


Understanding the Proton Mass in QCD

(A. Metz, Temple University)

- Motivation
- Energy momentum tensor (EMT) and its renormalization
- Mass decompositions (sum rules)
- Numerics for proton mass sum rules
- Summary and discussion

Based on: S. Rodini, A. Metz, B. Pasquini, JHEP 09 (2020) 067, arXiv:2004.03704

A. Metz, B. Pasquini, S. Rodini, PRD 102 (2020) 114042, arXiv:2006.11171

supported by the 

Motivation

- For decades, community has studied proton spin sum rule
(Jaffe, Manohar, 1989 / Ji, 1996)

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G^{\text{can}} + L_q^{\text{can}} + L_g^{\text{can}} = \frac{1}{2} \Delta\Sigma + L_q^{\text{kin}} + J_g^{\text{kin}}$$

- In comparison, less work has been done for mass sum rule
- Yet, different mass sum rules exist
- How do mass sum rules compare to each other ?
- What is impact of recent developments concerning renormalization of EMT ?
(Hatta, Rajan, Tanaka, 2018 / Tanaka, 2018)
- Important questions concerning mass decomposition
 1. mathematically valid mass sum rules ?
 2. optimal mass sum rule based on physics interpretation ?
 3. operator expressions for terms in mass sum rules ?

EMT: Definition

- Canonical EMT: Noether current of space-time translational invariance → conserved

$$\partial_\mu T_C^{\mu\nu}(x) = 0$$

- Symmetric (gauge invariant) EMT: definition (QCD)

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T_q^{\mu\nu} = \frac{i}{4} \bar{\psi} \gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}} \psi \quad \left(\gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}} = \gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu \right)$$

$$T_g^{\mu\nu} = -F^{\mu\alpha} F^\nu{}_\alpha + \frac{g^{\mu\nu}}{4} F^2$$

- Interpretation of components on main diagonal (for extended system)
 - T^{00} energy density
 - T^{jj} pressure
 - trace of EMT contains energy and pressure terms

EMT: Renormalization

(Hatta, Rajan, Tanaka, 2018 / Tanaka 2018)

- Total EMT not renormalized, but individual terms $T_i^{\mu\nu}$ require (extra) renormalization
- Operators that mix under renormalization

$$\begin{aligned}\mathcal{O}_1 &= -F^{\mu\alpha} F^\nu{}_\alpha & \mathcal{O}_2 &= g^{\mu\nu} F^2 \\ \mathcal{O}_3 &= \frac{i}{4} \bar{\psi} \gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}} \psi & \mathcal{O}_4 &= g^{\mu\nu} m \bar{\psi} \psi\end{aligned}$$

$$T^{\mu\nu} = \mathcal{O}_1 + \frac{\mathcal{O}_2}{4} + \mathcal{O}_3$$

- Mixing equations

$$\mathcal{O}_{1,R} = Z_T \mathcal{O}_1 + Z_M \mathcal{O}_2 + Z_L \mathcal{O}_3 + Z_S \mathcal{O}_4$$

$$\mathcal{O}_{2,R} = Z_F \mathcal{O}_2 + Z_C \mathcal{O}_4$$

$$\mathcal{O}_{3,R} = Z_\psi \mathcal{O}_3 + Z_K \mathcal{O}_4 + Z_Q \mathcal{O}_1 + Z_B \mathcal{O}_2$$

$$\mathcal{O}_{4,R} = \mathcal{O}_4$$

- Trace (anomaly) of EMT

(Adler, Collins, Duncan, 1977 / Nielsen, 1977 / Collins, Duncan, Joglekar, 1977 / ...)

$$T^\mu_\mu = (m\bar{\psi}\psi)_R + \gamma_m (m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R$$

- Quark and gluon contribution to trace of EMT

$$T^\mu_\mu = (T_{q,R})^\mu_\mu + (T_{g,R})^\mu_\mu$$

$$(T_{q,R})^\mu_\mu = (1 + \mathbf{y})(m\bar{\psi}\psi)_R + \mathbf{x}(F^2)_R$$

$$(T_{g,R})^\mu_\mu = (\gamma_m - \mathbf{y})(m\bar{\psi}\psi)_R + \left(\frac{\beta}{2g} - \mathbf{x}\right)(F^2)_R$$

\mathbf{x} and \mathbf{y} related to finite parts of renormalization constants \rightarrow choose scheme

- Different scheme choices

- MS scheme / $\overline{\text{MS}}$ scheme (Hatta, Rajan, Tanaka, 2018 / Tanaka 2018)

