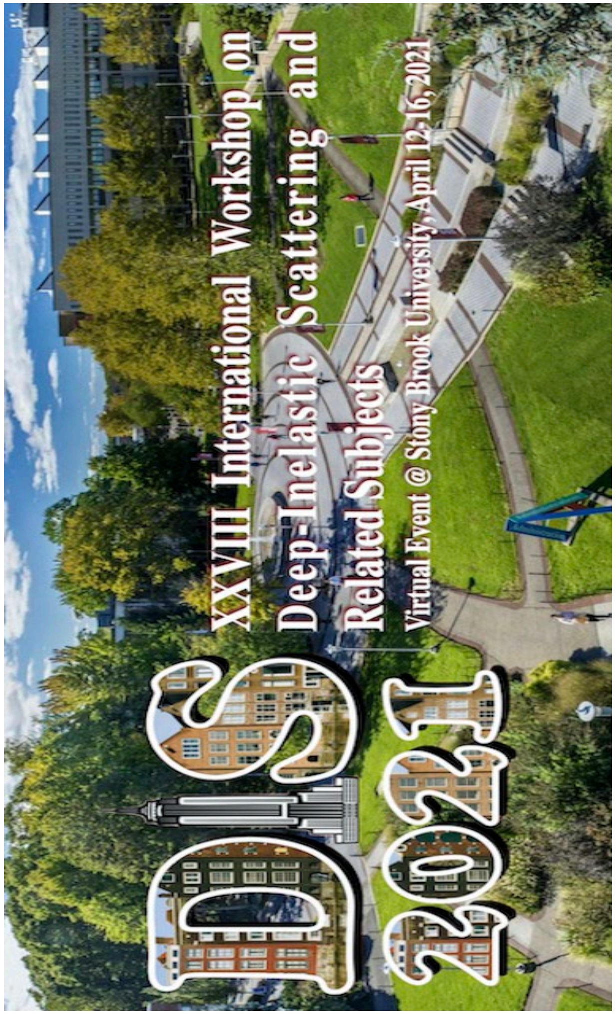

Λ POLARIZING FRAGMENTATION FUNCTION FROM BELLE e^+e^- DATA

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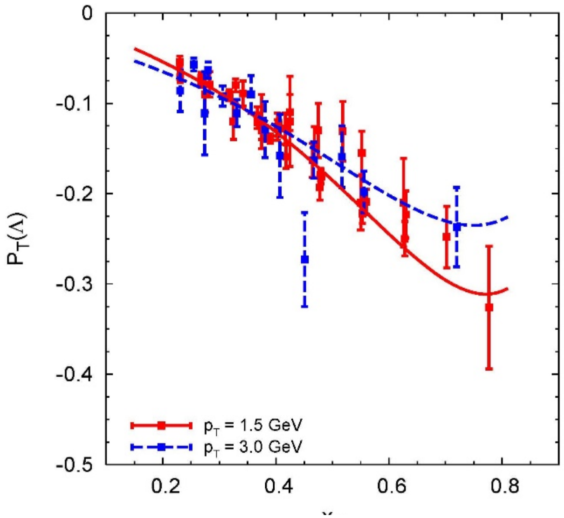
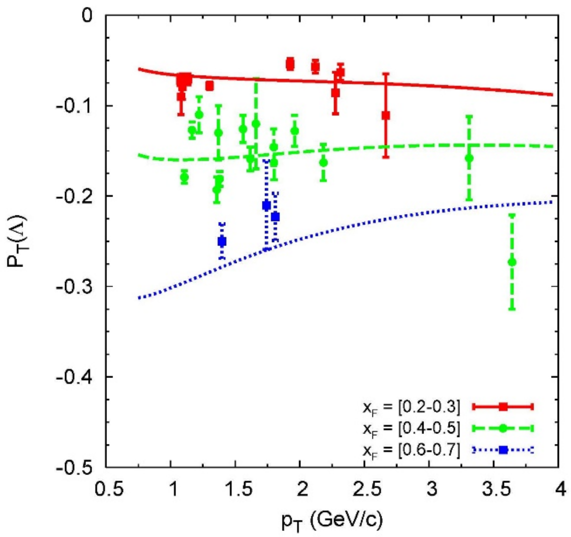
PRD **102**, 054001 (2020)

OUTLINE

- ❑ Λ polarization: a challenge and a tool
- ❑ TMD fragmentation functions for Spin-½ hadrons within the helicity formalism
- ❑ $e^+e^- \;\to\; \Lambda\, h + X$ within a TMD scheme
  - $e^+e^- \;\to\; \Lambda(\mathrm{jet}) + X$
- ❑ Fit of Belle data and extraction of the polarizing fragmentation function
- ❑ Predictions for EIC
- ❑ Concluding remarks





