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$\cos(2\phi_h)$ asymmetry in J/ψ production in unpolarized ep collision

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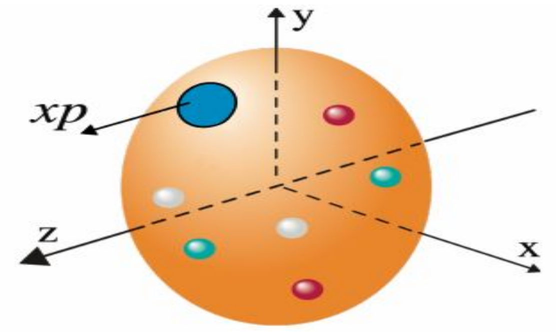
Transverse Momentum Dependent (TMD) Distributions

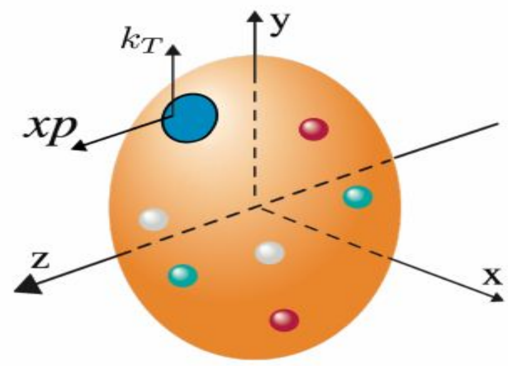


1D

Collinear pdf $f(x)$

Universality

$$lp \to lX$$



3D

TMD pdf $f(x, k_\perp)$

Non-trivial Universality

$$lp \to lhX$$

$$pp \to hX$$

Operator definition of Gluon TMDs

The unpolarised gluon TMDs are defined as Fourier transform of forward matrix elements of bilocal products of the gluon field strength.

$$\mathscr{F}(x,k_\perp) = 2\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 2P^+} e^{ixP^+\xi^- - ik_\perp.\xi_\perp} \left\langle P \,|\, \mathrm{Tr}[F^{i+}(0)U^{[\mathscr{C}]}F^{i+}(\xi)U^{[\mathscr{C}']} \,|\, P \right\rangle$$

Parameterization of gluon correlator at "Leading Twist" is

$$\Phi_g^{\mu\nu}(x,\mathbf{k}_\perp^2) = -\frac{1}{2x}\left\{ \underbrace{g_T^{\mu\nu} f_1^g(x,\mathbf{k}_\perp^2)}_{\text{Unpolarized gluon TMD}} - \left(\frac{k_\perp^\mu k_\perp^\nu}{M_h^2} + g_T^{\mu\nu}\frac{\mathbf{k}_\perp^2}{2M_h^2}\right) \underbrace{h_1^{\perp g}(x,\mathbf{k}_\perp^2)}_{\text{Linearly polarized gluon TMD}} \right\}$$

[P. Mulders and J. Rodrigues, PRD 63, 094021 (2001)]

Linearly polarised gluon distribution

Linearly polarized gluon distributions were first introduced by [Mulders and Rodrigues, PRD 63, 094021 (2001)]

- *It affects unpolarised cross section and cause azimuthal asymmetries: cos2ϕ, cos4ϕ.*
- *It's a time-reversal even function and in small-x region, it can be WW type or Dipole distribution depending on gauge link.*
- *It can be probed in Drell-Yan process and SIDIS process.*
- *No experimental investigation has been carried out to extract the $h_1^{\perp g}$ until now .*
- *Theoretical upper bound* $\frac{\mathbf{k}_\perp^2}{2M_h^2}|h_1^{\perp g}(x,\mathbf{k}_\perp^2)| \leq f_1^g(x,\mathbf{k}_\perp^2)$

Quarkonium production is an effective way to extract $h_1^{\perp g}$

Quarkonium production models

Color Singlet Model

Initial $c\bar{c}$ pair should have the same spin, orbital and color quantum numbers as that of the final quarkonium.

