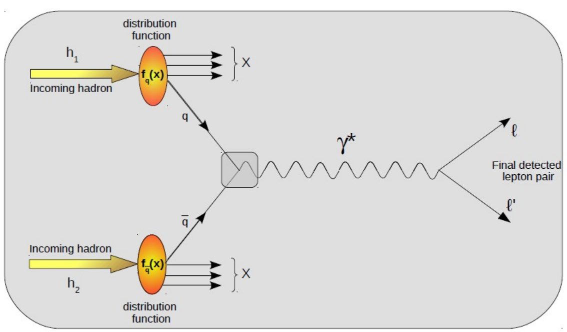
Phenomenological extraction of a universal TMD fragmentation function from single hadron production in e^+e^- annihilations

M. Boglione
In collaboration with O. Gonzalez and A. Simonelli

Where do we learn about TMDs?

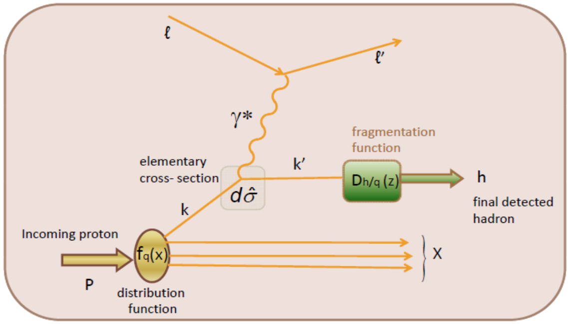
Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x,k_\perp) \otimes f_{\bar{q}}(x,k_\perp) \otimes \hat{\sigma}^{q\bar{q}\to\ell\ell'}$$

Allows extraction of distribution functions

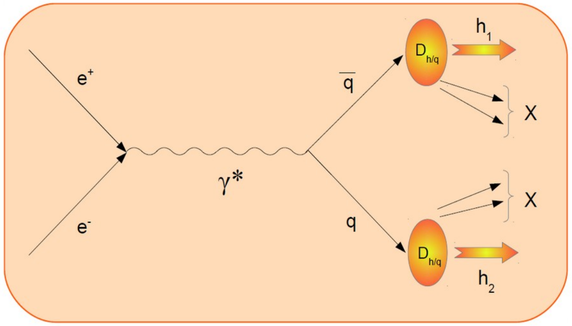
Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction of distribution and fragmentation functions

$e^+ e^- \to h_1 h_2 X$

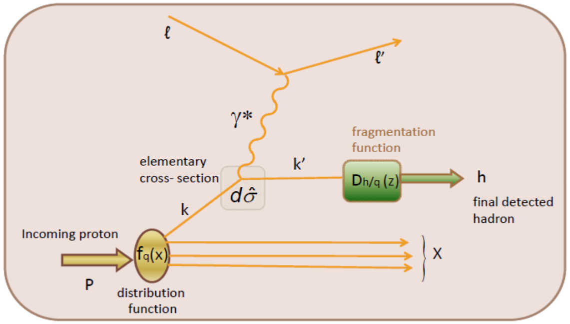


$$\sigma_{h1\,h2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

Allows extraction of fragmentation functions

Where do we learn about TMDs?

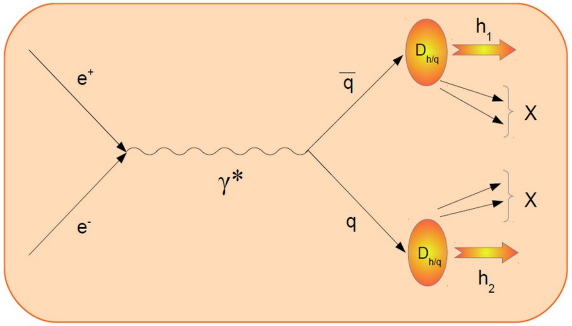
Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction of distribution and fragmentation functions