A LOOK BACK…

$$P_T(\Lambda) = \frac{d\sigma^{AB\to\Lambda^\uparrow X} - d\sigma^{AB\to\Lambda^\downarrow X}}{d\sigma^{AB\to\Lambda^\uparrow X} + d\sigma^{AB\to\Lambda^\downarrow X}}$$

- pA collisions (late 70's)

 - fixed target exp.s - $E \backsim 400$ GeV
- Λ self-analyzing: a powerful tool
- Transverse Λ polarization w.r.t. the production plane
  - large and negative
  - increasing up to $p_T \sim 1$ GeV then a plateau
  - increasing with x_F in the plateau regime
  - $\bar{\Lambda}$ no polarization
  - Almost zero in collinear pQCD at leading twist

- TMD polarizing fragmentation function Boer, Jakob, Mulders 1997

  unpolarized quark fragmenting into a transversely polarized spin-1/2 hadron

- First attempt to describe $P_T(\Lambda)$ data within a phenomenological TMD model

  Anselmino, Boer, UD, Murgia 2001

- Twist-three approach: Kanazawa, Koike 2001

- 2019: Belle data for $P_T(\Lambda)$ in $e^+e^- \to \Lambda^\uparrow h + X$ and $e^+e^- \to \Lambda^\uparrow(\text{jet}) + X$

  Guan et al. (Belle Coll.) 2019

QUARK TMD FFS FOR SPIN-1/2 HADRONS

- Close analogy with the distribution sector
- Within the helicity formalism (*like a quark-hadron correlator*)

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h}\,\hat{D}_{h/q,s_q}(z,\mathbf{p}_\perp) = \sum_{\lambda_q,\lambda'_q} \rho^{q}_{\lambda_q,\lambda'_q}\,\hat{D}^{\lambda_h,\lambda'_h}_{\lambda_q,\lambda'_q}(z,\mathbf{p}_\perp)$$

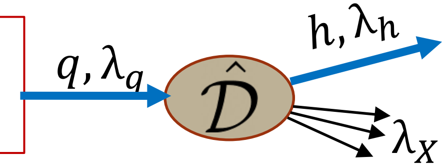
$$\sum\!\!\!\!\!\!\!\int_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_h,\lambda_X;\lambda_q}(z,\mathbf{p}_\perp)\,\hat{\mathcal{D}}^{*}_{\lambda'_h,\lambda_X;\lambda'_q}(z,\mathbf{p}_\perp)$$

$$\rho^{h,S_h}_{\lambda_h,\lambda'_h} = \frac{1}{2}\begin{pmatrix} 1+P^h_Z & P^h_X - iP^h_Y \\ P^h_X + iP^h_Y & 1-P^h_Z \end{pmatrix}$$

Hadron helicity density matrix

polarization components

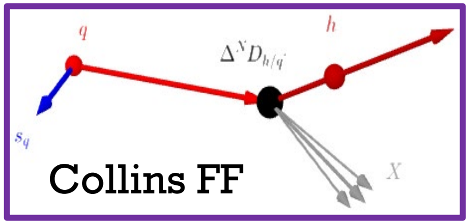
product of fragm. amplitudes



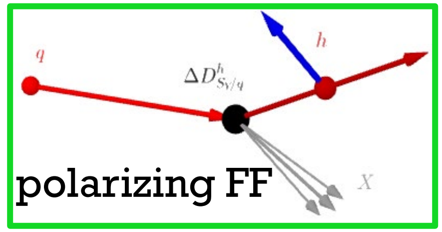
$$D_{h/q}(z) = \frac{1}{2}\sum_{\lambda_q,\lambda_h}\int d^2\mathbf{p}_\perp\, D^{\lambda_h,\lambda_h}_{\lambda_q,\lambda_q}(z,\mathbf{p}_\perp)$$

Collinear unpolarized fragm. function

8 independent TMD Fragmentation Functions



T-odd & chiral-odd



T-odd & chiral-even

$$\Delta D_{h^\uparrow/q}(z,p_\perp) = \frac{p_\perp}{z m_h} D_{1T}^{\perp q}(z,p_\perp)$$

UNIVERSAL

		Hadron		
	Pol. States	**U**	**L**	**T**
Quark	**U**	$\hat{D}_{h/q}$		$\Delta\hat{D}^h_{S_Y/q}$
	L		$\Delta\hat{D}^{h/q}_{S_Z/s_L}$	$\Delta\hat{D}^{h/q}_{S_X/s_L}$
	T	$\Delta^N D_{h/q^\uparrow}$	$\Delta\hat{D}^{h/q}_{S_Z/s_T}$	$\Delta\hat{D}^{h/q}_{S_X/s_T}$ / $\Delta^-\hat{D}^{h/q}_{S_Y/s_T}$