- D1 scheme: $\mathbf{x} = 0, \mathbf{y} = \gamma_m$

- D2 scheme: $\mathbf{x} = \mathbf{y} = 0$

D-type schemes look natural

EMT and Proton Mass

- Forward matrix element of total EMT

$$\langle T^{\mu\nu} \rangle \equiv \langle P | T^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$

- $\langle T^{\mu\nu}(x) \rangle$ neither depends on space-time point x nor on hadron spin

- Forward matrix element of quark and gluon parts $T_{i,R}^{\mu\nu}$

$$\langle T_{i,R}^{\mu\nu} \rangle = 2P^\mu P^\nu A_i(0) + 2M^2 g^{\mu\nu} \bar{C}_i(0)$$

- form factors A_i and \bar{C}_i satisfy

$$A_q(0) + A_g(0) = 1 \quad \bar{C}_q(0) + \bar{C}_g(0) = 0$$

- in forward limit, matrix elements of EMT fully determined by two form factors
- any mass sum rule for the proton related to at most two independent numbers (emphasized in Lorcé, 2017)

- Trace of EMT and proton mass (here $n = \frac{1}{2M}$, depends on normalization of state)

$$n \langle T^\mu_\mu \rangle = M$$

- T^{00} and proton mass (in rest frame)

$$n \langle T^{00} \rangle = M$$

- Working with QCD Hamiltonian

$$\int d^3\mathbf{x} T^{00} = \int d^3\mathbf{x} \mathcal{H}_{\text{QCD}} = H_{\text{QCD}}$$

$$\frac{\langle H_{\text{QCD}} \rangle}{\langle P|P \rangle} \Big|_{\mathbf{P}=0} = M$$

- Mass sum rules discussed below based on decomposition of $\langle T^\mu_\mu \rangle$ or $\langle T^{00} \rangle$ into quark and gluon parts

Two-Term Sum Rule by Hatta, Rajan, Tanaka

(Hatta, Rajan, Tanaka, JHEP 12, 008 (2018) / Tanaka, JHEP 01, 120 (2019))

- Sum rule based on decomposition of T^μ_μ

$$M = \bar{M}_q + \bar{M}_g = n \left(\langle (T_{q,R})^\mu_\mu \rangle + \langle (T_{g,R})^\mu_\mu \rangle \right)$$

- Recall operators

$$(T_{q,R})^\mu_\mu = (1 + \mathbf{y})(m\bar{\psi}\psi)_R + \mathbf{x}(F^2)_R$$

$$(T_{g,R})^\mu_\mu = (\gamma_m - \mathbf{y})(m\bar{\psi}\psi)_R + \left(\frac{\beta}{2g} - \mathbf{x}\right)(F^2)_R$$

- Using D-type schemes

$$(T_{q,R})^\mu_\mu|_{\mathbf{D1}} = (1 + \gamma_m)(m\bar{\psi}\psi)_R \quad (T_{g,R})^\mu_\mu|_{\mathbf{D1}} = \frac{\beta}{2g}(F^2)_R$$

$$(T_{q,R})^\mu_\mu|_{\mathbf{D2}} = (m\bar{\psi}\psi)_R \quad (T_{g,R})^\mu_\mu|_{\mathbf{D2}} = \gamma_m(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R$$

Two-Term Sum Rule by Lorcé

(Lorcé, EPJC 78, 120 (2018))

- Sum rule based on decomposition of T^{00}

$$M = U_q + U_g = n \left(\langle T_{q,R}^{00} \rangle + \langle T_{g,R}^{00} \rangle \right)$$

- Renormalized operators discussed below
- Relation to EMT form factors for two-term sum rules

$$U_i = M(A_i(0) + \bar{C}_i(0))$$

$$\bar{M}_i = M(A_i(0) + 4\bar{C}_i(0))$$

- $U_i \neq \bar{M}_i$ obviously (\bar{M}_i mixture of energy and pressure ($\sim \bar{C}_i(0)$) terms)
- $U_q + U_g = \bar{M}_q + \bar{M}_g$ because $\bar{C}_q(0) + \bar{C}_g(0) = 0$

- Two-term sum rules have one independent term

(Modified) Four-Term Sum Rule by Ji

(Ji, PRL 74, 1071 (1995) and PRD 52, 271 (1995) / our papers)

- Sum rule based on decomposition of T^{00} into traceless part and trace part

$$T^{\mu\nu} = \underbrace{\bar{T}^{\mu\nu}}_{\text{traceless part}} + \underbrace{\hat{T}^{\mu\nu}}_{\text{trace part}} \quad \hat{T}^{\mu\nu} = \frac{1}{4} g^{\mu\nu} T^\alpha_\alpha \quad \bar{T}^{\mu\nu} = T^{\mu\nu} - \hat{T}^{\mu\nu}$$

