Transversity through Dihadron:

a constrained fit

DIS2021

virtual

WG5: Spin Physics

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"FORDECYT-PRONACES"

dgapa
Dirección General de Asuntos del Personal Académico

Based on 1912.03289 [Eur.Phys.J.C 80 (2020) 5, 465]

with J. Benel (UNI, Peru) and R. Ferro (Mainz U.)

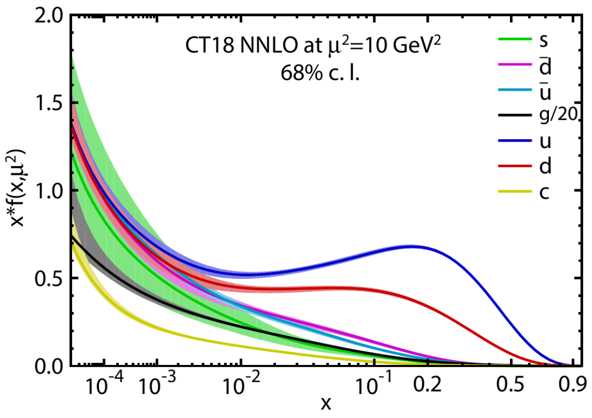
Instituto de Física UNAM

Leading-twist parton distribution functions

$$\phi_{ij}(k;p,s) = 2\pi \sum_X \int \frac{d^3\mathbf{P}_X}{2E_X} \delta^4(p-k-P_X) \langle p,s|\bar{\psi}_j(0)|X\rangle\langle X|\psi_i(0)|p,s\rangle$$

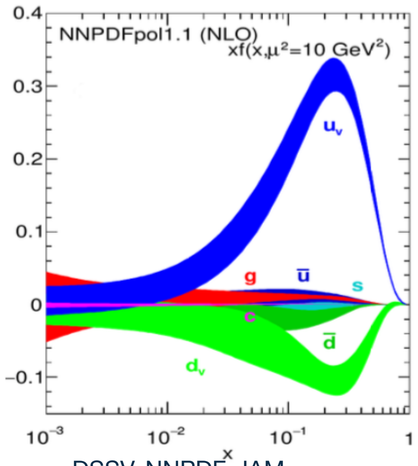
$$\boxed{\phi(x,s) = \frac{1}{2}\left[\mathbf{f_1}(x)\not{n}_+ + s_L g_1(x)\gamma^5\not{n}_+ + h_1 i\sigma_{\mu\nu}\gamma^5 n_+^\mu s_T^\nu\right]}$$

