



Reweighting the quark Sivers function with STAR jet data

Carlo Flore

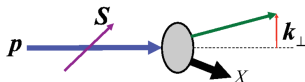
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**DIS 2021
(virtual meeting)**

13 Apr 2021

M. Boglione, U. D'Alesio, CF,
J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin,
Phys. Lett. B 815 (2021) 136135

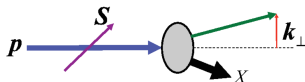
Introduction - the Sivers function



D.W. Sivers, PRD 41 (1990) 83 & PRD 43 (1991) 261

$$\begin{aligned}
 & f_{a/p\uparrow}(x, \mathbf{k}_{\perp}) - f_{a/p\uparrow}(x, -\mathbf{k}_{\perp}) \\
 &= \Delta^N f_{a/p\uparrow}(x, k_{\perp}^2) \frac{(\hat{\mathbf{p}} \times \mathbf{k}_{\perp}) \cdot \mathbf{S}}{|\mathbf{k}_{\perp}|} \quad (TO - CA) \\
 &= -\frac{2|\mathbf{k}_{\perp}|}{M} f_{1T}^{\perp a}(x, k_{\perp}^2) \frac{(\hat{\mathbf{p}} \times \mathbf{k}_{\perp}) \cdot \mathbf{S}}{|\mathbf{k}_{\perp}|} \quad (Amsterdam)
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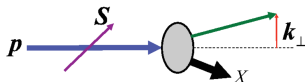


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- genuine TMD distribution

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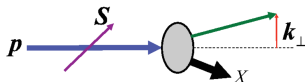


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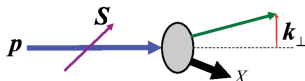
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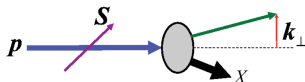


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- in the GPM, Sivers effect generates single spin asymmetries (SSAs) in $p^{\uparrow} p \rightarrow \text{jet } X$

Introduction - formalism

U. D'Alesio, F. Murgia PRD 70 (2004) 074009; M. Anselmino *et al.*, PRD 73 (2006) 014020;
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- Inclusive jet production in $p^\uparrow p$ collisions can be described in the **GPM**, where a **factorized formulation in terms of TMDs is assumed as a starting point**
- a **color gauge invariant formulation of GPM (CGI-GPM)** was developed, with inclusion of initial and final state interaction; **process dependence is recovered**
- A_N in $p^\uparrow p \rightarrow \text{jet } X$:

$$A_N \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \frac{d\Delta\sigma}{2d\sigma},$$

with

$$d\Delta\sigma^{\text{CGI-GPM}} = \frac{2\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a dx_b}{x_a x_b} d^2\mathbf{k}_{\perp a} d^2\mathbf{k}_{\perp b} \left(-\frac{k_{\perp a}}{M_p} \right) f_{1T}^{\perp a}(x_a, \mathbf{k}_{\perp a}) \cos \varphi_a \\ \times f_{b/p}(x_b, \mathbf{k}_{\perp b}) H_{ab \rightarrow cd}^{\text{Inc}} \delta(\hat{s} + \hat{t} + \hat{u})$$

and

$$d\sigma = \frac{\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a dx_b}{x_a x_b} d^2\mathbf{k}_{\perp a} d^2\mathbf{k}_{\perp b} f_{a/p}(x_a, \mathbf{k}_{\perp a}) f_{b/p}(x_b, \mathbf{k}_{\perp b}) H_{ab \rightarrow cd}^U \delta(\hat{s} + \hat{t} + \hat{u}).$$

[note: $d\Delta\sigma^{\text{GPM}} = d\Delta\sigma^{\text{CGI-GPM}} [H_{ab \rightarrow cd}^{\text{Inc}} \rightarrow H_{ab \rightarrow cd}^U]$]

The reweighting method

W.T. Giele, S. Keller PRD 58 (1998) 094023; R.D. Ball *et al.*, NPB 849 (2011) 112;
N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020; H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

