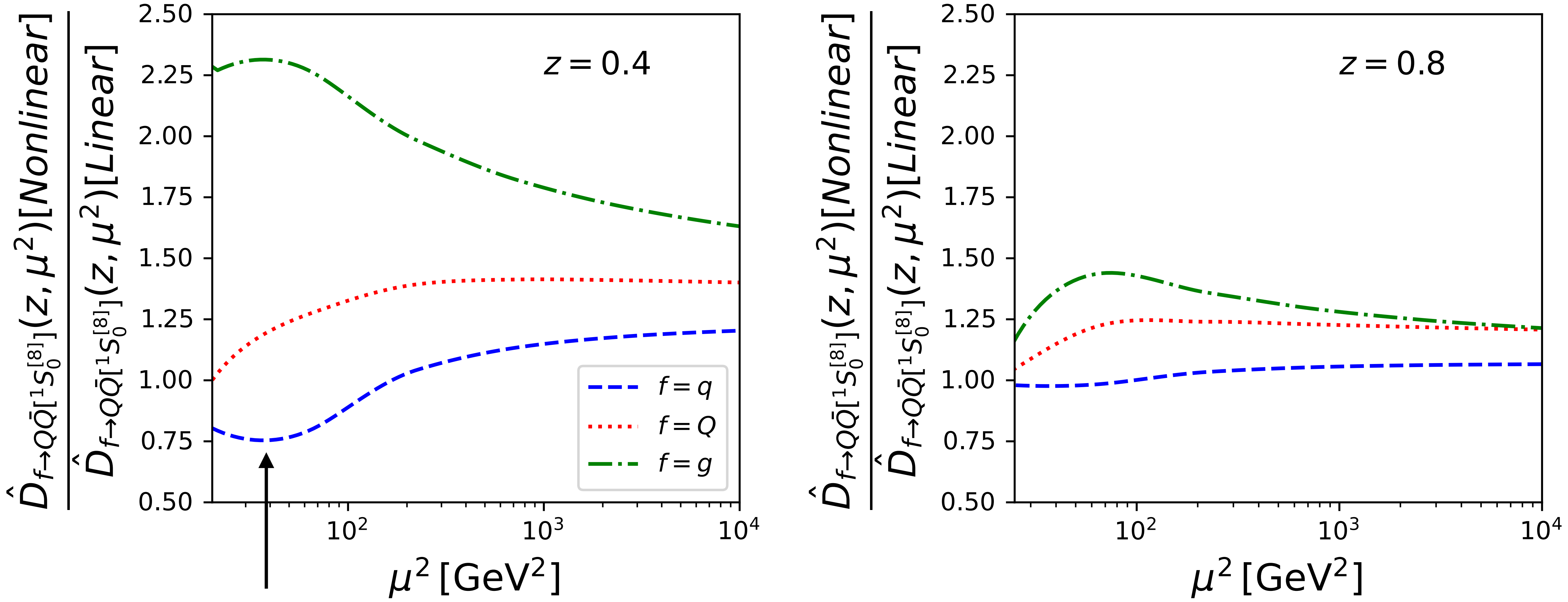


$f \to Q\bar{Q}[^3S_1^{[8]}]$

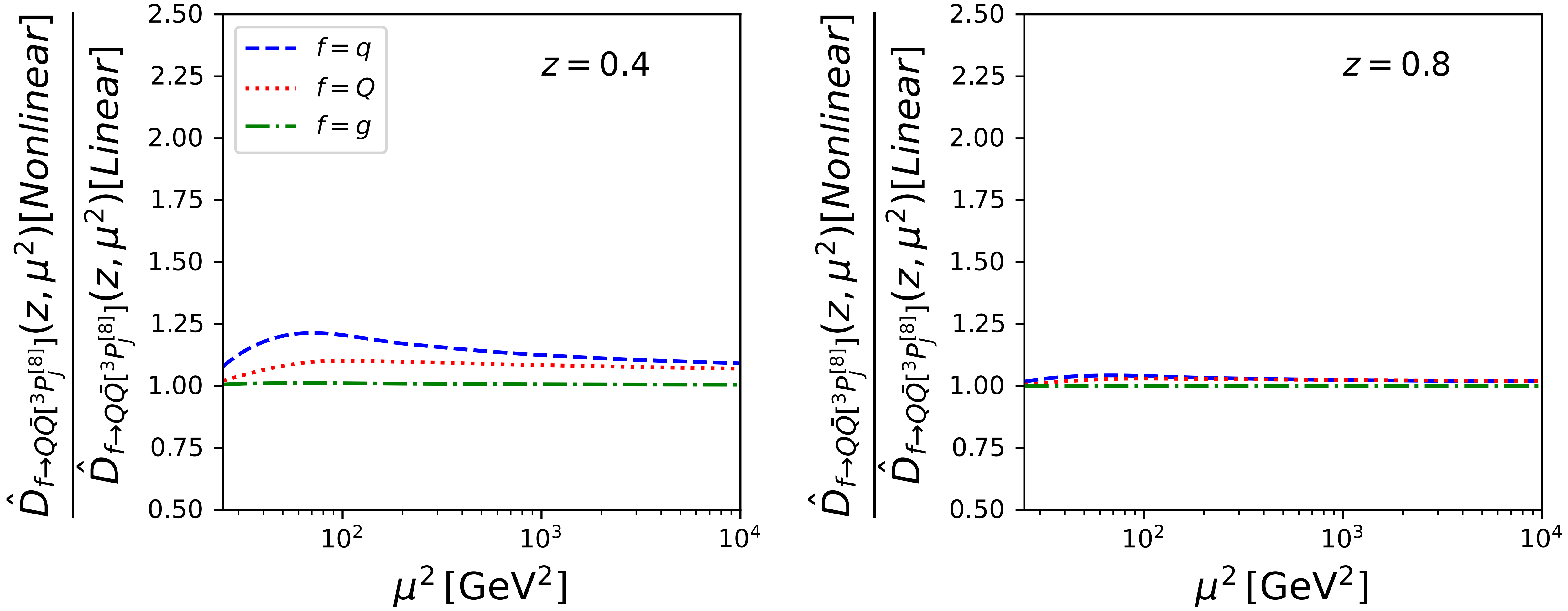


$f \to Q\bar{Q}[^1S_0^{[8]}]$

$\mu_0 = 3m,\ \mu_\Lambda = m$

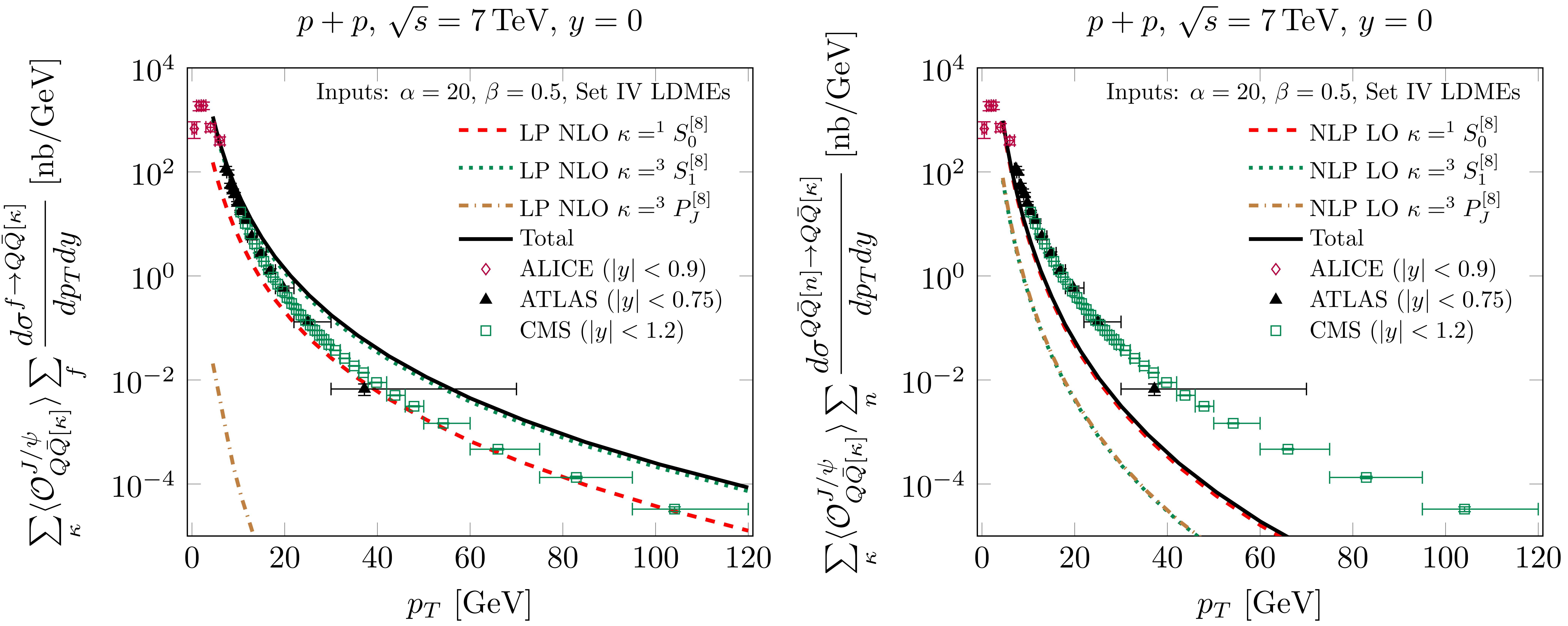


$f \to Q\bar{Q}[^3P_J^{[8]}]$



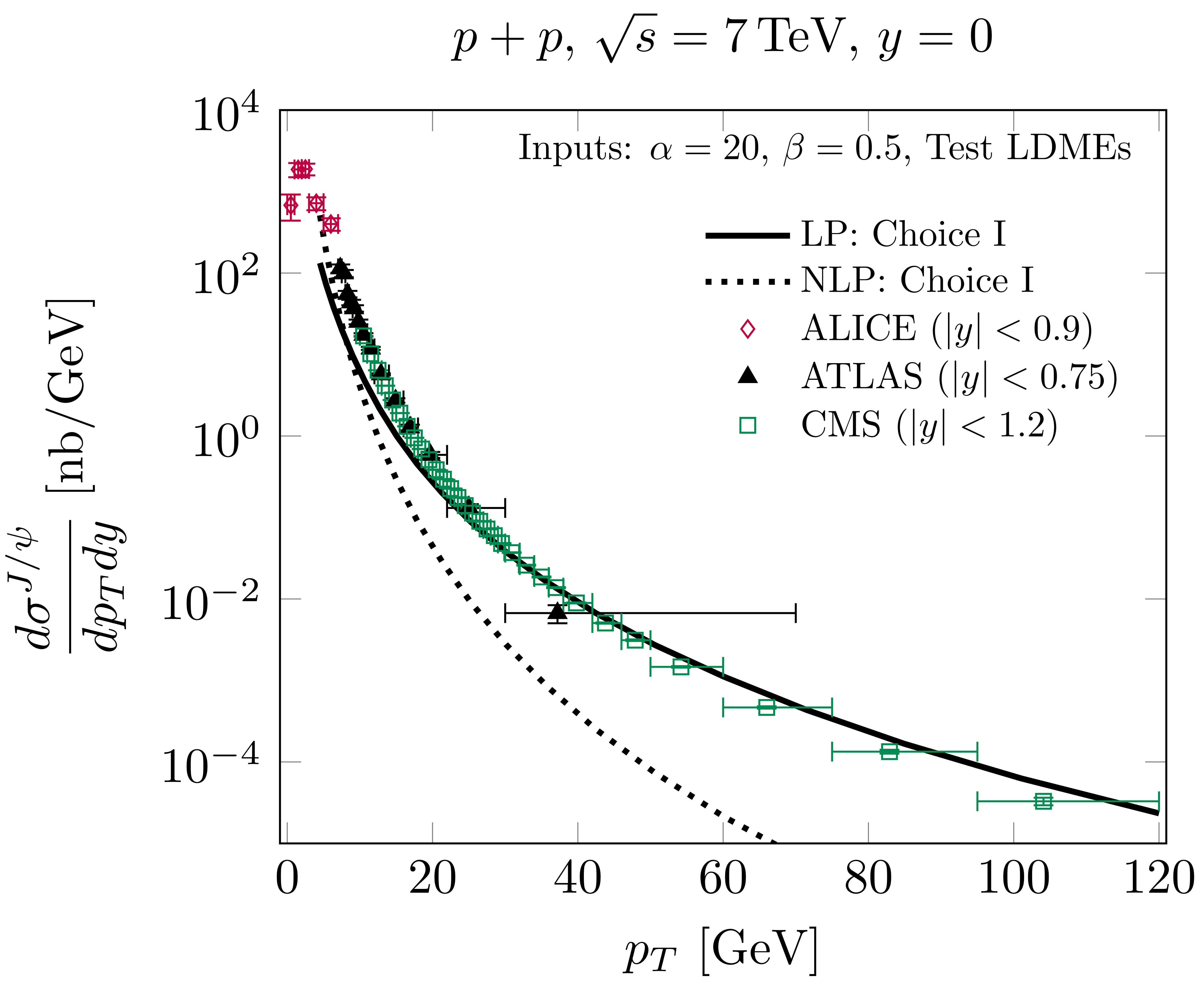
wrong interpolated points obtained by using LHAPDF interpolator.

Hadronic J/ψ production: Exercises



	$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2} GeV^3	$\langle\mathcal{O}(^3P_0^{[8]})\rangle$ 10^{-2} GeV^5
Set I (Butenschoen *et al.*)	1.32	3.04	0.16	-0.91
Set II (Chao *et al.*)	1.16	8.9	0.30	1.26
Set III (Gong *et al.*)	1.16	9.7	-0.46	-2.14
Set IV (Bodwin *et al.*)	-	9.9	1.1	1.1

LP and NLP contributions in hadronic J/ψ production



	$\langle \mathcal{O}(^3S_1^{[1]}) \rangle$ GeV^3	$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}(^3P_0^{[8]}) \rangle$ 10^{-2} GeV^5
Choice I	1.16	1.0	0.10	0.10
Choice II	1.16	10.0	0	0
Choice III	1.16	0	1.0	1.0
Choice IV	1.16	0	0	0

LP $\gg$ NLP at high $p_T : p_\perp > 10\,\text{GeV}$

NLP $\gtrsim$ LP at moderate $p_T : p_\perp \lesssim 10\,\text{GeV} = \mathcal{O}(2m)$