

$N^3\text{LO}$ extraction of the Sivers function

from SIDIS, Drell-Yan, and W/Z data

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I present recent extraction of Sivers function
from the global fit of SIDIS and DY data
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Outline

- ▶ Theory input
- ▶ Data input
- ▶ Fit quality and uncertainty estimation
- ▶ Some results
 - ▶ Sivers function
 - ▶ Qiu-Sterman function
 - ▶ Tomography in momentum space



Sivers asymmetry within TMD factorization

Many factors cancel
including the hard
coefficient function.

$$A_{UT}^{\sin(\phi_h - \phi_S)} = -M \frac{\sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi} b J_1\left(\frac{b|P_{hT}|}{z}\right) R(b, Q) f_{1T, q \leftarrow h_1}^\perp(x, b) D_{1, q \rightarrow h_2}(z, b)}{\sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi} J_0\left(\frac{b|P_{hT}|}{z}\right) R(b, Q) f_{1, q \leftarrow h_1}(x, b) D_{1, q \rightarrow h_2}(z, b)}$$



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All numerics is done by
Artemide

TMD evolution
independent nonperturbative

SV19
DY+SIDIS fit
[Scimemi, AV, 1912.06532]



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Role of perturbation theory in TMD factorization in ζ -prescription

In ζ -prescription NP evolution and NP TMD *exactly* decorrelated

$$\frac{d\sigma}{dP.S.} \simeq \sigma_0 \, |C_V(Q, \mu)|^2 \int d^2b e^{i(bq_T)} R[\underline{D}(b)](Q, \mu) f_{1T}^\perp(x, b) \, \underline{D}(z, b)$$

PT input:

→ hard coef.func.

→ μ -evolution

both N³LO

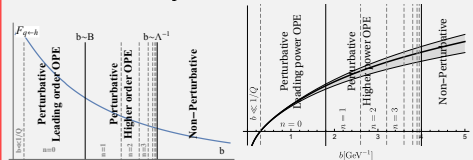
$$= a_s^3 + a_s^4 [\Gamma_{cusp}]$$

No restrictions
for Siverson function
at special ζ -line

Additional PT input:

→ N³LO small- b for CS-kernel

→ NNLO for unpol.TMDs

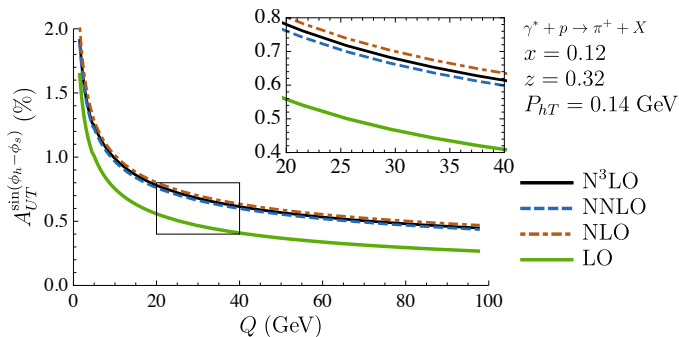


Such scheme is consistent only in a “fixed reference scale scheme”, e.g. ζ -prescription
but not in a “variable reference scale scheme”, e.g. CSS-scheme



Perfect perturbative convergence!

$$\mathbf{N^3LO} = \frac{C_V \text{ (cancel)}}{\alpha_s^3} \left| \frac{\gamma_V}{\alpha_s^3} \right| \left| \frac{\Gamma_{cusp}}{\alpha_s^4} \right| \left| \frac{\text{CS-kernel at small-b}}{\alpha_s^3} \right| \left| \frac{\text{unpol.TMD at small-b}}{\alpha_s^2} \right|$$



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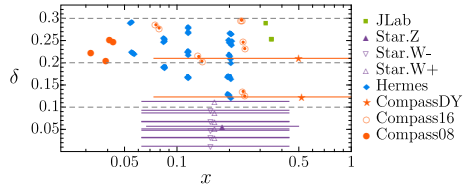
TMD factorization is valid only at $q_T/Q \ll 1$
 we use $q_T < 0.3Q$ ($q_T = p_T/z$)
 $\langle Q \rangle > 2\text{GeV}$

☹ Too many data do not fulfill requirement
 ► COMPASS and JLab “1D” binning
 (integral over p_T or z)

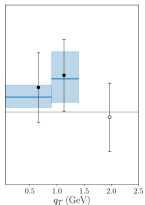
😊 The latest HERMES 3D

😊 All points from DY (except 1) are valid

Only p_T -differential points are taken



Example: πDY



last point:
 $4.3 < Q < 8.5\text{GeV}$,
 $1.4 < q_T < 5.0\text{GeV}$

► SIDIS = 63 pt.