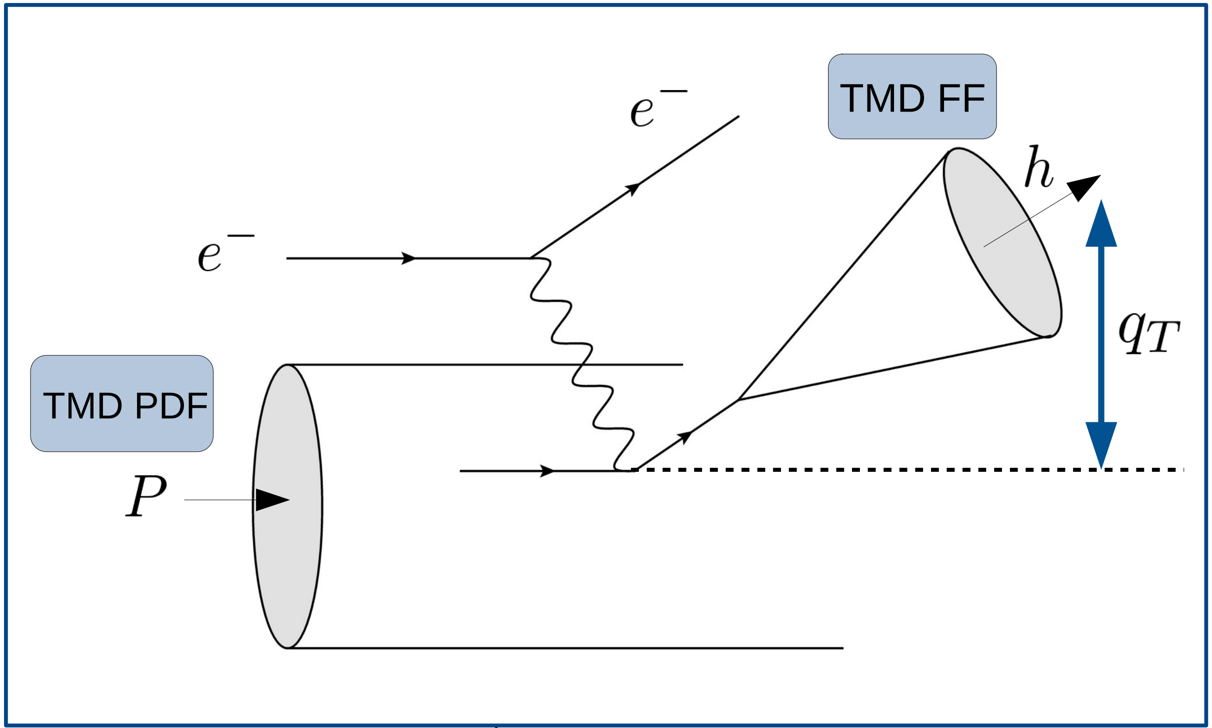
$e^+ e^- \to h_1\, h_2\, X$



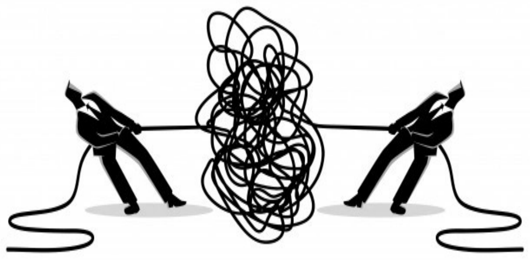
$$\sigma_{h1\,h2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

Allows extraction of fragmentation functions

SIDIS: $e\,p \to h\,X$



In e^+e^- cross sections, distribution and fragmentation TMDs are convoluted.
How can they be disentangled?