- Decomposition of \bar{T}^{00} and \hat{T}^{00} into quark and gluon parts

$$\mathcal{H}'_q = \bar{T}_{q,R}^{00} = (\psi^\dagger i\mathbf{D} \cdot \boldsymbol{\alpha} \psi)_R + (m\bar{\psi}\psi)_R - \frac{1+y}{4}(m\bar{\psi}\psi)_R - \frac{x}{4}(F^2)_R$$

$$\mathcal{H}'_m = \hat{T}_{q,R}^{00} = \frac{1+y}{4}(m\bar{\psi}\psi)_R + \frac{x}{4}(F^2)_R$$

$$\mathcal{H}'_g = \bar{T}_{g,R}^{00} = \frac{1}{2}(E^2 + B^2)_R + \frac{y - \gamma_m}{4}(m\bar{\psi}\psi)_R - \frac{1}{4}\left(\frac{\beta}{2g} - x\right)(F^2)_R$$

$$\mathcal{H}'_a = \hat{T}_{g,R}^{00} = \frac{\gamma_m - y}{4}(m\bar{\psi}\psi)_R + \frac{1}{4}\left(\frac{\beta}{2g} - x\right)(F^2)_R$$

- summing four terms provides mass

$$M = n \left(\langle \mathcal{H}'_q \rangle + \langle \mathcal{H}'_m \rangle + \langle \mathcal{H}'_g \rangle + \langle \mathcal{H}'_a \rangle \right)$$

- $\mathcal{H}'_{q,m,g,a}$ mixture of energy and pressure terms (Lorcé, 2017)

- Form suitable linear combinations of $\mathcal{H}'_{q,m,g,a}$ to obtain “nice” terms $\mathcal{H}_{q,m,g,a}$
 - one must satisfy

$$\mathcal{H}_q + \mathcal{H}_m + \mathcal{H}_g + \mathcal{H}_a = \mathcal{H}'_q + \mathcal{H}'_m + \mathcal{H}'_g + \mathcal{H}'_a$$

- M expressed in terms of linear combinations $\mathcal{H}_{q,m,g,a}$

$$M = n \left(\langle \mathcal{H}_q \rangle + \langle \mathcal{H}_m \rangle + \langle \mathcal{H}_g \rangle + \langle \mathcal{H}_a \rangle \right) = M_q + M_m + M_g + M_a$$

- Final form of sum rule (“modified Ji sum rule”)

$$\mathcal{H}_q = (\psi^\dagger i \mathbf{D} \cdot \boldsymbol{\alpha} \psi)_R \quad \text{quark kinetic plus potential energy}$$

$$\mathcal{H}_m = (m \bar{\psi} \psi)_R \quad \text{quark mass term}$$

$$\mathcal{H}_g = \frac{1}{2} (E^2 + B^2)_R \quad \text{gluon energy}$$

- three (instead of four) nontrivial terms only
- sum rule has two independent terms
- re-shuffling of terms provides “clean” interpretation

- Comparison with Ji's original work (Ji, 1995)

$$(\mathcal{H}_q)_{[Ji]} = (\psi^\dagger i\mathbf{D} \cdot \boldsymbol{\alpha} \psi)_R$$

$$(\mathcal{H}_m)_{[Ji]} = \left(1 + \frac{\gamma_m}{4}\right) (m\bar{\psi}\psi)_R$$

$$(\mathcal{H}_g)_{[Ji]} = \frac{1}{2}(E^2 + B^2)_R$$

$$(\mathcal{H}_a)_{[Ji]} = \frac{\beta}{8g}(F^2)_R$$

- sum rules differ by terms in red ($\frac{1}{4}$ of trace anomaly at operator level)
- mismatch due to difference in computation of traceless part \bar{T}^{00} ,
or due to difference in meaning of (some) operators (Ji, Liu, 2021 / Ji, 2021)

- Comparison with Lorcé's two-term decomposition (Lorcé, 2017)

$$M = U_q + U_g = n \left(\langle T_{q,R}^{00} \rangle + \langle T_{g,R}^{00} \rangle \right)$$

$$T_{q,R}^{00} = (m\bar{\psi}\psi)_R + (\psi^\dagger i\mathbf{D} \cdot \boldsymbol{\alpha} \psi)_R$$

$$T_{g,R}^{00} = \frac{1}{2}(E^2 + B^2)_R$$

- “modified Ji sum rule” can be considered refinement of two-term sum rule by Lorcé

