# TMD structure at the EIC using jets

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In Collaboration with
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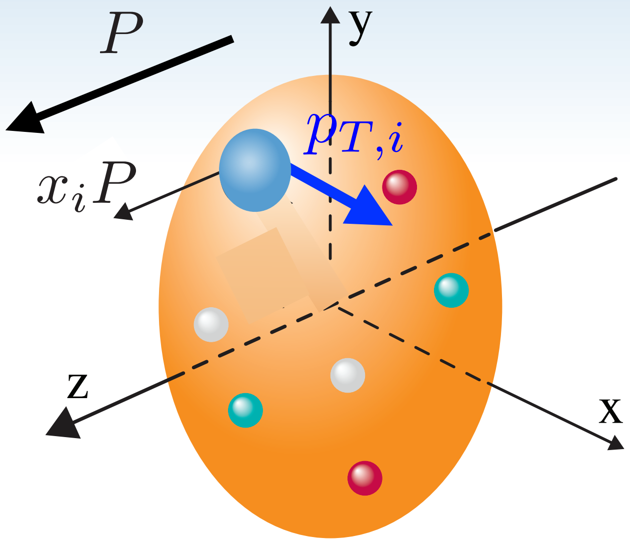
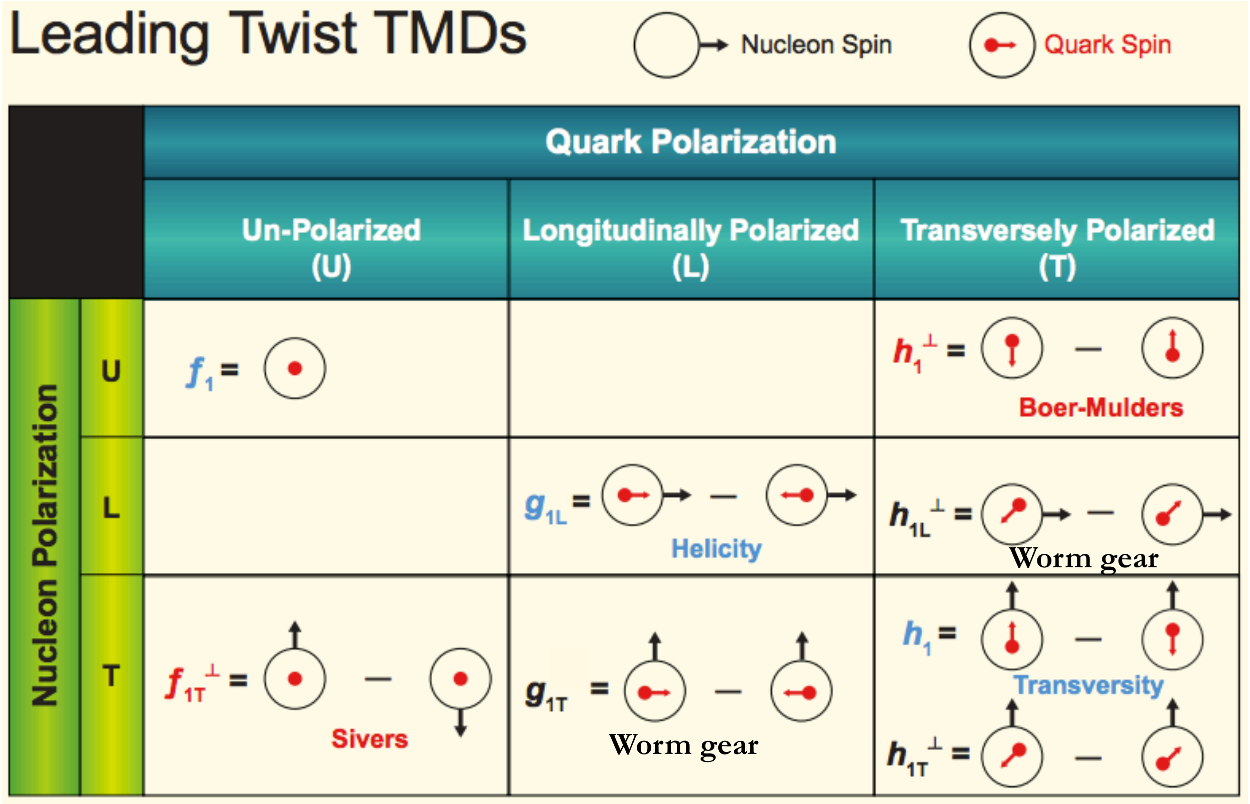
# TMD structure

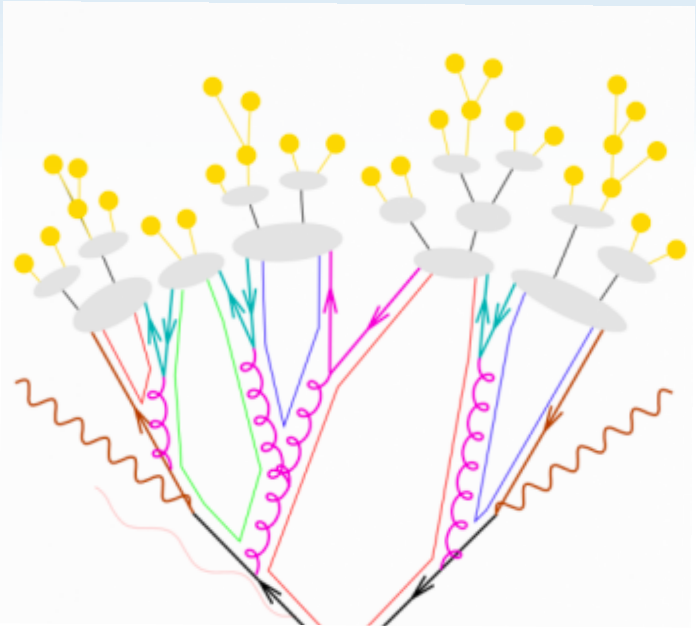




Leading Twist TMDs ⊙→ Nucleon Spin ⊙ Quark Spin

| | | Quark Polarization | | |
|---|---|---|---|---|
| | | Un-Polarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
| Nucleon Polarization | U | $f_1$ = ⊙ | | $h_1^\perp$ = ⊙ − ⊙<br>Boer-Mulders |
| | L | | $g_{1L}$ = ⊙→ − ⊙→<br>Helicity | $h_{1L}^\perp$ = ⊙→ − ⊙→<br>Worm gear |
| | T | $f_{1T}^\perp$ = ⊙ − ⊙<br>Sivers | $g_{1T}$ = ⊙ − ⊙<br>Worm gear | $h_1$ = ⊙ − ⊙<br>Transversity<br>$h_{1T}^\perp$ = ⊙ − ⊙ |