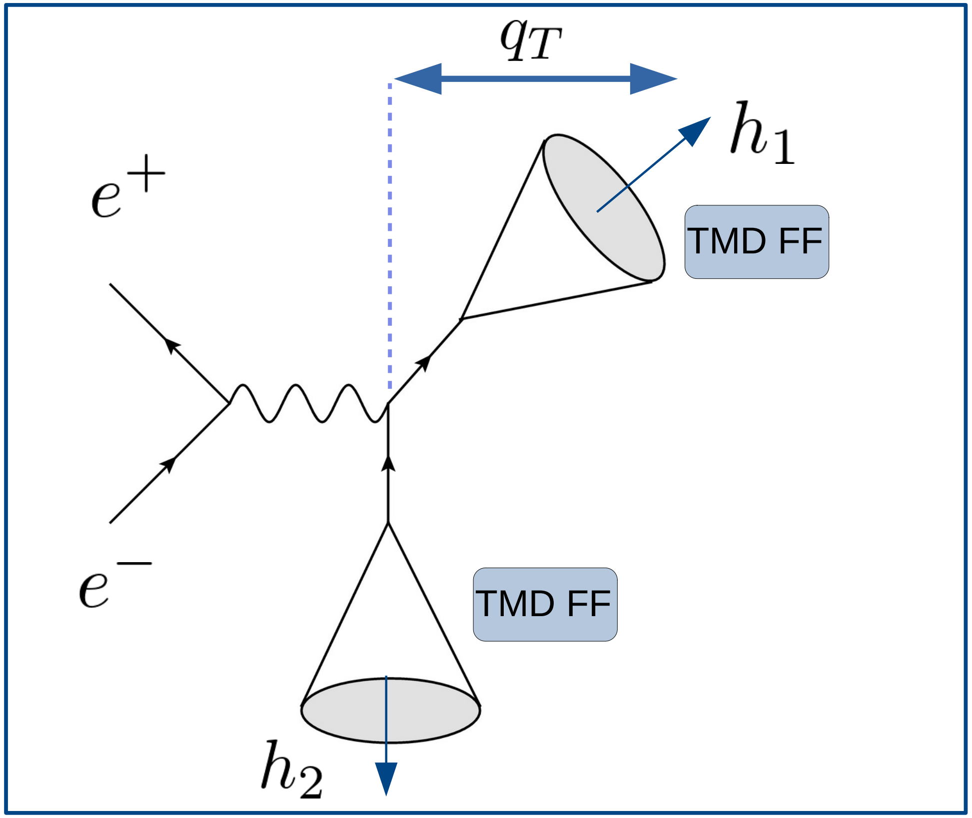


$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\rm sidis} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, D(b_T)$$

3D-picture of partons inside the target hadron

3D-picture of partons hadronizing into the detected hadron

e$^+$e$^-$ annihilations in two hadrons: $e^+ e^- \to h_1 h_2 X$

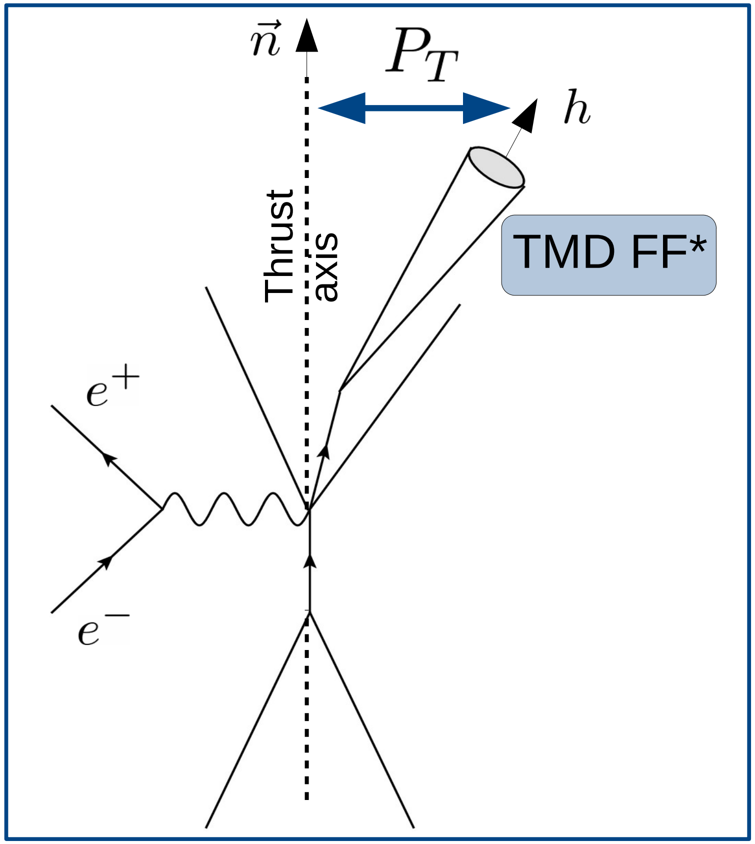


In e$^+$e$^-$ cross sections, distribution and fragmentation TMDs are convoluted.
How can they be disentangled?

$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{2-h}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, D_1(b_T)\, D_2(b_T)$$

3D-picture of the **hadronization** of partons into hadrons

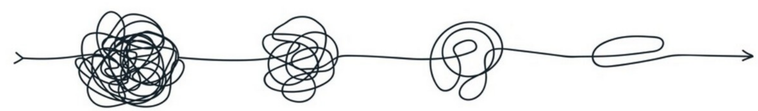
e⁺e⁻ annihilations in one hadron: $e^+e^- \to h\,X$



$$\frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^\star(P_T)$$

3D-picture of the **hadronization** of partons into hadrons

In $e^+e^- \to h\,X$ cross sections, only one fragmentation TMD appears



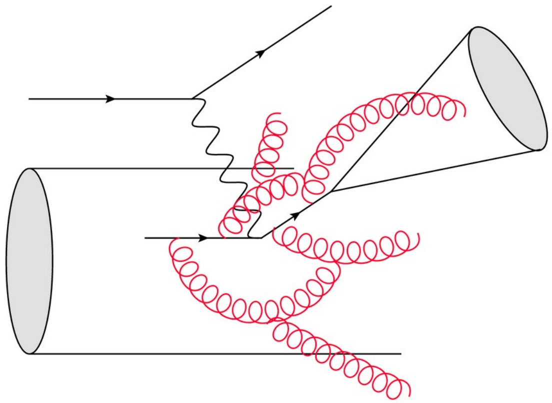
One of the cleanest ways to access TMD Fragmentation Functions***...**

BUT

D*(P_T) is not the same as D(P_T) !!!

