Zero modes
&
Matching for the twist-3 PDFs

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In Collaboration with
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Aurora Scapellato (Adam Mickiewicz U./ Temple U.)
Fernanda Steffens (Bonn U.)

Quick introduction on twist-3 PDFs

Twist-2 PDFs

Order of contribution: $\mathcal{O}(1)$

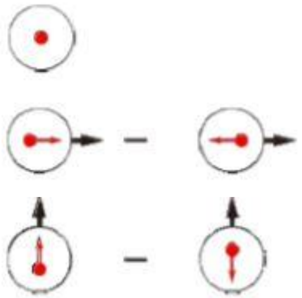
PDFs	Dirac structure
$f_1(x)$	$\Gamma = \gamma^+$
$g_1(x)$	$\Gamma = \gamma^+\gamma_5$
$h_1(x)$	$\Gamma = i\sigma^{i+}\gamma_5$

Density interpretation:

$f_1(x)$

$g_1(x)$

$h_1(x)$



Twist-3 PDFs

Order of contribution: $\mathcal{O}(1/\mathrm{Q})$

Jaffe, Ji (PRL 67, 552)/ Jaffe, Ji (Nucl. Phys. B 375, 527)

PDFs	Dirac structure
$e(x)$	$\Gamma = 1$
$g_T(x)$	$\Gamma = \gamma_\perp^i\gamma_5$
$h_L(x)$	$\Gamma = i\sigma^{+-}\gamma_5$

No density interpretation:

Twist-3 PDFs

$\downarrow$

qgq correlation

Burkardt (arXiv: 0810.3589)

$\int dx\, x^2 g_T(x) \rightarrow \perp$ **force**

$\int dx\, x^2 e(x) \rightarrow \perp$ **force**

Sketch of quasi-PDF approach

Light-cone (standard) correlator

$$F^{[\Gamma]}(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}}z^-$
- **<u>Cannot</u> be computed on Euclidean lattice**

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Correlator for quasi-PDFs (Ji, 2013)

$$F_{\mathrm{Q}}^{[\Gamma]}(x;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **Can be computed on Euclidean lattice**

- **Quasi-PDF approach made it possible to directly extract light-cone PDFs from lattice QCD**

Sketch of quasi-PDF approach

Sample calculations for twist-2 PDF f_1 in Quark Target Model (QTM)

What if I calculate f_1 , with, a completely spatial correlator & with γ^3 in the Infinite Momentum Frame (IMF)?

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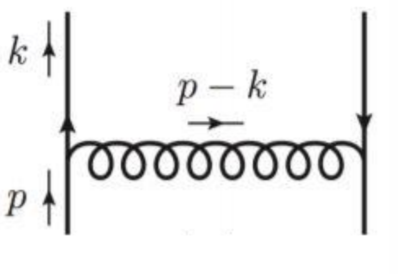

$$f_1(x) = -\frac{i g^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\overline{u}\,\gamma^\nu \left(\not{k} + \cancel{m_q}\right) \gamma^3 \left(\not{k} + \cancel{m_q}\right) \gamma^\mu\right]}{(k^2 - \cancel{m_q^2} + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)}\, \delta\left(x - \frac{\boldsymbol{k^3}}{\boldsymbol{p^3}}\right) \frac{1}{\boldsymbol{p^3}}$$

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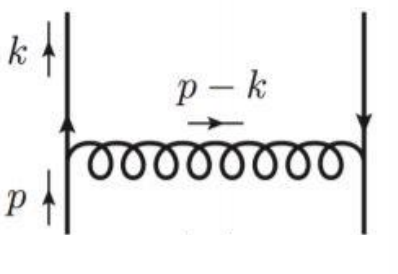

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- $k \to (k^0,\, k_\perp,\, k^3)$

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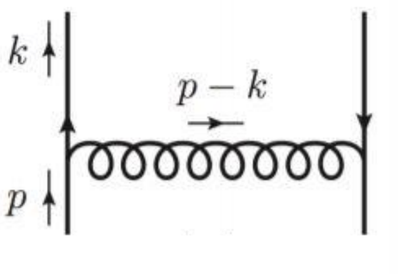

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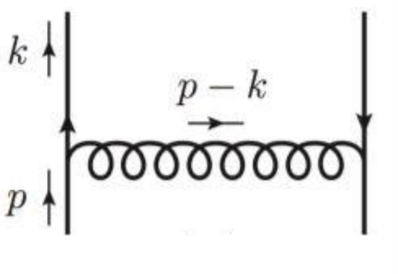

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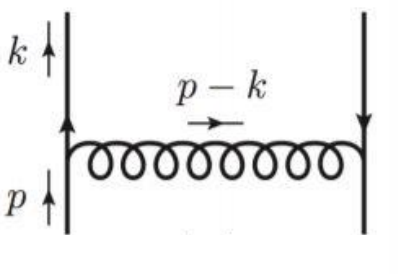

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$$\int_{\mathbf{0}}^{\boldsymbol{\infty}} dk_\perp$$

$$\boxed{f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\mathbf{UV}} + \ln\frac{\mu^2}{x\boldsymbol{m_g^2}} - 2\right)}$$

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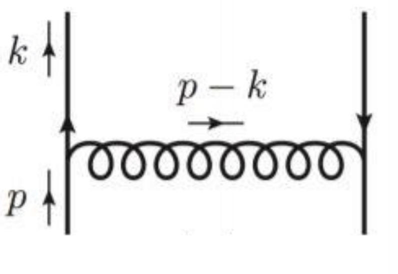

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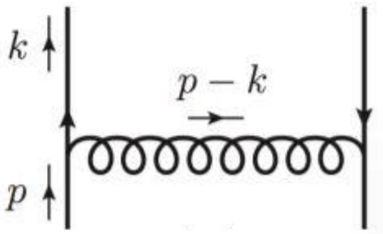

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What if I keep p^3 finite & repeat this calculation?

- $\int dk^0 \to$ Residue theorem
- Take $p^3 \to \infty$
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$\int_{\mathbf{0}}^{\boldsymbol{\infty}} dk_\perp$

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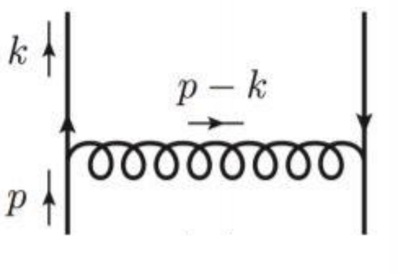

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- Keeping p^3 finite, perform $\int d^2 k_\perp$

$$\boxed{f_1(x, p^3) = \frac{\alpha_s C_F}{2\pi} \begin{cases} (1-x)\,\ln\dfrac{x}{x-1} + 1 & x > 1 \\[2ex] (1-x)\,\ln\dfrac{4(1-x)p_3^2}{\boldsymbol{m_g^2}} + x & 0 < x < 1 \\[2ex] (1-x)\,\ln\dfrac{x-1}{x} - 1 & x < 0 \end{cases}}$$

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Support outside "physical" region $0 < x < 1$

$$\int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\overline{u}\,\gamma^\nu\,(\not{k} + \cancel{m_q})\,\gamma^3\,(\not{k} + \cancel{m_q})\,\gamma^\mu\right]}{(k^2 - \cancel{m_q^2} + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)}\,\delta\!\left(x - \frac{\boldsymbol{k^3}}{\boldsymbol{p^3}}\right)\frac{1}{\boldsymbol{p^3}}$$

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$\int_0^\infty dk_\perp$

Sketch of quasi-PDF approach

IR singularities of quasi-PDFs & light-cone PDFs are same

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Divergences will manifest after we integrate over x

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$$\int_0^\infty dk_\perp$$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

- **Matching formula**

$$q_{\mathrm{Q}}(x;P^3) = \int_{-1}^{+1} \frac{d\,y}{|y|} C\left(\frac{x}{y}\right) q(y) + \mathcal{O}\left(\frac{M^2}{(P^3)^2}\right)$$

(Scale dependence omitted)

(Xiong, Ji, Zhang, Zhao, 2013/ Stewart, Zhao, 2017/
Izubuchi, Ji, Jin, Stewart, Zhao, 2018/
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- **One-loop matching coefficient**

$$C(x;P^3) = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\left[q_{\mathrm{Q}}(x;P^3) - q(x)\right]$$