Quark **TMDPDF** inside spin-$\frac{1}{2}$ hadron



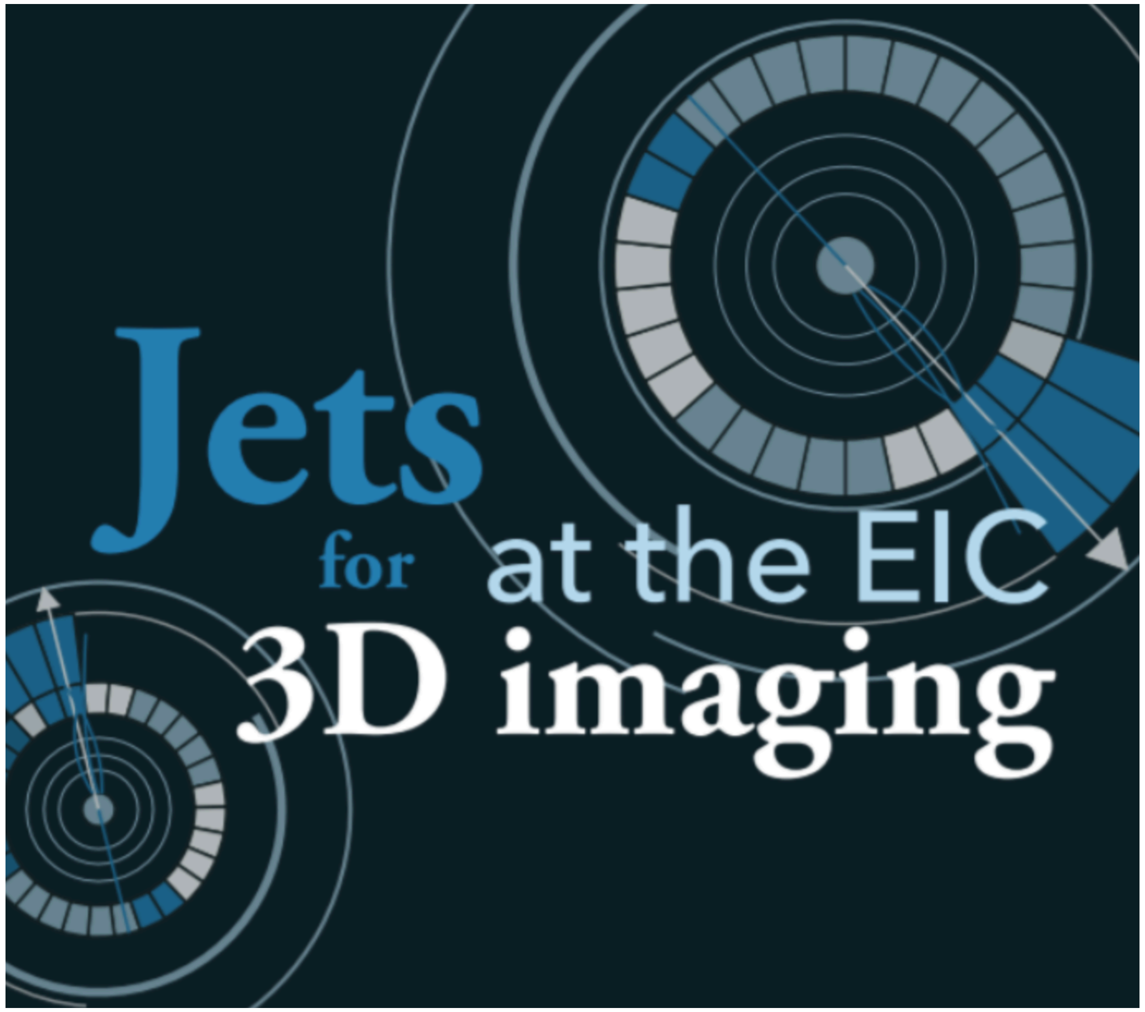
Quark polarization

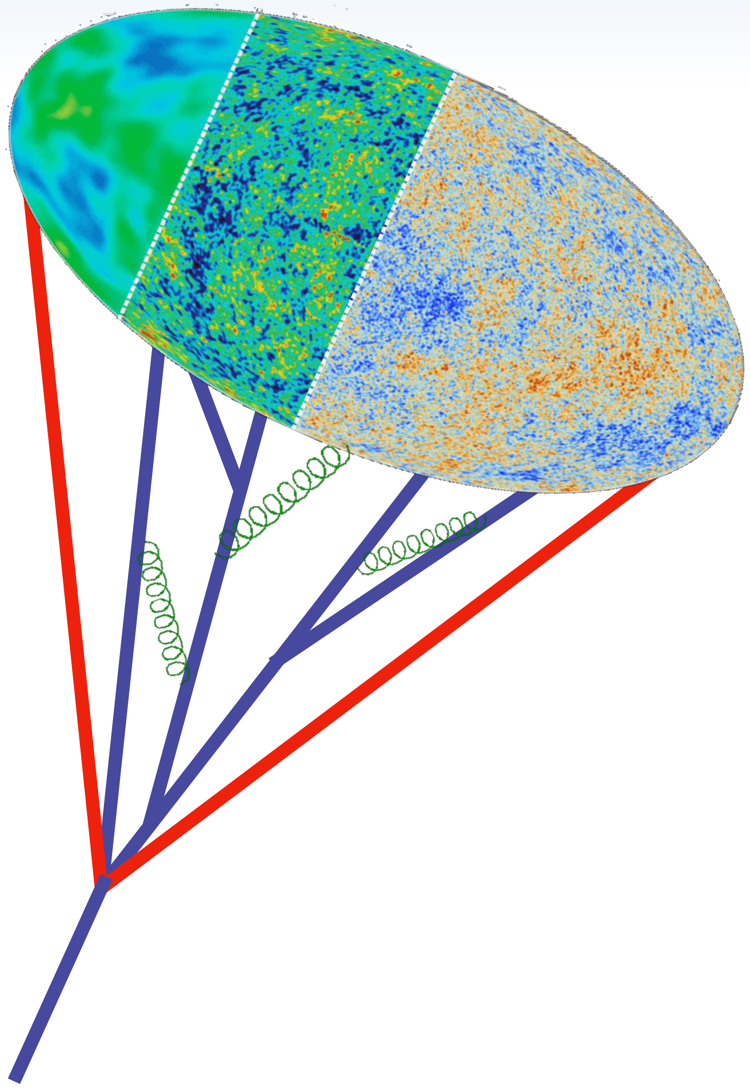
Hadron polarization

| | U | L | T |
|---|---|---|---|
| U | $D^{h/q}$ | | $H^{\perp h/q}$ |
| L | | $G^{h/q}$ | $H_L^{\perp h/q}$ |
| T | $D_T^{\perp h/q}$ | $G_T^{h/q}$ | $H^{h/q}$ $H_T^{\perp h/q}$ |

Quark **TMDFF** inside spin-$\frac{1}{2}$ hadron

# Study of hadron structures



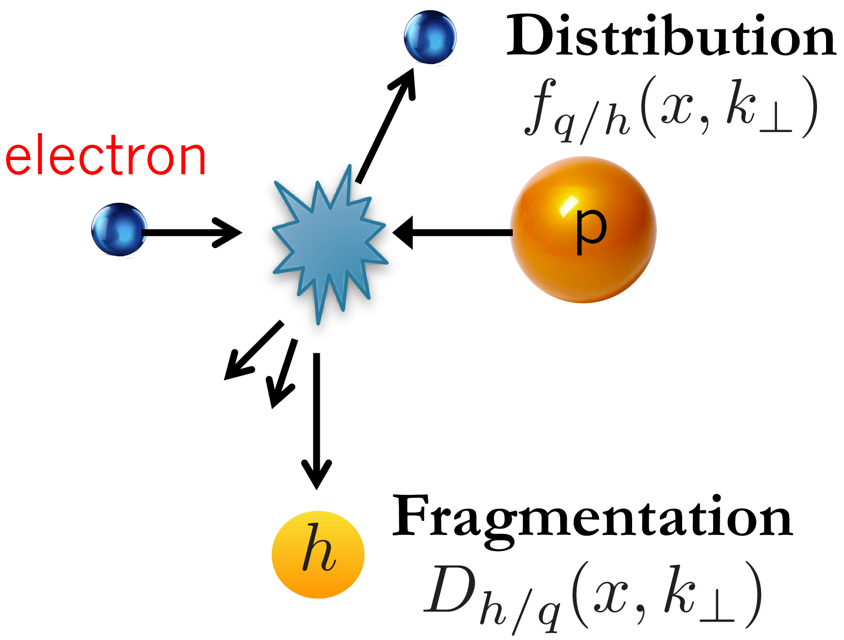


"Can we use jets at the EIC to probe TMD structure?"

# Standard processes

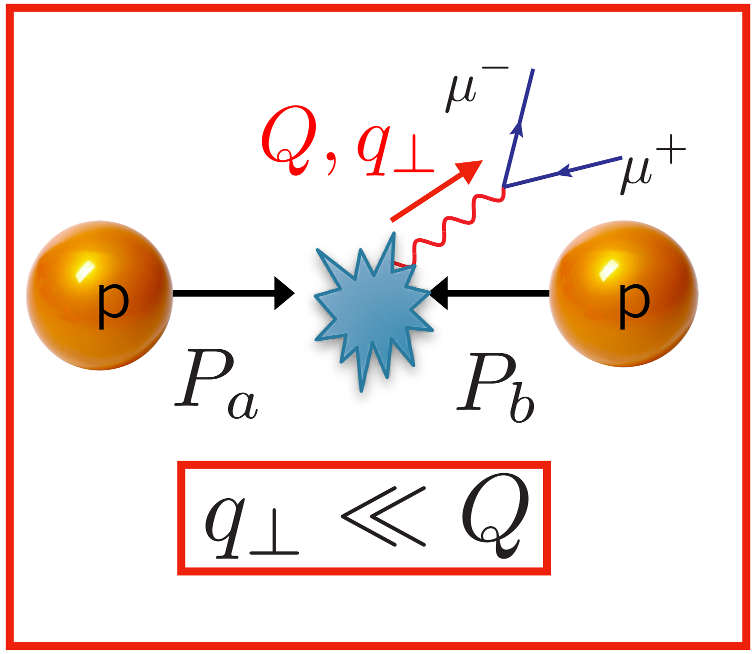
## Semi-Inclusive DIS (SIDIS)

$\sigma \sim f_{q/P}(x,k_\perp) D_{h/q}(x,k_\perp)$



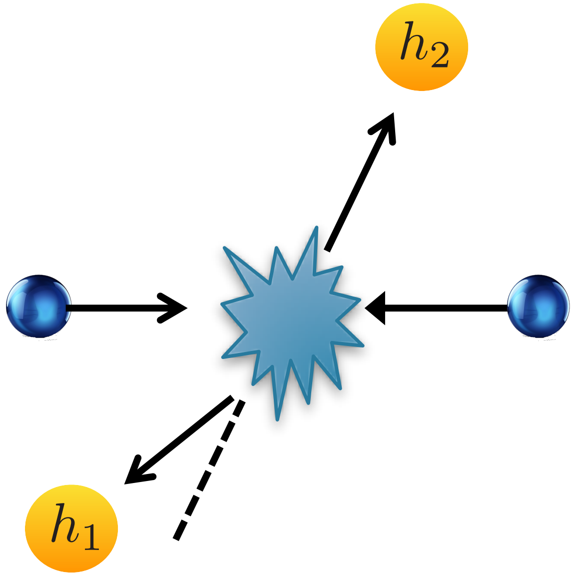
## Drell-Yan

$\sigma \sim f_{q/P}(x,k_\perp) f_{\bar{q}/P}(x,k_\perp)$



## Dihadrons in $e^+e^-$

$\sigma \sim D_{h_1/q}(x,k_\perp) D_{h_2/q}(x,k_\perp)$



*(CSS) Collin, Soper, Sterman `81-`85*
*Ji, Ma, Yuan ` 04*
*Becher, Neubert, Wilhelm`11-`13*
*Echevarria, Idilbi, Scimemi `11-`14*
…

# Beyond the standard processes

- Many other imaginable processes with sensitivity to the TMD structure

$$PP \to J_1 + J_2 + X,\quad PP \to J + V + X,\quad PP \to J({\color{red}h}) + X, \ldots$$

**LHC / RHIC**

$$e\,P \to e + J + X\quad e\,P \to Q + \bar{Q} + X,\quad e\,P \to J({\color{red}h}) + X, \ldots$$