► DY = 13 pt.

► Total=76 pt.

Different flavor combinations!

Mainly middle-x

Ansatz for Sivers function

$$f_{1T;q \leftarrow h}^\perp(x, b) = N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q, \epsilon_q)} \exp\left(-\frac{r_0 + x r_1}{\sqrt{1 + r_2 x^2} b^2} b^2\right)$$

$u, d, s, sea(=\bar{u}, \bar{d}, \bar{s})$

Common for all flavors
Similar to unpol.

12 free parameters

- ▶ $\{r_0, r_1, r_2\}$ – TMD part
- ▶ $\{N_{u,d}, \beta_{u,d}, \epsilon_{u,d}\}$ – valence quarks
- ▶ $\{N_{s,sea}, \beta_s = \beta_{sea}\}$ – rest

No restrictions for parameters

- ▶ Data driven extraction
- ▶ Positivity (almost satisfied)



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$$f_{1T;q \leftarrow h}^\perp(x, b) = N_q \frac{(1-x)x^{\alpha}\beta_q(1+\epsilon_q x)}{n(\beta_q, \epsilon_q)} \exp\left(-\frac{r_0 + x r_1}{\sqrt{1 + r_2 x^2} b^2}\right)$$

$u, d, s, sea(=\bar{u}, \bar{d}, \bar{s})$

Sea is not constrained

$\beta_q = \beta_{sea}, \quad \epsilon_s = \epsilon_{sea} = 0$

No restrictions
typical fit $\alpha \in [0, 5]$

fix $\alpha = 1$

Common for all flavors
Similar to unpol.

12 free parameters

- ▶ $\{r_0, r_1, r_2\}$ – TMD part
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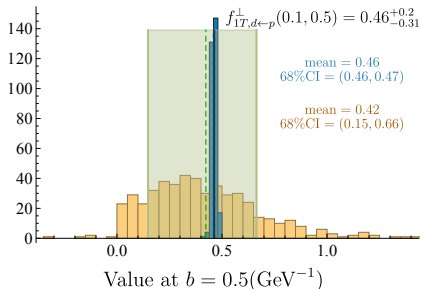
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Poor data = large uncertainties



- ▶ Uncertainties estimated by replica method
 - ▶ Fitting 300 replicas of pseudo data
- ▶ Large and (often) asymmetric uncertainties
- ▶ Uncertainty due to unpol.TMD are non-negligible but much smaller then due to data

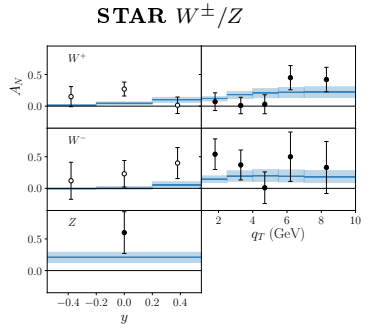
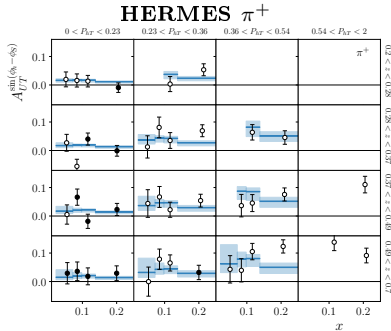


	$\chi^2/N_{pt}[SIDIS]$	$\chi^2/N_{pt}[DY]$	$\chi^2/N_{pt}[total]$
Fit only SIDIS	0.87	1.23	0.93
			
Fit SIDIS+DY	0.88	0.90	0.88

No principal problems with SIDIS+DY
(reminder: large error-bars)



Example of data description



Filled points = in fit,

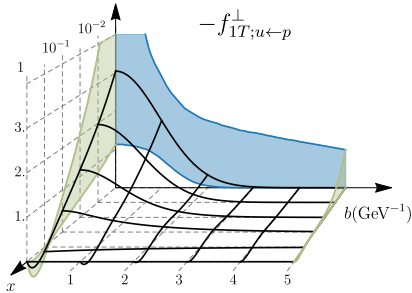
Open point = prediction

Actually, we can explain more data (up to $q_T < 0.4Q$ in SIDIS)