[Chang 80, Berger, Jones 81, Baier,Ruckl 81, Schuler 94,Lansberg 11]

Color Evaporation Model (CEM)

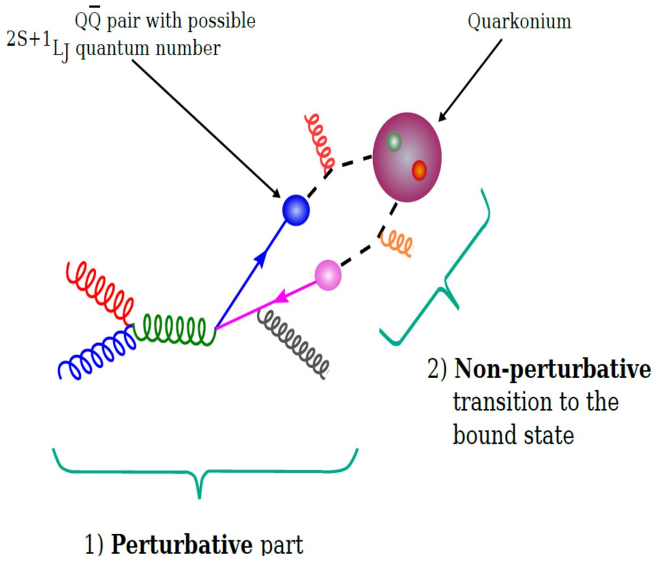
Probability of forming any quarkonium state is independent of the spin , orbital and color quantum numbers of the heavy quark pair.

[Fritzsch 77, Halzen 77, Gluck,Owens, Reya 78]

Non-relativistic QCD (NRQCD)

The heavy $c\bar{c}$ pair can be produced in color singlet (CS) state or (CO) state

[Bodwin, Braaten, Lepage 95]



Non-Relativistic QCD (NRQCD)

In the rest frame of bound state, the relative momenta of two quarks is small compared to their mass, and that allows to non-relativistic approach called NRQCD.

NRQCD FACTORISATION

[G. T. Bodwin et al, PRD 51 (1995)]

$$d\sigma^{ab\to J/\psi} = \sum_n d\hat{\sigma}\left[ab \to c\bar{c}(n)\right] \langle 0|\mathcal{O}_n^{J/\psi}|0\rangle$$

Non-perturbative Long distance Matrix Elements(LDMEs)

Perturbative short distance coefficients

Describe the conversion of $c\bar{c}[n]$ *state into final state* J/ψ

Cross sections in a particular color, angular momentum and spin state "n": ${}^{2S+1}L_J^{[1,8]}$, *calculated by perturbative QCD*

${}^3S_1^{[1]}$ *(singlet), and* ${}^3P_J^{[8]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$ *(octets)*

$h_{1\perp}^{g}$ in J/ψ Production

The Leading order process contributing to the cos2ϕ asymmetry in $\gamma^* + g \rightarrow c + \bar{c}$

Contributes at $z=1$, where z is energy fraction of $\gamma*$ carried by J/ψ in proton rest frame

[A. Mukherjee and S. Rajesh, EPJC 77, 854 (2017)]

It was extended to the kinematical region $z<1$ in the CS model.

[RK and A. Mukherjee; Phys.Rev.D99(2019)]

In this project we further extended it and incorporated the NRQCD based CO contributions to J/ψ production mechanism.

$$e\ p \rightarrow e'\ J/\psi\ X$$

Using the TMD factorization

$$d\sigma = \frac{1}{2s}\frac{d^3 l'}{(2\pi)^3 2E_l'}\frac{d^3 P_h}{(2\pi)^3 2E_{P_h}}\int\frac{d^3 p_g}{(2\pi)^3 2E_g}\int dx d^2\mathbf{k}_\perp (2\pi)^4\delta(q+k-P_h-p_g)$$
$$\frac{1}{Q^4}\mathcal{L}^{\mu\mu'}(l,q)\Phi^{\nu\nu'}(x,\mathbf{k}_\perp)\,\mathcal{M}_{\mu\nu}(\mathcal{M}_{\mu'\nu'})^*$$