**EIC**

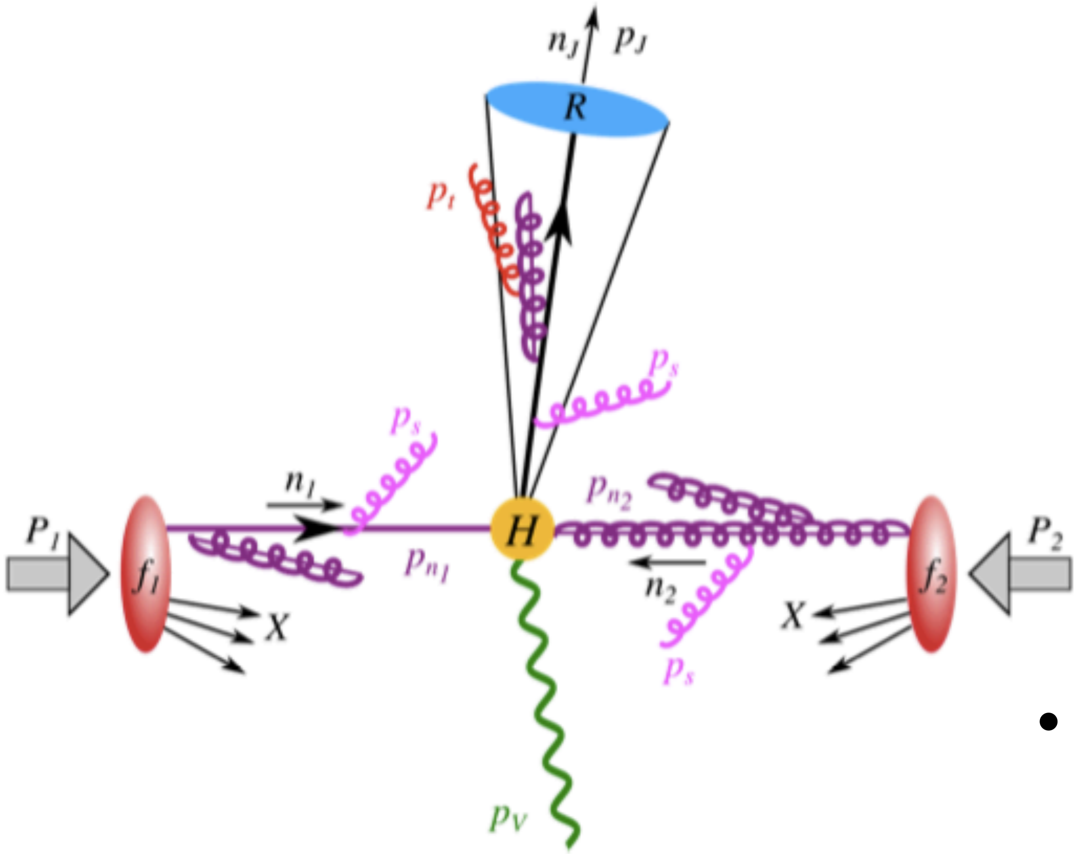


*Fig. from Chien, Shao, Wu `19*

- Many experiments sensitive to such processes
- Standard processes have low sensitivity to gluon TMDs.
- Standard processes sensitive to two TMDs simultaneously; many involving jets will only be sensitive to a single TMD.

1) Inclusive jet production

$$e\,P \to J({\color{red}h}) + X$$

2) Lepton + jet imbalance

$$e\,P \to e + J + X$$

3) Lepton + jet imbalance with hadron in jet

$$e\,P \to e + J({\color{red}h}) + X$$

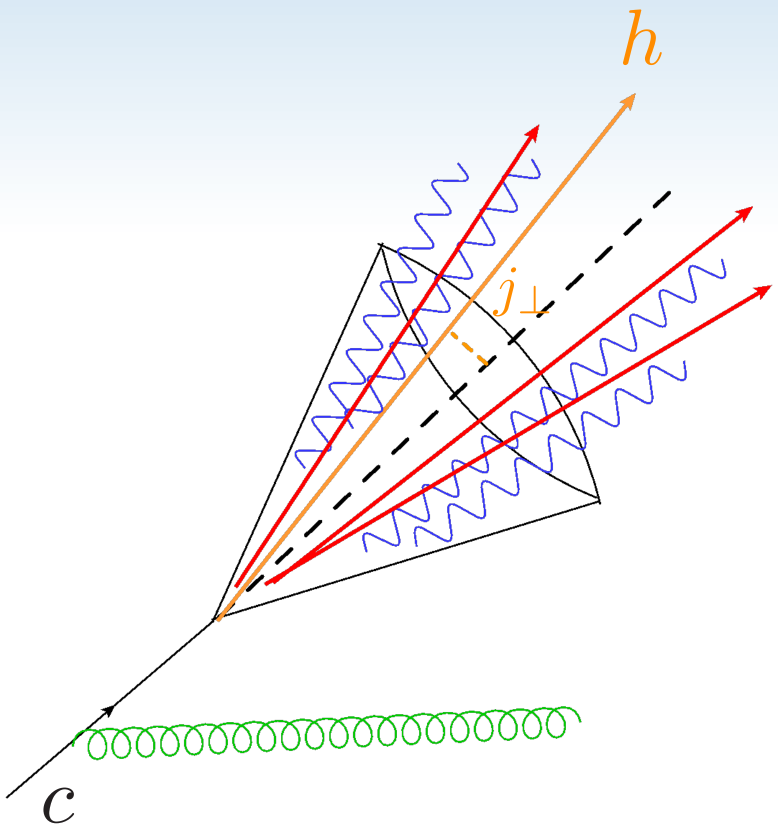
# Hadron inside inclusive jet production

**Unpolarized case:**

**(replace $pp$ with $ep$ for EIC)**

$$\frac{d\sigma^{pp\to \text{jet}(h)X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c^h(z_h, \boldsymbol{j}_\perp)$$

$$\Lambda_{\text{QCD}} \qquad p_T \qquad p_T R \quad \Lambda_{\text{QCD}}$$



**where** $z = p_T^J / p_T^c$

$z_h = p_T^h / p_T^J$

$$\boxed{\frac{d\sigma^{pp\to hX}}{dp_T d\eta} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \boxed{D_c^h}}$$

$$\Lambda_{\text{QCD}} \qquad p_T \qquad \Lambda_{\text{QCD}}$$

**where** $z = p_T^h / p_T^c$

*Procura, Stewart `10*
*Arleo, Fontannaz, Guillet, Nguyen `14*
*Kaufmann, Mukherjee, Vogelsang `15*
*Kang, Ringer, Vitev `16*
*Dai, Kim, Leibovich `16*

# Hadron inside inclusive jet production

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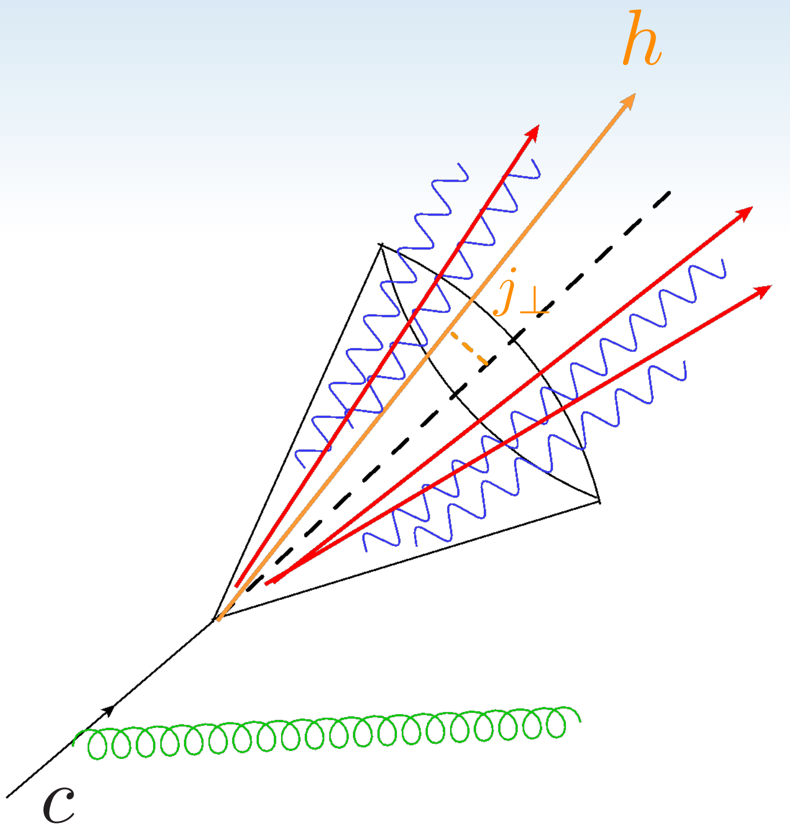
$\Lambda_{\mathrm{QCD}}$ $p_T$ $p_T R$

$\Lambda_{\mathrm{QCD}}$



**where**

$$z = p_T^J/p_T^c$$
$$z_h = p_T^h/p_T^J$$

(including polarized jet fragmentation functions)

**TMD Fragmentation Functions (TMDFFs)**

Quark polarization / Hadron polarization

| | U | L | T |
|---|---|---|---|
| **U** | $D^{h/q}$ | | $H^{\perp h/q}$ |
| **L** | | $G^{h/q}$ | $H_L^{\perp h/q}$ |
| **T** | $D_T^{\perp h/q}$ | $G_T^{h/q}$ | $H^{h/q}$ $H_T^{\perp h/q}$ |