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- it allows to assess the impact of new data sets on existing PDFs extractions, avoiding a new global fit
- used both in the **Bayesian framework** and in the **Hessian approach**
- we apply for the first time the Bayesian reweighting to the Siverson function:

$$\Delta^N f_{q/p^\dagger}(x, k_\perp) = \frac{4M_p k_\perp}{\langle k_\perp^2 \rangle_S} \Delta^N f_{q/p^\dagger}^{(1)}(x) \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

where $q = u, d$, and where $\Delta^N f_{q/p^\dagger}^{(1)}(x)$ is the Siverson first $k_\perp$ -moment:

$$\begin{aligned} \Delta^N f_{q/p^\dagger}^{(1)}(x) &= \int d^2 \mathbf{k}_\perp \frac{k_\perp}{4M_p} \Delta^N f_{q/p^\dagger}(x, k_\perp) \equiv -f_{1T}^{\perp(1)q}(x) \\ &= N_q (1-x)^{\beta_q} \end{aligned}$$

parametrization from **M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, JHEP 07 (2018) 148**

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

BAYESIAN REWEIGHTING:

- consider a model for a TMD depending on $\mathbf{a} = \{a_1, \dots, a_n\}$ parameters with prior probability distribution $\pi(\mathbf{a})$
[if no prior knowledge exists, $\pi(\mathbf{a})$ is flat]

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- for a set of data $\mathbf{y} = y_1, \dots, y_{N_{\text{dat}}}$, χ^2 is defined as

$$\chi^2[\mathbf{a}, \mathbf{y}] = \sum_{i,j=1}^{N_{\text{dat}}} (y_i[\mathbf{a}] - y_j) C_{ij}^{-1} (y_j[\mathbf{a}] - y_i)$$

[if only uncorrelated uncertainties: $\chi^2[\mathbf{a}, \mathbf{y}] = \sum_{i=1}^{N_{\text{dat}}} (y_i[\mathbf{a}] - y_i)^2 / \sigma_i^2$]

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- best fit $\mathbf{a}_0$ with associated χ_0^2 ; uncertainty represented by $\mathbf{a}_k$ parameter sets ($k = 1, \dots, N_{\text{set}}$) with $\chi_k^2 \in [\chi_0^2, \chi_0^2 + \Delta\chi^2]$ given by

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[we adopt $\Delta\chi^2$ for 5 params at 2 σ CL, i.e. $\Delta\chi^2 = 11.31$]

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- $2 \cdot 10^5$ $\mathbf{a}_k$ sets produced adopting Markov Chain MC with Metropolis-Hastings algorithm

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$$\mathbb{E}[\mathcal{O}] \simeq \sum_k w_k \mathcal{O}(\mathbf{a}_k) \quad \mathbb{V}[\mathcal{O}] \simeq \sum_k w_k (\mathcal{O}(\mathbf{a}_k) - \mathbb{E}[\mathcal{O}])^2$$

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median as central value, uncertainties at **2 σ CL**

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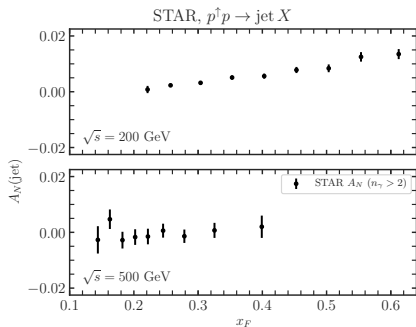
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Let's reweight the Siverts function with STAR data!

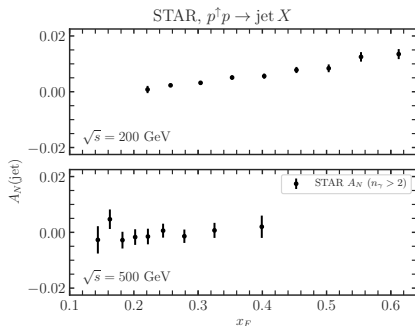
Reweighting the Sivers function (I)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135;
STAR Coll., arXiv:2012.11428