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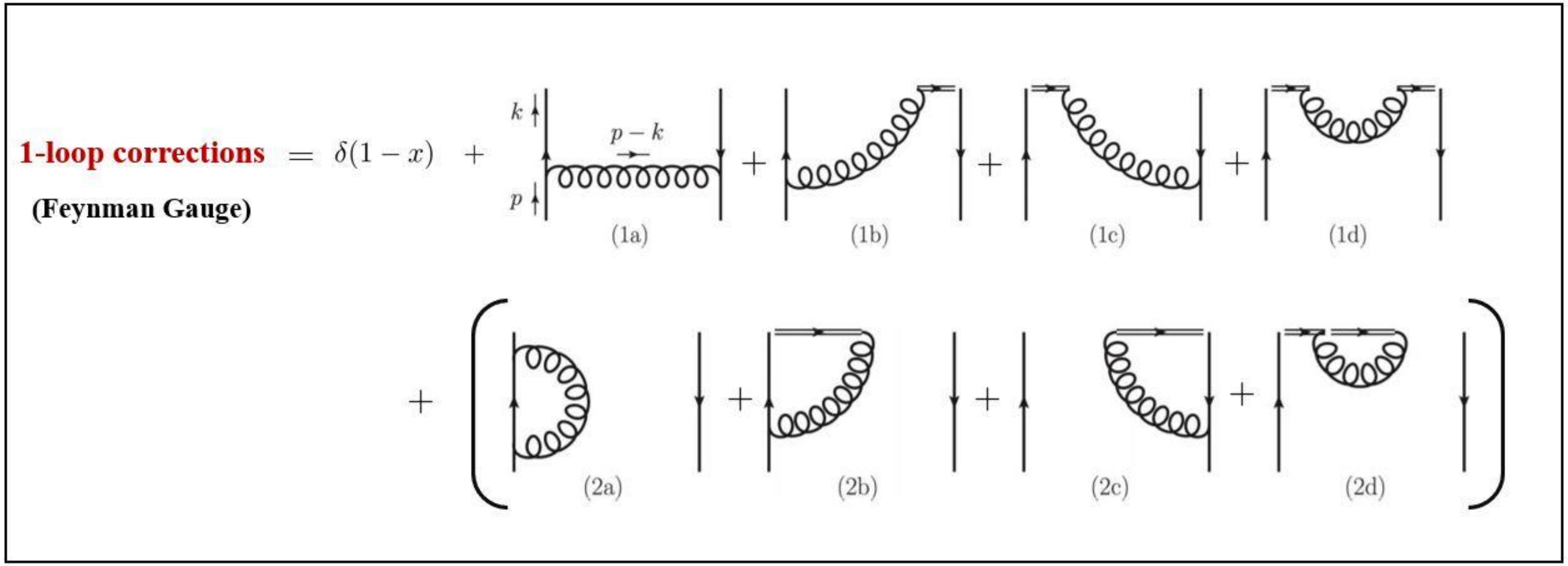
SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

- Matc

, 2017/



- One-

- Set up for our calculation

i. Feynman Gauge

ii. UV : $\int^{\infty} d^2k_\perp \longrightarrow \epsilon_{\rm UV}$

iii. IR : $\int_0 d^2k_\perp \longrightarrow \begin{cases} m_q \neq 0 \\ \epsilon_{\rm IR} \\ m_g \neq 0 \end{cases}$

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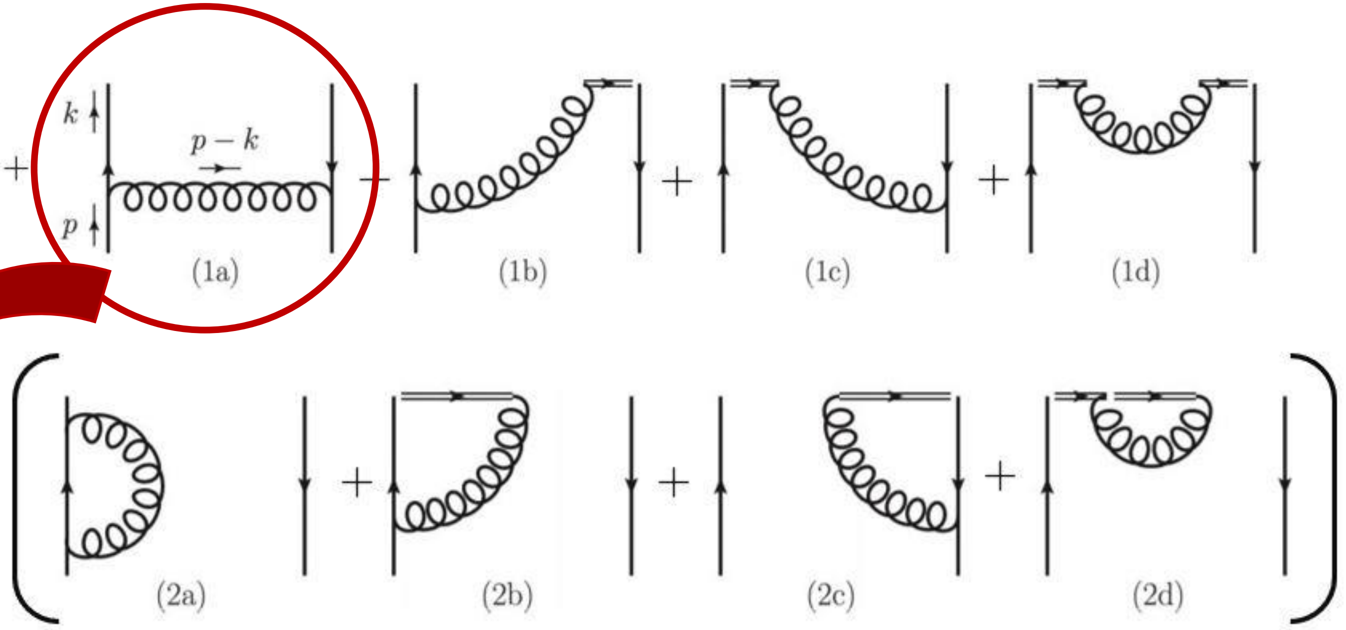
SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

- **Matc**

, 2017/

1-loop corrections $= \delta(1-x) +$

(Feynman Gauge)



- **One-**

- **Set up for our calculatio**

Ladder diagram: Origin of new features at twist-3

i. **Feynman Gauge**

ii. **UV :** $\int^{\infty} d^2k_\perp \longrightarrow \epsilon_{\rm UV}$

iii. **IR :** $\int_0 d^2k_\perp \longrightarrow \begin{cases} m_q \neq 0 \\ \epsilon_{\rm IR} \\ m_g \neq 0 \end{cases}$

<u>Case 1</u>: g_T & $g_{T,\mathbf{Q}}$

Matching for twist-3 PDF $g_T(x)$

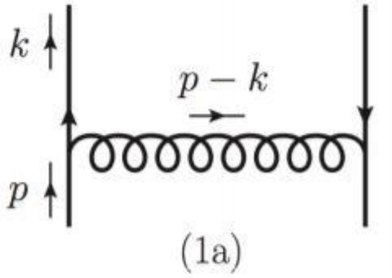
SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_T(x)$

Definition: $$\frac{M}{P^+} S_\perp^i \, g_T(x) = \Phi^{[\gamma^i \gamma_5]}$$

Hadron attributes: $(M,\, S_\perp^i,\, P^+)$

Quark Target Model (QTM)



$$\boxed{\frac{m_q}{p^+} s_\perp^i \, g_T^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\!\left(x-\frac{k^+}{p^+}\right)\frac{1}{p^+}}$$

- **One <u>cannot</u> set** m_q **to zero at the start in QTM calculations**
- **Extract linear terms in** m_q **& then set** $m_q = 0$ **, unless it is used as the IR regulator**

Matching for twist-3 PDF $g_T(x)$

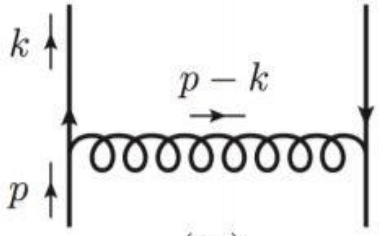
SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_T(x)$

Definition: $$\frac{M}{P^+} S_\perp^i \, g_T(x) = \Phi^{[\gamma^i \gamma_5]}$$

Hadron attributes: $(M, S_\perp^i, P^+)$

Quark Target Model (QTM)



