A new method to extract the valence transversity distributions

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Plan of the talk

Transversity Distribution

Weighted Single Spin Asymmetry $A_N^{\sin(\phi_h+\phi_S)}$

A suggested probe to transversity distribution: difference asymmetry

A new probe to transversity distribution

Numerical estimates

Transversity Distribution $h_1(x)$

- Transversity distribution, $h_1(x, Q^2)$, a fundamental and necessary information, apart from $q(x, Q^2)$ and $\Delta q(x, Q^2)$, to fully understand the quark structure of polarized hadron in collinear configuration.

- It is defined as : $h_1(x, Q^2) = q_\uparrow(x, Q^2) - q_\downarrow(x, Q^2)$, where $q_{\uparrow(\downarrow)}(x, Q^2)$ represents the number density of quarks with polarization parallel (anti-parallel) to the polarization of parent nucleon.

- $h_1(x)$ is a chiral-odd function which makes it decoupled and can not occur alone in a deep-inelastic scattering (DIS) process.

- In order to probe the transversity distribution, it has to be coupled with another chiral-odd function either in initial state or in final state.

- Transversely polarized proton-antiproton scattering is one of the promising channel to probe $h_1(x)$ by studying double transverse spin asymmetry.

M. Anselmino et al j.physletb.2004.05.029

- In SIDIS process, by studying weighted single spin asymmetry, $A_{UT}^{\sin(\phi+\phi_S)}$, we can probe $h_1(x)$, where it is coupled with Collins fragmentation function $H_1^\perp(z)$.

Formalism

- For a SIDIS process: $l + p^{\uparrow(\downarrow)} \to l' + h + X$, where unpolarized lepton beams collide with transversely polarized target proton, the cross section is given as

$$\frac{d\sigma^{\ell p(S_T)\to\ell' h X}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S} = \frac{2\alpha^2}{Q^4}\left\{\frac{1+(1-y)^2}{2} F_{UU} + \ldots + \left[(1-y)\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \ldots\right]\right\}$$

M. Anselmino et al. Phys. Rev. D83, 114019 (2011)

Where the SIDIS variables: x_B, z_h, y, Q and s have their usual meanings

- The structure functions, F_{UU} and F_{UT} in Gaussian parameterization are given by

For unpolarized case:

$$F_{UU} = \sum_q e_q^2\, f_{q/p}(x)\, D_{h/q}(z)\, \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

For transversely polarized target and unpolarized beam

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \sum_q e_q^2\, h_1^q(x)\, \Delta^N D_{h/q^\uparrow}(z)\, \sqrt{\frac{e}{2}}\, \frac{P_T}{M_C}\, \frac{\langle p_\perp^2\rangle_C^2}{\langle p_\perp^2\rangle}\, \frac{e^{-P_T^2/\langle P_T^2\rangle_T}}{\pi\langle P_T^2\rangle_T^2}$$

Transversity distribution

Collins function

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Single Spin Asymmetry $A_{UT}^{\sin(\phi_h+\phi_S)}$

- In SIDIS process, Single Spin asymmetry is defined as a weighted azimuthal moment:

$$A_{UT}^{W(\phi_h,\phi_S)} = 2\,\frac{\int d\phi_h\, d\phi_S\, \left[d\sigma^{\uparrow} - d\sigma^{\downarrow}\right]\, W(\phi_h,\phi_S)}{\int d\phi_h\, d\phi_S\, \left[d\sigma^{\uparrow} + d\sigma^{\downarrow}\right]}$$

$$d\sigma^{\uparrow} - d\sigma^{\downarrow} = \frac{2\alpha^2}{Q^4}\left\{2(1-y)\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \ldots\right\}$$

$$d\sigma^{\uparrow} + d\sigma^{\downarrow} = \frac{2\alpha^2}{Q^4}\left\{\left[1+(1-y)^2\right]F_{UU} + \ldots\right\}$$

- With the weight function $W(\phi_h,\phi_S) = \sin(\phi_h+\phi_S)$, the contribution to the SSA from the Collins functions is given by

$$A_{UT}^{\sin(\phi_h+\phi_S)} = \frac{2(1-y)\,F_{UT}^{\sin(\phi_h+\phi_S)}}{\left[1+(1-y)^2\right]F_{UU}} \equiv D_{NN}\frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}}$$

$$D_{NN} = 2(1-y)/[1+(1-y)^2]$$

A suggested probe to $h_1(x)$

- The SIDIS cross section can be rewritten in their notations forms as:

$$\sigma_t^{\pm} = \sigma_{0,t}^{\pm} + \sin(\phi_h + \phi_S)\, D_{NN}\, \sigma_{C,t}^{\pm} + \dots$$