Surviving in the collinear limit

$$e^+e^- \rightarrow h_1(P_1)h_2(P_2) + X$$

- Complete results for the azimuthal and polarization observables
  Boer, Jakob, Mulders 1997
- TMD factorization for small relative transverse momenta w.r.t. Q^2
  Collins 2011 - Echevarria, Idilbi Scimemi 2012
- Formulation within the helicity formalism (partonic interpretation)
  UD, Murgia, Zaccheddu 2021 (in preparation)*

* with full set of leading-twist quark and **gluon** TMD-FFs for spin-1/2 hadrons

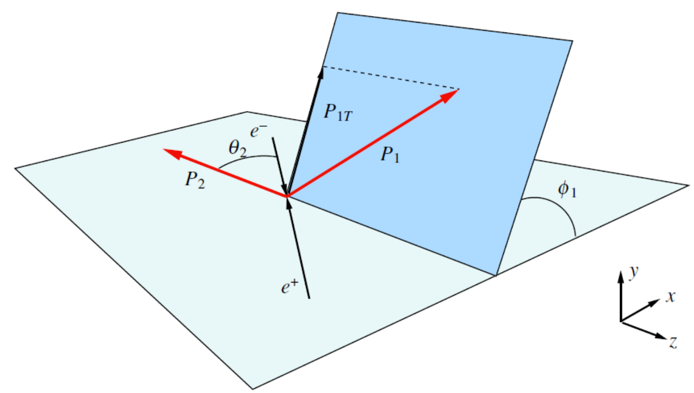
$$e^+e^- \to h_1(P_1)h_2(P_2) + X$$

Master formula

Fragmentation Funct.s

$$\rho^{h_1,S_1}_{\lambda_{h_1},\lambda'_{h_1}}\,\rho^{h_2,S_2}_{\lambda_{h_2},\lambda'_{h_2}}\,\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}} = \sum_{q_c,\bar q_d}\sum_{\{\lambda\}}\frac{1}{32\pi s}\frac{1}{4}\hat M_{\lambda_c\lambda_d,\lambda_a\lambda_b}\hat M^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}\hat D^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1})\hat D^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})$$

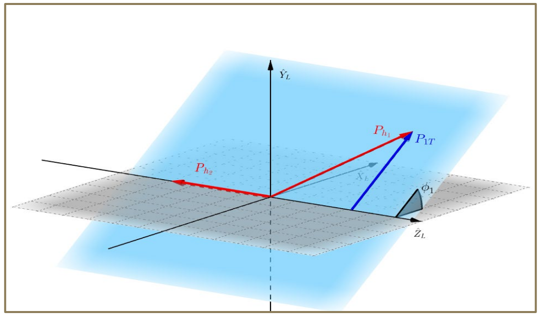
Helicity scatt. amplitude



$$\frac{d\sigma^{e^+e^-\to h_1(S_1)h_2(S_2)\,X}}{d\cos\theta dz_1 dz_2 d^2\mathbf{P}_{1T}}$$

$$= \int d^2\mathbf{p}_{\perp 2}\,d^2\mathbf{p}_{\perp 1}\,\delta^2(\mathbf{p}_{\perp 1} - \mathbf{P}_1 + z_{p_1}\,\beta_1\,\mathbf{q}_1)\,\frac{d\sigma^{e^+e^-\to h_1(S_1)h_2(S_2)\,X}}{d\cos\theta dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

TRANSVERSE Λ POLARIZATION



$$\mathcal{P}_T = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^{\text{unp}}},$$

- Measured along $\hat{\boldsymbol{n}} = -\boldsymbol{P}_{h_2} \times \boldsymbol{P}_{h_1}$ (hadron frame)
- Proper projection from the hadron helicity frame

Polarizing FF

$$P_Y^{h_1} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dPS_{12}} = \frac{3\pi\alpha^2}{2s} \sum_q e_q^2 \Big\{ (1+\cos^2\theta)\, \Delta D^{h_1}_{S_Y/q}(z_1, p_{\perp 1})\, D_{h_2/\bar{q}}(z_2, p_{\perp 2}) + \frac{1}{2}\sin^2\theta\, \Delta^- D^{h_1}_{S_Y/s_T}(z_1, p_{\perp 1})\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \cos(2\varphi_2 + \phi_{h_1}) \Big\}$$

$$dPS_{12} = dz_1 d^2\boldsymbol{p}_{\perp 1} dz_2 d^2\boldsymbol{p}_{\perp 2}$$

Collins FF

- By integrating over Φ_1 $\qquad d^2\mathbf{p}_{\perp 1} \to d^2\mathbf{P}_{T1}$
  - ✓ the transverse component along X is washed out
  - ✓ the contribution from the Collins FF is washed out