(1a)

$$\boxed{\frac{m_q}{p^+} s_\perp^i \, g_T^{(1a)}(x) = -\frac{i g^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u \, \bar{u} \, \gamma^\nu \, (\not{k} + m_q) \, \gamma^i \gamma_5 \, (\not{k} + m_q) \, \gamma^\mu\right]}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)} \, \delta\!\left(x - \frac{k^+}{p^+}\right) \frac{1}{p^+}}$$

Trace algebra $\Big|_{n=4-2\epsilon}$

$$g_T^{(1a)}(x) = -\frac{i g^2 C_F \mu^{2\epsilon}}{(2\pi)^n} p^+ \int_{-\infty}^{\infty} d^{n-2}k_\perp dk^- dk^+ \, \frac{2p^+ k^- + \ldots}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)} \, \delta\!\left(x - \frac{k^+}{p^+}\right) \frac{1}{p^+}$$

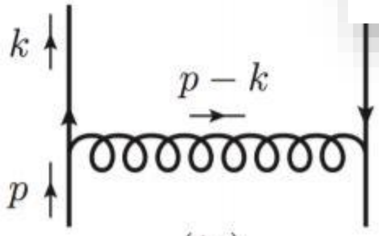
Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_T(x)$

Definition:

Twist-2	Twist-3
Numerator independent of k^-	Numerator has one power of k^-



(1a)

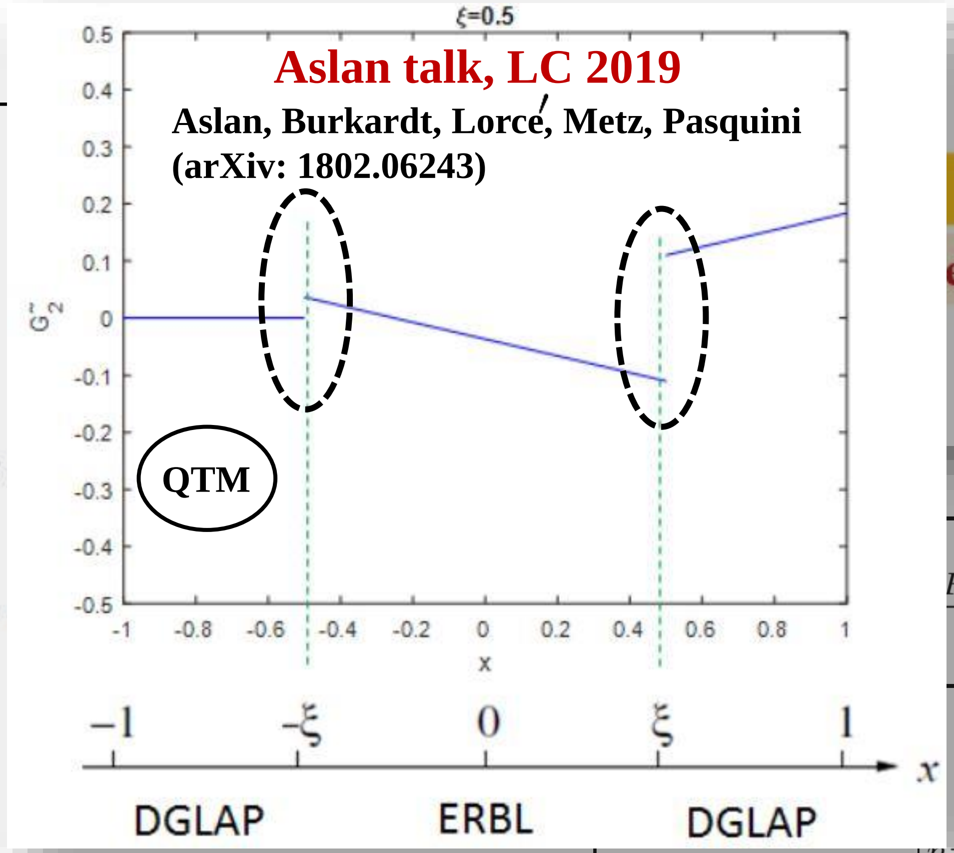
$$\boxed{\frac{m_q}{p^+}\, s_\perp^i\, g_T^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4}\int_{-\infty}^{\infty}\frac{d^n k}{(2\pi)^n}\frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\left(x-\frac{k^+}{p^+}\right)\frac{1}{p^+}}$$

$\downarrow$ **Trace algebra** $\Big|_{n=4-2\epsilon}$

$$g_T^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon}}{(2\pi)^n}p^+\int_{-\infty}^{\infty} d^{n-2}k_\perp dk^- dk^+ \frac{2p^+k^- + \dots}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\left(x-\frac{k^+}{p^+}\right)\frac{1}{p^+}$$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)



	Twist-3
ent of k^-	Numerator has one power of k^-

k

p

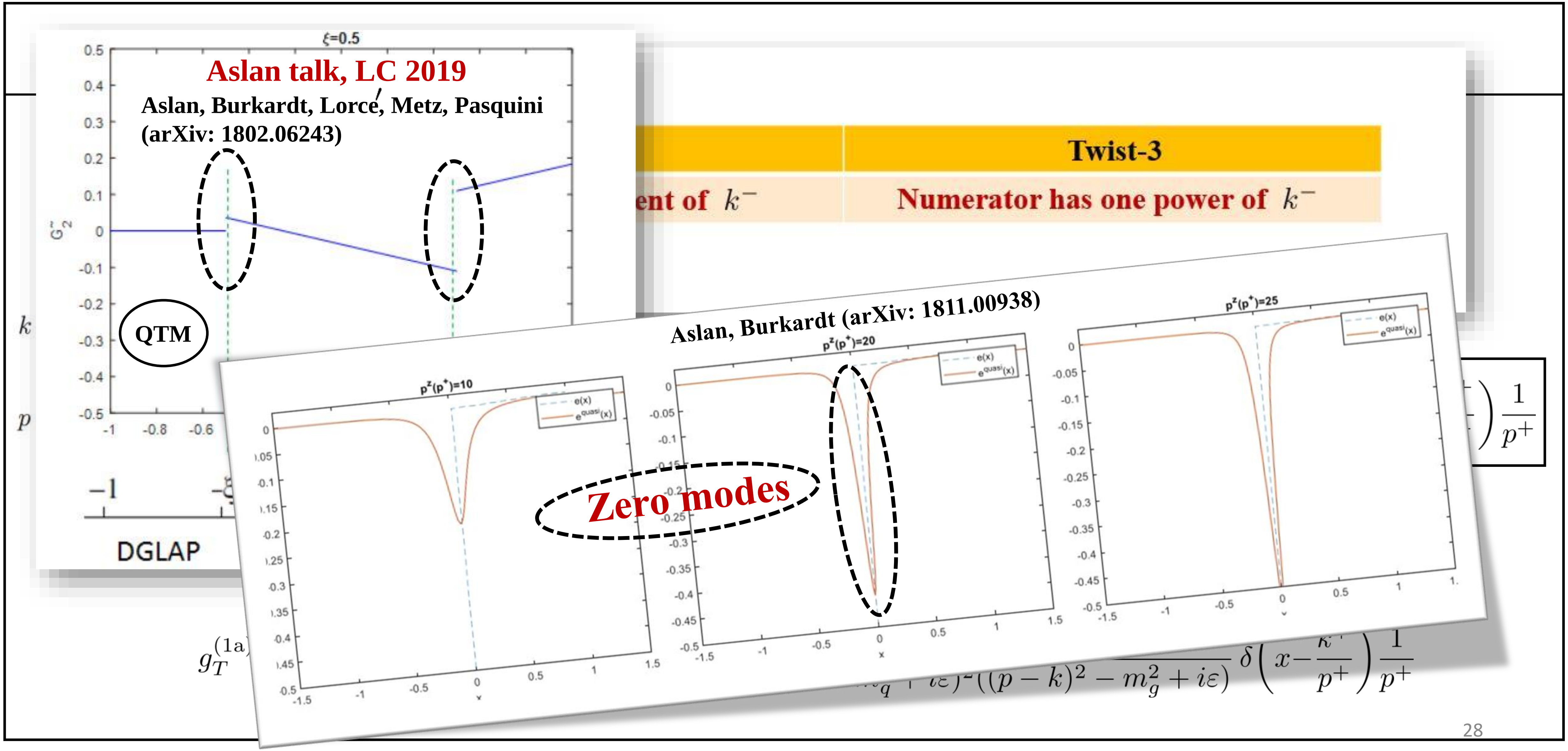
$$\ldots {}_F\mu^{2\epsilon} g_{\mu\nu} \over 4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\left(x-\frac{k^+}{p^+}\right)\frac{1}{p^+}$$