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- The observed hadrons are Π^+ *and* Π^- and the targets, $t = p, d$, are proton and deuteron.

$$\sigma_0 = Y F_{UU} = Y \sum_q e_q^2 f_{q/p}(x) D_{h/q}(z) A(P_T)$$

$$\sigma_C = Y F_{UT}^{\sin(\phi_h+\phi_S)} = Y \sum_q e_q^2 h_1^q(x) \Delta^N D_{h/q^{\uparrow}}(z) B(P_T)$$

Where $Y = \frac{\alpha^2}{Q^4}[1 + (1-y)^2]$ $\quad A(P_T) = \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$ $\quad B(P_T) = \sqrt{\frac{e}{2}}\,\frac{P_T}{M_C}\,\frac{\langle p_\perp^2\rangle_C^2}{\langle p_\perp^2\rangle}\,\frac{e^{-P_T^2/\langle P_T^2\rangle_T}}{\pi\langle P_T^2\rangle_T^2}$

- The measure of the Collins asymmetry is the ratio

$$A_C = \frac{\sigma_C}{\sigma_0} = \frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}} = \frac{1}{D_{NN}} A_{UT}^{\sin(\phi_h+\phi_S)}$$

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σ_C and σ_0 for $\Pi^{\pm}$ as final hadrons

- For Π^+ and Π^- as final hadron, σ_0 and σ_C can be given in terms of PDFs, favored and disfavored fragmentation and Collins functions and the transversity distribution as;

For proton as target:

$$\sigma^+_{C,p} \sim \left[(4h_1^u + h_1^{\bar{d}})\,\Delta^N D_{fav} + (4h_1^{\bar{u}} + h_1^d)\,\Delta^N D_{dis} + (h_1^s + h_1^{\bar{s}})\Delta^N D_{1,s}\right] B(P_T)$$
$$\sigma^-_{C,p} \sim \left[(4h_1^u + h_1^{\bar{d}})\,\Delta^N D_{dis} + (4h_1^{\bar{u}} + h_1^d)\,\Delta^N D_{fav} + (h_1^s + h_1^{\bar{s}})\Delta^N D_{1,s}\right] B(P_T)$$
$$\sigma^+_{0,p} \sim \left[(4f_1^u + f_1^{\bar{d}})\,D_{1,fav} + (4f_1^{\bar{u}} + f_1^d)\,D_{1,dis} + (f_1^s + f_1^{\bar{s}})\,D_{1,s}\right] A(P_T)$$
$$\sigma^-_{0,p} \sim \left[(4f_1^u + f_1^{\bar{d}})\,D_{1,dis} + (4f_1^{\bar{u}} + f_1^d)\,D_{1,fav} + (f_1^s + f_1^{\bar{s}})\,D_{1,s}\right] A(P_T)$$

For deuteron as target:

$$\sigma^+_{C,d} \sim \left[(h_1^u + h_1^d)\,(4\Delta^N D_{fav} + \Delta^N D_{dis}) + (h_1^{\bar{u}} + h_1^{\bar{d}})\,(\Delta^N D_{fav} + 4\Delta^N D_{dis}) + 2(h_1^s + h_1^{\bar{s}})\Delta^N D_{1,s}\right] B(P_T)$$
$$\sigma^-_{C,d} \sim \left[(h_1^u + h_1^d)\,(\Delta^N D_{fav} + 4\Delta^N D_{dis}) + (h_1^{\bar{u}} + h_1^{\bar{d}})\,(\Delta^N D_{dis} + 4\Delta^N D_{fav}) + 2(h_1^s + h_1^{\bar{s}})\Delta^N D_{1,s}\right] B(P_T)$$
$$\sigma^+_{0,d} \sim \left[(f_1^u + f_1^d)\,(4D_{1,fav} + D_{1,dis}) + (f_1^{\bar{u}} + f_1^{\bar{d}})\,(D_{1,fav} + 4D_{1,dis}) + 2(f_1^s + f_1^{\bar{s}})\,D_{1,s}\right] A(P_T)$$
$$\sigma^-_{0,d} \sim \left[(f_1^u + f_1^d)\,(4D_{1,dis} + D_{1,fav}) + (f_1^{\bar{u}} + f_1^{\bar{d}})\,(D_{1,dis} + 4D_{1,fav}) + 2(f_1^s + f_1^{\bar{s}})\,D_{1,s}\right] A(P_T)$$