FACTORIZED GAUSSIAN MODEL

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$\langle p_\perp^2\rangle_{\rm pol} = \frac{M_{\rm pol}^2}{M_{\rm pol}^2+\langle p_\perp^2\rangle}\,\langle p_\perp^2\rangle$$

$$\Delta D_{h^\uparrow/q}(z,p_\perp) = \Delta D_{h^\uparrow/q}(z)\,\frac{\sqrt{2e}\,p_\perp}{M_{\rm pol}}\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{\rm pol}}}{\pi\langle p_\perp^2\rangle}$$

$$\mathcal{P}_n(z_1,z_2) = \sqrt{\frac{e\pi}{2}}\,\frac{1}{M_{\rm pol}}\,\frac{\langle p_\perp^2\rangle_{\rm pol}^2}{\langle p_{\perp 1}^2\rangle}\,\frac{z_2}{\left\{[z_1(1-m_{h_1}^2/(z_1^2 s))]^2\langle p_{\perp 2}^2\rangle + z_2^2\langle p_\perp^2\rangle_{\rm pol}\right\}^{1/2}} \times \frac{\sum_q e_q^2\,\Delta D_{h_1^\uparrow/q}(z_1)D_{h_2/\bar q}(z_2)}{\sum_q e_q^2\,D_{h_1/q}(z_1)D_{h_2/\bar q}(z_2)} \qquad P^{h_1}\cdot\hat n$$

- ✓ Integrated over transverse momenta
- ✓ $h_1 = \Lambda$, h_2 light unpolarized hadron – inclusion of mass effects
- ✓ No TMD evolution (fixed-scale analysis)

$e^+e^- \to h_1(P_1)$ jet X

- Phenomelogically useful:
  - ✓ direct access to the intrinsic transverse momentum dependence
- Tricky from the theory side:
  - ✓ TMD factorization, universality w.r.t. 2-hadron case, role of soft factors

Kang, Shao, Zhao 2020 – Boglione,Simonelli 2021

Gamberg, Kang, Shao, Terry, Zhao 2021

today talks

- Use of a simplified TMD approach
  - ✓ check for consistency with 2-hadron case
  - ✓ see also Anselmino, Kishore, Mukherjee 2019

$$\boxed{\mathcal{P}_T(z_1, p_{\perp 1}) = \frac{\sum_q e_q^2\, \Delta D_{h_1^\uparrow/q}(z_1, p_{\perp 1})}{\sum_q e_q^2\, D_{h_1/q}(z_1, p_{\perp 1})}}$$

✓ Orthogonal to the thrust plane $\hat{\mathbf{T}} \times \mathbf{P}_{h_1}$

FIT OF BELLE DATA

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5\ - 0.9]$ excluded → 96/128 data points
- Λ(jet) : $z_\Lambda = [0.5\ - 0.9]$ excluded → 24/32 data points

- **Unpol. FFs**: DSS07 for π/K , AKK08 for Λ

$$\langle p_\perp^2 \rangle = 0.2\ \text{GeV}^2$$

$$\boxed{D_{\Lambda/\bar{q}}(z_p) = (1 - z_p) D_{\Lambda/q}(z_p)}$$

flavour separation

- **Polarizing FF**

$$\Delta D_{\Lambda^\uparrow/q}(z) = N_q z^{a_q}(1-z)^{b_q} \frac{(a_q + b_q)^{(a_q+b_q)}}{a_q^{a_q} b_q^{b_q}} D_{\Lambda/q}(z)$$

$$|N_q| \leq 1$$ Positivity bound

FIT OF BELLE DATA

$N_u,\ N_d,\ N_s,\ N_{\rm sea},\ a_s,\ b_u,\ b_{\rm sea}$ $\langle p_\perp^2\rangle_{\rm pol}$