Scattering amplitude

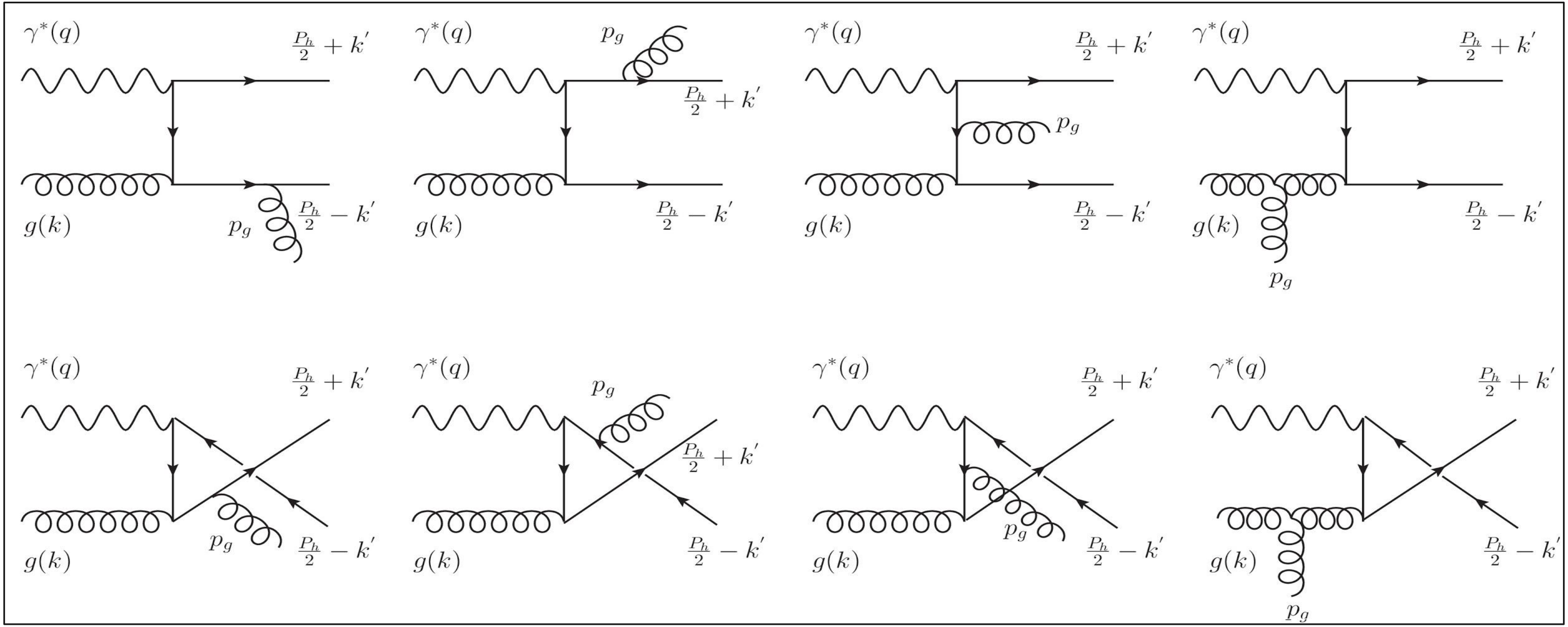
Gluon correlator

$$\boxed{\Phi_g^{\nu\nu'}(x,\mathbf{k}_\perp) = -\frac{1}{2x}\left\{g_\perp^{\nu\nu'} f_1^g(x,\mathbf{k}_\perp^2) - \left(\frac{k_\perp^\nu k_\perp^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'}\frac{\mathbf{k}_\perp^2}{2M_p^2}\right) h_1^{\perp g}(x,\mathbf{k}_\perp^2)\right\}}$$

Leptonic tensor

$$\boxed{\mathcal{L}^{\mu\mu'}(l,q) = e^2\left(-g^{\mu\mu'}Q^2 + 2(l^\mu l'^{\mu'} + l^{\mu'} l'^{\mu})\right)}$$

$$\gamma^* + g \to c + \bar{c} + g$$



Feynman Diagrams

Scattering Amplitude

[D. Boer and C. Pisano, PRD 86,094007 (2012)]

$$\mathcal{M}\left(\gamma^* g \to Q\bar{Q}\left[{}^{2S+1}L_j^{(1,8)}\right](P_h) + g\right) = \sum_{L_z S_z} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \Psi_{LL_z}(\mathbf{k}')\langle LL_z; SS_z | JJ_z\rangle$$
$$Tr[O(q,k,P_h,k')\mathcal{P}_{SS_z}(P_h,k')]$$