Unpolarized PDF



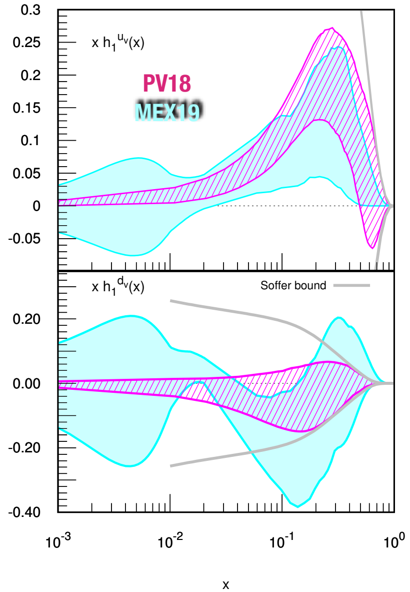
CTEQ, NNPDF, MSHT

Helicity PDF

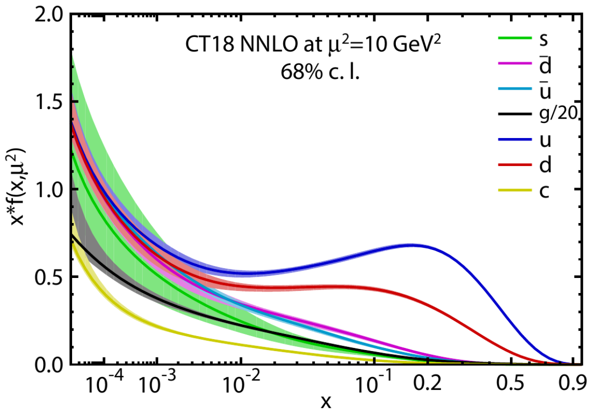


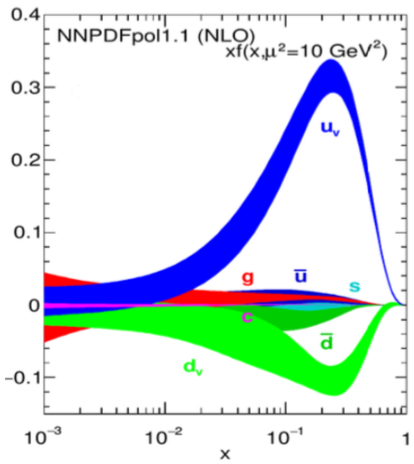
DSSV, NNPDF, JAM

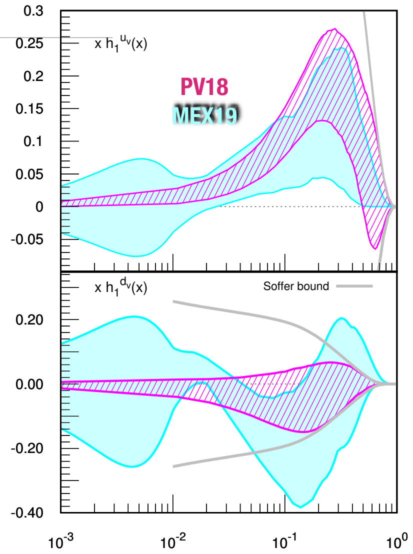
Transversity PDF

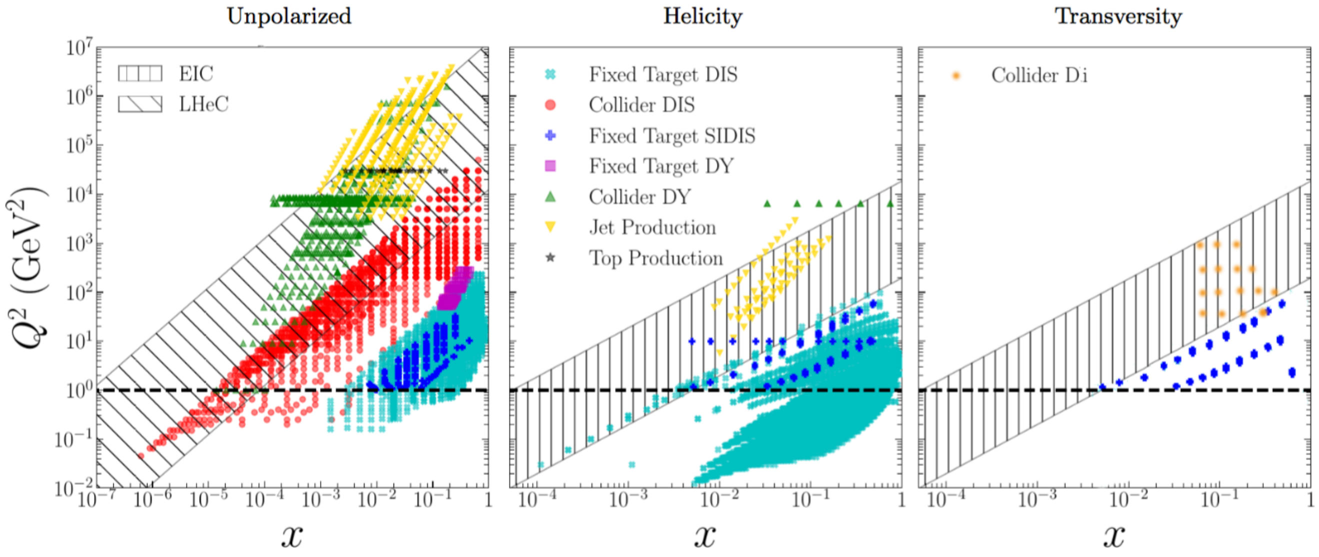


PDF kinematics coverage: present and future experiments





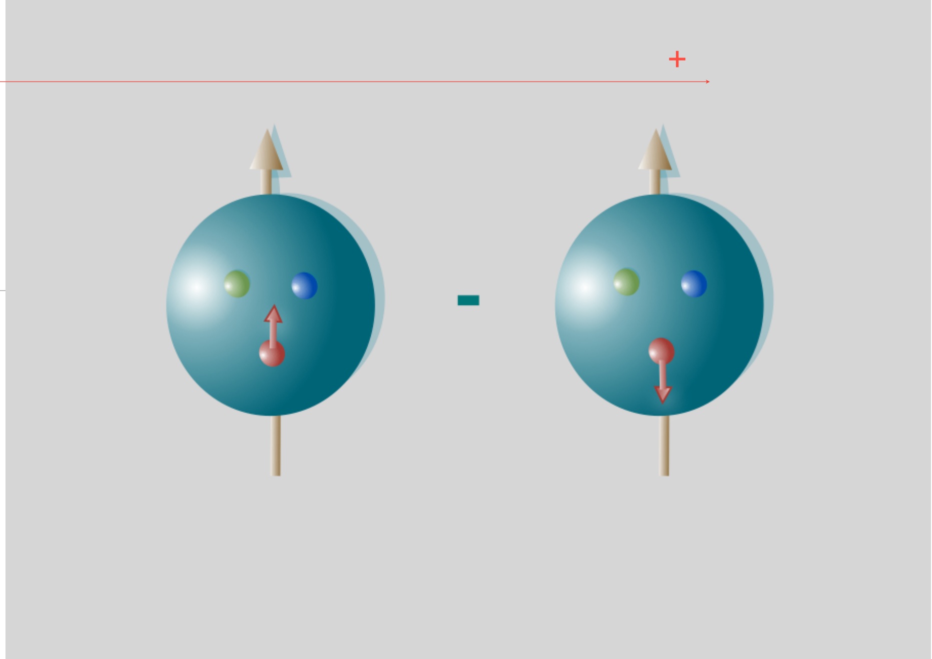




- **Transversity** is the least known twist-2 PDF;
- chiral-odd: appears with other nonpert. object;
- reduced kinematics coverage *wrt* f_1 and g_1;
- Error treatment must reflect
  - the first principle constraints: positivity [large x]
  - integrability [small x]

PDFLattice [2006.08636]

Constraints on $h_1(x)$



Constraints based on first principles for the Transversity

- Support, e.g. $x \in [0,1]$
- Endpoint behavior, e.g. $h_1(x=1)=0$
- Positivity bounds: Soffer bound

$$|h_1^q(x)| \leq \frac{1}{2}\left(f_1^q(x) + g_1^q(x)\right)$$

Characteristics of the transversity PDF

- Can be assessed in semi-inclusive processes with a chiral-odd partner
- No constraining sum rule
- Its first Mellin moment gives the tensor charge

Data

Dihadron production data

- SIDIS @HERMES, COMPASS
- pp↑ @RHIC
- e^+e^- @Belle

Single hadron production data

- SIDIS @HERMES, COMPASS & JLab
- pp↑ @RHIC, more DY and A_N
- e^+e^- @Belle

Ingredients of a fit

- data;
  - first principles; → untouchable, not assumptions.*
    - theoretical framework;
      - QCD-based assumptions;
        - parametrization
          - error treatment. → reconciles all previous points in an *elegant* and *physical* way.

*But, the implementation of the first-principle constraints might be data-driven, *e.g.* positivity for g_1 and h_1.

How the uncertainties/tensions between data and first principles are taken care of is a signature of the fit.

Imposing the positivity on the functional form

A crucial step and a physical statement

1. Parametrization that obeys the positivity bounds.

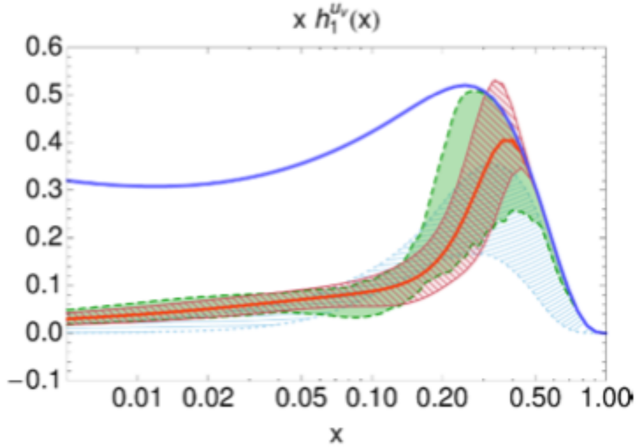
$$|h_1^q(x)| \leq \frac{1}{2}\left(f_1^q(x) + g_1^q(x)\right)$$

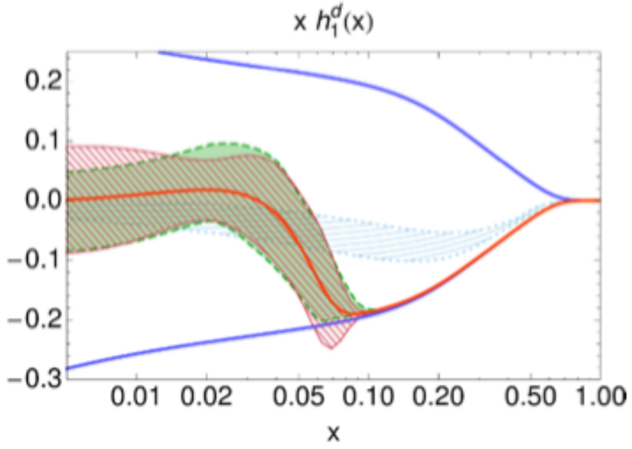
Its expression is based on the analyses of the unpolarized f_1 and the helicity g_1 PDFs.

$$x\, h_1(x) = \frac{F(x)}{|F(x)|} \times x \, \text{Soffer Bound} \qquad \text{with} \qquad F(x) \propto x^{\alpha}(1-x)^{\beta}$$

Good fits are obrained, with a clear **saturation** of the bound.