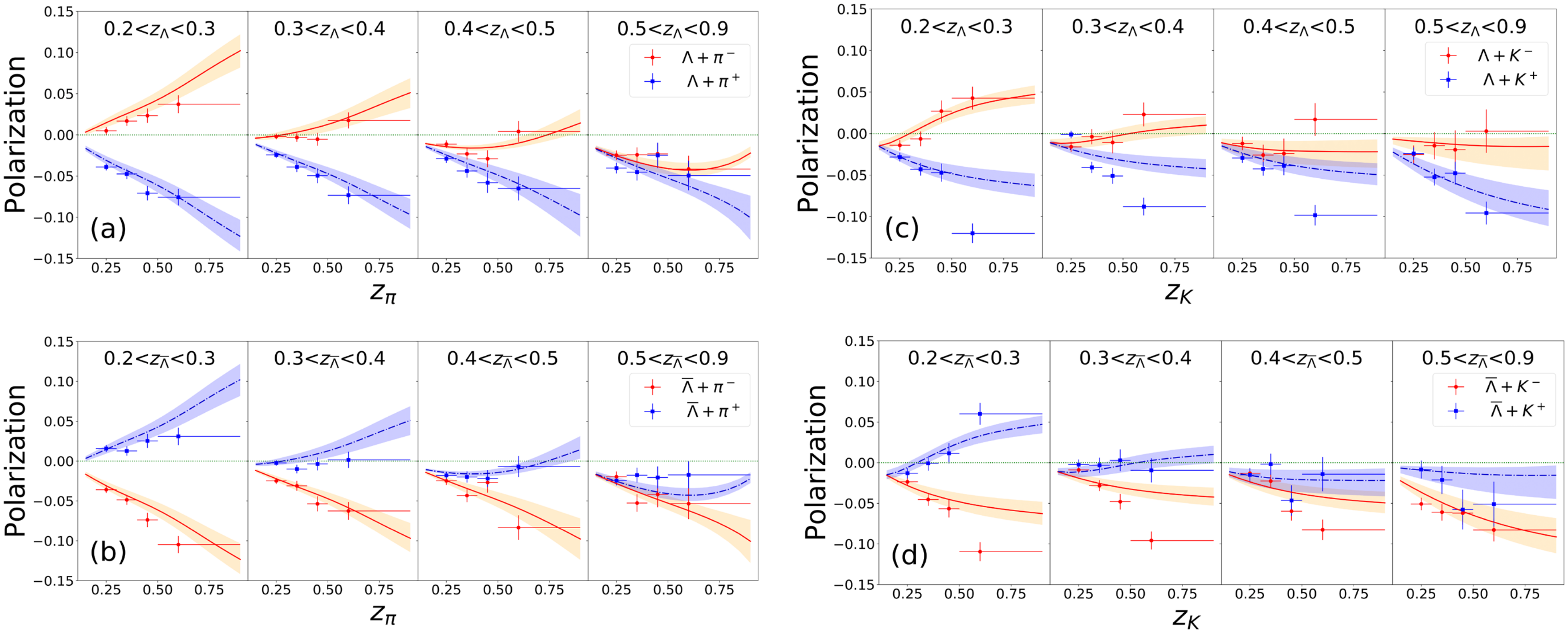
Full data fit : $\chi^2_{\rm dof} = 1.94$

$N_u = 0.47^{+0.32}_{-0.20}$	$N_d = -0.32^{+0.13}_{-0.13}$
$N_s = -0.57^{+0.29}_{-0.43}$	$N_{\rm sea} = -0.27^{+0.12}_{-0.20}$
$a_s = 2.30^{+1.08}_{-0.91}$	
$b_u = 3.50^{+2.33}_{-1.82}$	$b_{\rm sea} = 2.60^{+2.60}_{-1.74}$
$\langle p_\perp^2\rangle_{\rm pol} = 0.10^{+0.02}_{-0.02}$ GeV^2	

$\Lambda + \pi/K$ data fit : $\chi^2_{\rm dof} = 1.26$

Similar analysis (associated production) Callos, Kang, Terry 2020

Associated production [full-data analysis]

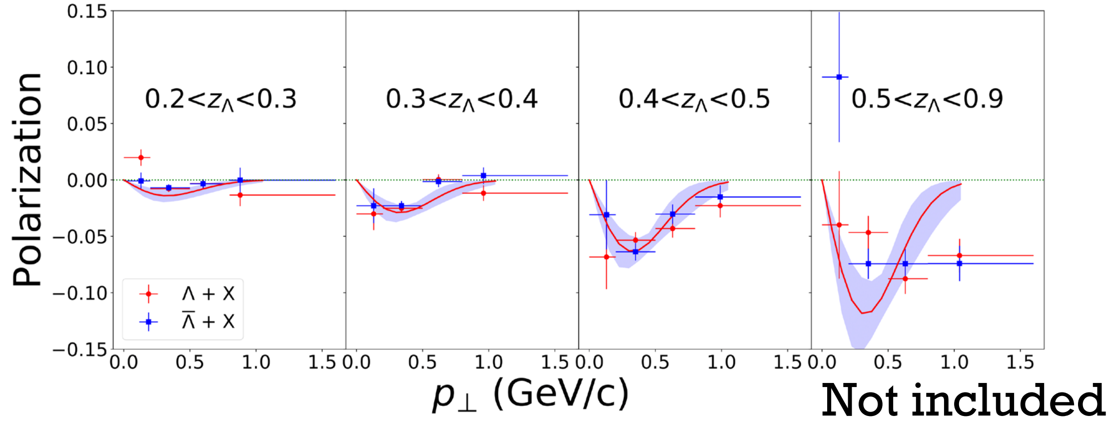


$\chi^2_{\rm point} = 1.55/0.8$

$\chi^2_{\rm point} = 1.61/1.5$

- simpler fits with only two pFFs (*u*=*d* and *s* and no sea) ➡ higher χ^2
- pion well described (also including the highest *z* bin)
- kaon, some tension (data, FFs?)
- Shaded areas correspond to a 2σ uncertainty (250K sets) $\Delta\chi^2 = 15.79$