Sivers function

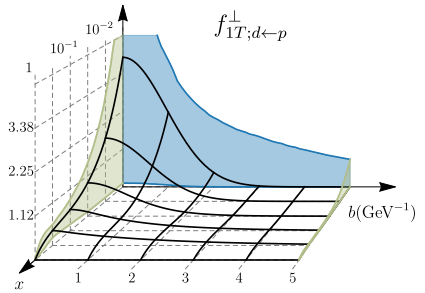
u-quark

$$-f_{1T}^{\perp; u \leftarrow p}$$



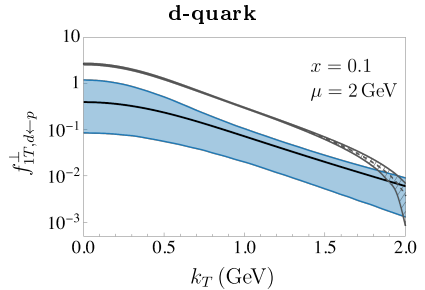
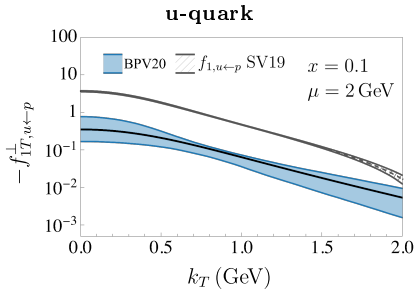
d-quark

$$f_{1T}^{\perp; d \leftarrow p}$$

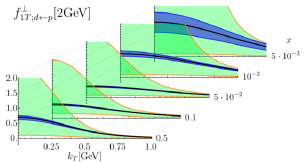


- ▶ Notably huge uncertainties
- ▶ Not sign definite
- ▶ Hope for data by JLab12GeV, and EIC →

Sivers function



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Extraction is only the first step...

- ▶ Sivers function in position/momentum spaces
- ▶ **Qiu-Sterman function**
- ▶ Positivity constrain
- ▶ Limits of TMD approach
- ▶ Analysis of the sign-change relation
- ▶ **Tomographic picture**

→[M.Bury,A.Prokudin,AV,2103.03270]



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Extracting Qiu-Sterman function

[Scimemi, Tarasov, AV, 1901.04519]

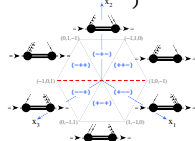
$$f_{1T,q\leftarrow h}^\perp(x,b) = -\pi \left\{ T_q(-x,0,x;\mu) + a_s(\mu) \left[-2\mathbf{L}_\mu P \otimes T - C_F \frac{\pi^2}{6} T(-x,0,x;\mu) + \right. \right. \quad (4.8)$$

$$\left. \int_{-1}^1 d\xi \int_0^1 dy \delta(x-y\xi) \left(-\frac{\bar{y}}{N_c} T_q(-\xi,0,\xi;\mu) + \frac{3y\bar{y}}{2\xi} G^{(+)}(-\xi,0,\xi;\mu) \right) + \mathcal{O}(a_s^2) \right] + \mathcal{O}(b^2) \Big\},$$

QS-function arise in the limit of small-b

- ▶ QS-function is twist-3 (complicated evolution)
- ▶ Not only QS-function, but entire $T(x_1, x_2, x_3)$
- ▶ Mixture with gluons

How to resolve it?



- ▶ Fix $\mu = c_0/b$ ($\mathbf{L}_\mu = 0$)
- ▶ Fix b small ($b^2 \sim 0$)
- ▶ Invert the formula



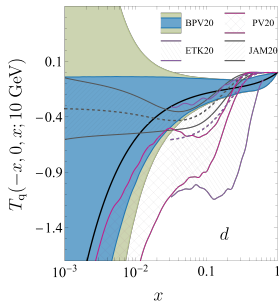
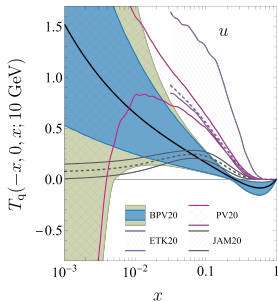
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Extracting Qiu-Sterman function

$$b = 0.11/\text{GeV} \Rightarrow \mu = 10\text{GeV}$$

$$T_q(-x, 0, x; \mu_b) = -\frac{1}{\pi} \left(1 + C_F a_s(\mu_b) \frac{\pi^2}{6} \right) f_{1T; q \leftarrow h}^\perp(x, b) \\ - \frac{a_s(\mu_b)}{\pi} \int_x^1 \frac{dy}{y} \left[\frac{\bar{y}}{N_c} f_{1T; q \leftarrow h}^\perp \left(\frac{x}{y}, b \right) + \frac{3y^2 \bar{y}}{2x} G^{(+)} \left(-\frac{x}{y}, 0, \frac{x}{y}; \mu_b \right) \right] + \mathcal{O}(a_s^2) + \mathcal{O}(b^2)$$

