Twist-3 PDFs from lattice QCD

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Twist-3 PDFs

PDFs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

Leading twist: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3:

- no density interpretation,
- contain important information about qgq correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements difficult due to their suppressed $\mathcal{O}(1/Q)$ kinematical behavior.

Our work:

- (Shohini's talk) **matching for twist-3 PDF $g_T(x)$**
  proved factorization at 1-loop
  extracted the matching coefficient between quasi and light-cone
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
- (this talk) **lattice extraction of isovector combination $g_T^{u-d}(x)$**
  test of Wandzura-Wilczek (WW) approximation
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 111501(R)
- (Shohini's talk) **role of zero-mode contributions + matching for twist-3 chiral-odd PDFs $h_L(x)$ and $e(x)$**
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 114025
- (this talk) **lattice extraction of isovector combination $h_L^{u-d}(x)$**
  test of Wandzura-Wilczek (WW) approximation
  S. Bhattacharya et al., in preparation

Reminder of quasi-PDFs lattice procedure

spatial correlation in a boosted nucleon
$\langle N(P_3)|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(P_3)\rangle$
lattice computation of bare ME

↓

renormalization
intermediate RI scheme
conversion to $\mathrm{M}\overline{\mathrm{MS}}$ scheme
(incl. evolution to $\mu = 2$ GeV)

↓

reconstruction of x-dependence
z-space$\rightarrow$$x$-space
Backus-Gilbert

↓

matching to light cone
$\mathrm{M}\overline{\mathrm{MS}} \rightarrow \overline{\mathrm{MS}}$

↓

light-cone PDF

Lattice matrix element:

$$\mathcal{M}(P,z) = \langle P|\,\overline{\psi}(0,z)\,\Gamma\,W(z)\,\psi(0,0)\,|P\rangle\,.$$

Dirac structure: $g_T(x)$: $\Gamma = \gamma^x\gamma^5$ or $\gamma^y\gamma^5$
$h_L(x)$: $\Gamma = \sigma_{xy}$

Quasi-distribution:

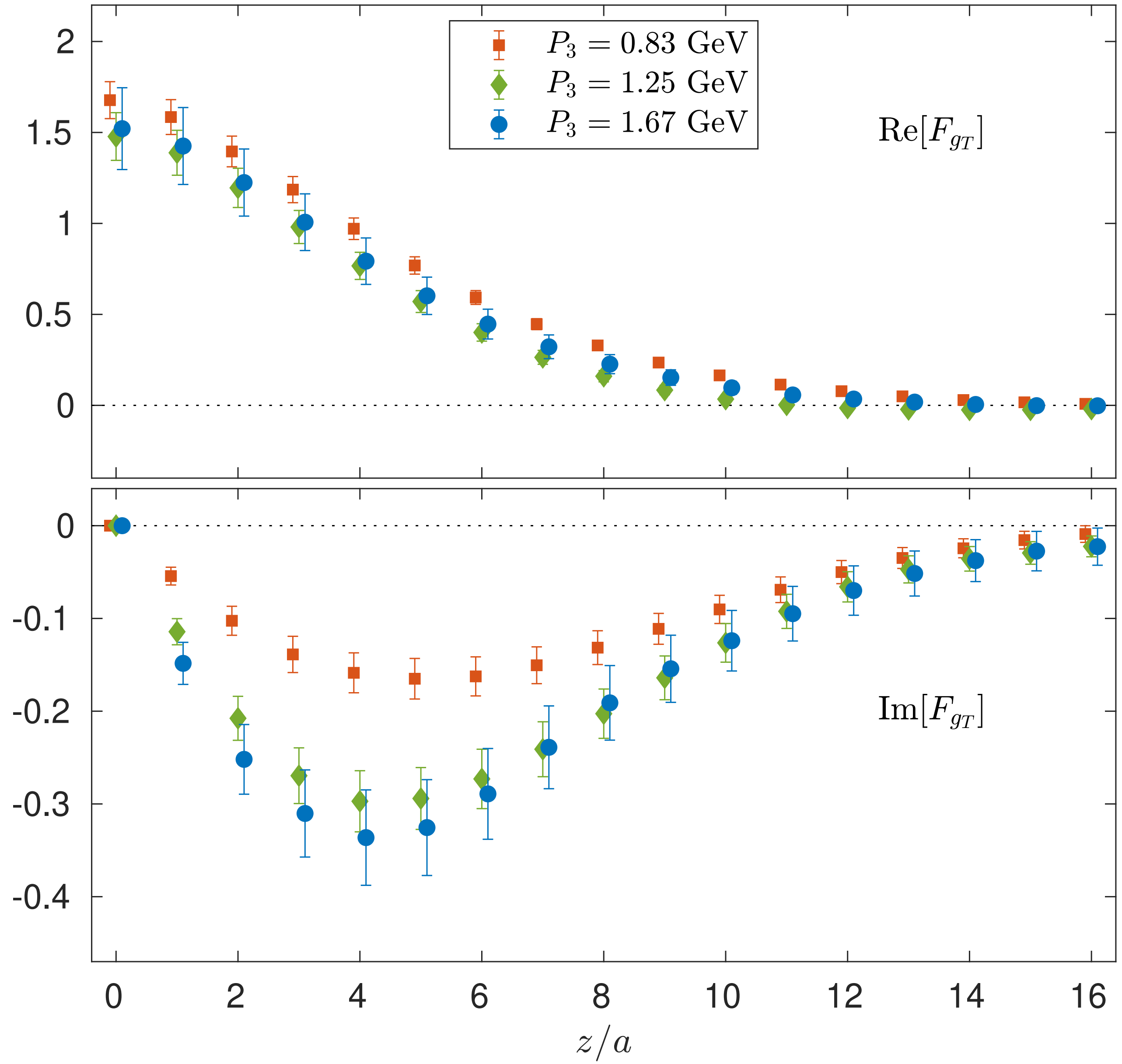
$$\widetilde{q}(x,\Lambda,P_3) = 2P_3 \int_{-\infty}^{+\infty} \frac{dz}{4\pi}\, e^{-ixP_3z}\, F(P_3,z)$$

$F(P_3,z)$ – $\mathcal{M}(P,z)$ + kinematic & renormalization factors

Lattice setup:

- fermions: $N_f = 2+1+1$ TM fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.726$
- $a \approx 0.093$ fm, $m_\pi \approx 260$ MeV
- $32^3 \times 64$, $L = 3$ fm, $m_\pi L = 4$

Bare matrix elements (g_T)