Non-relativistic orbital angular momentum bound-state wave function

Clebsch-Gordon coefficients

$$O(q,k,P_h,k') = \sum_{m=1}^{8} \mathcal{C}_m O_m(q,k,P_h,k')$$

Spin projection operator

$$\mathcal{P}_{SS_z}(P_h,k') = \sum_{s_1,s_2} \left\langle \frac{1}{2}s_1; \frac{1}{2}s_2 | SS_z \right\rangle v\left(\frac{P_h}{2} - k', s_1\right) \bar{u}\left(\frac{P_h}{2} + k', s_2\right),$$
$$= \frac{1}{4M^{3/2}}(-\not{P}_h + 2\not{k}' + M)\Pi_{SS_z}(\not{P}_h + 2\not{k}' + M) + \mathcal{O}(k'^2)$$

${}^{3}S_{1}^{(8)}$ Amplitude

$$\mathcal{M}[{}^3S_1^{(8)}](P_h,k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\sqrt{2}}{2} d_{abc} \mathrm{Tr}\left[\sum_{m=1}^{3} O_m(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}\right],$$

$$\sum_{m=1}^{3} O_m(0) = g_s^2(ee_c)\varepsilon^{\mu}_{\lambda_a}(k)\varepsilon^{\nu}_{\lambda_b}(q)\varepsilon^{\rho*}_{\lambda_g}(p_g)\left[\frac{\gamma_\nu(\not{P}_h - 2\not{q} + M)\gamma_\mu(-\not{P}_h - 2\not{p}_g + M)\gamma_\rho}{(\hat{s}-M^2)(\hat{u}-M^2)} + \frac{\gamma_\rho(\not{P}_h + 2\not{p}_g + M)\gamma_\nu(-\not{P}_h + 2\not{k} + M)\gamma_\mu}{(\hat{s}-M^2)(\hat{t}-M^2)} + \frac{\gamma_\nu(\not{P}_h - 2\not{q} + M)\gamma_\rho(-\not{P}_h + 2\not{k} + M)\gamma_\mu}{(\hat{t}-M^2)(\hat{u}-M^2)}\right].$$

${}^{3}S_{1}^{(1)}$ Amplitude

$$\mathcal{M}[{}^3S_1^{(1)}](P_h,k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\delta_{ab}}{\sqrt{N_c}} \mathrm{Tr}\left[\sum_{m=1}^{3} O_m(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}\right],$$

${}^{1}S_{0}^{(8)}$ Amplitude

$$\mathcal{M}[{}^1S_0^{(8)}](P_h,k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\sqrt{2}}{2} i f_{abc} \mathrm{Tr}\left[\,(O_1(0) - O_2(0) - O_3(0) + 2O_4(0))\,(-\not{P}_h + M)\gamma^5\right]$$

$$O_4(0) = g_s^2(ee_c)\varepsilon^{\mu}_{\lambda_a}(k)\varepsilon^{\nu}_{\lambda_b}(q)\varepsilon^{\rho*}_{\lambda_g}(p_g)\frac{\gamma_\nu(\not{P}_h - 2\not{q} + M)\gamma^\sigma}{\hat{u}(\hat{u}-M^2)}\mathcal{T}_{\mu\rho\sigma}(k,p_g)$$

Three gluon vertex

${}^{3}p_{J(1,2,3)}^{(8)}$ Amplitude

$$\mathcal{M}[{}^3P_J^{(8)}](P_h,k) = \frac{\sqrt{2}}{2} f_{abc}\sqrt{\frac{3}{4\pi}} R_1'(0) \sum_{L_z S_z} \varepsilon^{\alpha}_{L_z}(P_h)\langle 1L_z; 1S_z | J J_z\rangle$$
$$\mathrm{Tr}\Big[\,(O_{1\alpha}(0) - O_{2\alpha}(0) - O_{3\alpha}(0) + 2O_{4\alpha}(0))\,\mathcal{P}_{SS_z}(0) + (O_1(0) - O_2(0) - O_3(0) + 2O_4(0))\,\mathcal{P}_{SS_z\alpha}(0)\Big]$$