$n=4-2\epsilon$

$$g_T^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon}}{(2\pi)^n} p^+ \int_{-\infty}^{\infty} d^{n-2}k_\perp dk^- dk^+ \frac{2p^+k^- + \ldots}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\left(x-\frac{k^+}{p^+}\right)\frac{1}{p^+}$$

Matching for twist-3 PDF $g_T(x)$

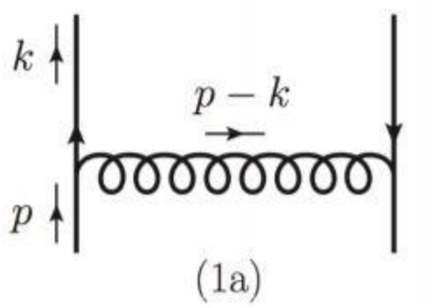
SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)



Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

General structure for the ladder-diagram result: LC $g_T(x)$



$$g_T^{(1a)} = g_{T(\mathrm{s})}^{(1a)} + g_{T(\mathrm{c})}^{(1a)}$$

Singular term $\propto \delta(x)$

Canonical term

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Close look at singular term

Result after $\int d^{n-2}k_\perp$ **:** $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + m_q^2)}$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Close look at singular term

Result after $\int d^{n-2}k_\perp$ **:** $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + m_q^2)}$

i. $m_q \neq 0$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Close look at singular term

Result after $\int d^{n-2}k_\perp$ **:** $g^{(1a)}_{T(\mathrm{s})}(x) = -\alpha_s C_F \delta(x) \underbrace{(4-n)}_{\propto\, \epsilon_{\mathrm{UV}}} \mu^{2\epsilon} \int \underbrace{\frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + m_q^2)}}_{\propto\, \frac{1}{\epsilon_{\mathrm{UV}}}}$

i. $m_q \neq 0$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Close look at singular term

Result after $\int d^{n-2}k_\perp$ **:** $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + m_q^2)}$

i. $m_q \neq 0$	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{m_q \neq 0} = -\frac{\alpha_s C_F}{2\pi}\delta(x)$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Close look at singular term

Result after $\int d^{\,n-2}k_\perp$ **:** $g_{T(\mathrm{s})}^{(1a)}(x) \;=-\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon}\int \frac{d^{\,n-2}k_\perp}{(2\pi)^{n-2}}\,\frac{1}{(k_\perp^2+\cancel{m_q^2})}$

i. $m_q \neq 0$	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{m_q\neq 0} = -\frac{\alpha_s C_F}{2\pi}\delta(x)$
ii. ϵ_{IR}	

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Close look at singular term

Result after $\int d^{n-2}k_\perp$ **:** $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \underbrace{\int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + \cancel{m_q^2})}}$

$$\delta(x)\,\epsilon_{\mathrm{UV}}\,\frac{1}{\epsilon_{\mathrm{UV}}} - \delta(x)\,\epsilon_{\mathrm{IR}}\,\frac{1}{\epsilon_{\mathrm{IR}}}$$

i. $m_q \neq 0$	$g_{T(\mathrm{s})}^{(1a)}\big\vert_{m_q\neq 0} = -\frac{\alpha_s C_F}{2\pi}\delta(x)$
ii. ϵ_{IR}	

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Close look at singular term

Result after $\int d^{n-2}k_\perp$: $g_{T(\mathrm{s})}^{(1a)}(x) = -\alpha_s C_F \delta(x)\,(4-n)\,\mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{1}{(k_\perp^2 + \cancel{m_q^2})}$

i. $m_q \neq 0$	$g_{T(\mathrm{s})}^{(1a)}\big\|_{m_q \neq 0} = -\frac{\alpha_s C_F}{2\pi}\delta(x)$
ii. ϵ_{IR}	$g_{T(\mathrm{s})}^{(1a)}\big\|_{\epsilon_{\mathrm{IR}}} = 0$

- **Appearance of zero modes is an IR scheme-dependent feature**

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Results for canonical part:

$$g_{T(\mathrm{c})}(x) \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^-}{2\pi} \frac{2k^2 + 2k_\perp^2 + 2m_q^2 - (4-n)m_g^2}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)}$$

i. $m_q \neq 0$	$g_{T(\mathrm{c})}^{(1\mathrm{a})}(x)\Big\rvert_{m_q} = \frac{\alpha_s C_F}{2\pi}\left(x\,\mathcal{P}_{\mathrm{UV}} + x\ln\frac{\mu_{\mathrm{UV}}^2}{(1-x)^2 m_q^2} + \frac{x^2-2x-1}{1-x}\right)$
ii. $m_g \neq 0$	$g_{T(\mathrm{c})}^{(1\mathrm{a})}(x)\Big\rvert_{m_g} = \frac{\alpha_s C_F}{2\pi}\left(x\,\mathcal{P}_{\mathrm{UV}} + x\ln\frac{\mu_{\mathrm{UV}}^2}{x m_g^2} + (1-x)\right)$
iii. ϵ_{IR}	$g_{T(\mathrm{c})}^{(1\mathrm{a})}(x)\Big\rvert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\left(x\,(\mathcal{P}_{\mathrm{UV}} - \mathcal{P}_{\mathrm{IR}}) + x\ln\frac{\mu_{\mathrm{UV}}^2}{\mu_{\mathrm{IR}}^2}\right)$

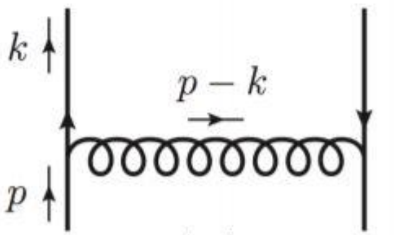
$$\mathcal{P}_{\mathrm{UV/IR}} = \frac{1}{\epsilon_{\mathrm{UV/IR}}} + \ln 4\pi - \gamma_E$$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

Definition: $$\frac{M}{P^3} S_\perp^i \, g_{T,\mathrm{Q}}(x) = \Phi^{[\gamma^i \gamma_5]}$$



(1a)

$$\boxed{\frac{m_q}{p^3} g_{T,\mathrm{Q}}^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\overline{u}\,\gamma^\nu\,(\not{k} + m_q)\,\gamma^i\gamma_5\,(\not{k} + m_q)\,\gamma^\mu\right]}{(k^2 - m_q^2 + i\varepsilon)^2((p-k)^2 - m_g^2 + i\varepsilon)} \,\delta\!\left(x - \frac{k^3}{p^3}\right) \frac{1}{p^3}}$$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

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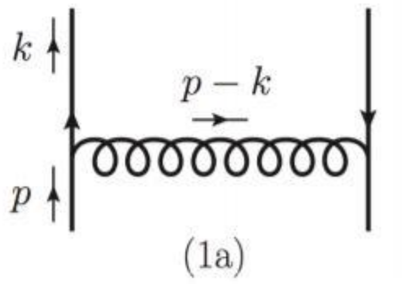


$$\frac{m_q}{p^3} g_{T,\mathrm{Q}}^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\!\left(x-\frac{k^3}{p^3}\right)\frac{1}{p^3}$$

$g_{T,\mathrm{Q(s)}}^{(1a)}$

Split into singular & canonical parts

$g_{T,\mathrm{Q(c)}}^{(1a)}$

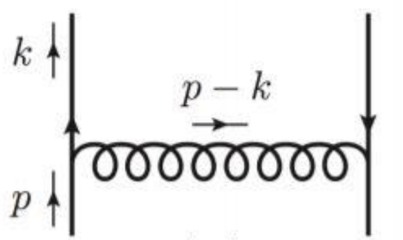
$$g_{T,\mathrm{Q}} \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^0}{2\pi} \frac{(4-n)}{(k^2-m_q^2+i\varepsilon)^2} + \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^0}{2\pi} \frac{2k^2+2k_\perp^2+2m_q^2-(4-n)m_g^2}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}$$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

Definition: $$\frac{M}{P^3} S_\perp^i \, g_{T,\mathrm{Q}}(x) = \Phi^{[\gamma^i\gamma_5]}$$