Inclusive production [full-data analysis]

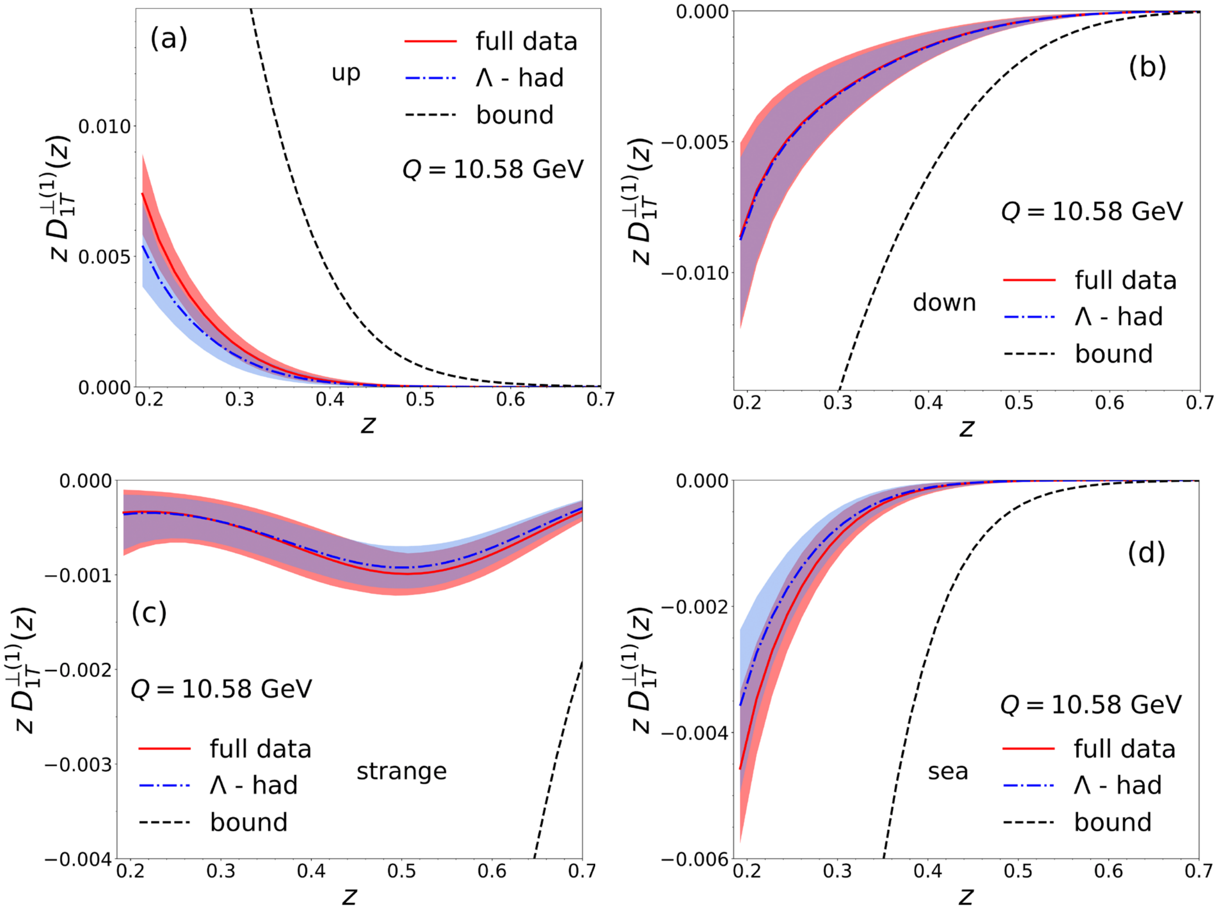


$$\chi^2_{\rm point} = 2.75$$

- Less good description
- Theory
  - ✓ Polarization = 0 at $p_\perp$= 0
  - ✓ $P(\Lambda) = P(\bar{\Lambda})$

- not *clearly* visible in the data

$$D_{1T}^{\perp(1)}(z) = \int d^2\mathbf{p}_\perp \frac{p_\perp}{2zm_h} \Delta D_{h^\uparrow/q}(z, p_\perp)$$ First moment of the polarizing FF



Consistency between full-data and $\Lambda - h$ extractions both for the central lines and the overlapping uncertainty bands!!!