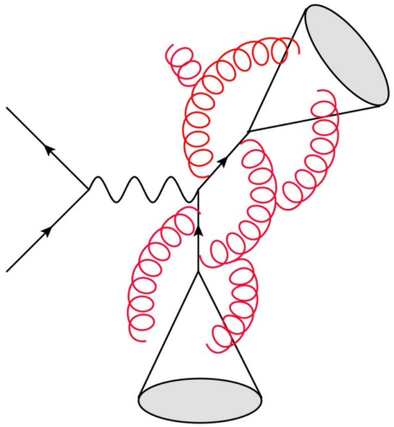
Soft Gluon contribution

SIDIS



$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\rm sidis} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, D(b_T)$$

Double hadron production



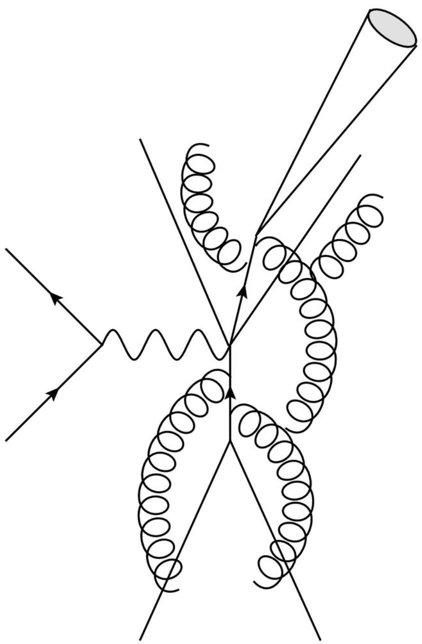
$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{2-h}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, D_1(b_T)\, D_2(b_T)$$

Soft Gluon Factor:
- Non-Perturbative contribution
- Evenly shared by the TMDs

Soft Gluons

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \to hX$$



$$\frac{d\sigma}{dP_T} = d\widehat{\sigma} \otimes D^\star(P_T)$$

Soft Gluon Factor:

- Perturbative (computable) contribution (soft thrust function in the partonic cross section).
- The TMD FF***** is **free** from any soft gluon contributions

D(P_T) and D*(P_T) are different,
BUT
the relation between D and D* is known!

We can perform combined analyses and disentangle non-perturbative terms.

Relation between FF and FF*

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$D = D^{\star}\sqrt{M_S}$$

SQUARE ROOT DEFINITION

Usual definition of TMDs.
Soft Gluon Factor contributing to the cross section are included in the two TMDS and equally shared between them.

FACTORIZATION DEFINITION

Purely collinear TMD, totally free from any soft gluon contribution.

SOFT MODEL

The Soft Gluon Factor appearing in the cross section (process dependent) is **not** included in the TMD

- Same for Drell-Yan, SIDIS and 2-hadron production. (2-h class universality).
- Non-perturbative function (phenomenology).

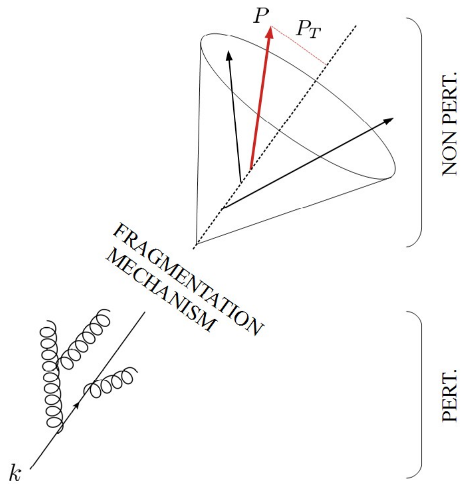
$e^+e^- \to hX$ cross section

M. Boglione, A. Simonelli, JHEP 02 (2021) 076

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z} \frac{d\widehat{\sigma}_f}{dz_h/z\, dT} D_{1,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\, Q^2)$$