Recent global fit shows that pp data slightly release that saturation
[Bacchetta & Radici, PRL 120 (2018)], **PAV18**





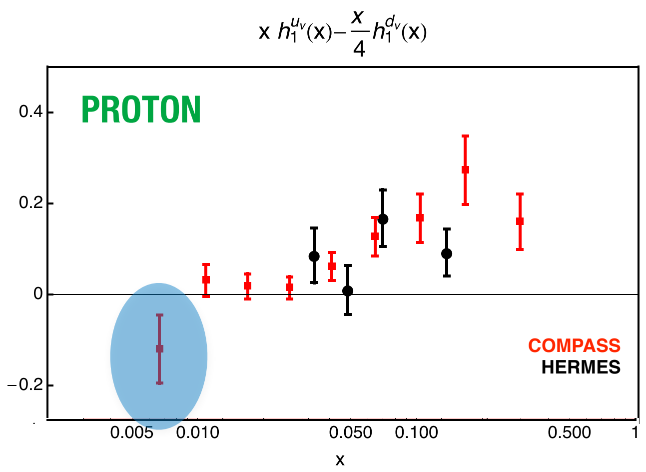
[Bacchetta, A.C., Radici, JHEP1303]
[Radici, Bacchetta, A.C., Guagnelli, JHEP1505]
PAV15

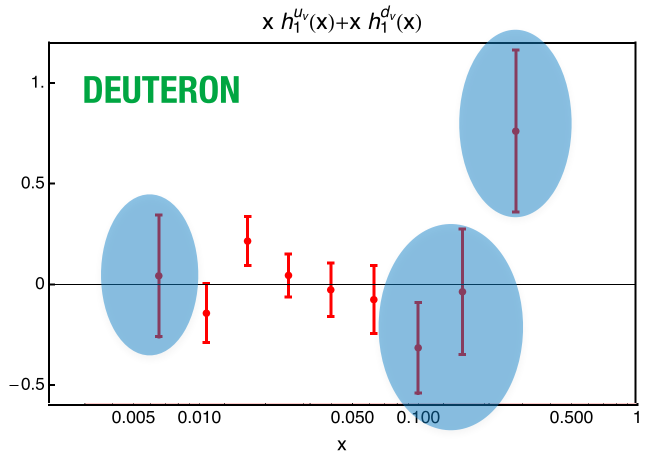
Room for improvement?

Radici, AC, Bacchetta & Guagnelli
JHEP 1505 (2015) 123

1. COMPASS data: the deuteron

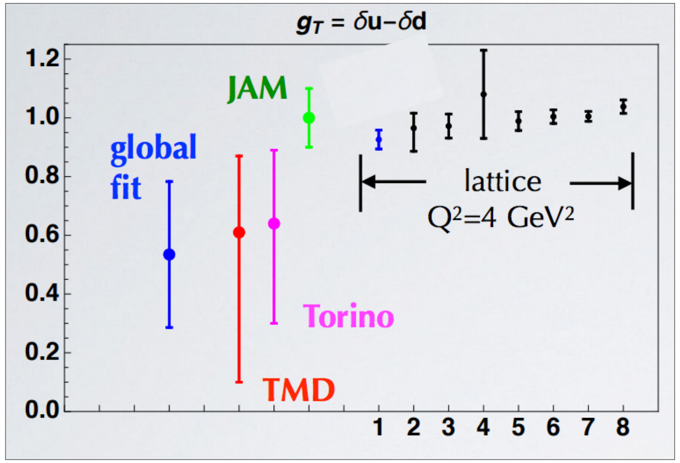
Tension with Soffer bound





2. The tensor charge: "discrepancy" with lattice evaluations

$$g_T(Q^2) = \int_0^1 dx \left\{ h_1^{u_V}(x, Q^2) - h_1^{d_V}(x, Q^2) \right\}$$



From M. Radici at PDFLattice 2019

Room for improvement?

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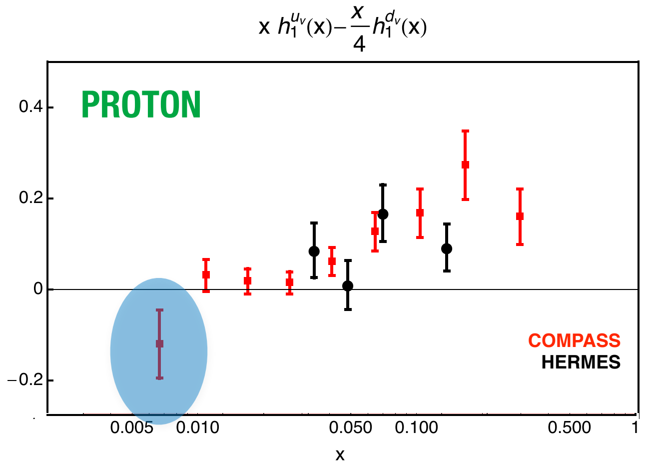