(1a)

$$\frac{m_q}{p^3} g_{T,\mathrm{Q}}^{(1a)}(x) = -\frac{ig^2 C_F \mu^{2\epsilon} g_{\mu\nu}}{4} \int_{-\infty}^{\infty} \frac{d^n k}{(2\pi)^n} \frac{\mathrm{Tr}\left[u\,\bar{u}\,\gamma^\nu\,(\not{k}+m_q)\,\gamma^i\gamma_5\,(\not{k}+m_q)\,\gamma^\mu\right]}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}\,\delta\!\left(x-\frac{k^3}{p^3}\right)\frac{1}{p^3}$$

$g_{T,\mathrm{Q(s)}}^{(1a)}$

Split into singular & canonical parts

$g_{T,\mathrm{Q(c)}}^{(1a)}$

$$g_{T,\mathrm{Q}} \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^0}{2\pi} \frac{(4-n)}{(k^2-m_q^2+i\varepsilon)^2} + \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \frac{dk^0}{2\pi} \frac{2k^2+2k_\perp^2+2m_q^2-(4-n)m_g^2}{(k^2-m_q^2+i\varepsilon)^2((p-k)^2-m_g^2+i\varepsilon)}$$

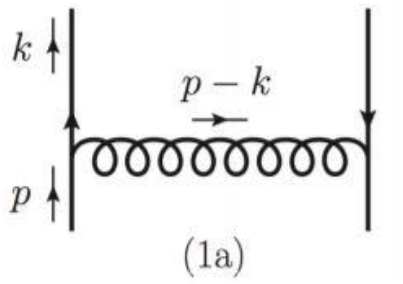
Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

Close look at singular term

$$g^{(1a)}_{T,\mathrm{Q(s)}} \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \underbrace{\int \frac{dk^0}{2\pi} \frac{(4-n)}{(k^2 - m_q^2 + i\varepsilon)^2}}_{\displaystyle \frac{(4-n)}{(k_\perp^2 + x^2 p_3^2 + m_q^2)^{3/2}}}$$



Matching for twist-3 PDF $g_T(x)$

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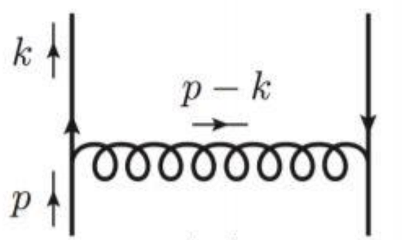

(1a)

i. $m_q \neq 0$

Matching for twist-3 PDF $g_T(x)$

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Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

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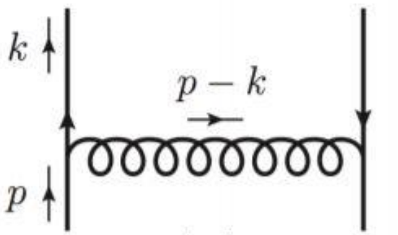
$$\frac{(4-n)}{(k_\perp^2 + x^2 p_3^2 + m_q^2)^{3/2}} \propto \epsilon$$



(1a)

i. $m_q \neq 0$

Matching for twist-3 PDF $g_T(x)$

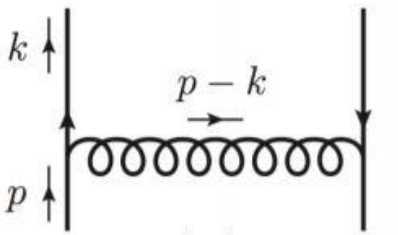
SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

Close look at singular term

$$g^{(1a)}_{T,\mathrm{Q(s)}} \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \underbrace{\int \frac{dk^0}{2\pi} \frac{(4-n)}{(k^2 - m_q^2 + i\varepsilon)^2}}_{\frac{(4-n)}{(k_\perp^2 + x^2 p_3^2 + m_q^2)^{3/2}}}$$

$$\frac{(4-n)}{(k_\perp^2 + x^2 p_3^2 + m_q^2)^{3/2}} \quad \propto \epsilon$$



(1a)

i. $m_q \neq 0$	$g^{(1a)}_{T,\mathrm{Q(s)}}\big\vert_{m_q\neq 0} = 0$

Matching for twist-3 PDF $g_T(x)$

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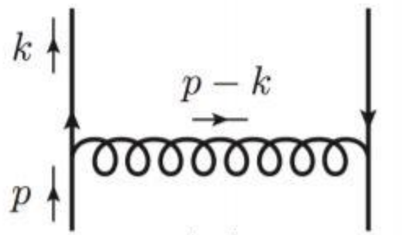


(1a)

i. $m_q \neq 0$	$g^{(1a)}_{T,\mathrm{Q(s)}}\big\rvert_{m_q \neq 0} = 0$
ii. ϵ_{IR}	

Matching for twist-3 PDF $g_T(x)$

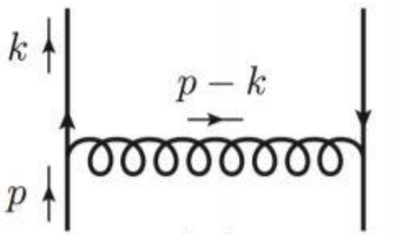
SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

Close look at singular term

$$g^{(1a)}_{T,\mathrm{Q(s)}} \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \underbrace{\int \frac{dk^0}{2\pi} \frac{(4-n)}{(k^2 - m_q^2 + i\varepsilon)^2}}_{\frac{(4-n)}{(k_\perp^2 + x^2 p_3^2 + \cancel{m_q^2})^{3/2}}}$$

- **UV-finite**



(1a)

i. $m_q \neq 0$	$g^{(1a)}_{T,\mathrm{Q(s)}}\big\vert_{m_q \neq 0} = 0$
ii. ϵ_{IR}	

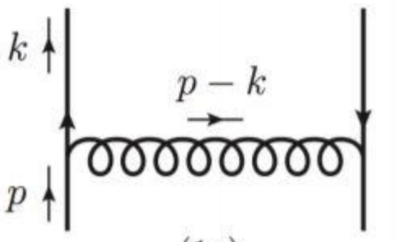
Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

Close look at singular term

$$g^{(1a)}_{T,\mathrm{Q(s)}} \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \underbrace{\int \frac{dk^0}{2\pi} \frac{(4-n)}{(k^2 - m_q^2 + i\varepsilon)^2}}_{\frac{(4-n)}{(k_\perp^2 + x^2 p_3^2 + \cancel{m_q^2})^{3/2}}}$$



(1a)

- **UV-finite**
- **IR-finite except at** $x = 0$

i. $m_q \neq 0$	$g^{(1a)}_{T,\mathrm{Q(s)}}\big\rvert_{m_q \neq 0} = 0$
ii. ϵ_{IR}	

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

Close look at singular term

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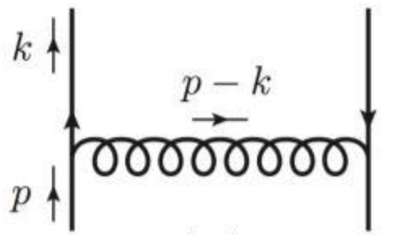

(1a)

- **UV-finite**
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$$\int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \frac{(4-n)}{(k_\perp^2 + x^2 p_3^2)^{3/2}} = \frac{2^{-1+2\epsilon_{\mathrm{IR}}} \pi^{-3/2+\epsilon_{\mathrm{IR}}} \Gamma(1/2 + \epsilon_{\mathrm{IR}})}{(p^3)^{1+2\epsilon_{\mathrm{IR}}}} \frac{1}{|x|^{1+2\epsilon_{\mathrm{IR}}}}$$

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Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

We derive:

$$\frac{1}{|x|^{1+2\,\epsilon_{\mathrm{IR}}}} = -\frac{\delta(x)}{\epsilon_{\mathrm{IR}}} + \left[\frac{1}{|x|}\right]_{+(0)} + \mathcal{O}(\epsilon_{\mathrm{IR}}) \qquad -1 < x < 1$$

$\epsilon_{\mathrm{IR}} < 0$

(See also Izubuchi et. al., arXiv: 1801.0391)

$$\underbrace{\int \frac{dk^0}{2\pi} \frac{(4-n)}{(k^2 - m_q^2 + i\varepsilon)^2}}_{\displaystyle \frac{(4-n)}{(k_\perp^2 + x^2 p_3^2 + \cancel{m_q^2})^{3/2}}}$$

- **UV-finite**
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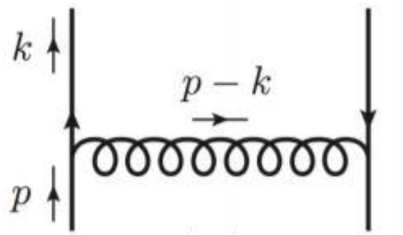
Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