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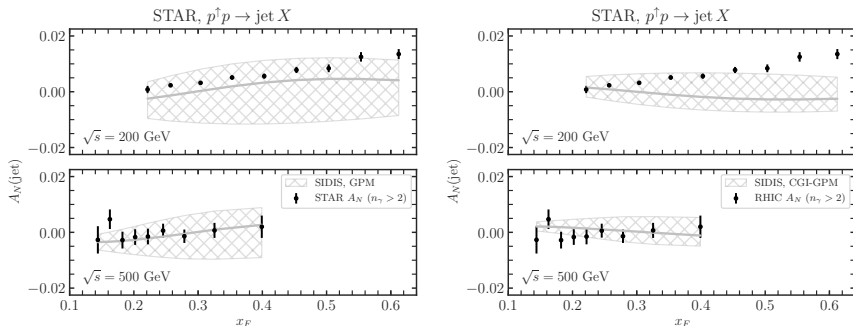
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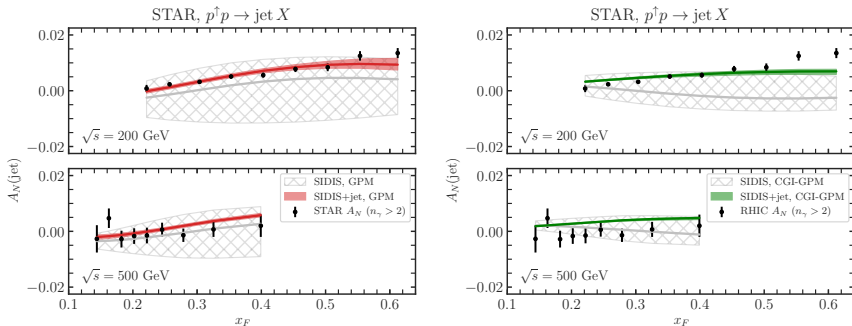
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- **GPM** (left) and **CGI-GPM** (right) predictions for SIDIS only describe the data within errors parametrization from **M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, JHEP 07 (2018) 148**

Reweighting the Sivvers function (I)

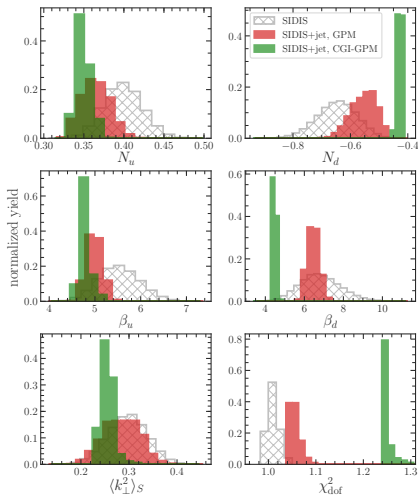
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135;
STAR Coll., arXiv:2012.11428



- STAR data is for **electromagnetic jets**; $n_\gamma > 2$ not contaminated by single photon and isolated π^0 production; **wide x_F coverage** ($0.1 \lesssim x_F \lesssim 0.6$), complementary to SIDIS
- **GPM** (left) and **CGI-GPM** (right) predictions for SIDIS only describe the data within errors parametrization from **M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, JHEP 07 (2018) 148**
- reweighted curves show **reduced uncertainties**

Reweighting the Siverson function (II)

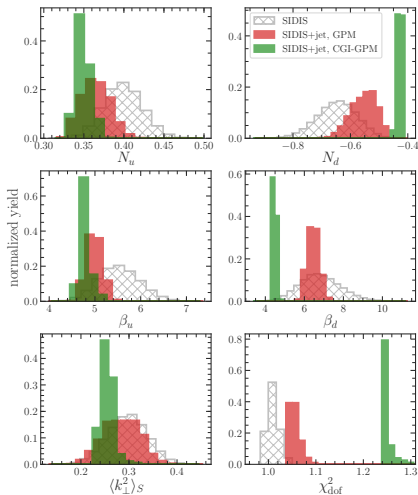
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$$\Delta^N f_{q/p^\uparrow}^{(1)}(x) \equiv -f_{1T}^{\perp(1)q}(x) = N_q (1-x)^{\beta_q} \quad (q = u, d)$$

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M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