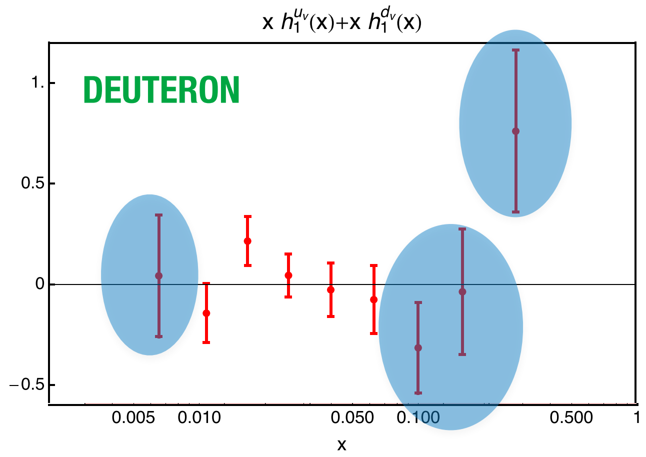

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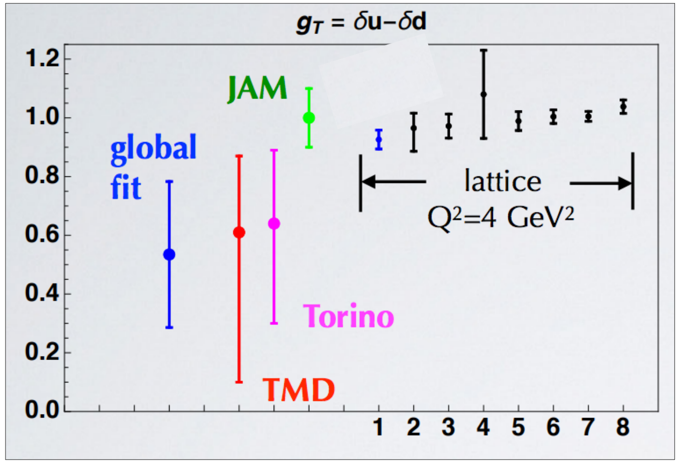
g_T is relevant for search of physics beyond the Standard Model

[Bhattacharya et al, Phys.Rev.D 85]
[AC et al, Phys.Rev.Lett.115]
[Liu et al, Phys.Rev.D97]



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Opportunities for improvement

Radici, AC, Bacchetta & Guagnelli
JHEP 1505 (2015) 123

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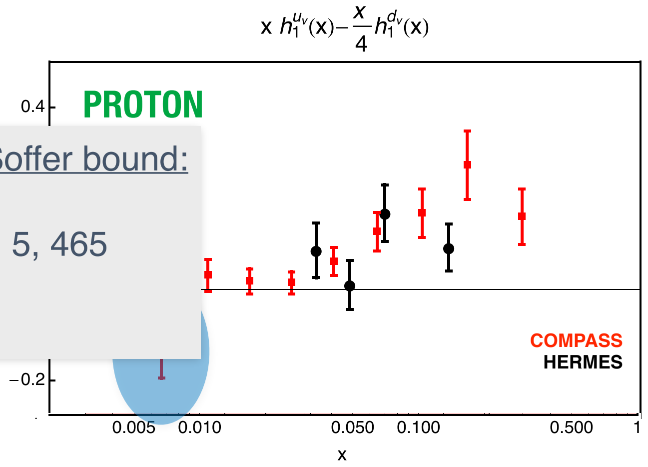
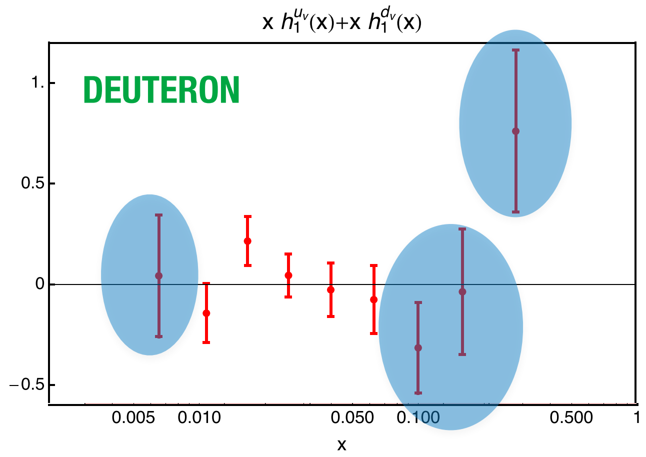




<u>Studies of impact of the implementation of the Soffer bound:</u>

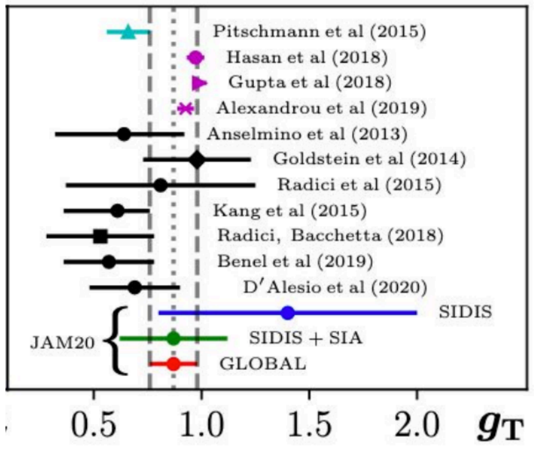
1. this work: Benel et al, Eur.Phys.J.C 80 (2020) 5, 465
2. d'Alesio et al, Phys.Lett.B 803 (2020) 135347

2. The tensor charge: "discrepancy" with lattice evaluations

<u>Total relaxation of the Soffer bound:</u>

1. Cammarota et al, Phys. Rev. D 102

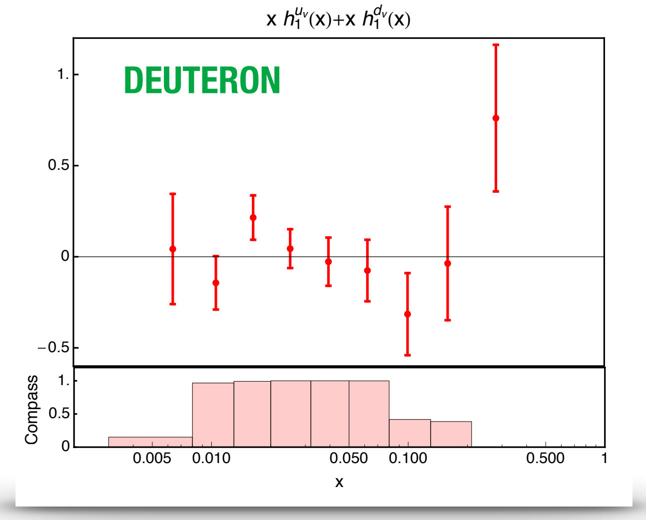
No apparent reconciliation between the positivity and lattice!