Where the PDFs and favored and disfavored fragmentation and Collins functions are given as

$$f_{u/p} = f_{d/n} \equiv f_1^u \qquad f_{d/p} = f_{u/n} \equiv f_1^d \qquad f_{\bar{u}/p} = f_{\bar{d}/n} \equiv f_1^{\bar{u}} \qquad f_{\bar{d}/p} = f_{\bar{u}/n} \equiv f_1^{\bar{d}}$$
$$f_{s/p} = f_{s/n} \equiv f_1^s \qquad f_{\bar{s}/p} = f_{\bar{s}/n} \equiv f_1^{\bar{s}}$$
$$D_{\pi^+/u} = D_{\pi^-/d} = D_{\pi^+/\bar{d}} = D_{\pi^-/\bar{u}} \equiv D_{1,fav}$$
$$D_{\pi^+/\bar{u}} = D_{\pi^-/\bar{d}} = D_{\pi^+/d} = D_{\pi^-/u} \equiv D_{1,dis} \qquad D_{\pi^{\pm}/s,\bar{s}} \equiv D_{1,s}$$
$$\Delta^N D_{\pi^+/u^{\uparrow}} = \Delta^N D_{\pi^-/d^{\uparrow}} = \Delta^N D_{\pi^+/\bar{d}^{\uparrow}} = \Delta^N D_{\pi^-/\bar{u}^{\uparrow}} \equiv \Delta^N D_{fav}$$
$$\Delta^N D_{\pi^+/\bar{u}^{\uparrow}} = \Delta^N D_{\pi^-/\bar{d}^{\uparrow}} = \Delta^N D_{\pi^+/d^{\uparrow}} = \Delta^N D_{\pi^-/u^{\uparrow}} \equiv \Delta^N D_{dis} \qquad \Delta^N D_{\pi^{\pm}/s,\bar{s}} = \Delta^N D_{1,s}$$

π Difference Asymmetries A_D

- The difference asymmetries for positively and negatively charged hadron (pions) is defined as

$$A_{D,t} = \frac{\sigma^{+}_{C,t} - \sigma^{-}_{C,t}}{\sigma^{+}_{0,t} + \sigma^{-}_{0,t}}$$

For the proton target:

$$A_{D,p} = \frac{(\Delta^N D_{1,fav} - \Delta^N D_{1,dis})(4h_1^{u_v} - h_1^{d_v})B(P_T)}{[(D_{1,fav} + D_{1,dis})(4f_1^u + 4f_1^{\bar{u}} + f_1^d + f_1^{\bar{d}}) + 2(f_1^s + f_1^{\bar{s}})D_{1,s}]A(P_T)}$$

For the deuteron target:

$$A_{D,d} = 3\frac{(\Delta^N D_{1,fav} - \Delta^N D_{1,dis})(h_1^{u_v} + h_1^{d_v})B(P_T)}{[5(D_{1,fav} + D_{1,dis})(f_1^u + f_1^{\bar{u}} + f_1^d + f_1^{\bar{d}}) + 4(f_1^s + f_1^{\bar{s}})D_{1,s}]A(P_T)}$$

- The ratio of these difference asymmetries; $R_{D,d/p} \equiv \frac{A_{D,d}}{A_{D,p}}$ cancels the Collins function and remaining with functions of unpolarized PDFs and FFs and the transversity distribution.

$$R_{D,d/p} = 3\left[\frac{(4f_1^u + 4f_1^{\bar{u}} + f_1^d + f_1^{\bar{d}})\,(D_{1,fav} + D_{1,dis}) + 2(f_1^s + f_1^{\bar{s}})\,D_{1,s}}{5(f_1^u + f_1^{\bar{u}} + f_1^d + f_1^{\bar{d}})\,(D_{1,fav} + D_{1,dis}) + 4(f_1^s + f_1^{\bar{s}})\,D_{1,s}}\right]\frac{h_1^{u_v} + h_1^{d_v}}{4h_1^{u_v} - h_1^{d_v}}$$

- A_D can also be extracted from the measured Collins asymmetry data as

$$A_{D,t} = \frac{\sigma^{+}_{0,t}}{\sigma^{+}_{0,t} + \sigma^{-}_{0,t}}A^{+}_{C,t} - \frac{\sigma^{-}_{0,t}}{\sigma^{+}_{0,t} + \sigma^{-}_{0,t}}A^{-}_{C,t}$$