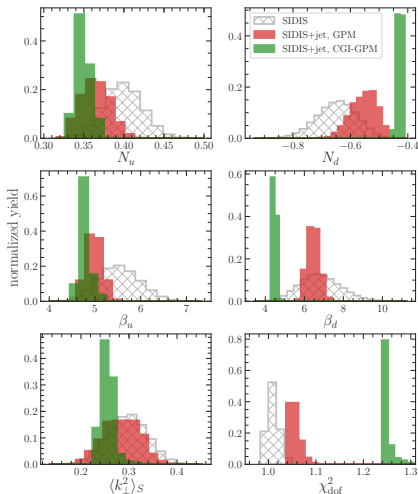


- $\langle k_{\perp}^2 \rangle_S$ Gaussian width does not vary much

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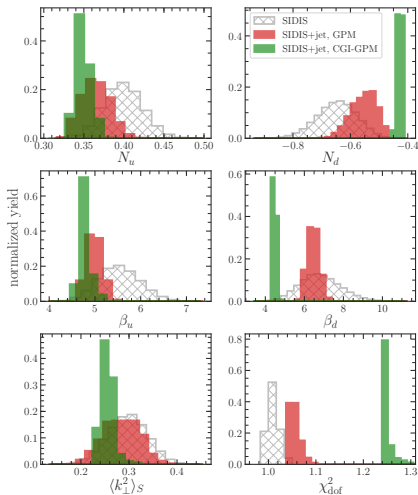


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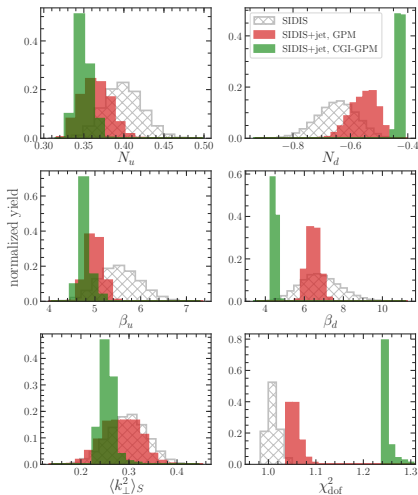


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M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

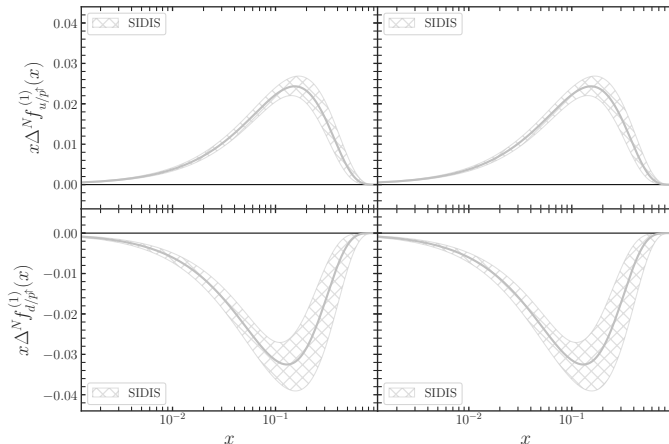


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- χ_{dof}^2 after the reweighting for $N_{\text{dat}}^{\text{SIDIS+jet}} = 238$ slightly favors the GPM approach

$$\Delta N_{q/p\uparrow}^{f(1)}(x) \equiv -f_{1T}^{\perp(1)q}(x) = N_q (1-x)^{\beta_q} \quad (q = u, d)$$

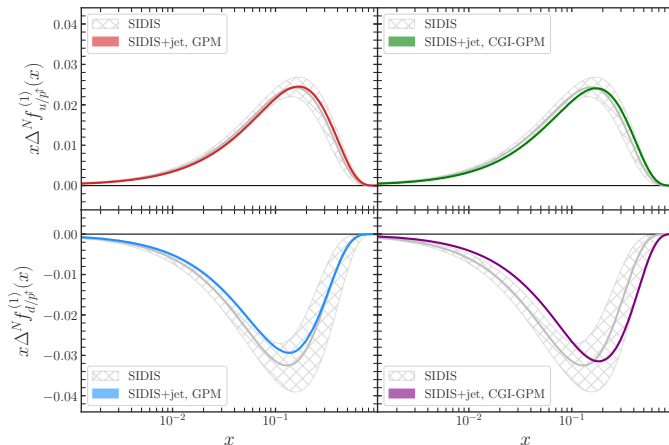
Reweighting the Sivers function (III)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