Close look at singular term

$$g^{(1a)}_{T,\mathrm{Q(s)}} \approx \alpha_s C_F \mu^{2\epsilon} \int \frac{d^{n-2}k_\perp}{(2\pi)^{n-2}} \int \underbrace{\frac{dk^0}{2\pi} \frac{(4-n)}{(k^2-m_q^2+i\varepsilon)^2}}_{\frac{(4-n)}{(k_\perp^2+x^2p_3^2+m_q^2)^{3/2}}}$$



(1a)

i. $m_q \neq 0$	$g^{(1a)}_{T,\mathrm{Q(s)}}\big\vert_{m_q\neq 0} = 0$
ii. ϵ_{IR}	$g^{(1a)}_{T,\mathrm{Q(s)}}\big\vert_{\epsilon_{\mathrm{IR}}} = \dfrac{\alpha_s C_F}{2\pi}\delta(x)$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Calculation of ladder diagram for $g_{T,\mathrm{Q}}(x)$

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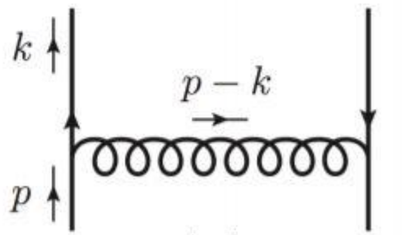

(1a)

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- **Just as in LC case, existence of zero modes in quasi-PDF is IR scheme-dependent**

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Results for canonical part:

i. $m_q \neq 0$	$$g^{(1a)}_{T,\mathrm{Q(c)}}(x)\Big\vert_{m_q} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x\,\ln\frac{x}{x-1} - 1 & x>1 \\ x\,\ln\frac{p_3^2}{m_q^2} + x\ln\frac{4x}{1-x} + 1 - 2x + \frac{2}{x-1} & 0<x<1 \\ x\,\ln\frac{x-1}{x} + 1 & x<0 \end{cases}$$
ii. $m_g \neq 0$	$$g^{(1a)}_{T,\mathrm{Q(c)}}(x)\Big\vert_{m_g} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x\,\ln\frac{x}{x-1} - 1 & x>1 \\ x\,\ln\frac{p_3^2}{m_g^2} + x\ln 4(1-x) + 1 - 2x & 0<x<1 \\ x\,\ln\frac{x-1}{x} + 1 & x<0 \end{cases}$$
iii. ϵ_{IR}	$$g^{(1a)}_{T,\mathrm{Q(c)}}(x)\Big\vert_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x\,\ln\frac{x}{x-1} - 1 & x>1 \\ -x\,\mathcal{P}_{\mathrm{IR}} + x\,\ln\frac{4x(1-x)p_3^2}{\mu_{\mathrm{IR}}^2} - x & 0<x<1 \\ x\,\ln\frac{x-1}{x} + 1 & x<0 \end{cases}$$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, M… D 102 (2020)

Results f…

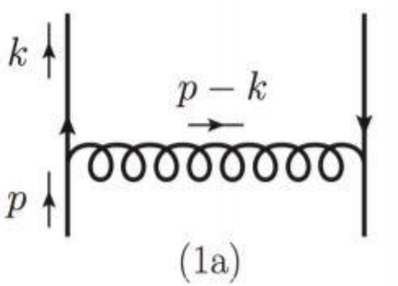
IR singularities present in the "physical region" $0 < x < 1$ only

i.	$m_q \neq 0$	$$g^{(1a)}_{T,\mathrm{Q(c)}}(x)\Big|_{m_q} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x \ln\frac{x}{x-1} - 1 & x > 1 \\ x \ln\frac{p_3^2}{m_q^2} + x \ln\frac{4x}{1-x} + 1 - 2x + \frac{2}{x-1} & 0 < x < 1 \\ x \ln\frac{x-1}{x} + 1 & x < 0 \end{cases}$$
ii.	$m_g \neq 0$	$$g^{(1a)}_{T,\mathrm{Q(c)}}(x)\Big|_{m_g} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x \ln\frac{x}{x-1} - 1 & x > 1 \\ x \ln\frac{p_3^2}{m_g^2} + x \ln 4(1-x) + 1 - 2x & 0 < x < 1 \\ x \ln\frac{x-1}{x} + 1 & x < 0 \end{cases}$$
iii.	ϵ_{IR}	$$g^{(1a)}_{T,\mathrm{Q(c)}}(x)\Big|_{\epsilon_{\mathrm{IR}}} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x \ln\frac{x}{x-1} - 1 & x > 1 \\ -x\,\mathcal{P}_{\mathrm{IR}} + x \ln\frac{4x(1-x)p_3^2}{\mu_{\mathrm{IR}}^2} - x & 0 < x < 1 \\ x \ln\frac{x-1}{x} + 1 & x < 0 \end{cases}$$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Agreement of the IR singularities between light-cone PDF & quasi-PDF



Matching for twist-3 PDF $g_T(x)$

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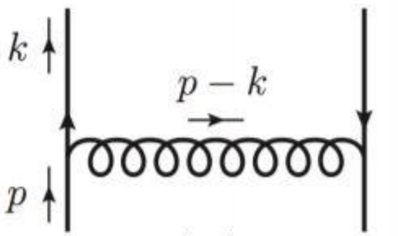

(1a)

- **Singular terms:** Coefficient of singular terms are IR finite

Light-cone PDF	Quasi-PDF
$g_{T(\mathrm{s})}^{(1a)}(x) = \begin{cases} g_{T(\mathrm{s})}^{(1a)}(x)\big\vert_{m_q\neq 0} = -\dfrac{\alpha_s C_F}{2\pi}\delta(x) \\ g_{T(\mathrm{s})}^{(1a)}(x)\big\vert_{\epsilon_{\mathrm{IR}}} = 0 \end{cases}$	$g_{T,\mathrm{Q(s)}}^{(1a)}(x) = \begin{cases} g_{T,\mathrm{Q(s)}}^{(1a)}(x)\big\vert_{m_q\neq 0} = 0 \\ g_{T,\mathrm{Q(s)}}^{(1a)}(x)\big\vert_{\epsilon_{\mathrm{IR}}} = \dfrac{\alpha_s C_F}{2\pi}\delta(x) \end{cases}$

Matching for twist-3 PDF $g_T(x)$

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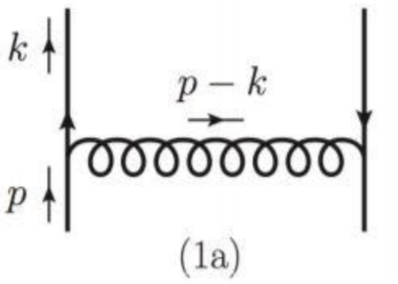
Agreement of the IR singularities between light-cone PDF & quasi-PDF

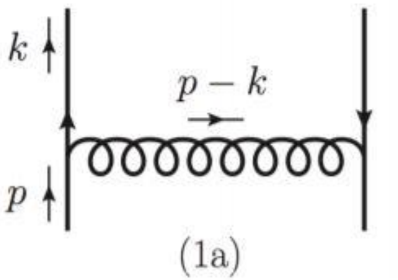


- **Singular terms:** Coefficient of singular terms are IR finite
- **Canonical terms:**

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Agreement of the IR singularities between light-cone PDF & quasi-PDF



- **Singular terms:** **Coefficient of singular terms are IR finite**
- **Canonical terms:**

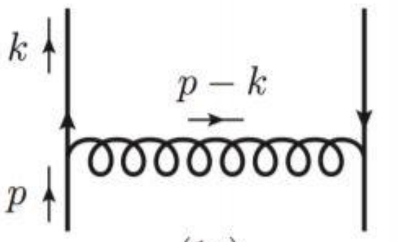
Quark mass

Light-cone PDF	$g_{T(\mathrm{c})}^{(1\mathrm{a})}(x)\Big\vert_{m_q} = \frac{\alpha_s C_F}{2\pi}\left(x\ln\frac{\mu_{\mathrm{UV}}^2}{m_q^2} + x\ln\frac{1}{(1-x)^2} + x\,\mathcal{P}_{\mathrm{UV}} + \frac{x^2-2x-1}{1-x}\right)$
Quasi-PDF	$g_{T,\mathrm{Q(c)}}^{(1\mathrm{a})}(x)\Big\vert_{m_q} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x\ \ln\frac{x}{x-1} - 1 & x>1 \\ x\ \ln\frac{p_3^2}{m_q^2} + x\ln\frac{4x}{1-x} + 1 - 2x + \frac{2}{x-1} & 0<x<1 \\ x\ \ln\frac{x-1}{x} + 1 & x<0 \end{cases}$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Agreement of the IR singularities between light-cone PDF & quasi-PDF