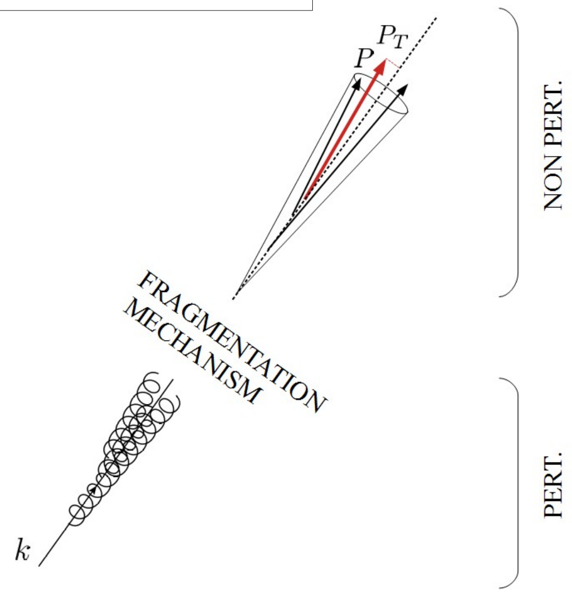
Only fermions contribute, the fragmenting gluon is suppressed by $\mathcal{O}(1-T)$

The TMD FF acquires a dependence on thrust through its rapidity cut-off.



2-jet limit

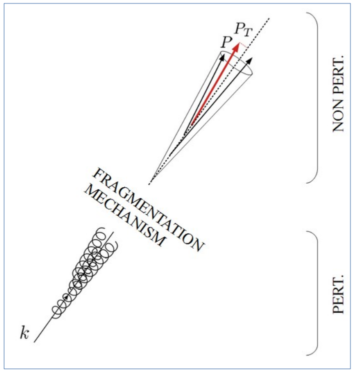
$T \to 1$



Partonic cross section (NLO)

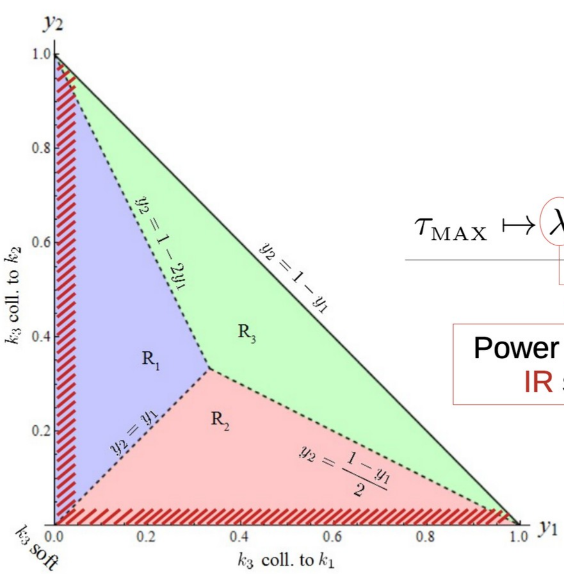
M. Boglione, A. Simonelli, JHEP 02 (2021) 076

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z} \underbrace{\frac{d\widehat{\sigma}_f}{dz_h/z\, dT}} D_{1,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\,Q^2)$$



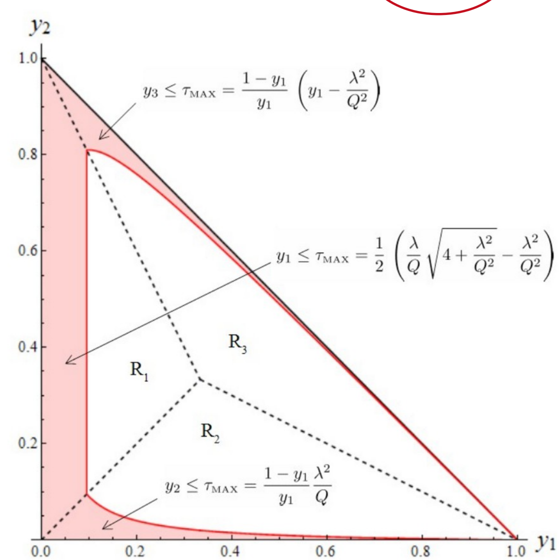
Topology cut-off $\tau = 1 - T \le \tau_{\rm MAX}$

2-jet limit $\longleftrightarrow$ $\tau_{\rm MAX} \to 0$ $\longleftrightarrow$ $\lambda \to 0$



$\tau_{\rm MAX} \mapsto \lambda \ge k_T$

Power counting
IR scale



Partonic cross section (NLO)