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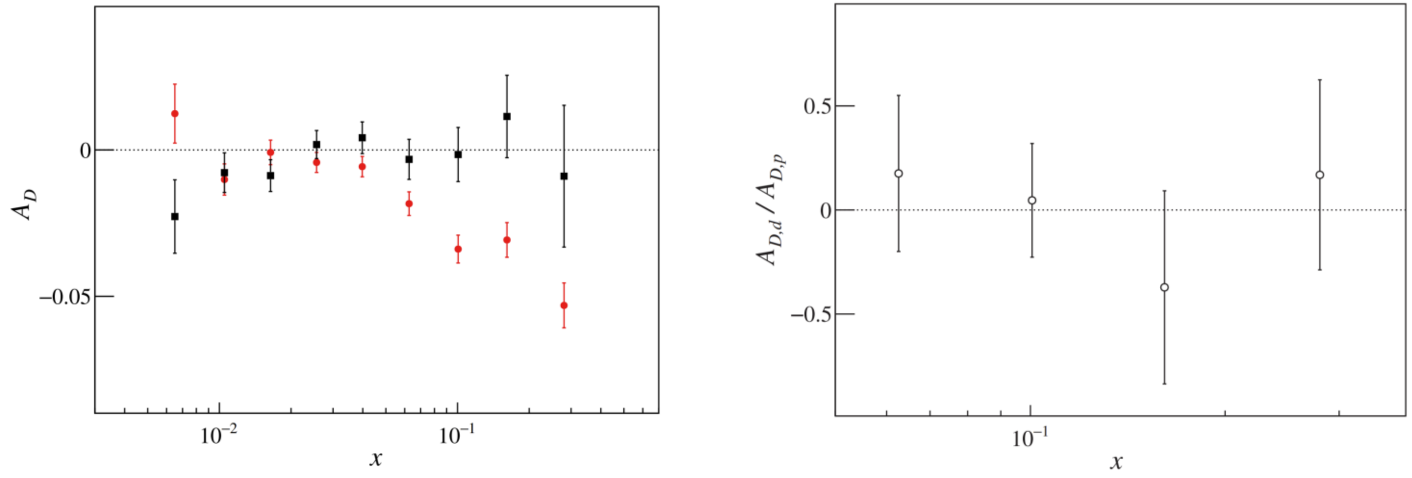
Extraction of $h_1^{d_v}/h_1^{u_v}$

- Extracted A_D from COMPASS data for $A_C^{\mp}$ with their statistical uncertainties.

- Extracted the ratio of d-quark and u-quark transversity distribution, $h_1^{d_v}/h_1^{u_v}$, from the equation of $R_{D,d/p}$, assuming one knows the unpolarized PDFs and FFs.



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- A_D's are small quantities, with large relative errors, as they are related to a ratio $\sigma_C/\sigma_0 \sim (h_1 \Delta^N D)/f_1 D$.

- Ratio of A_D's is bound to have huge uncertainties which can be seen in the plots.

A new probe to $h_1(x)$

- Motivated from the measurement of $\sin(\phi_h + \phi_S)$ modulation of SIDIS cross section directly relates with σ_C, if we measure this for different targets and make observations for both Π^+ and Π^- in final state, we can define a ratio of difference of weighted polarized cross section as

$$R_{C,d/p} \equiv \frac{\sigma_{C,d}^{+} - \sigma_{C,d}^{-}}{\sigma_{C,p}^{+} - \sigma_{C,p}^{-}}$$

- R_C does not have any dependance on Collins functions as well as unpolarized PDFs and FFs

$$R_{C,d/p} = 3\frac{h_1^{u_v} + h_1^{d_v}}{4h_1^{u_v} - h_1^{d_v}} = 3\frac{1 + \frac{h_1^{u_v}}{h_1^{d_v}}}{4 - \frac{h_1^{u_v}}{h_1^{d_v}}}$$

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- Unlike A_D, R_C is not ratio of small quantities. The numerator and denominator of R_C are simply proportional to $\sigma_C \sim h_1 \Delta^N D$.

Numerical estimates

- With the given parameterizations and the best fit free parameters for $u - quark$ and $d - quark$ transversity distributions and Collins functions extracted from HERMESS, COMPASS and Belle data.