**TMD Jet Fragmentation Functions (TMDJFFs)**

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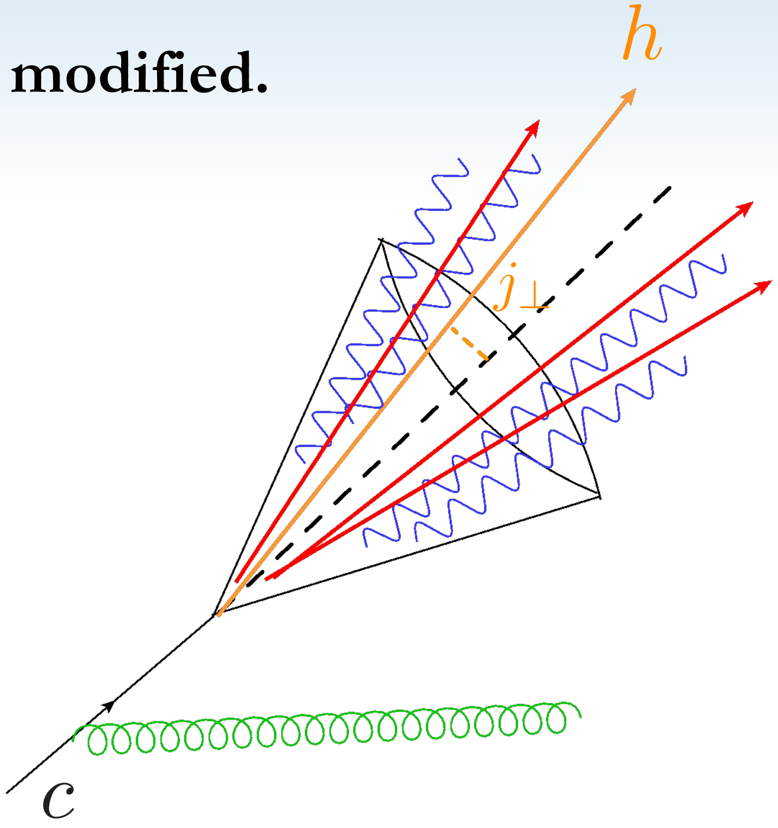
| **T** | $\mathcal{D}_T^{\perp h/q}$ | $\mathcal{G}_T^{h/q}$ | $\mathcal{H}^{h/q}$ $\mathcal{H}_T^{\perp h/q}$ |

# TMD hadron in jet (TMDJFFs)

- Still hard-collinear factorization structure other than jet function modified.

$$\frac{d\sigma^{pp\to \text{jet}(h)X}}{dp_T d\eta dz_h d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c^h(z_h, \boldsymbol{j}_\perp)$$



**When** $\Lambda_{\rm QCD} \lesssim j_\perp \ll p_T R, \qquad \lambda \sim j_\perp/p_T$

collinear $\quad k_c \sim p_T(\lambda^2, 1, \lambda)$

soft $\quad k_s \sim p_T(\lambda R, \lambda/R, \lambda)$

$$\mathcal{G}_c^h(z, z_h, \omega_J R, \boldsymbol{j}_\perp, \mu) = \mathcal{H}_{c\to i}(z, \omega_J R, \mu) \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{\lambda}_\perp \delta^2\left(z_h \boldsymbol{\lambda}_\perp + \boldsymbol{k}_\perp - \boldsymbol{j}_\perp\right)$$

$$\times D_{h/i}(z_h, \boldsymbol{k}_\perp, \mu, \nu) S_i(\boldsymbol{\lambda}_\perp, \mu, \nu R)$$

$$= \mathcal{C}_{c\to i}(z, p_T R, \mu) \underbrace{\hat{D}_{h/i}(z_h, \boldsymbol{j}_\perp; \mu = p_T R)}$$

**Standard subtracted TMDFFs, say in SIDIS**
**Shown to be true perturbatively at NLO**

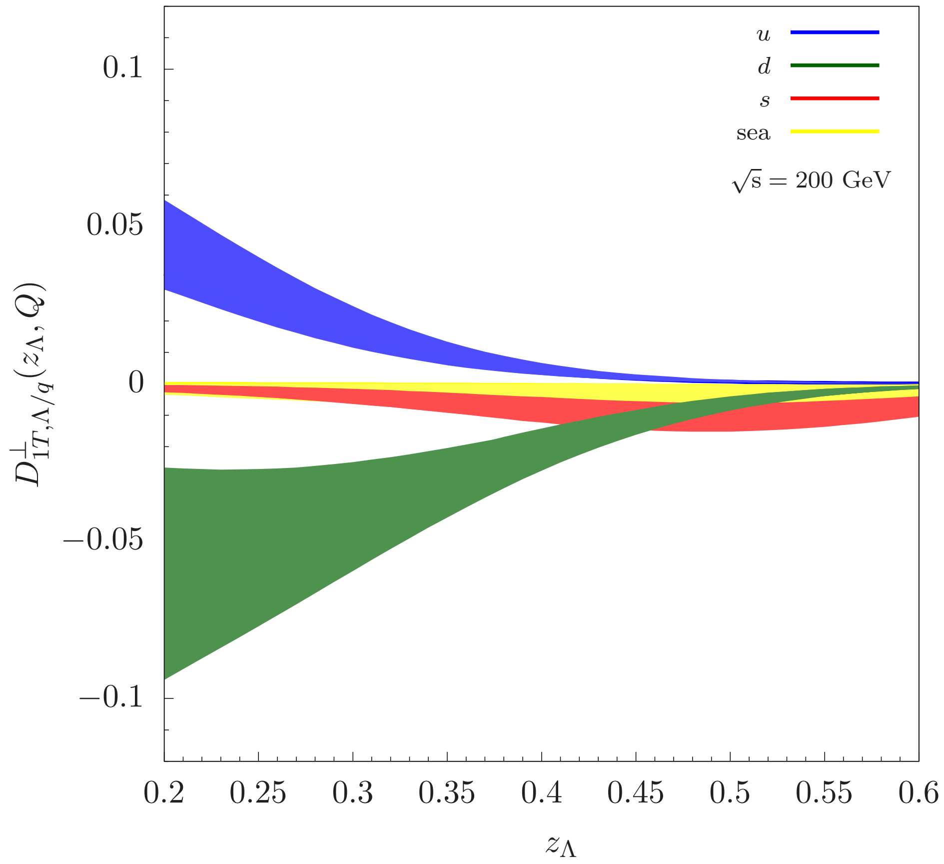
- **TMDJFFs can be related to TMDFFs**

*Kang, Liu, Ringer, Xing, `17*
*Kang, KL, Zhao, `20*

# Polarizing JFF

- **Polarizing FF fits from Belle data**

*Callos, Kang, Terry, `20*



**TMD Fragmentation Functions (TMDFFs)**

Quark polarization

Hadron polarization

| | U | L | T |
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| **U** | $D^{h/q}$ | | $H^{\perp h/q}$ |
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**Polarizing FF**

**TMD Jet Fragmentation Functions (TMDJFFs)**

Quark polarization

Hadron polarization

| | U | L | T |
|---|---|---|---|
| **U** | $\mathcal{D}^{h/q}$ | | $\mathcal{H}^{\perp h/q}$ |
| **L** | | $\mathcal{G}^{h/q}$ | $\mathcal{H}_L^{\perp h/q}$ |
| **T** | $\mathcal{D}_T^{\perp h/q}$ | $\mathcal{G}_T^{h/q}$ | $\mathcal{H}^{h/q}$ $\mathcal{H}_T^{\perp h/q}$ |