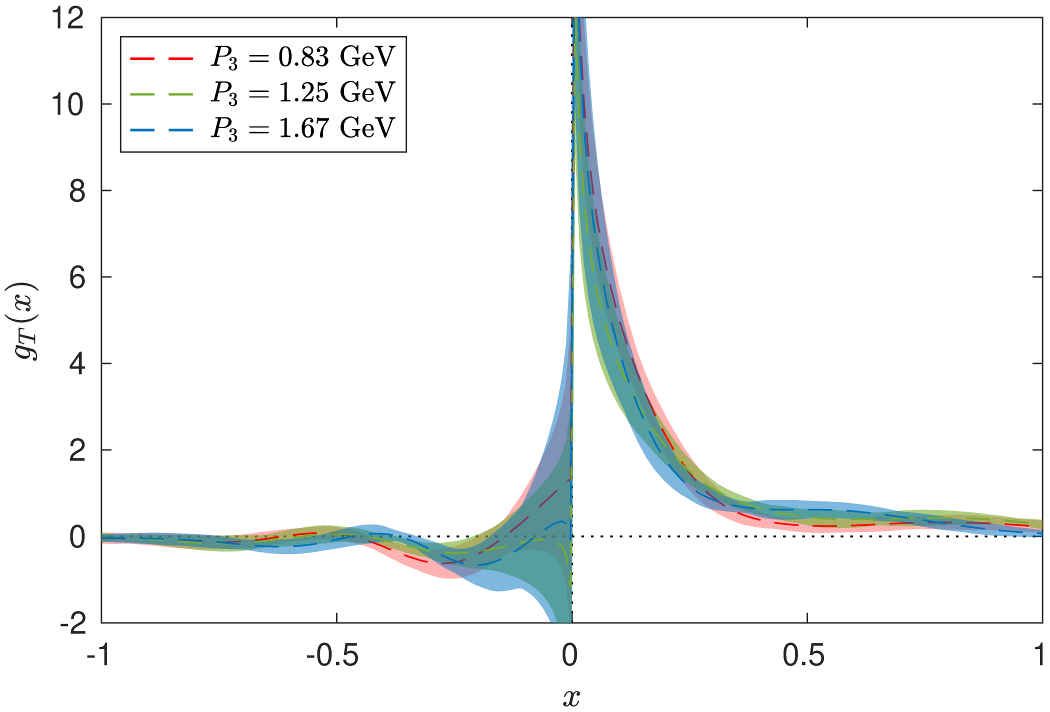


similar for h_L

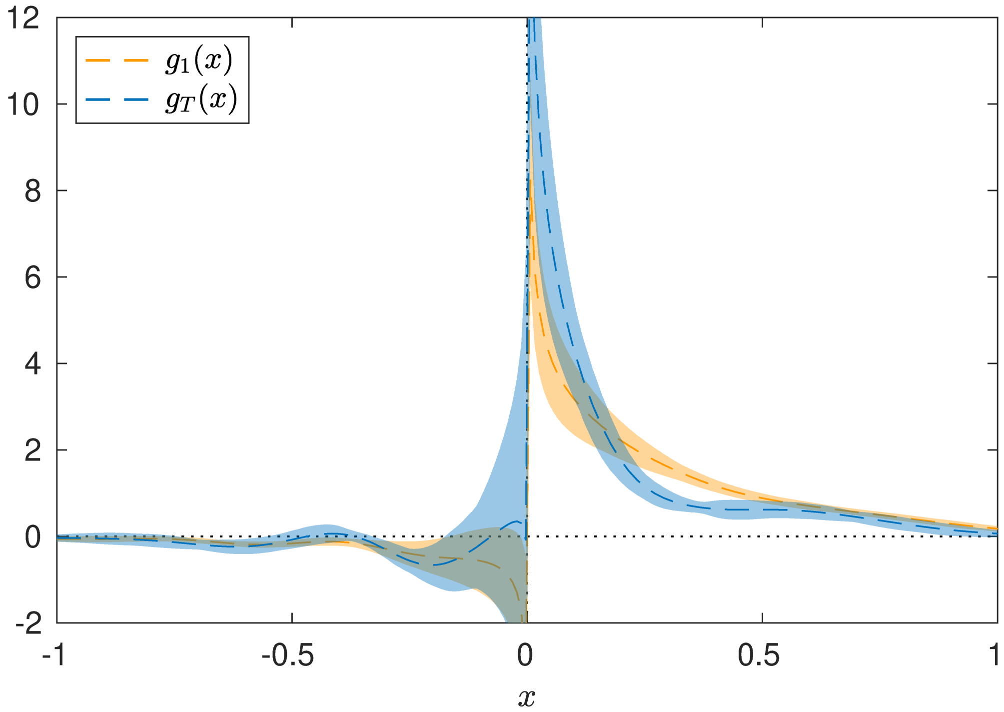
Matching to light-cone distributions (see Shohini's talk)