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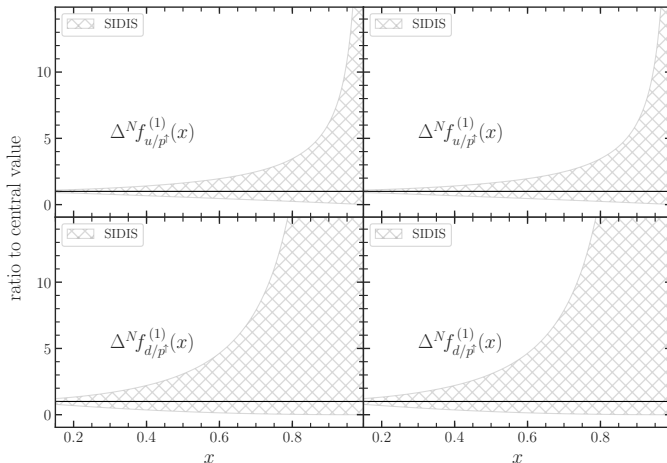
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



drastic reduction of uncertainties

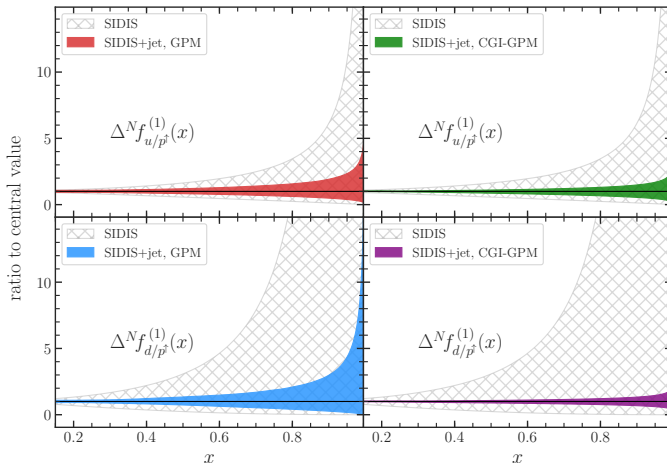
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STAR data allows to constrain the Sivers function at large x !

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- Forthcoming measurements from **COMPASS**, **JLab** and **the EIC** will play **a crucial role in unravelling the nucleon structure in its full complexity**

Thank you

Backup

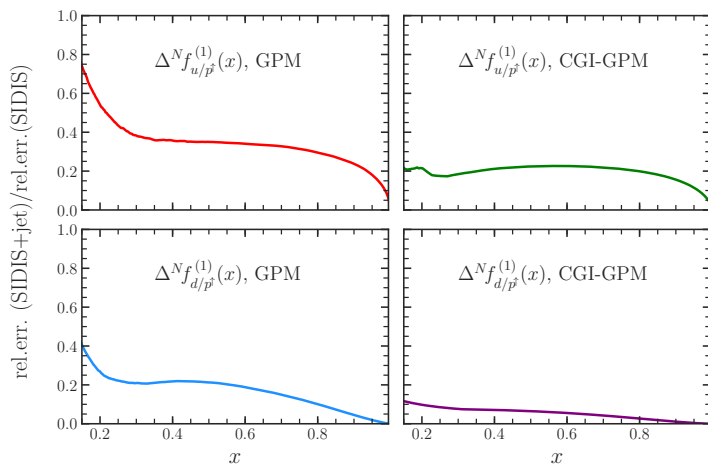
Reweighting the Sivers function - parameters

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

	SIDIS	SIDIS+jet, GPM	SIDIS+jet, CGI-GPM
N_u	$0.40^{+0.05}_{-0.04}$	$0.36^{+0.04}_{-0.03}$	$0.35^{+0.02}_{-0.01}$
N_d	$-0.65^{+0.13}_{-0.15}$	$-0.55^{+0.07}_{-0.10}$	$-0.43^{+0.01}_{-0.02}$
β_u	$5.52^{+0.93}_{-0.83}$	$4.98^{+0.34}_{-0.30}$	$4.79^{+0.28}_{-0.19}$
β_d	$6.77^{+2.29}_{-1.85}$	$6.45^{+0.63}_{-0.52}$	$4.48^{+0.17}_{-0.13}$
$\langle k_{\perp}^2 \rangle_S$	$0.30^{+0.08}_{-0.08}$	$0.28^{+0.07}_{-0.07}$	$0.26^{+0.03}_{-0.02}$
χ^2_{dof}	$1.01^{+0.03}_{-0.02}$	$1.05^{+0.03}_{-0.01}$	$1.25^{+0.04}_{-0.01}$

Reweighting the Siverts function - relative errors

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Ratio of relative errors