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$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z} \underbrace{\frac{d\widehat{\sigma}_f}{dz_h/z\, dT}} D_{1,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\,Q^2)$$

$$\boxed{\frac{d\widehat{\sigma}_f}{dz\, dT} = \left[-\sigma_B e_f^2 N_C \frac{\alpha_S(Q)}{4\pi} C_F \delta(1-z) \left[\frac{3+8\log\tau}{\tau}\right] + \mathcal{O}\left(\alpha_S(Q)^2\right)\right] e^{-\frac{\alpha_S(Q)}{4\pi}\, 3C_F (\log\tau)^2 + \mathcal{O}\left(\alpha_S(Q)^2\right)}}$$

k_T is naturally constrained by kinematics, this helps in fixing the lambda cut-off

$$\begin{cases} k_T \le \lambda \\ k_T \le \sqrt{\tau} Q \end{cases} \qquad \boxed{\lambda = \sqrt{\tau} Q}$$

TMD Fragmentation Function

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z} \frac{d\widehat{\sigma}_f}{dz_h/z\, dT} \underbrace{D_{1,\,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\, Q^2)}$$

Fourier Transform of:

Collinear FFs

$$\widetilde{D}_{1,\,\pi^\pm/f}(z,\, b_T;\, Q,\, \tau Q^2) = \frac{1}{z^2} \sum_k \left[\boxed{d_{\pi^\pm/k}} \otimes \mathcal{C}_{k/f}\right] (\mu_b) \times$$

$$\times \exp\left\{ \frac{1}{4}\, \widetilde{K}\, \log \frac{\tau Q^2}{\mu_b^2} + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D - \frac{1}{4}\, \gamma_K\, \log \frac{\tau Q^2}{\mu'^2} \right] \right\} \times$$

$$\times \underbrace{(M_D)_{f,\,\pi^\pm}\,(z,\, b_T)}\, \exp\left\{ -\frac{1}{4}\, \underbrace{g_K(b_T)}\, \log\left(\tau\, \frac{Q^2}{M_H^2} \right) \right\}$$

Perturbative part (NLL)

Non-Perturbative part Pheno Model

Embeds the non-perturbative, long-range behavior of the TMD FF

Universal, independent of the TMD definition used

Phenomenological parametrization

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, work in progress

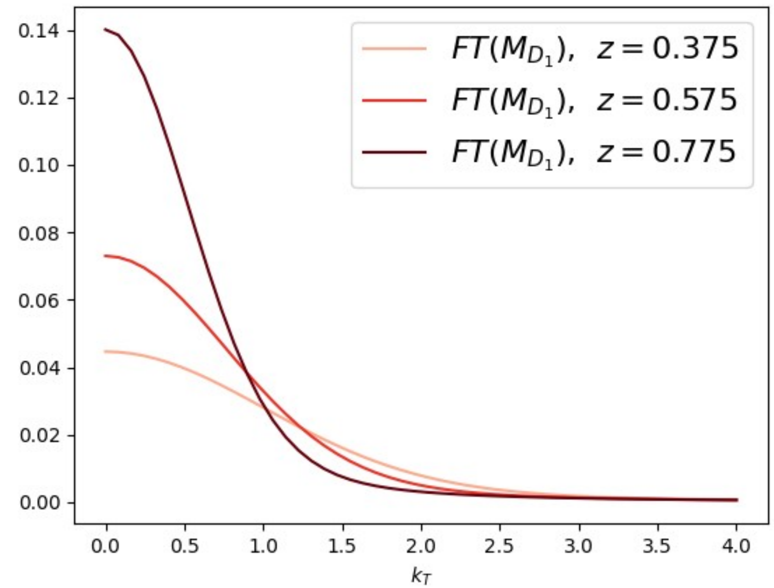
$g_K(b_T) = a\, b_T^2$ → Usually parametrized as a quadratic (but not necessarily) → Relevant to the T - behavior of the cross section

$$(M_D)_{f,\,\pi^\pm}(z,\, b_T) = z^{-\rho b_T^2} \frac{2^{2-p}}{\Gamma(p-1)} (b_T\, m)^{p-1} K_{p-1}(b_T\, m)$$