(1a)

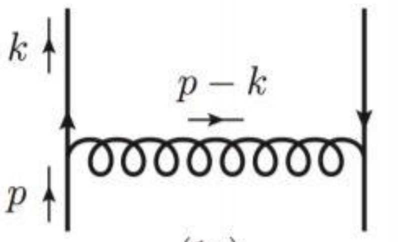
- **Singular terms:** **Coefficient of singular terms are IR finite**
- **Canonical terms:**

Quark mass	**Light-cone PDF**	$g^{(1a)}_{T(c)}(x)\Big\vert_{m_q} = \dfrac{\alpha_s C_F}{2\pi}\left(x\ln\dfrac{\mu^2_{\text{UV}}}{m_q^2} + x\ln\dfrac{1}{(1-x)^2} + x\,\mathcal{P}_{\text{UV}} + \dfrac{x^2-2x-1}{1-x}\right)$
	Quasi-PDF	$g^{(1a)}_{T,\text{Q(c)}}(x)\Big\vert_{m_q} = \dfrac{\alpha_s C_F}{2\pi}\begin{cases} x\ \ln\frac{x}{x-1} - 1 & x>1 \\ x\ \ln\frac{p_3^2}{m_q^2} + x\ln\frac{4x}{1-x} + 1 - 2x + \frac{2}{x-1} & 0<x<1 \\ x\ \ln\frac{x-1}{x} + 1 & x<0 \end{cases}$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Agreement of the IR singularities between light-cone PDF & quasi-PDF



(1a)

- **Singular terms:** **Coefficient of singular terms are IR finite**
- **Canonical terms:**

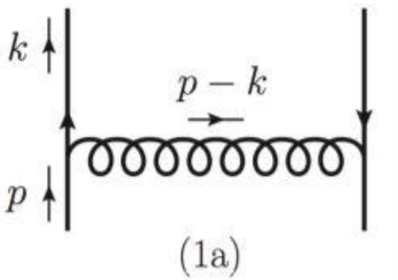
Gluon mass

Light-cone PDF	$g^{(1a)}_{T(c)}(x)\Big\vert_{m_g} = \frac{\alpha_s C_F}{2\pi}\left(x\ln\frac{\mu^2_{\rm UV}}{m_g^2} + x\ln\frac{1}{x} + x\,\mathcal{P}_{\rm UV} + (1-x)\right)$
Quasi-PDF	$g^{(1a)}_{T,{\rm Q(c)}}(x)\Big\vert_{m_g} = \frac{\alpha_s C_F}{2\pi}\begin{cases} x\ln\frac{x}{x-1} - 1 & x>1 \\ x\ln\frac{p_3^2}{m_g^2} + x\ln 4(1-x) + 1 - 2x & 0<x<1 \\ x\ln\frac{x-1}{x} + 1 & x<0 \end{cases}$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Agreement of the IR singularities between light-cone PDF & quasi-PDF



- **Singular terms:** **Coefficient of singular terms are IR finite**
- **Canonical terms:**

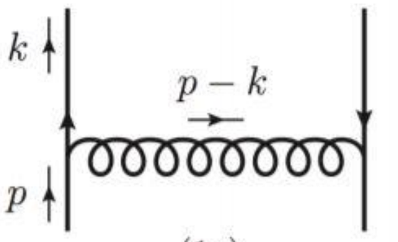
DR for IR

Light-cone PDF	$g_{T(\mathrm{c})}^{(1\mathrm{a})}(x)\Big\vert_{\epsilon_{\mathrm{IR}}} = \dfrac{\alpha_s C_F}{2\pi}\left(-x\,\mathcal{P}_{\mathrm{IR}} + x\,\mathcal{P}_{\mathrm{UV}} + x\ln\dfrac{\mu_{\mathrm{UV}}^2}{\mu_{\mathrm{IR}}^2}\right)$
Quasi-PDF	$g_{T,\mathrm{Q}(\mathrm{c})}^{(1\mathrm{a})}(x)\Big\vert_{\epsilon_{\mathrm{IR}}} = \dfrac{\alpha_s C_F}{2\pi}\begin{cases} x\ln\dfrac{x}{x-1} - 1 & x>1 \\ -x\,\mathcal{P}_{\mathrm{IR}} + x\ln\dfrac{4x(1-x)p_3^2}{\mu_{\mathrm{IR}}^2} - x & 0<x<1 \\ x\ln\dfrac{x-1}{x} + 1 & x<0 \end{cases}$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Agreement of the IR singularities between light-cone PDF & quasi-PDF



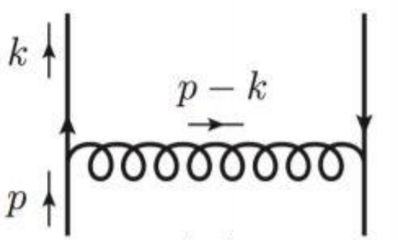
(1a)

- **Singular terms:** **Coefficient of singular terms are IR finite**
- **Canonical terms:** **IR singularities agree for all 3 regulators**

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Agreement of the IR singularities between light-cone PDF & quasi-PDF



(1a)

- **Singular terms:** **Coefficient of singular terms are IR finite**
- **Canonical terms:** **IR singularities agree for all 3 regulators**

- **Other diagrams can be calculated just like in the twist-2 case**
- **Diagram by diagram the IR singularities agree, which is at the heart of quasi-PDF approach**
- **Matching is possible for $g_T(x)$**

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Matching in $\overline{\rm MS}$ scheme

$$
C^{(1)}_{\overline{\rm MS}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} 0 & \xi > 1 \\ \delta(\xi) & 0 < \xi < 1 \\ 0 & \xi < 0 \end{cases} + \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ - \frac{3}{2\xi} & \xi > 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ - \frac{3}{2(1-\xi)} & \xi < 0 \end{cases}
$$

$$
+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(-\frac{1}{2} + \frac{3}{2}\ln\frac{\mu^2}{4p_3^2}\right)
$$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Matching in $\overline{\mathrm{MS}}$ scheme

$$C^{(1)}_{\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0 & \xi > 1 \\ \delta(\xi) & 0 < \xi < 1 \end{cases} + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ - \frac{3}{2\xi} & \xi > 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ & \xi < 0 \end{cases}$$

$$+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(-\frac{1}{2} + \frac{3}{2}\ln\frac{\mu^2}{4p_3^2}\right)$$

Matching coefficient for $g_T(x)$ is independent of IR scheme

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Matching in $\overline{\mathrm{MS}}$ scheme

$$C^{(1)}_{\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0 & \xi > 1 \\ \delta(\xi) & 0<\xi<1 \\ 0 & \xi<0 \end{cases} + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{\xi}{1-\xi}+\frac{3}{2\xi}\right]_+ - \frac{3}{2\xi} & \xi>1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2}+\frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi}-\frac{\xi}{1-\xi}+\frac{3}{2(1-\xi)}\right]_+ - \frac{3}{2(1-\xi)} & \xi<0 \end{cases}$$

$$+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(-\frac{1}{2}+\frac{3}{2}\ln\frac{\mu^2}{4p_3^2}\right)$$

- **Convolution integrals** $q(x) = \int_{-\infty}^{\infty}\frac{d\xi}{|\xi|}C(\xi)\tilde{q}\left(\frac{x}{\xi}\right)$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Matching in $\overline{\mathrm{MS}}$ scheme

$$C^{(1)}_{\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0 & \xi > 1 \\ \delta(\xi) & 0 < \xi < 1 \\ 0 & \xi < 0 \end{cases} + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ - \frac{3}{2\xi} & \xi > 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ - \frac{3}{2(1-\xi)} & \xi < 0 \end{cases}$$

$$+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(-\frac{1}{2} + \frac{3}{2}\ln\frac{\mu^2}{4p_3^2}\right)$$