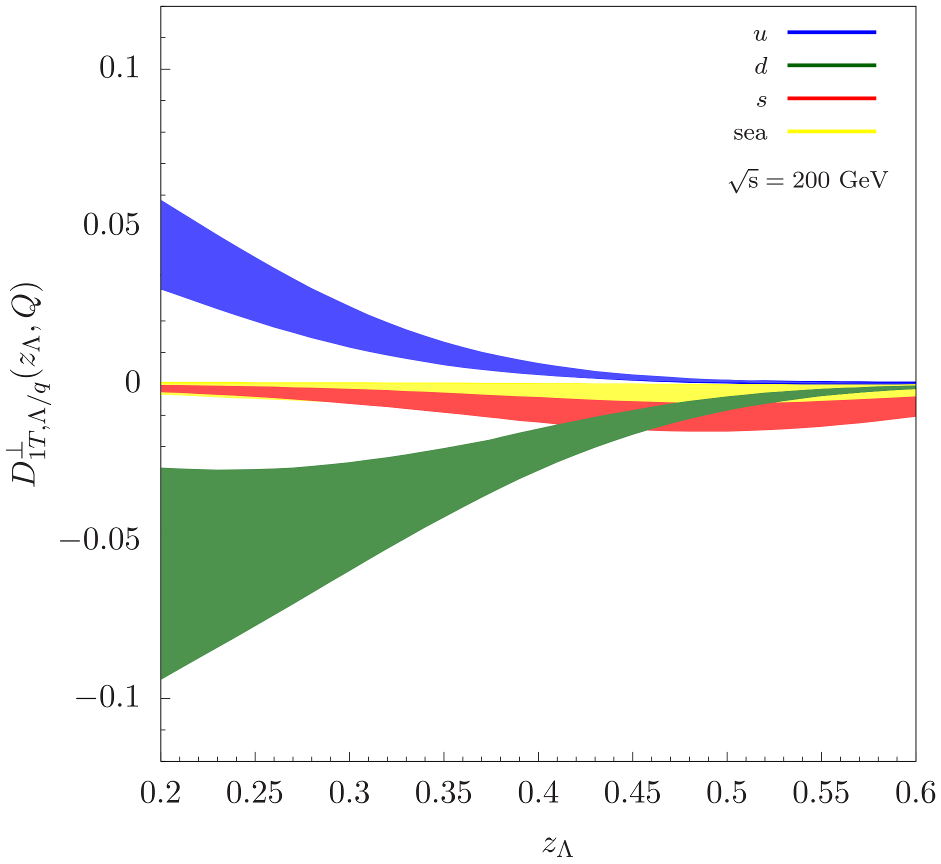
**Polarizing JFF**

*Kang, KL, Zhao, `20*

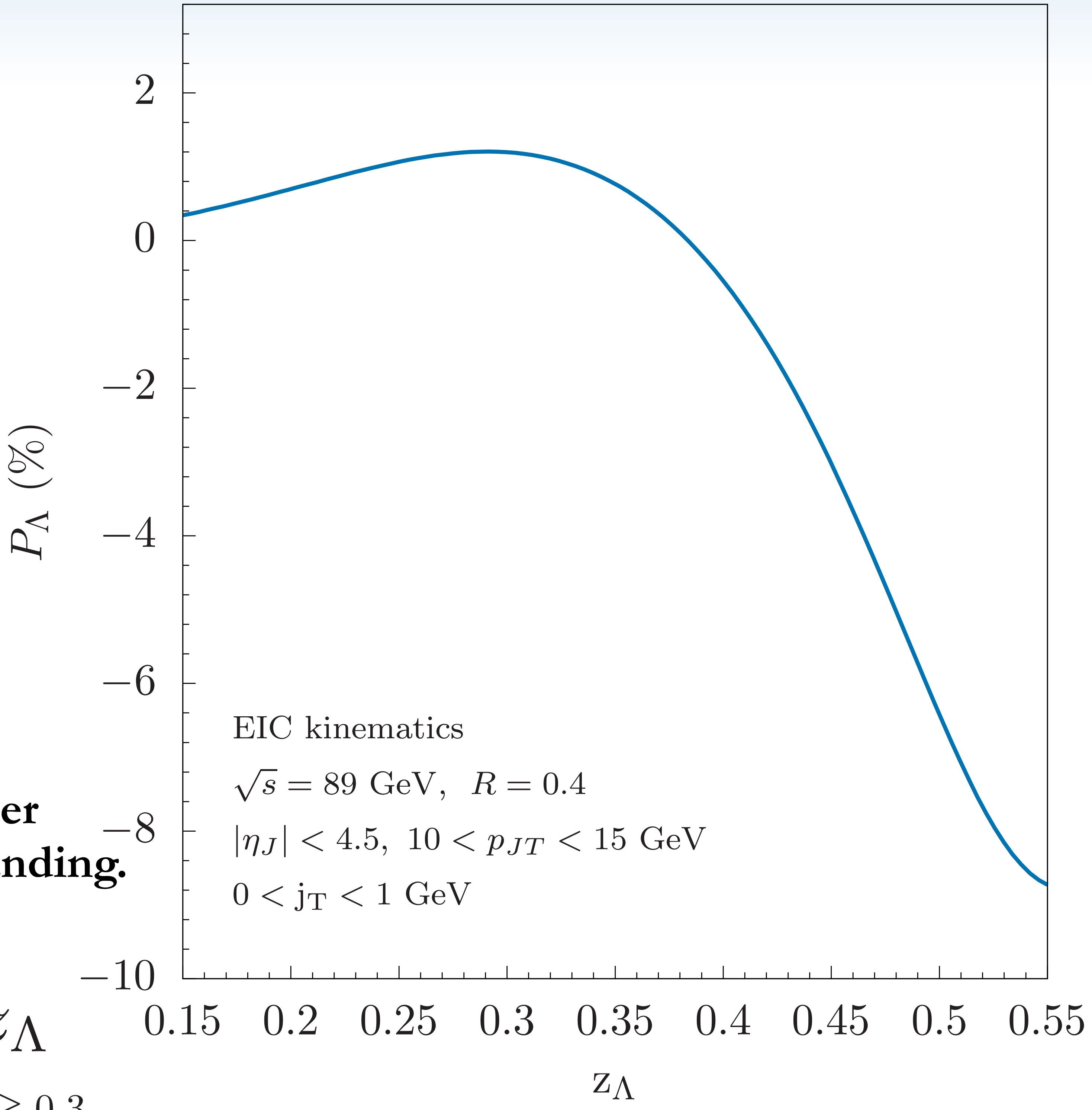
# Polarizing JFF

- Polarizing FF fits from Belle data

*Callos, Kang, Terry, `20*



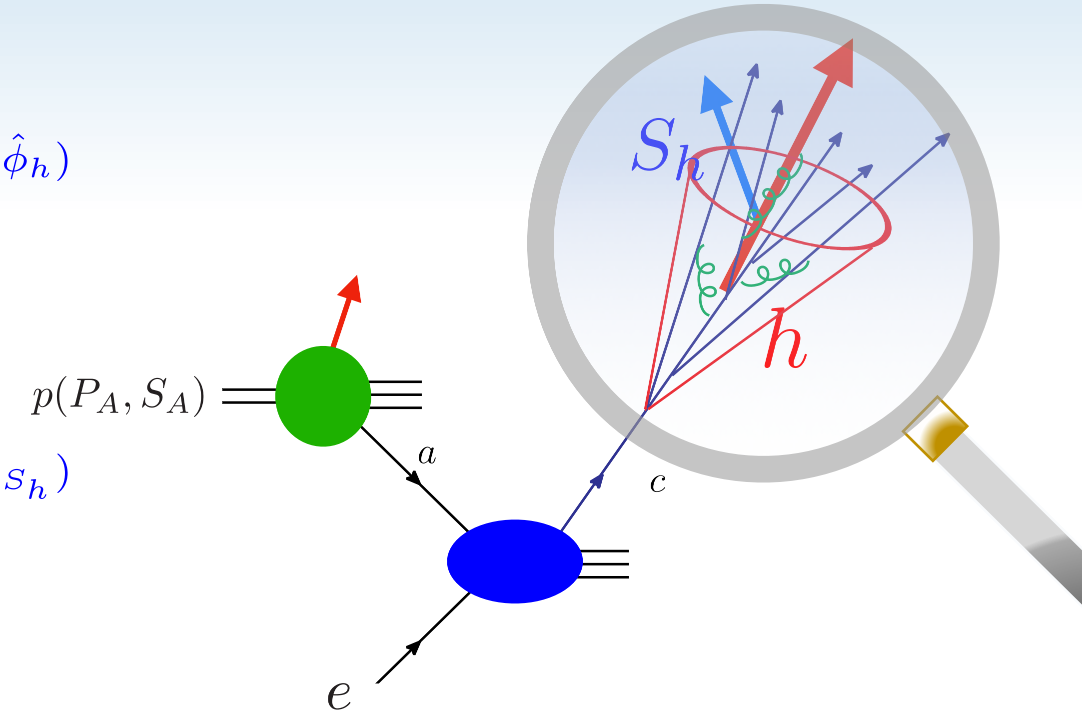
- Differential information in $z_\Lambda$ gives better constraint on data / qualitative understanding.
- Predictions at the EIC kinematics
  - Positive from up quark PFF at small $z_\Lambda$
  - Negative from down quark PFF at $z_\Lambda \gtrsim 0.3$



*Kang, KL, Zhao, `20*

# Azimuthal angular dependence

$$\begin{aligned}
\frac{d\sigma^{p(S_A)+p/e\to(\text{jet}\,h(S_h))X}}{dp_{JT}\,d\eta_J\,dz_h\,d^2\boldsymbol{j}_\perp} &= F_{UU,U} + |\boldsymbol{S}_T|\sin(\phi_{S_A}-\hat\phi_h)F_{TU,U}^{\sin(\phi_{S_A}-\hat\phi_h)} \\
&+ \Lambda_h\left[\lambda F_{LU,L} + |\boldsymbol{S}_T|\cos(\phi_{S_A}-\hat\phi_h)F_{TU,L}^{\cos(\phi_{S_A}-\hat\phi_h)}\right] \\
&+ |\boldsymbol{S}_{h\perp}|\Big\{\sin(\hat\phi_h-\hat\phi_{S_h})F_{UU,T}^{\sin(\hat\phi_h-\hat\phi_{S_h})} + \lambda\cos(\hat\phi_h-\hat\phi_{S_h})F_{LU,T}^{\cos(\hat\phi_h-\hat\phi_{S_h})} \\
&\quad + |\boldsymbol{S}_T|\Big(\cos(\phi_{S_A}-\hat\phi_{S_h})F_{TU,T}^{\cos(\phi_{S_A}-\hat\phi_{S_h})} \\
&\quad + \cos(2\hat\phi_h-\hat\phi_{S_h}-\phi_{S_A})F_{TU,T}^{\cos(2\hat\phi_h-\hat\phi_{S_h}-\phi_{S_A})}\Big)\Big\},
\end{aligned}$$



$$F_{S_A S_B,S_h}$$

$\uparrow$

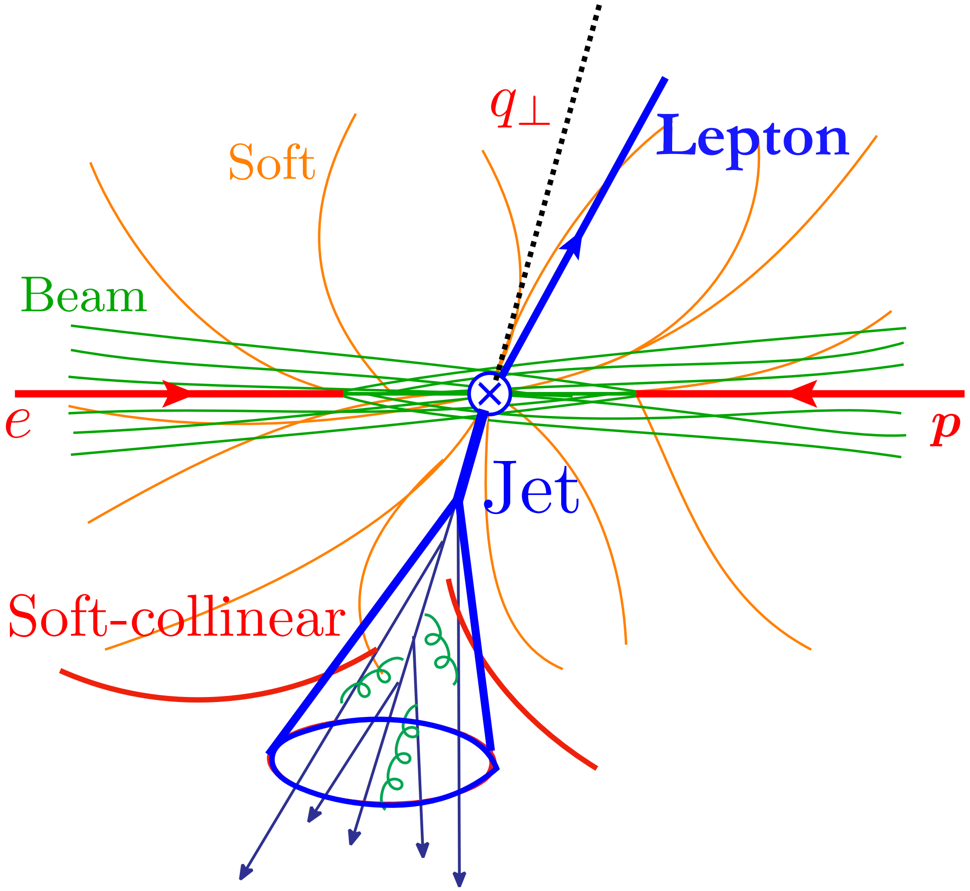
Polarization of $A, B, h$

- **Different structures come with different characteristic angular dependence.**