Overview: Comparison of Sum Rules

- Two-term decomposition of $\langle T^\mu_\mu \rangle$ (in D2 scheme)

$$M = \bar{M}_q + \bar{M}_g = n \left(\langle (m\bar{\psi}\psi)_R \rangle + \left\langle \gamma_m(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R \right\rangle \right)$$

- two scale-independent terms

- Two-term decomposition of $\langle T^{00} \rangle$

$$M = U_q + U_g = n \left(\langle (m\bar{\psi}\psi)_R + (\psi^\dagger i\mathbf{D} \cdot \boldsymbol{\alpha} \psi)_R \rangle + \left\langle \frac{1}{2}(E^2 + B^2)_R \right\rangle \right)$$

- two scale-dependent terms

- Three-term decomposition of $\langle T^{00} \rangle$

$$M = M_q + M_m + M_g = n \left(\langle (m\bar{\psi}\psi)_R \rangle + \langle (\psi^\dagger i\mathbf{D} \cdot \boldsymbol{\alpha} \psi)_R \rangle + \left\langle \frac{1}{2}(E^2 + B^2)_R \right\rangle \right)$$

- one scale-independent term, and two scale-dependent terms

- Relation between matrix elements

$$\left\langle (\psi^\dagger i\mathbf{D} \cdot \boldsymbol{\alpha} \psi)_R + \frac{1}{2}(E^2 + B^2)_R \right\rangle = \left\langle \gamma_m(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R \right\rangle$$

- one can speak about contribution from trace anomaly **or** from parton energies
- a sum rule with contributions from trace anomaly **and** parton energies does not appear naturally
- relation between matrix elements confirmed in recent calculation for hydrogen atom (Sun, Sun, Zhou, arXiv:2012.09443)
- relation between matrix elements, not between operators

Numerical Results

- First input: parton momentum fractions a_i , related to traceless parton operators

$$\frac{3}{2} M^2 a_q = \langle \bar{T}_{q,R}^{00} \rangle \quad \frac{3}{2} M^2 a_g = \langle \bar{T}_{g,R}^{00} \rangle \quad (a_q + a_g = 1)$$

- Second input: quark mass term

$$2M^2 b = (1 + \gamma_m) \langle (m\bar{\psi}\psi)_R \rangle \rightarrow 2M^2 (1 - b) = \frac{\beta}{2g} \langle (F^2)_R \rangle$$

- direct input on trace anomaly (from experiment and/or LQCD) would be useful

- Example: “modified Ji sum rule” in terms of a_i and b

$$M_q = \frac{3}{4} M a_q + \frac{1}{4} M \left(\frac{(y-3)b}{1+\gamma_m} + x(1-b) \frac{2g}{\beta} \right)$$

$$M_m = M \frac{b}{1+\gamma_m}$$

$$M_g = \frac{3}{4} M a_g + \frac{1}{4} M \left[\frac{(\gamma_m - y)b}{1+\gamma_m} + \left(1 - x \frac{2g}{\beta} \right) (1-b) \right]$$

- Momentum fractions from CT18NNLO parameterization (at $\mu = 2 \text{ GeV}$)

$$a_q = 0.586 \pm 0.013 \quad a_g = 1 - a_q = 0.414 \pm 0.013$$

- Quark mass term from sigma terms

$$\sigma_u + \sigma_d = \sigma_{\pi N} = \frac{\langle P | \hat{m} (\bar{u}u + \bar{d}d) | P \rangle}{2M} \quad \sigma_s = \frac{\langle P | m_s \bar{s}s | P \rangle}{2M} \quad \sigma_c = \frac{\langle P | m_c \bar{c}c | P \rangle}{2M}$$

- **Scenario A:** sigma terms from phenomenology

(Alarcon et al, 2011, 2012 / Hoferichter et al, 2015)

$$\sigma_{\pi N}|_{\text{ChPT}} = (59 \pm 7) \text{ MeV} \quad \sigma_s|_{\text{ChPT}} = (16 \pm 80) \text{ MeV}$$

- **Scenario B:** sigma terms from lattice QCD

(Alexandrou et al, 2019)

$$\sigma_{\pi N}|_{\text{LQCD}} = (41.6 \pm 3.8) \text{ MeV} \quad \sigma_s|_{\text{LQCD}} = (39.8 \pm 5.5) \text{ MeV}$$

$$\sigma_c|_{\text{LQCD}} = (107 \pm 22) \text{ MeV}$$

- main difference between scenarios: including or not σ_c

- Scheme dependence, for “modified Ji sum rule” (at $\mu = 2 \text{ GeV}$)