Asymmetry Calculations

Final expression of the differential cross section

$$\frac{d\sigma}{dydx_B dzd^2P_{h\perp}} = d\sigma^U(\phi_h) + d\sigma^T(\phi_h),$$

$$d\sigma^U(\phi_h) = \frac{1}{256\pi^4}\frac{1}{x_B^2 s^3 y^2 z(1-z)}\int k_\perp dk_\perp \{(A_0 + A_1\cos(\phi_h) + A_2\cos(2\phi_h))f_1^g(x,k_\perp^2)\}$$

$$d\sigma^T(\phi_h) = \frac{1}{256\pi^4}\frac{1}{x_B^2 s^3 y^2 z(1-z)}\int dk_\perp \frac{k_\perp^3}{M_p^2}\{(B_0 + B_1\cos(\phi_h) + B_2\cos(2\phi_h))h_1^{\perp g}(x,k_\perp^2)\}$$

We are interested in small-x region, we neglected higher terms in x_B

$$\langle cos(2\phi_h)\rangle = \frac{\int d\phi_h cos(2\phi_h) d\sigma}{\int d\phi_h d\sigma}$$

$$\boxed{\langle cos(2\phi_h)\rangle \propto \frac{\int k_\perp dk_\perp \left(A_2 f_1^g(x,k_\perp^2) + \frac{k_\perp^2}{M_p^2} B_2 h_1^{\perp g}(x,k_\perp^2)\right)}{\int k_\perp dk_\perp \left(A_0 f_1^g(x,k_\perp^2) + \frac{k_\perp^2}{M_p^2} B_0 h_1^{\perp g}(x,k_\perp^2)\right)}}$$

Gaussian parameterization

$$f_1^g(x,\mathrm{k}_\perp^2) = f_1^g(x,\mu)\frac{1}{\pi\langle k_\perp^2\rangle}e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$h_1^{\perp g}(x,\mathrm{k}_\perp^2) = \frac{M_p^2 f_1^g(x,\mu)}{\pi\langle k_\perp^2\rangle^2}\frac{2(1-r)}{r}e^{1-\frac{k_\perp^2}{r\langle k_\perp^2\rangle}}$$

Collinear PDFs

*MSTW2008 pdfs

[The European Physical Journal C 63, 189 (2009)]

McLerran Venugopalan(MV) parameterization

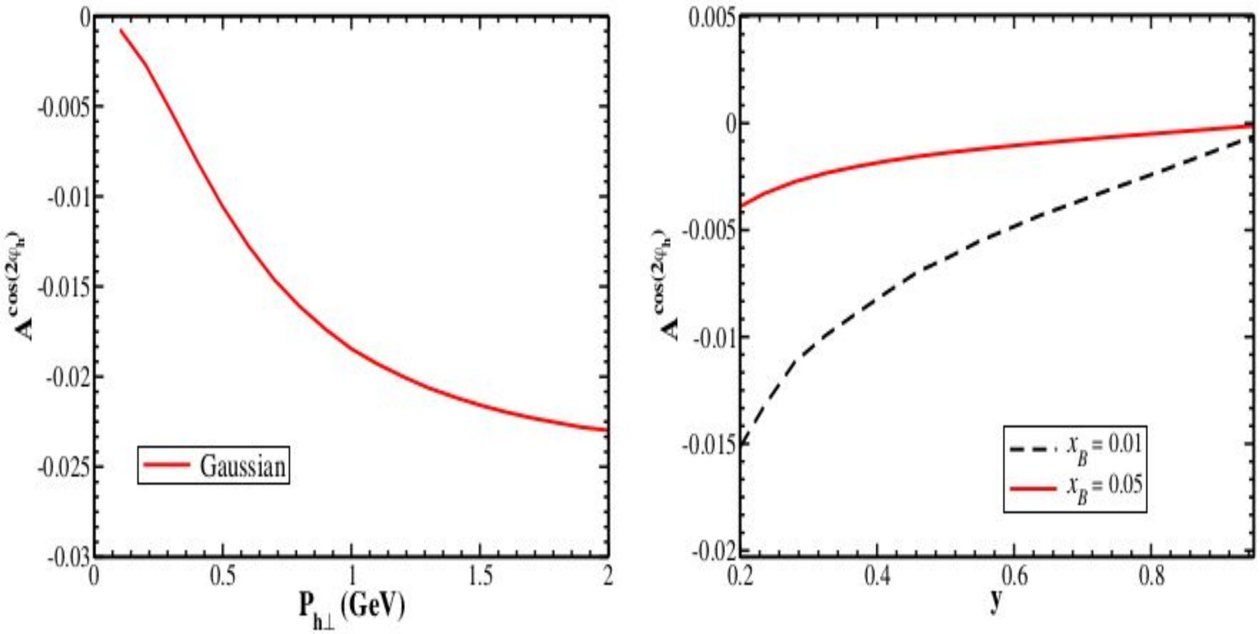
[McLerran and Venugopalan, PRD (1994)]