*Kang, KL, Zhao, `20*

# Lepton + Jet imbalance

- **One of the simplest process** $e + P \to e + \text{Jet} + X$



$$q_\perp \equiv |\vec{p}_{e\perp} + \vec{p}_{J\perp}|, \qquad p_\perp \equiv |\vec{p}_{e\perp} - \vec{p}_{J\perp}|/2$$

$q_\perp \ll p_\perp$, **sensitive to the large logs of** $\ln(q_\perp/p_\perp)$ **and TMD structures of the hadrons.**

$$q_\perp = p_{X,\perp} = |\vec{k}_{c,\perp} + \vec{k}_{gs,\perp} + \vec{k}_{sc,\perp}|$$

**Giving relevant modes :** $(+,-,\perp)$ $\qquad \lambda = q_\perp/p_\perp$

$n$-collinear $\quad k_n \sim p_\perp(\lambda^2, 1, \lambda)_{n\bar{n}}$

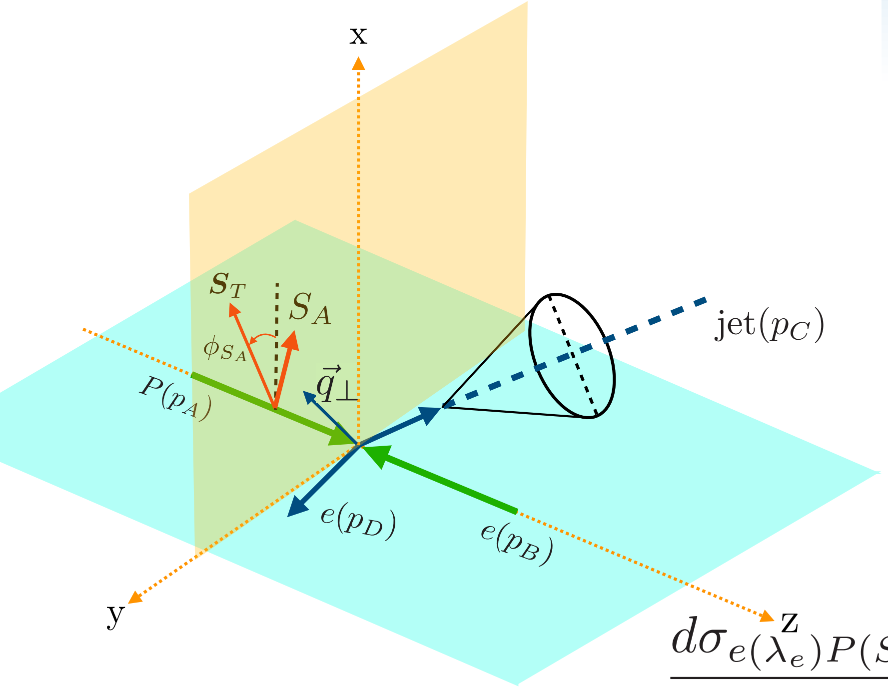
global soft $\quad k_{gs} \sim p_\perp(\lambda, \lambda, \lambda)$

soft-collinear $\quad k_{sc} \sim p_\perp R(\lambda R, \lambda/R, \lambda)_{n_J,\bar{n}_J}$

$n_J$-collinear $\quad k_J \sim p_\perp(R^2, 1, R)_{n_J,\bar{n}_J}$

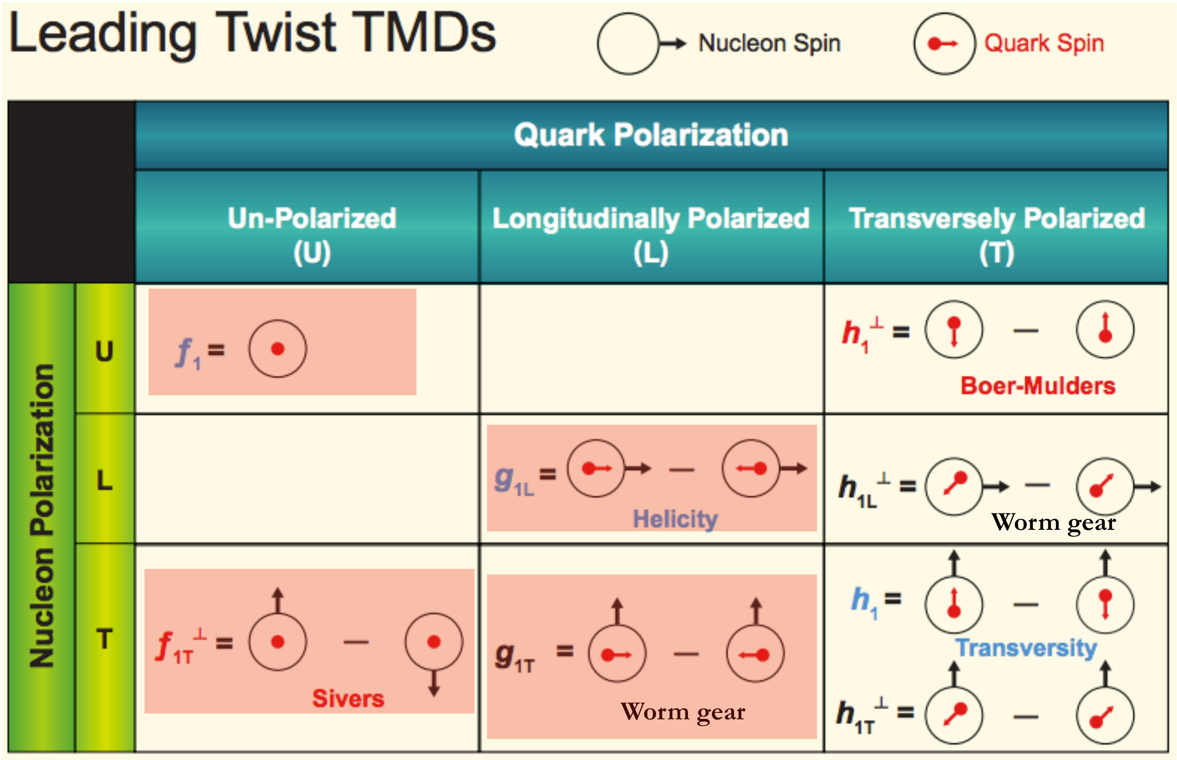
*Liu, Ringer, Vogelsang, Yuan `18, `20*
*Arratia, Kang, Prokudin, Ringer `20*

# Lepton + Jet imbalance



$$\frac{d\sigma_{eP\to e+\text{jet}}}{dp_\perp dq_\perp} = \int \prod_i^3 d^2k_{i\perp} H(Q)\,\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}-q_\perp) \times f_a(x,\vec{k}_{1\perp})\,S^{\text{global}}(\vec{k}_{2\perp})\,S_{J_c}(\vec{k}_{3\perp})\,J_c(p_\perp R)$$

$$\frac{d\sigma_{e(\lambda_e)P(S)\to e+\text{jet}}}{dp_\perp dq_\perp} = \overset{\sim f_1}{\boxed{F_{UU}}} + \lambda_p\lambda_e \overset{\sim g_{1L}}{\boxed{F_{LL}}} + S_T\left\{\sin(\phi_{S_A}-\phi_q)\underset{\sim f_{1T}^\perp}{\boxed{F_{TU}^{\sin(\phi_{S_A}-\phi_q)}}} + \lambda_e\cos(\phi_{S_A}-\phi_q)\underset{\sim g_{1T}}{\boxed{F_{TL}^{\cos(\phi_{S_A}-\phi_q)}}}\right\},$$