Imposing the positivity by the fitting procedure

2. Guide the positivity bounds at large x

→increase flexibility of the fitting procedure by adding extra physical considerations



A. Weight by the probability of the positivity bound given the data

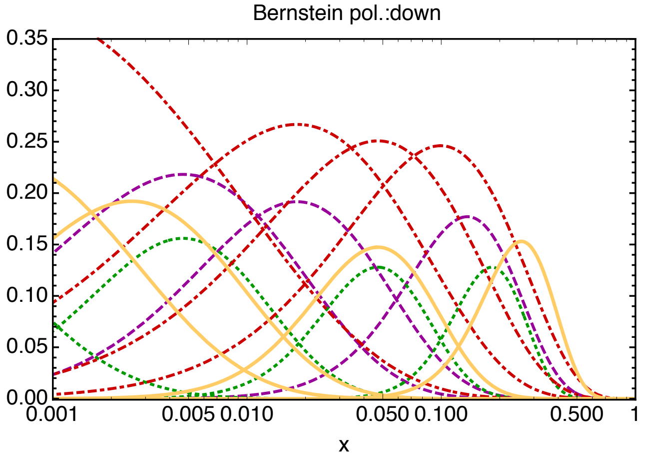
$$\chi_r^2\left(\{p^I\}\right) = \sum_j w_j \frac{\left[x_j h_{1\,\text{theo}}^{p/D}\left(x_j;\{p^I\}\right) - \left(x h_1^{p/D}(x)\right)_{j\,,r\,\text{data}}\right]^2}{\sigma_j^2}$$

Overall the Soffer bound and the data are compatible — $P\,(\text{inside}|\ \text{data}) = \int_0^1 dt\,(1-t)\,\text{Distr}(t) = 0.95$

Weight given from a point-by-point case — $P_j\,(\text{inside}|\ \text{data}) = \int_{(1-P(a_j|data))}^1 dt\,\text{Distr}(t)$

Imposing the positivity by the fitting procedure

2. Implicitly guide the positivity bounds at large x



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B. Parametrize the x-dependence ; repeat N times (Bootstrap method)

$$x\, h^{q_v}_{1,i}\left(x; p^q_{i,k}\right) = x^{1.25} \sum_{k=\{\kappa_{q,i}\}} p^q_{i,k}\, B_{k,n_i}\left(g(x)\right)$$

Small-x region controlled by x^a, *a* fixed* —modulated by Bernstein polynomials. *In agreement with [Kovchegov & Sievert,Phys.Rev.D99]

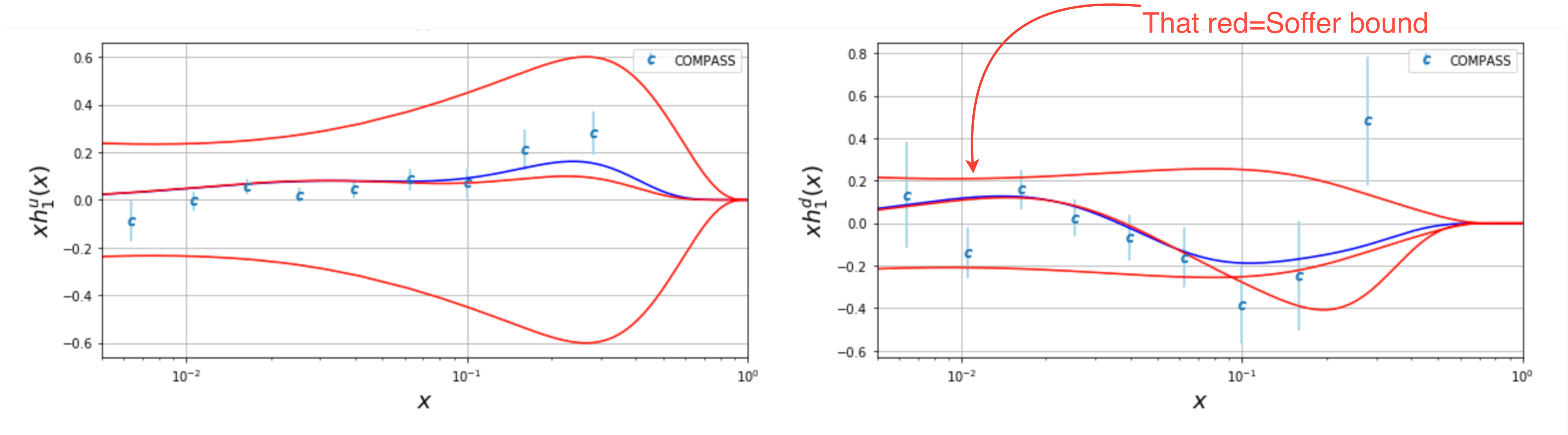
Large-x region mimicking a QCD-based fall-off.

Imposing the positivity by the fitting procedure

2. Implicitly guide the positivity bounds at large x

A. Weight

B. Param



Example for one replica, not final fit.