Saturation scale

$$f_1^g(x,\mathrm{k}_\perp^2) = \frac{S_\perp C_F}{\alpha_s\pi^3}\int dr\frac{J_0(k_\perp r)}{r}\left(1-\exp^{\frac{-r^2}{4}Q_{sg}^2(r)}\right)$$

$$h_1^{\perp g}(x,\mathrm{k}_\perp^2) = \frac{2S_\perp C_F}{\alpha_s\pi^3}\frac{M_P^2}{k_\perp^2}\int dr\frac{J_2(k_{\perp r})}{r\log\left(\frac{1}{r^2\Lambda_{QCD}^2}\right)}\left(1-\exp^{\frac{-r^2}{4}Q_{sg}^2(r)}\right)$$

Numerical Estimates for EIC

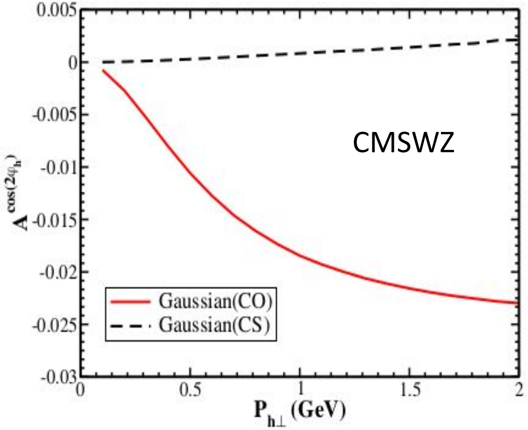


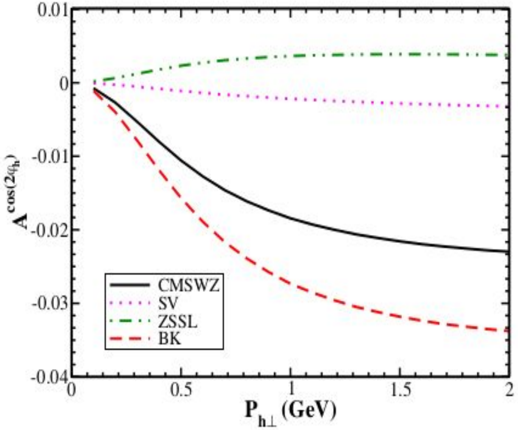
Asymmetry plotted using Gaussian parameterizaion of TMDs

We used CMSWZ set of LDMEs

[CMSWZ: Phys Rev letters 108, 242004(2012)]