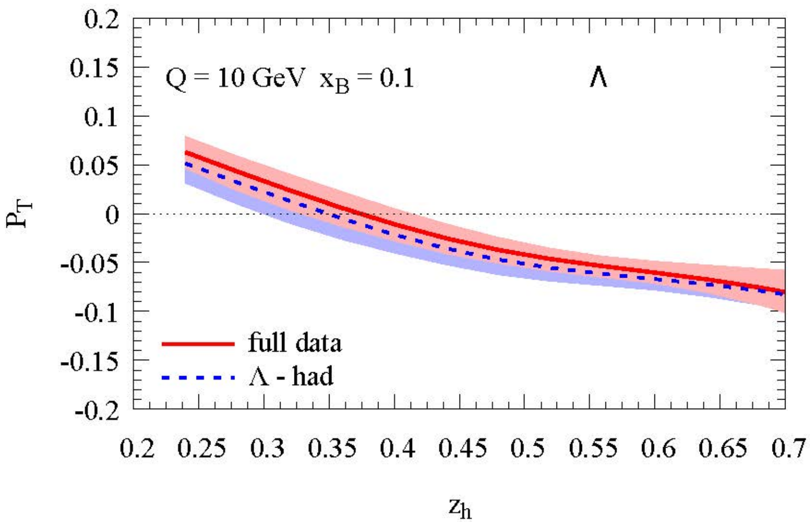
TRANSVERSE Λ POLARIZATION IN SIDIS

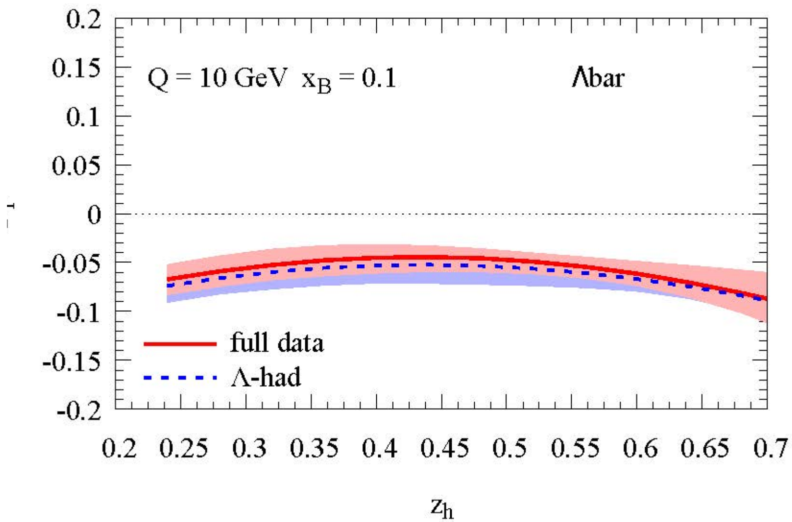
$$P_T(x_B, z_h) = \frac{\sqrt{2e\pi}}{2M_p}\frac{\langle p_\perp^2\rangle_{\rm pol}^2}{\langle p_\perp^2\rangle}\frac{1}{\sqrt{\langle p_\perp^2\rangle_{\rm pol} + \xi_p^2\langle k_\perp^2\rangle}} \times \frac{\sum_q e_q^2 f_{q/p}(x_B)\Delta D_{\Lambda^\uparrow/q}(z_h)}{\sum_q e_q^2 f_{q/p}(x_B) D_{\Lambda/q}(z_h)}$$

Transverse w.r.t.
the target-Λ plane

$$\xi_p = z_h\left(1 - \frac{m_h^2}{z_h^2 Q^2}\frac{x_B}{1-x_B}\right)$$

EIC kinematics





CONCLUDING REMARKS

- Fit of Belle e^+e^- data and extraction of the polarizing FF within a TMD factorization scheme
- Associated Λ-hadron and inclusive Λ production [consistency]
- Clear separation in flavour
  - ✓ three different valence polFFs (up, down and strange)
  - ✓ relative sign between up and down polFFs
  - ✓ role of the polFF from sea quarks
- extraction of the $p_\perp$ dependence within a Gaussian parametrization
- SIDIS measurements @EIC important to complement these results

THANKS for the ATTENTION

BACK-UP SLIDES

Uncertainty band

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - F(x_i; \boldsymbol{a})}{\sigma_i} \right)^2$$