$$G^{(+)} = \pm |T_d + T_u|$$



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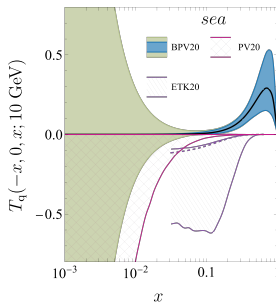
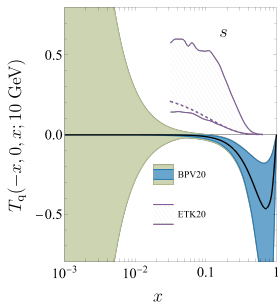
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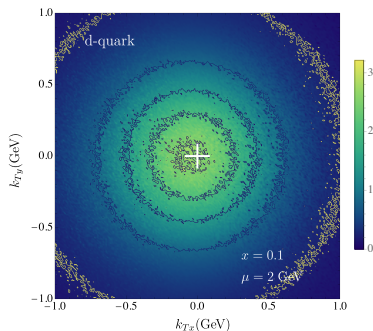
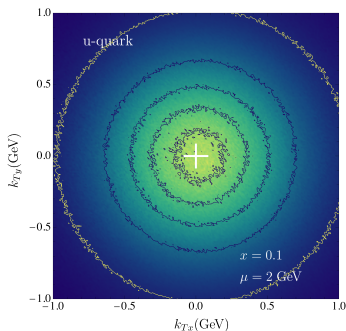
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Tomography in momentum space

$$\rho_{1;q \leftarrow h^\uparrow}(x, \mathbf{k}_T, \mathbf{S}_T, \mu) = f_{1;q \leftarrow h}(x, k_T; \mu, \mu^2) - \frac{k_{Tx}}{M} f_{1T;q \leftarrow h}^\perp(x, k_T; \mu, \mu^2)$$



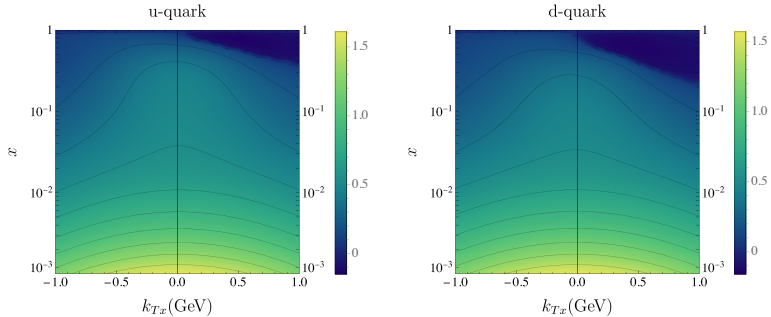
color fluctuations $\Leftrightarrow$ uncertainties



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Tomography in momentum space

$$\rho_{1;q \leftarrow h^\dagger}(x, \mathbf{k}_T, \mathbf{S}_T, \mu) = f_{1;q \leftarrow h}(x, k_T; \mu, \mu^2) - \frac{k_{Tx}}{M} f_{1T;q \leftarrow h}^\perp(x, k_T; \mu, \mu^2)$$



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Conclusion

Sivers function from DY+SIDIS

- ▶ $N^3\text{LO}$ theory (ζ -prescription)
- ▶ Unbiased ansatz for Sivers function
- ▶ Conservative cut for data
- ▶ Agreement between SIDIS and DY (accounting large error-bands)
- ▶ Many supplementary studies

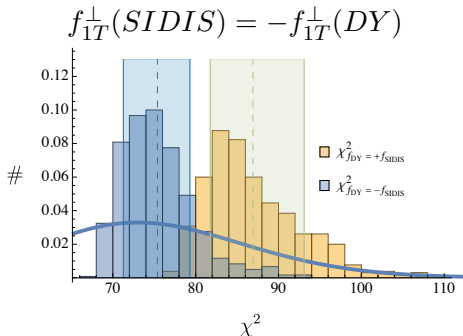
Results available at

- ▶ **artemide**
- ▶ also with **TMDlib2** (set BPV20)



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Check sign-change



$$f_{1T}^\perp(sea) \rightarrow -f_{1T}^\perp(sea)$$

Main source for large DY asymmetry are anti-quarks!