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$$\Delta^N D_{h/q^\uparrow}(z) = 2\mathcal{N}_q^C(z) D_{h/q}(z)$$

$$\mathcal{N}_q^C(z) = N_q^C z^\alpha (1-z)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}$$

$$h_1^q(x) = \frac{1}{2}\mathcal{N}_q^T(x)[f_{q/p}(x) + \Delta q(x)]$$

$$\mathcal{N}_q^T(z) = N_q^T x^\rho (1-x)^\sigma \frac{(\rho+\sigma)^{(\rho+\sigma)}}{\rho^\rho \sigma^\sigma}$$

Helicity distribution

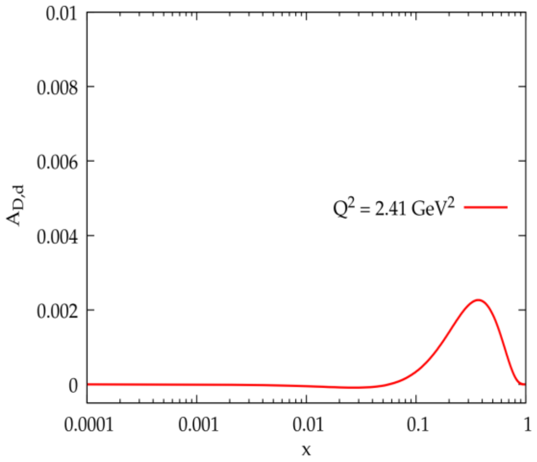
MSTW2008 PDFs used : Eur. Phys. J. C 63, 189 (2009).

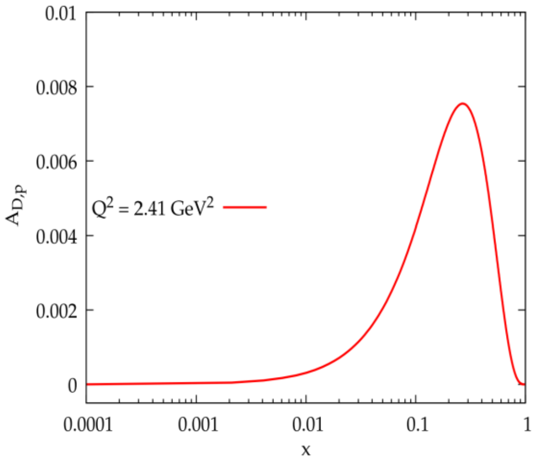
Pion fragmentation function taken from : Phys. Rev. D 91, 014035 (2015).

Helicity distribution taken from : Phys. Rev. D 63, 094005 (2001).

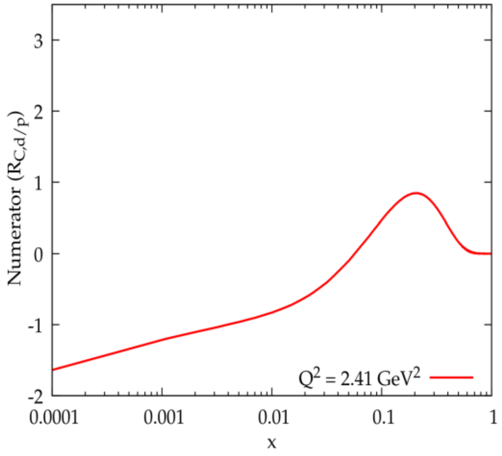
Numerical Estimates

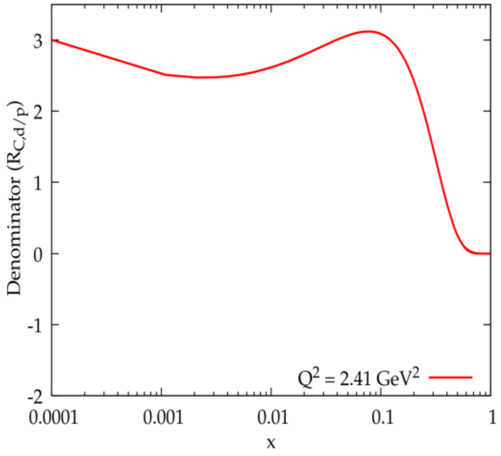
- Plots for difference asymmetries vs x. P_T and z are integrated over the ranges $0 < P_T < 5\,GeV$, $0.1 < z < 1$.





- Plots for numerator and denominator of R_C vs x. z is integrated over the range $0.1 < z < 1$.





Conclusion

- We saw a probe to transversity distribution using the concept of difference asymmetry for different target (proton and deuteron) with positive and negative hadrons (pions) in final state.
- A_D's are small quantity with large relative errors. Taking its ratio which cancels out the Collins function dependance, is bound to have large uncertainties.
- The ratio R_C can be estimated from the $\sin(\phi_h + \phi_S)$ modulation of transversely polarized SIDIS cross section.
- We have a simple partonic interpretation of R_C which directly probes the ratio $h_1^{d_v}/h_1^{u_v}$.
- This could be measured during the next deuteron COMPASS run, or during ongoing Jlab 12 experiment or can be measured at future EIC.