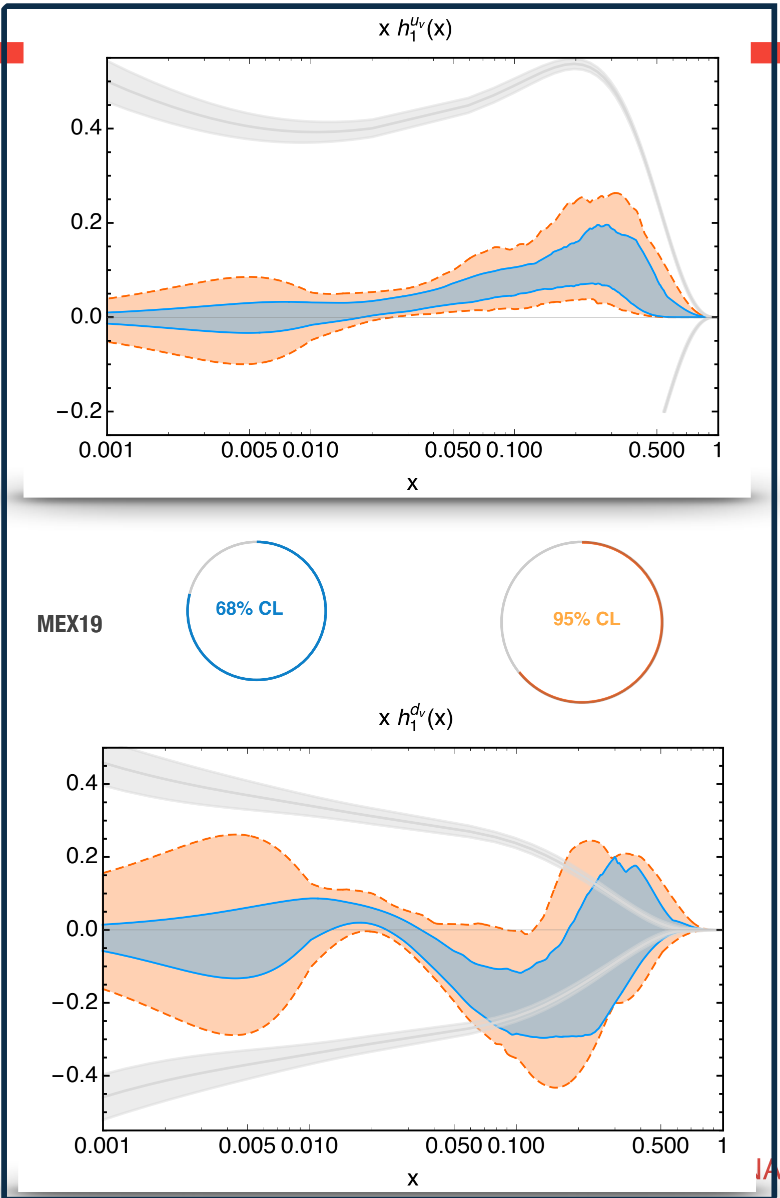
Red= first minimization

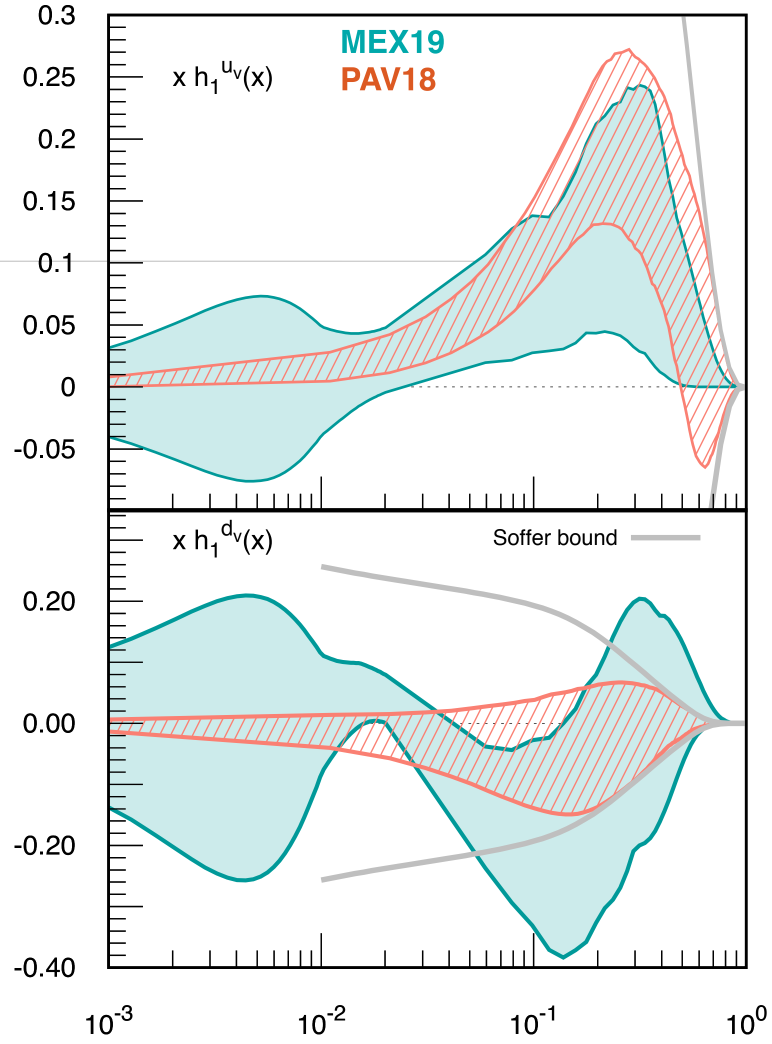
Blue=second/constrained minimization

C. Constrain the falloff of the PDF at large-x with Lagrange multipliers

$$
\begin{aligned}
C_{i,j}^{d_V}\left(p_{i,k}^{d\,II}\right) &= h_{1,i}^{q_V}\left(x_j; p_{i,k}^{d\,II}\right) < \epsilon_j \\
C_{i,j}^{d_V}\left(p_{i,k}^{d\,II}\right) &= h_{1,i}^{q_V}\left(x_j; p_{i,k}^{d\,II}\right) > -\epsilon_j
\end{aligned}
$$

Constrained fit of the transversity PDF





Data-induced relaxation of the bound brings more flexibility at large x.

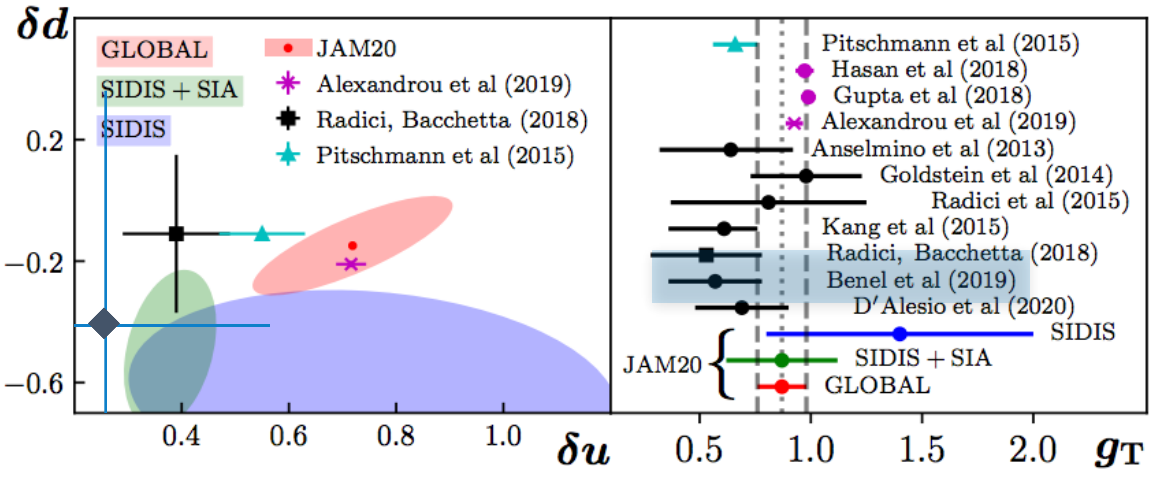
Chosen of functional form allows for a wider uncertainty at both small and large x.

Balanced interplay between constraints and data.