- **Convolution integrals** $q(x) = \int_{-\infty}^{\infty}\frac{d\xi}{|\xi|}C(\xi)\tilde{q}\left(\frac{x}{\xi}\right)$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Matching in $\overline{\mathrm{MS}}$ scheme

$$\approx \frac{3}{2}\ln\xi$$

$$C^{(1)}_{\overline{\mathrm{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0 & \xi > 1 \\ \delta(\xi) & 0 < \xi < 1 \\ 0 & \xi < 0 \end{cases} + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ - \frac{3}{2\xi} & \xi > 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ - \frac{3}{2(1-\xi)} & \xi < 0 \end{cases}$$

$$+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(-\frac{1}{2} + \frac{3}{2}\ln\frac{\mu^2}{4p_3^2}\right)$$

$$\approx \frac{3}{2}\ln\xi$$

Problems with $\overline{\mathrm{MS}}$

- **Convolution integrals** $q(x) = \int_{-\infty}^{\infty}\frac{d\xi}{|\xi|}C(\xi)\tilde{q}\left(\frac{x}{\xi}\right)$
- **Mismatch in norm:** $\int_{-\infty}^{\infty} dx\, \tilde{q}^{\mathrm{MS}}(x,\mu,p^3) \neq \int_0^1 dx\, q^{\mathrm{MS}}(x,\mu)$

Matching for twist-3 PDF $g_T(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Matching in $\mathrm{M\overline{MS}}$ scheme

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Introduce $\mathrm{M\overline{MS}}$ **(Alexandrou et. al., arXiv: 1902.00587)**

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Matching implemented by ETM Collaboration in lattice QCD

See Krzysztof's talk for our first lattice QCD result for $g_T(x)$

Case 2:

$$\begin{cases} e \\ h_L \end{cases} \quad \& \quad \begin{cases} e_{\mathbf{Q}} \\ h_{L,\mathbf{Q}} \end{cases}$$

Non-trivial role of zero-modes in matching for $e(x)$ & $h_L(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Light-cone PDF	Features
Example: $$h_{L(\mathrm{s})}^{(1a)}(x)\big|_{m_q} = -\frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu_{\mathrm{UV}}^2}{m_q^2} - 1\right)$$	• **Zero modes are unavoidable**

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Quasi-PDF	**Features**
Example: $h^{(1a)}_{L,\mathrm{Q(s)}}(x)\Big\vert_{m_q} = -\frac{\alpha_s C_F}{2\pi}\frac{p^3}{\sqrt{x^2 p_3^2 + m_q^2}}$	• **Seemingly different looking IR pole structure**

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Quasi-PDF	**Features**
Example: $h_{L,\mathrm{Q(s)}}^{(1a)}(x)\big\vert_{m_q} = -\frac{\alpha_s C_F}{2\pi}\frac{p^3}{\sqrt{x^2 p_3^2 + m_q^2}}$	• Seemingly different looking IR pole structure • Do quasi-PDFs and LC PDFs share same IR physics?

Non-trivial role of zero-modes in matching for $e(x)$ & $h_L(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Treatment of IR singularity for quasi-PDFs (non-zero quark mass)

$$h^{(1a)}_{L,\mathrm{Q(s)}}(x)\Big|_{m_q} = -\frac{\alpha_s C_F}{2\pi}\frac{1}{\sqrt{x^2+\eta^2}} \qquad \text{where,} \quad \eta^2 = \frac{m_q^2}{p_3^2} \quad -1 < x < 1$$

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$$h^{(1a)}_{L,\mathrm{Q(s)}}(x) \approx -\frac{\alpha_s C_F}{2\pi}\frac{1}{x}$$

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Recall:

$$h^{(1a)}_{L\mathrm{(s)}}(x)\Big|_{m_q} = -\frac{\alpha_s C_F}{2\pi}\boldsymbol{\delta(x)}\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\boldsymbol{\mu^2_{\mathrm{UV}}}}{\boldsymbol{m_q^2}} - 1\right)$$

$$h^{(1a)}_{L,\mathrm{Q(s)}}(x) \approx -\frac{\alpha_s C_F}{2\pi}\frac{1}{x}$$

- **Doing a twist-expansion before we calculate the matching coefficient gives rise to an <u>incorrect conclusion</u> of mismatch in the IR between quasi & LC PDFs!**

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Recall:

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Incorrect approach	Correct approach
$h^{(1a)}_{L,\mathrm{Q(s)}}(x) \approx -\frac{\alpha_s C_F}{2\pi}\frac{1}{x}$	$\int_{-1}^{1} dx\, \frac{f(x)}{\sqrt{x^2+\eta^2}} = \int_{-1}^{1} dx\, f(x)\,\delta(x)\left(\ln\frac{4}{\eta^2}\right) + \int_{-1}^{1} dx\, f(x)\left[\frac{1}{\lvert x\rvert}\right]_{+[0]} + \mathcal{O}(\eta^2)$
• **Doing a twist-expansion before we calculate the matching coefficient gives rise to an <u>incorrect conclusion</u> of mismatch in the IR between quasi & LC PDFs!**	• **By convoluting with a well-behaved test-function, it is possible to isolate singularity at** $x=0$ • **Agreement in the IR poles between quasi & LC PDFs: Matching possible for** $e(x)$, $h_L(x)$

Non-trivial role of zero-modes in matching for $e(x)$ & $h_L(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens/ PRD 102 (2020)

Treatment of IR singularity for quasi-PDFs (non-zero quark mass)

Point $x = 0$ is extremely delicate for quasi-PDFs!

Incorrect approach	Correct approach
$h^{(1a)}_{L,\mathrm{Q(s)}}(x) \approx -\frac{\alpha_s C_F}{2\pi} \frac{1}{x}$	$\int_{-1}^{1} dx \, \frac{f(x)}{\sqrt{x^2+\eta^2}} = \int_{-1}^{1} dx \, f(x) \delta(x) \left(\ln \frac{4}{\eta^2} \right) + \int_{-1}^{1} dx \, f(x) \left[\frac{1}{\lvert x \rvert} \right]_{+[0]} + \mathcal{O}(\eta^2)$
• **Doing a twist-expansion before we calculate the matching coefficient gives rise to an incorrect conclusion of mismatch in the IR between quasi & LC PDFs!**	• **By convoluting with a well-behaved test-function, it is possible to isolate singularity at $x = 0$** • **Agreement in the IR poles between quasi & LC PDFs: Matching possible for $e(x)$, $h_L(x)$**

Matching for twist-3 PDF $h_L(x)$

SB, Cichy, Constantinou, Metz, Scapellato, Steffens (in preparation)

Matching in $\mathrm{M\overline{MS}}$ scheme

Singular part of matching coefficient

$$C^{(\mathrm{s})}_{\mathrm{M\overline{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \delta(1-\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4p_3^2}\right) & \xi > 1 \\ -\delta(\xi)\left(\ln\frac{4p_3^2}{\mu^2} + 1\right) - \mathrm{R}_0(|\xi|) & 0 < \xi < 1 \\ \delta(1+\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4p_3^2}\right) & \xi < 0 \end{cases}$$

$$R_0(|\xi|) = \left[\frac{1}{|\xi|}\right]_{+[0]} =$$

$$\theta(|\xi|)\,\theta(1-|\xi|)\lim_{\beta\to 0}\left[\frac{\theta(|\xi|-\beta)}{|\xi|} + \delta(|\xi|-\beta)\ln\beta\right]$$

Canonical part of matching coefficient

$$C^{(\mathrm{c})}_{\mathrm{M\overline{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \left[\frac{2}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{1}{1-\xi} + \frac{1}{\xi}\right]_+ & \xi > 1 \\ \left[\frac{2}{1-\xi}\ln\frac{4\xi(1-\xi)p_3^2}{\mu^2} + 2(1-\xi) - \frac{1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{2}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{1}{1-\xi} + \frac{1}{1-\xi}\right]_+ & \xi < 0 \end{cases}$$

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SB, Cichy, Constantinou, Metz, Scapellato, Steffens (in preparation)

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Matching coefficient for $h_L(x)$ (as well as for $e(x)$) is independent of IR scheme

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Summary & Outlook

- **Quasi-PDF approach has made it possible to directly access PDFs from lattice QCD**

- **Extracted matching coefficient for $g_T(x)$, $h_L(x)$ (& $e(x)$) for the first time**

- **Presence of singular zero-modes in perturbative results makes the extraction of matching coefficient non-trivial**

- **We laid the necessary theoretical foundation to deal with zero-modes in matching**

- **Explore very recent twist-3 matching results by Braun, Ji, Vladimirov (arXiv: 2103.12105)**