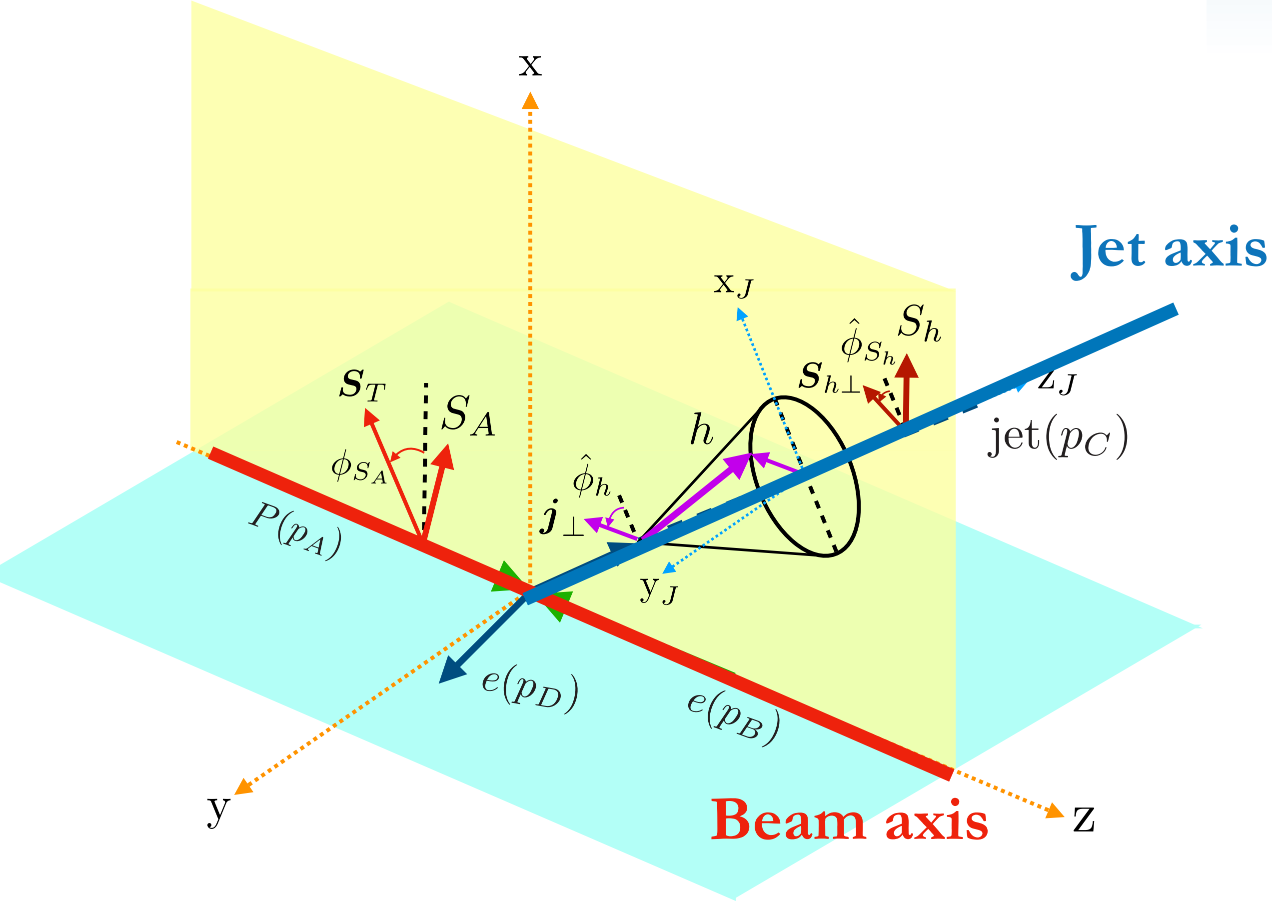
Leading Twist TMDs — Nucleon Spin — Quark Spin

| Nucleon Polarization | Quark Polarization: Un-Polarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
|---|---|---|---|
| U | $f_1$ = | | $h_1^\perp$ = − <br>Boer-Mulders |
| L | | $g_{1L}$ = − <br>Helicity | $h_{1L}^\perp$ = − <br>Worm gear |
| T | $f_{1T}^\perp$ = − <br>Sivers | $g_{1T}$ = − <br>Worm gear | $h_1$ = − <br>Transversity<br>$h_{1T}^\perp$ = − |

- **With jet, only sensitive to single TMDs (compared to standard processes)**
- **We do not get sensitivity to all TMDPDFs**

*Liu, Ringer, Vogelsang, Yuan `18, `20*
*Arratia, Kang, Prokudin, Ringer `20*
*Kang, KL, Shao, Zhao; In progress*

# Polarized Jet Fragmentation Functions and lepton + jet imbalance



- Observation of polarized hadron inside jet gives sensitivity to all TMDPDFs and TMDFFs. (analogous correlations to standard SIDIS)

- Sensitivity to two TMDs, but sensitive to $\vec{q}_\perp$ and $\vec{j}_\perp$ separately (advantage of two axes)

## Many characteristic correlations

$$
\begin{aligned}
\frac{d\sigma^{p(S_A)+e(\lambda_e)\to e+(\text{jet}\,h(S_h))+X}}{dy_l d^2\boldsymbol{l}_T d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} &= F_{UU,U} + \cos(\phi_q - \hat{\phi}_h) F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)} \\
&+ \lambda_p \Big\{ \lambda_e F_{LL,U} + \sin(\phi_q - \hat{\phi}_h) F_{LU,U}^{\sin(\phi_q-\hat{\phi}_h)} \Big\} \\
&+ S_T \Big\{ \sin(\phi_{S_A} - \phi_q) F_{TU,U}^{\sin(\phi_{S_A}-\phi_q)} + \lambda_e \cos(\phi_{S_A} - \phi_q) F_{TL,U}^{\cos(\phi_{S_A}-\phi_q)} \\
&\quad + \sin(\phi_{S_A} - \hat{\phi}_h) F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)} + \sin(2\phi_q - \hat{\phi}_h - \phi_{S_A}) F_{TU,U}^{\sin(2\phi_q-\hat{\phi}_h-\phi_{S_A})} \Big\} \\
&+ \Lambda_h \Big\{ \lambda_e F_{UL,L} + \sin(\phi_q - \hat{\phi}_h) F_{UU,L}^{\sin(\phi_q-\hat{\phi}_h)} + \lambda_p \Big[ F_{LU,L} + \cos(\phi_q - \hat{\phi}_h) F_{LU,L}^{\cos(\phi_q-\hat{\phi}_h)} \\
&\quad + S_T \Big[ \cos(\phi_{S_A} - \phi_q) F_{TU,L}^{\cos(\phi_{S_A}-\phi_q)} + \lambda_e \sin(\phi_{S_A} - \phi_q) F_{TL,L}^{\sin(\phi_{S_A}-\phi_q)} \\
&\quad + \cos(\phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)} + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_h)} \Big] \Big\} \\
&+ S_{h\perp} \Big\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} \\
&\quad + \sin(\phi_q - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\phi_q-\hat{\phi}_{S_h})} + \sin(\hat{\phi}_{S_h} + \phi_q - 2\hat{\phi}_h) F_{UU,T}^{\sin(\hat{\phi}_{S_h}+\phi_q-2\hat{\phi}_h)} \\
&\quad + \lambda_p \Big[ \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cos(\phi_q - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\phi_q-\hat{\phi}_{S_h})} \\
&\qquad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{LU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} \Big] \\
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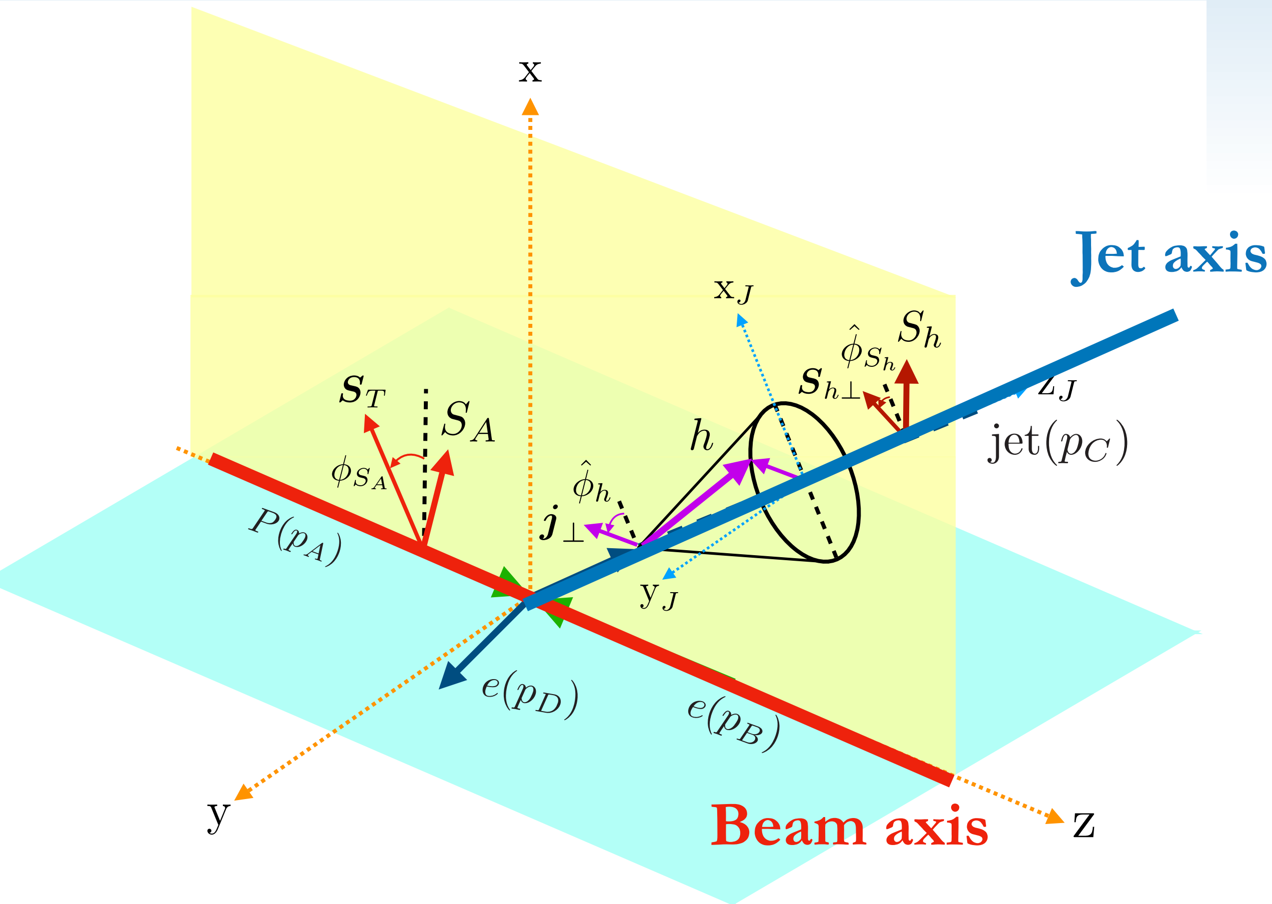
&\quad + \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_{S_A} - \phi_q) F_{TU,T}^{\sin(\phi_{S_A}-\hat{\phi}_h)\sin(\phi_q-\phi_{S_A})} \\
&\quad + \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_{S_A} - \phi_q)) F_{TU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_{S_A}-\phi_q)} \\
&\quad + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_{S_h})} \\
&\qquad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} + 2\phi_q - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}+2\phi_q-\phi_{S_A})} \\
&\quad + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_{S_A} - \phi_q) F_{TL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})\sin(\phi_{S_A}-\phi_q)} \\
&\quad + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_{S_A} - \phi_q)) F_{TL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})\cos(\phi_{S_A}-\phi_q)} \Big] \Big\},
\end{aligned}
$$