Comparison with lattice

1. Tensor charge

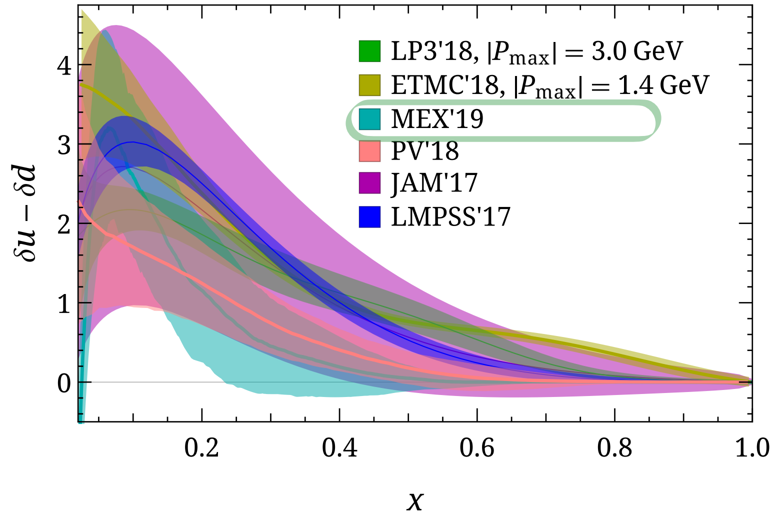
Cammarota et al, Phys. Rev. D 102



◆ this work (Benel et al)

2. Quasi/Pseudo PDFs

PDFLattice [2006.08636]



It is possible to get an **agreement with lattice g_T** with more flexibility in the functional form.

δu is the most problematic and yet most determined.

Comparison with lattice

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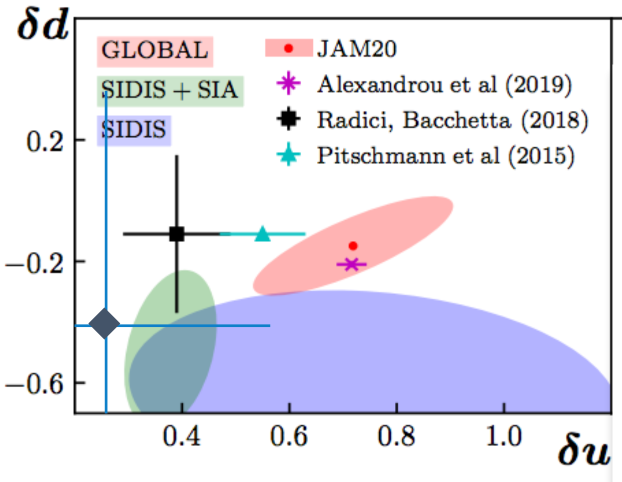
Cammarota et al, Phys. Rev. D 102



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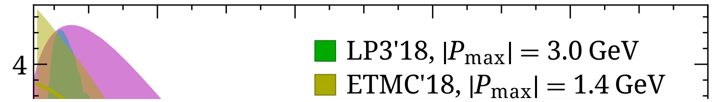
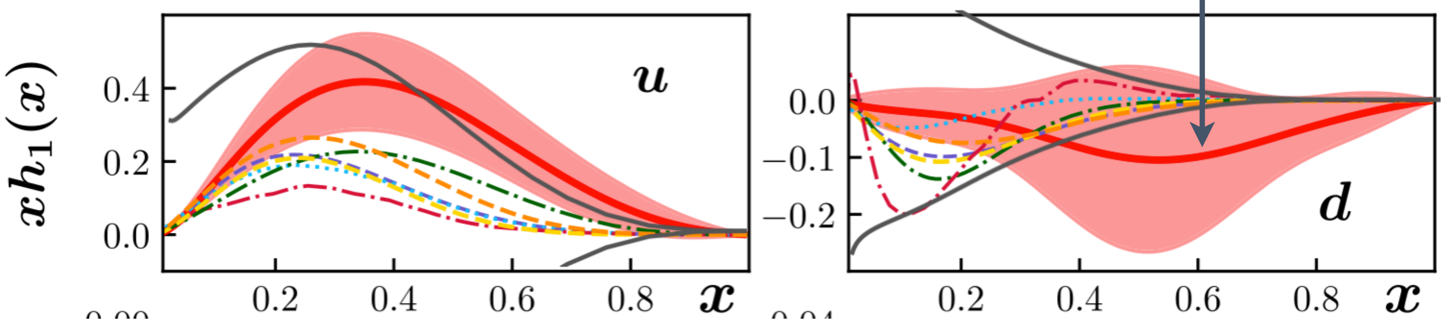


A too good agreement with lattice seems to come at the cost of violation of positivity bounds.



Soffer bound @2.4GeV² in grey, JAM20@4GeV² in red, MEX19 in pink, PAV18 in cyan.

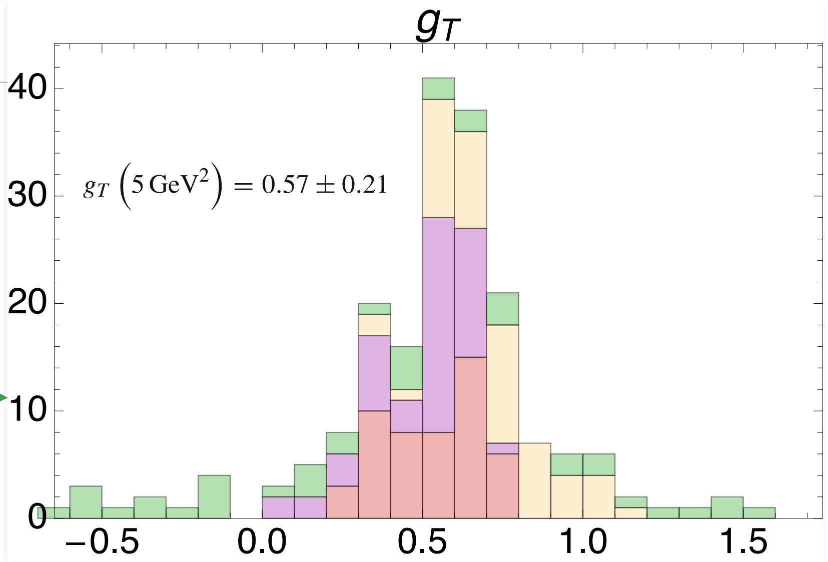
It is possible to get an **agreemer**

δu is the most problematic and yet most determined.

Comparison with lattice

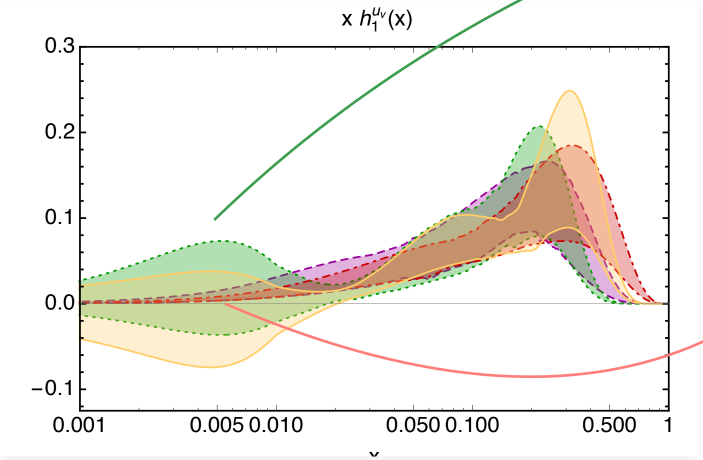
It is possible to get an **agreement with lattice g_T** with more flexibility in the functional form.