		MS	$\overline{\text{MS}}_1$	$\overline{\text{MS}}_2$	D1	D2
Scenario A	M_q	0.309 ± 0.044	0.194 ± 0.033	0.178 ± 0.032	0.362 ± 0.045	0.357 ± 0.051
	M_m	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080
	M_g	0.555 ± 0.036	0.669 ± 0.047	0.686 ± 0.048	0.502 ± 0.035	0.507 ± 0.029
Scenario B	M_q	0.234 ± 0.006	0.135 ± 0.003	0.120 ± 0.003	0.286 ± 0.006	0.272 ± 0.008
	M_m	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023
	M_g	0.517 ± 0.017	0.617 ± 0.020	0.631 ± 0.020	0.465 ± 0.017	0.479 ± 0.015

- considerable numerical scheme dependence
- qualitatively, similar results for other sum rules
- scheme dependence no new phenomenon

- Numerics for sum rule by Hatta, Raban, Tanaka (MS scheme)

		$O(\alpha_s^1)$	$O(\alpha_s^2)$	$O(\alpha_s^3)$
Scenario A	\bar{M}_q	-0.113 ± 0.102	-0.120 ± 0.105	-0.115 ± 0.107
	\bar{M}_g	1.051 ± 0.102	1.057 ± 0.105	1.053 ± 0.107
Scenario B	\bar{M}_q	0.032 ± 0.030	0.030 ± 0.031	0.035 ± 0.030
	\bar{M}_g	0.906 ± 0.030	0.908 ± 0.030	0.903 ± 0.030

- perturbative expansion very stable (applies for all sum rules, and for all schemes)
- \bar{M}_q can become negative

- Numerics for two-term sum rule by Lorcé (MS scheme)

		$O(\alpha_s^1)$	$O(\alpha_s^2)$	$O(\alpha_s^3)$
Scenario A	U_q	0.384 ± 0.035	0.383 ± 0.036	0.384 ± 0.036
	U_g	0.554 ± 0.035	0.556 ± 0.036	0.555 ± 0.036
Scenario B	U_q	0.420 ± 0.016	0.420 ± 0.017	0.421 ± 0.017
	U_g	0.518 ± 0.016	0.518 ± 0.017	0.517 ± 0.017

- very roughly, quark and gluon energies contribute equally to proton mass
- in $\overline{\text{MS}}$ scheme, contribution from gluon energy somewhat larger

- Numerics for “modified Ji sum rule” (MS scheme)

		$O(\alpha_s^1)$	$O(\alpha_s^2)$	$O(\alpha_s^3)$
Scenario A	M_q	0.311 ± 0.043	0.310 ± 0.043	0.309 ± 0.044
	M_m	0.073 ± 0.080	0.073 ± 0.079	0.074 ± 0.080
	M_g	0.554 ± 0.035	0.556 ± 0.036	0.555 ± 0.036
Scenario B	M_q	0.237 ± 0.006	0.235 ± 0.006	0.234 ± 0.006
	M_m	0.183 ± 0.023	0.184 ± 0.022	0.187 ± 0.023
	M_g	0.518 ± 0.016	0.518 ± 0.017	0.517 ± 0.017

- $M_g = U_g \rightarrow$ discussion for gluon part like for sum rule by Lorcé
- M_q dominates over M_m , but feature less significant if σ_c included
- precise determination of M_m important for proton mass decomposition
- contribution of M_m is $\sim 8\%$ for Scenario A, $\sim 20\%$ for Scenario B
 \rightarrow (much) larger than $\sim 1\%$ which is frequently attributed to Higgs mechanism

Summary and Discussion

- Several sum rules for proton mass exist that are based on EMT (T^{00} or T^{μ}_{μ})
- We arrived at three-term sum rule
 - terms have “clean” interpretation
 - sum rule can be considered refinement of two-term sum rule by Lorcé
- Ji’s original four-term decomposition
 - existing phenomenology for Ji’s decomposition not affected by our work
 - we can’t fully reproduce operator definitions of Ji’s decomposition
 - interpretation of some terms (admixture from pressure) ?
- Phenomenology of sum rules
 - form factors $A_i(0)$ and $\bar{C}_i(0)$ known to good accuracy
 - phenomenology for all mass sum rules largely fixed (different from spin sum rules)
 - yet, measurement of trace anomaly ($\langle F^2 \rangle$) still useful \rightarrow new input for $\bar{C}_g(0)$