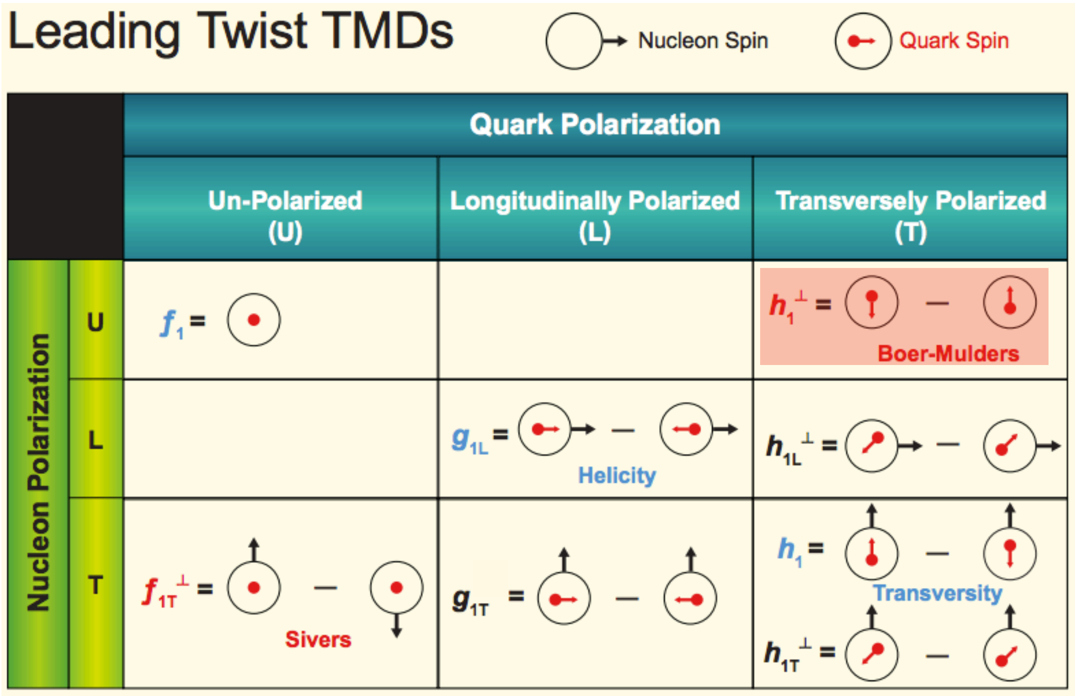
*Arratia, Kang, Prokudin, Ringer `20*
*Kang, KL, Shao, Zhao; In progress*

# Phenomenology : $A^{\cos(\phi_q-\hat{\phi}_h)}$



$$A^{\cos(\phi_q-\hat{\phi}_h)} \equiv \frac{F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}(q_\perp, j_\perp)}{F_{UU,U}(q_\perp, j_\perp)} \sim \frac{h_1^\perp(q_\perp)H_1^\perp(j_\perp)}{f_1(q_\perp)D_1(j_\perp)}$$

- Boer-Mulders and Collins functions sensitive to transverse momentum measured with respect to different axes.
- "Separation" of the incoming and outgoing dynamics.


Leading Twist TMDs ⊙→ Nucleon Spin ⊙ Quark Spin

| Nucleon Polarization | Quark Polarization: Un-Polarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
|---|---|---|---|
| U | $f_1$ = ⊙ | | $h_1^\perp$ = ⊙ − ⊙<br>Boer-Mulders |
| L | | $g_{1L}$ = ⊙→ − ⊙→<br>Helicity | $h_{1L}^\perp$ = ⊙→ − ⊙→ |
| T | $f_{1T}^\perp$ = ⊙ − ⊙<br>Sivers | $g_{1T}$ = ⊙ − ⊙ | $h_1$ = ⊙ − ⊙<br>Transversity<br>$h_{1T}^\perp$ = ⊙ − ⊙ |

Quark polarization

| Hadron polarization | U | L | T |
|---|---|---|---|
| U | $D^{h/q}$ | | $H^{\perp h/q}$ Collins |
| L | | $G^{h/q}$ | $H_L^{\perp h/q}$ |
| T | $D_T^{\perp h/q}$ | $G_T^{h/q}$ | $H^{h/q}$ $H_T^{\perp h/q}$ |

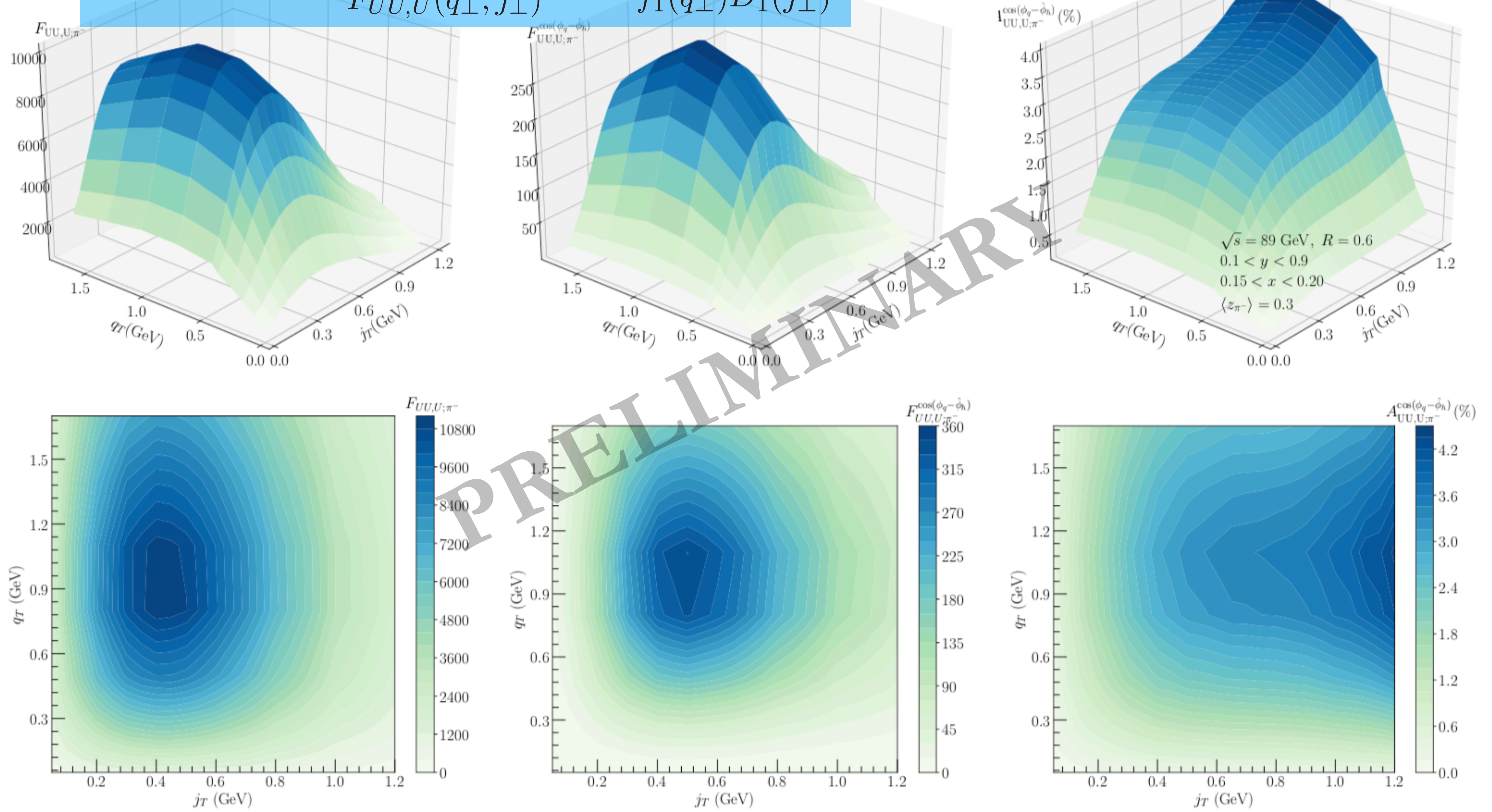
*Arratia, Kang, Prokudin, Ringer `20*
*Kang, KL, Shao, Zhao; In progress*

# Unpolarized π in jet (Boer-Mulders, Collins)

**$\pi^-$** $q_T$ [0,1.8], $j_T$ [0,1.2]

Denominator Numerator Ratio

$$A^{\cos(\phi_q-\hat{\phi}_h)} \equiv \frac{F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}(q_\perp,j_\perp)}{F_{UU,U}(q_\perp,j_\perp)} \sim \frac{h_1^\perp(q_\perp)H_1^\perp(j_\perp)}{f_1(q_\perp)D_1(j_\perp)}$$



Parametrization from *Barone, Melis, Prokudin `10*

*Kang, KL, Shao, Zhao; In progress*

# Thank you!