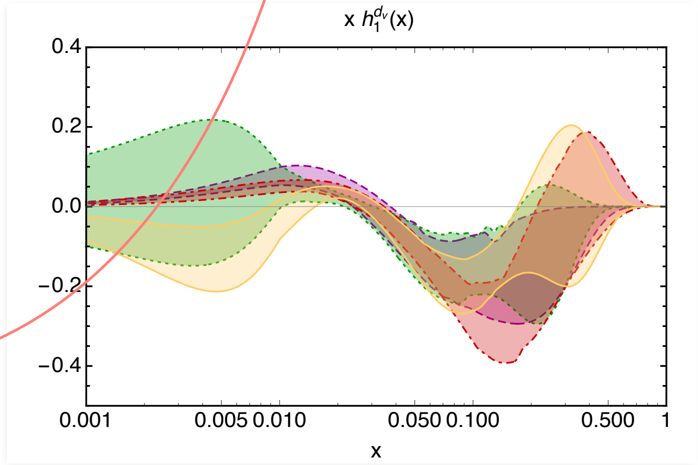
Stacked histogram of contribution from the four functional forms



Small x contributes to the uncertainty on the tensor charge:

- a narrower uncertainty results in a narrower distribution of g_T
- a wider uncertainty results in a wider distribution of g_T

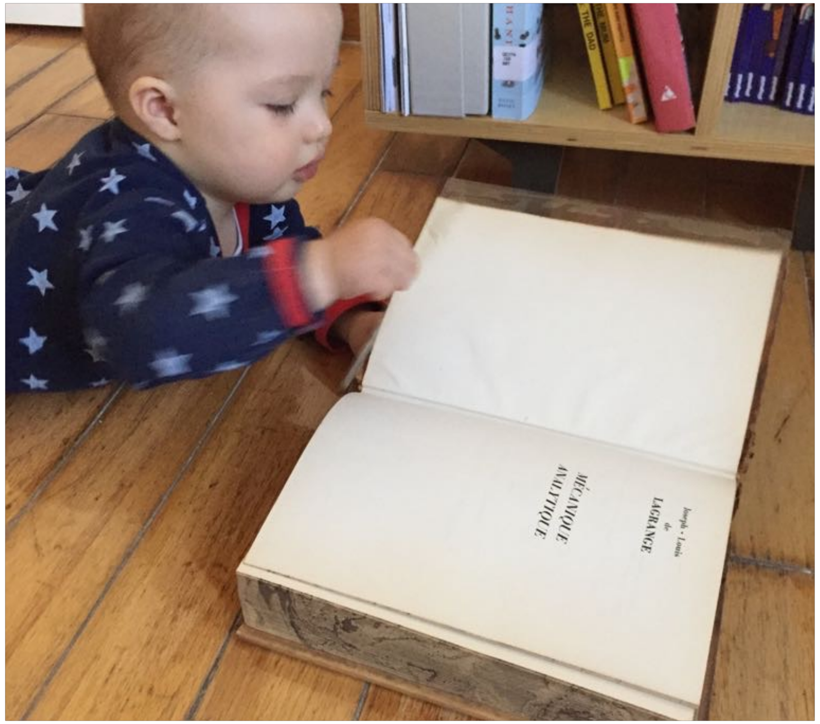




EIC is expected to contribute greatly,

both to single- and di-hadron channels.

Conclusions



Constrained fit of the transversity PDF

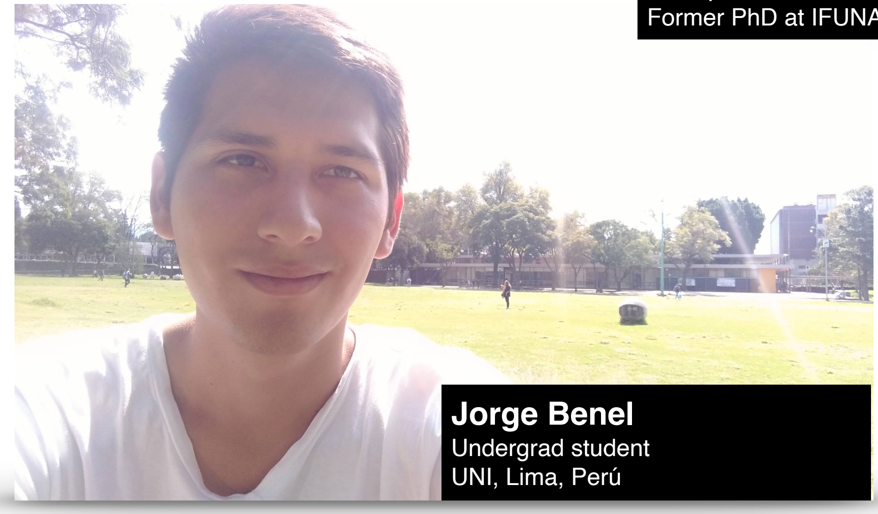
- Reflects tension with Soffer Bound through statistical reweighting
- Necessity of additional information at larger-x values
  - guide through constraints → Lagrange multipliers
- Bootstrap is tricking the parameter space: to be improved
- Treatment of uncertainties shows that there could be a compatibility wth both positivity and lattice results

Mexican transversity

J. Benel, A. Courtoy, R. Ferro-Hernandez

1912.03289
Eur.Phys.J.C 80 (2020) 5, 465

Rodolfo Ferro
Now postdoc at Mainz
Former PhD at IFUNAM



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HADRON 2021

19TH INTERNATIONAL CONFERENCE ON HADRON SPECTROSCOPY AND STRUCTURE

Hosted by UNAM (Mexico City) from 26th of July to 1st of August 2021

May 1, 2021: Registration and abstract submission opening

www.nucleares.unam.mx/hadron2021

The main topics of this conference include:

- Meson spectroscopy
- Baryon spectroscopy
- Exotic hadrons and candidates
- Hadron decays, production and interactions
- Analysis tools
- QCD and hadron structure
- Hadrons in hot and nuclear environment including hypernuclei

Conveners:

- Carlota Andrés
- Charlotte van Hulse
- Cristina Aguilar
- Martin Hentschinski