Power-law model

$\mathcal{FT}\{M_D\}$ reminiscent of a propagator in k_T space

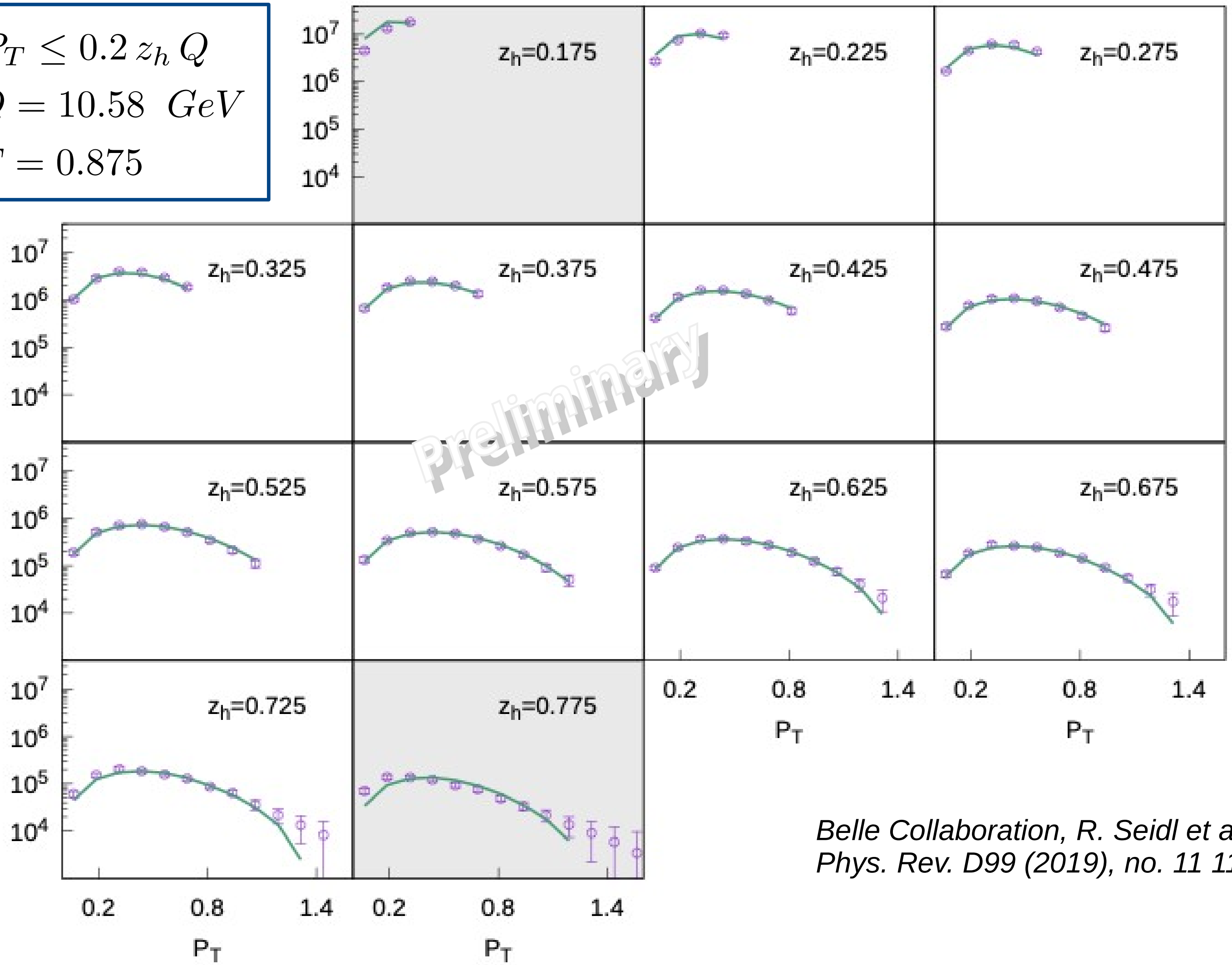
$$\frac{1}{(k_T^2 + m^2)^p}$$



Phenomenological results – z dependence

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, work in progress

$P_T \le 0.2\, z_h\, Q$
$Q = 10.58 \;\; GeV$
$T = 0.875$



$$\chi^2_{dof} = 0.9$$

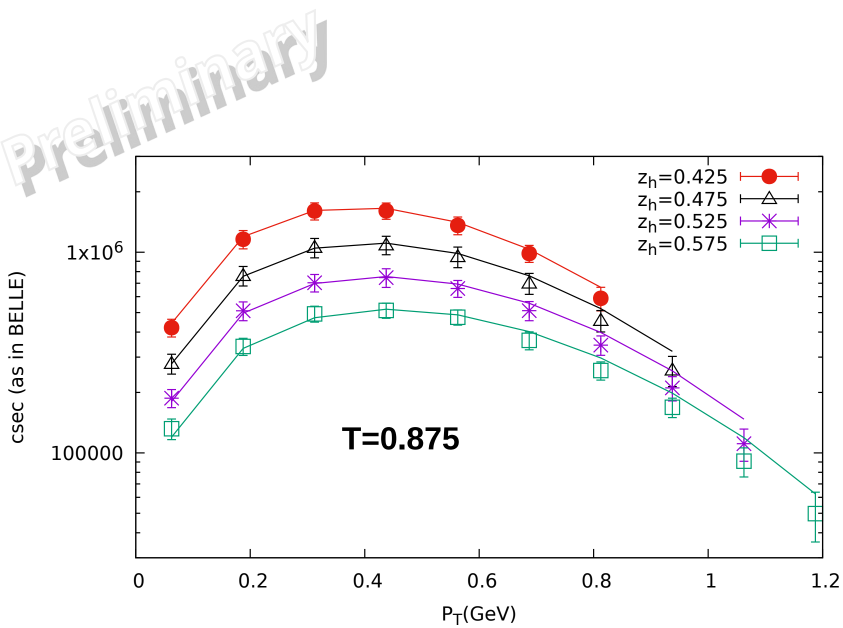
n. of fitted data: 89

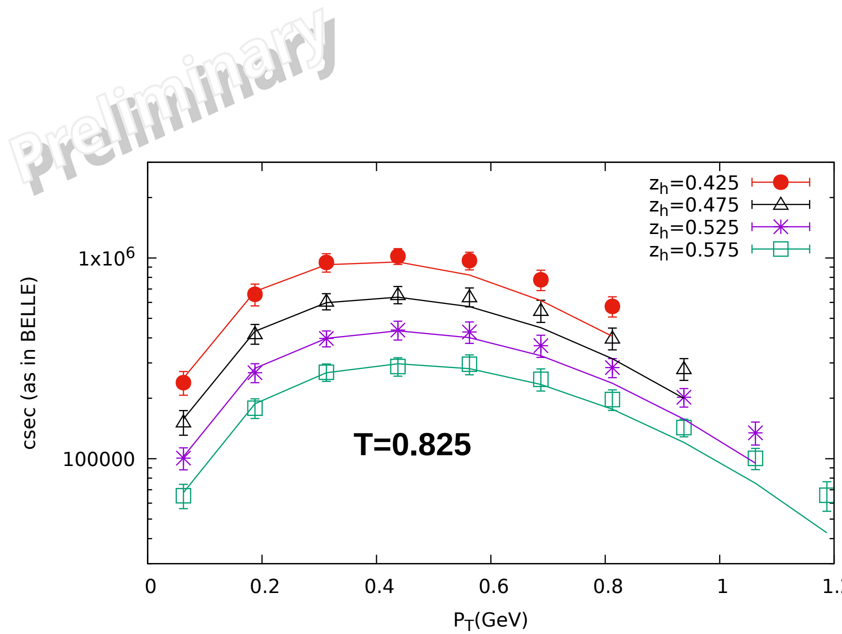
Data in the shaded boxes are not included in this fit

Belle Collaboration, R. Seidl et al., Phys. Rev. D99 (2019), no. 11 112006

Phenomenological results – T dependence

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, work in progress



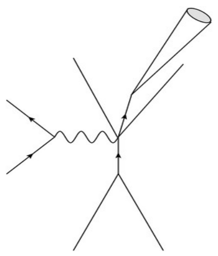


Belle Collaboration, R. Seidl et al.,
Phys. Rev. D99 (2019), no. 11 112006

$$\boxed{g_K(b_T) = a\, b_T^2}$$

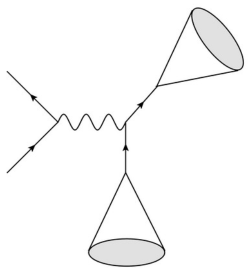
We are testing different b_T behaviors of g_K
(linear, quadratic, logarithmic, etc ...)

Outlook



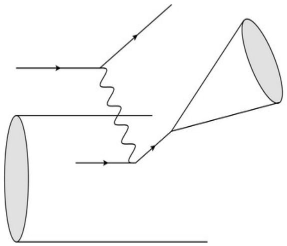
1. $e^+ e^- \to h\, X$

   Extraction of the unpolarized TMD FF, D*, for charged pions from BELLE data (using factorization definition)



2. $e^+ e^- \to h_1 h_2\, X$

   Two non-perturbative functions:
   D*, known from step 1
   Soft Model M_S, obtained as ratio: $M_S = D/D^\star$



3. SIDIS

   Three non-perturbative functions in the cross section
   D*, known from step 1.
   Soft Model M_S, known from step 2.
   Extraction of the TMD PDF, F* (in the factorization definition, F*≠F).

Outlook

The Soft Factor acquires a central role

The focus of phenomenological analyses moves from the TMDs considered as a whole, to the Soft Factor contribution (which encloses the full process dependent part of the TMD).

Can the TMD tangle finally be disentangled ?