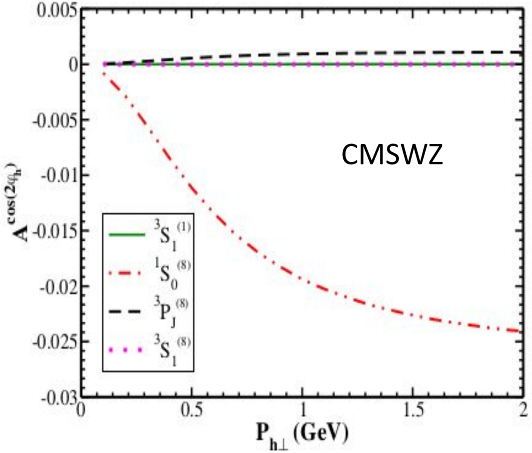
Numerical Estimates for EIC

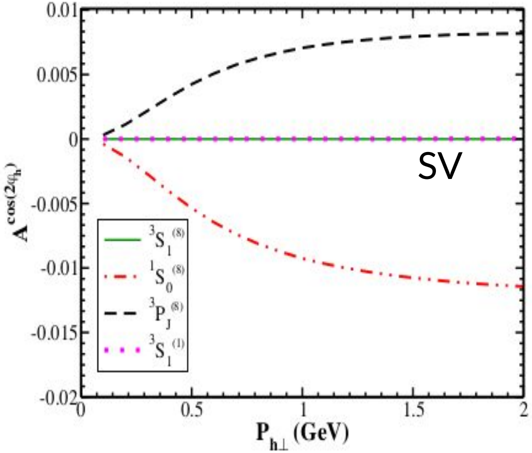




At EIC

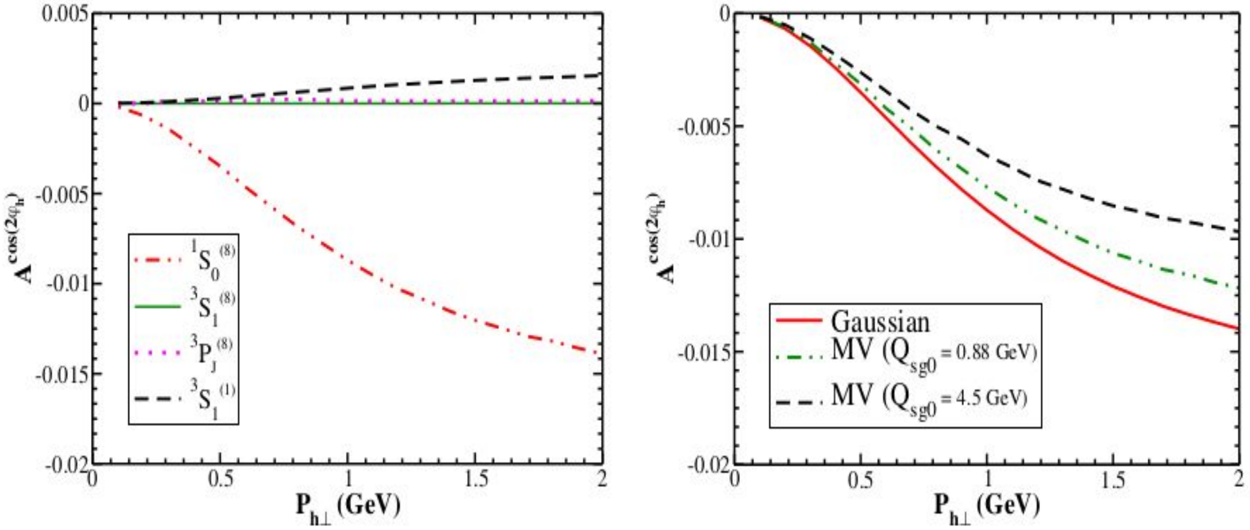
$$\sqrt{s} = 100\ GeV$$
$$Q^2 = 15\ GeV^2$$





[CMSWZ: Phys Rev letters 108, 242004(2012)]
[SV: Phys Rev C 87, 044905 (2013)]
[ZSSL: Phys Rev letters 114, 092006 (2015)]
[BK: Phys Rev D 84, 051501 (2011)]

Numerical Estimates for EIC



$$\sqrt{s} = 150\ GeV$$
$$x = 0.01$$
$$z = 0.7$$

Asymmetry plotted using MV parameterizaion of TMDs

CSMWZ set of LDMEs for the above plots.

Conclusion

- Asymmetry depends on the choice of LDMEs.
- The asymmetry is larger for smaller values of x_B.
- The asymmetry found in Gaussian model is more as compared to MV model.
- We obtain a small but sizable $\cos(\phi_h)$ asymmetry. It could be a useful channel to probe the ratio of linearly polarized gluon TMD to unpolarized gluon TMD at EIC.

Thank You