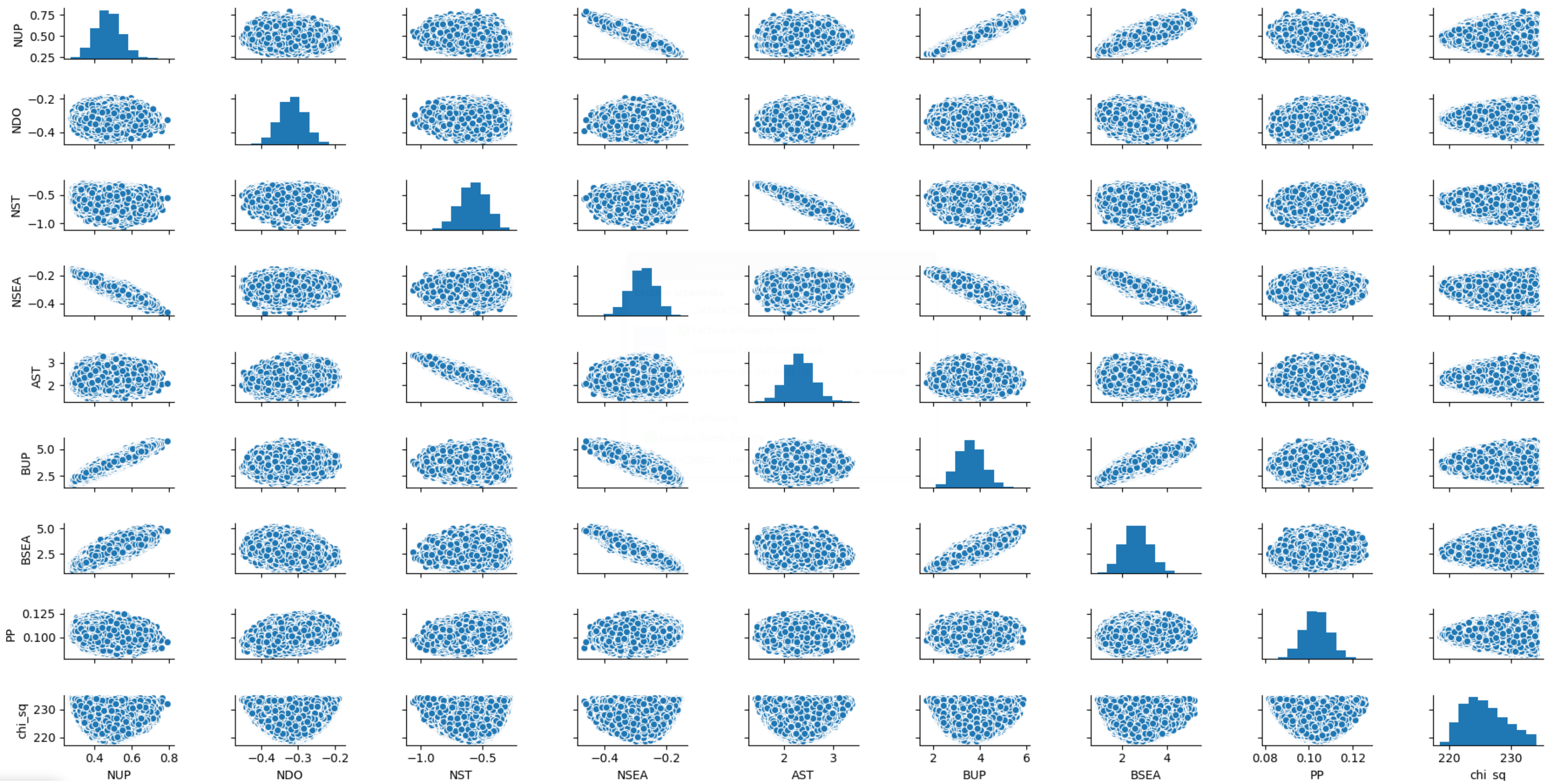
- N measurements y_i at known points x_i, with variance σ_i^2.
- $F(x_i; \boldsymbol{a})$ depends *non-linearly* on M unknown parameters a_i.
- Best fit: $\chi^2_{\rm min} \to \boldsymbol{a}_0$

Error band: all sets of parameters such that $\chi^2(\boldsymbol{a}_j) \leq \chi^2_{\rm min} + \Delta\chi^2$
- $\Delta\chi^2 = 1 \leftrightarrow 1$-$\sigma$: small errors, uncorrelated parameters, linearity, χ^2 parabolic
- $\Delta\chi^2$: fixed according to the coverage probability

$$P = \int_0^{\Delta\chi^2} \frac{1}{2\Gamma(M/2)} \left(\frac{\chi^2}{2}\right)^{(M/2)-1} \exp\left(-\frac{\chi^2}{2}\right) \mathrm{d}\chi^2 \, .$$

P= probability that true set of parameters falls inside the hypervolume

$[P = 0.68 \leftrightarrow 1$-$\sigma$, $P = 0.95 \leftrightarrow 2$-$\sigma]$


NUP
NDO
NST
NSEA
AST
BUP
BSEA
PP
chi_sq

Ratios of the first moments w.r.t. the bounds