generic	$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{\|\xi\|} \left[\delta(\xi-1) - \frac{\alpha_s C_F}{2\pi} C^{(1)}\left(\xi, \frac{\mu}{xP_3}\right)\right] \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$
$g_T(x)$	$C^{(1)}_{\overline{\rm MMS}}\left(\xi, \frac{\mu}{xP_3}\right) = \begin{cases} \left[\frac{\xi^2-2\xi-1}{1-\xi}\ln\frac{\xi-1}{\xi} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ & \xi>1, \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\left(\ln\frac{4(xP_3)^2}{\mu^2} + \ln(\xi(1-\xi))\right) + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1, \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ & \xi<0. \end{cases}$
$h_L(x)$	$C^{(1)}_{\overline{\rm MMS}}\left(\xi, \frac{\mu}{xP_3}\right) = C^{(1,{\rm s})}_{\overline{\rm MMS}}\left(\xi, \frac{\mu}{xP_3}\right) + C^{1,({\rm c})}_{\overline{\rm MMS}}\left(\xi, \frac{\mu}{xP_3}\right)$ $C^{1,({\rm s})}_{\overline{\rm MMS}}\left(\xi, \frac{\mu}{xP_3}\right) = \begin{cases} \delta(1-\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4(xP_3)^2}\right) & \xi>1 \\ -\delta(\xi)\left(\ln\frac{4(xP_3)^2}{\mu^2}+1\right) - {\rm R}_0(\|\xi\|) & -1<\xi<1 \\ \delta(1+\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4(xP_3)^2}\right) & \xi<-1 \end{cases}$ ${\rm R}_0(\|\xi\|) \equiv \left[\frac{1}{\|\xi\|}\right]_{+[0]} = \theta(\|\xi\|)\,\theta(1-\|\xi\|) \lim_{\beta\to0}\left[\frac{\theta(\|\xi\|-\beta)}{\|\xi\|} + \delta(\|\xi\|-\beta)\ln\beta\right]$ $C^{(1,{\rm c})}_{\overline{\rm MMS}}\left(\xi, \frac{\mu}{xP_3}\right) = \begin{cases} \left[\frac{2}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{1}{1-\xi} + \frac{1}{\xi}\right]_+ & \xi>1 \\ \left[\frac{2}{1-\xi}\ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2} + 2(1-\xi) - \frac{1}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\frac{2}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{1}{1-\xi} + \frac{1}{1-\xi}\right]_+ & \xi<0 \end{cases}$

Nucleon boost dependence
(after matching)
(quasi-g_T reconstructed with BG)



Twist-2 g_1 vs. twist-3 g_T
(at the largest boost)



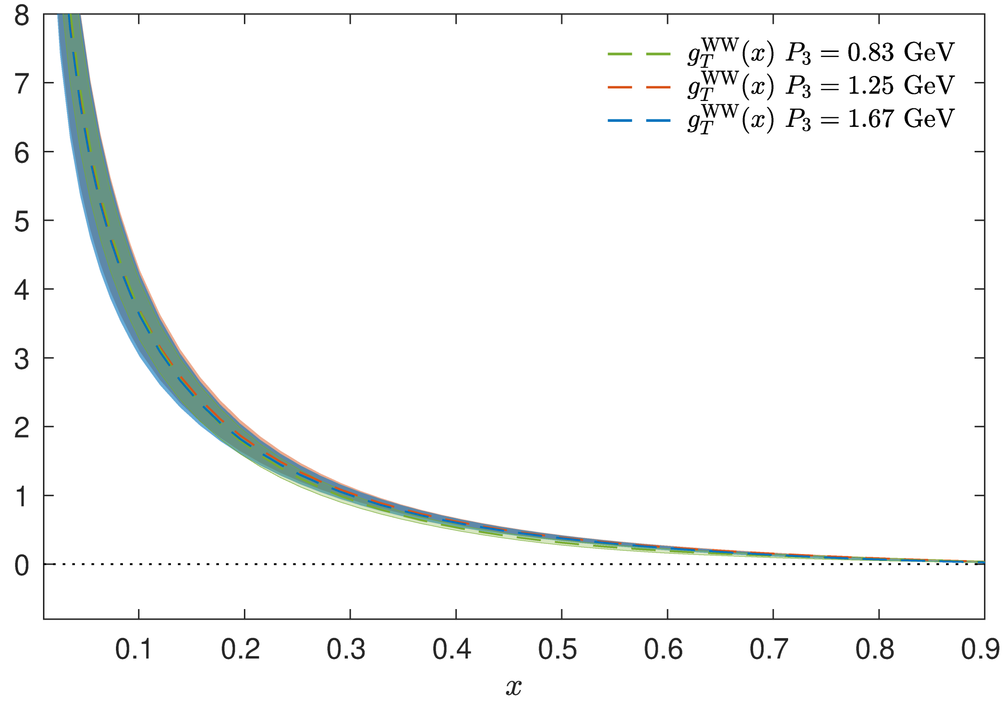
Burkhardt-Cottingham sum rule:

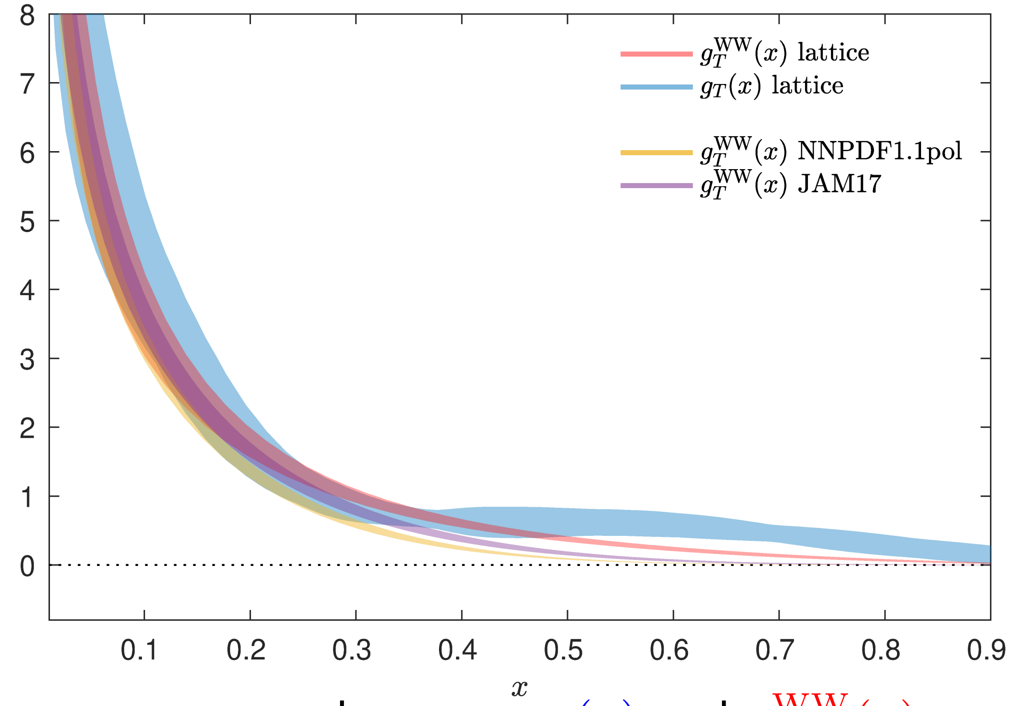
$$\int_{-1}^{1} dx g_T(x) = \int_{-1}^{1} dx g_1(x)$$

satisfied in our data.

WW approximation: twist-3 $g_T(x)$ fully determined by twist-2 $g_1(x)$:

$$g_T(x) \approx g_T^{\mathrm{WW}}(x) = \int_x^1 \frac{dy}{y} g_1(y)$$





agreement between $g_T(x)$ and $g_T^{\mathrm{WW}}(x)$
for $x \lesssim 0.5$ within uncertainties
still: possible violation up to 30-40%
interestingly, similar possible violation (15-40%)
in experimental data analysis by JLab:

A. Accardi, A. Bacchetta, W. Melnitchouk, M. Schlegel, JHEP 11 (2009) 093

Role of singular terms in h_L matching

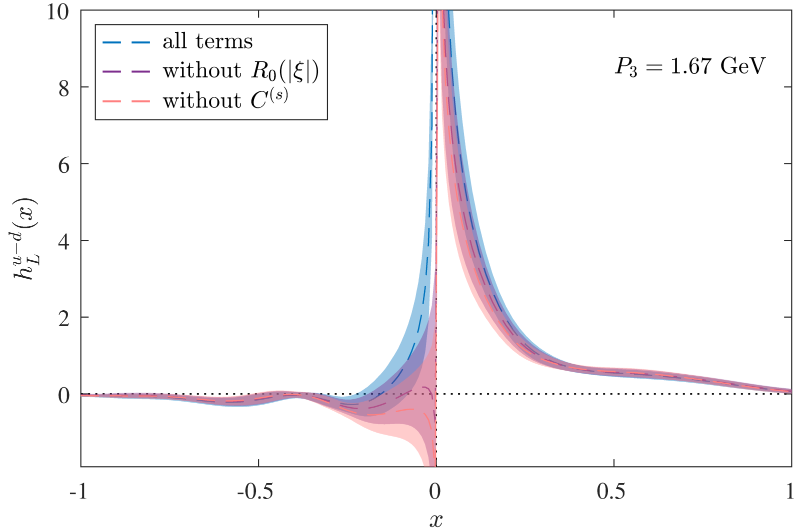
$$C^{(1)}_{\overline{\mathrm{MMS}}}\left(\xi, \frac{\mu}{xP_3}\right) = C^{(1,\mathrm{s})}_{\overline{\mathrm{MMS}}}\left(\xi, \frac{\mu}{xP_3}\right) + C^{1,(\mathrm{c})}_{\overline{\mathrm{MMS}}}\left(\xi, \frac{\mu}{xP_3}\right)$$

$$C^{1,(\mathrm{s})}_{\overline{\mathrm{MMS}}}\left(\xi, \frac{\mu}{xP_3}\right) = \begin{cases} \delta(1-\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4(xP_3)^2}\right) & \xi > 1 \\ -\,\delta(\xi)\left(\ln\frac{4(xP_3)^2}{\mu^2} + 1\right) - \mathrm{R}_0(|\xi|) & -1 < \xi < 1 \\ \delta(1+\xi)\left(\frac{1}{2} - \frac{1}{2}\ln\frac{\mu^2}{4(xP_3)^2}\right) & \xi < -1 \end{cases}$$

$$\mathrm{R}_0(|\xi|) \equiv \left[\frac{1}{|\xi|}\right]_{+[0]} = \theta(|\xi|)\,\theta(1-|\xi|)\lim_{\beta\to 0}\left[\frac{\theta(|\xi|-\beta)}{|\xi|} + \delta(|\xi|-\beta)\ln\beta\right]$$

$$C^{(1,\mathrm{c})}_{\overline{\mathrm{MMS}}}\left(\xi, \frac{\mu}{xP_3}\right) = \begin{cases} \left[\frac{2}{1-\xi}\ln\frac{\xi}{\xi-1} + \frac{1}{1-\xi} + \frac{1}{\xi}\right]_+ & \xi > 1 \\ \left[\frac{2}{1-\xi}\ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2} + 2(1-\xi) - \frac{1}{1-\xi}\right]_+ & 0 < \xi < 1 \\ \left[\frac{2}{1-\xi}\ln\frac{\xi-1}{\xi} - \frac{1}{1-\xi} + \frac{1}{1-\xi}\right]_+ & \xi < 0 \end{cases}$$

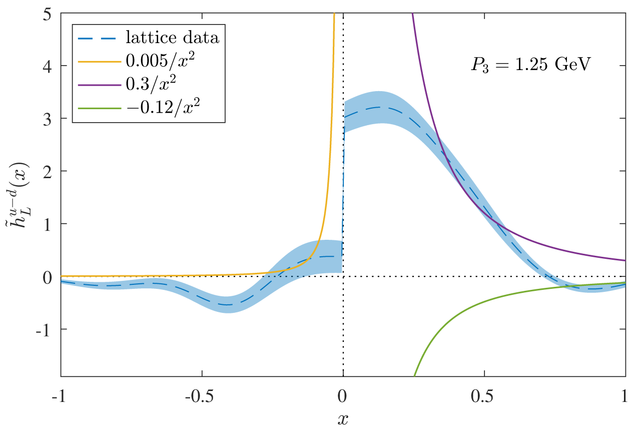
Excluding some terms
(at the largest boost)
(quasi-h_L reconstructed with BG)

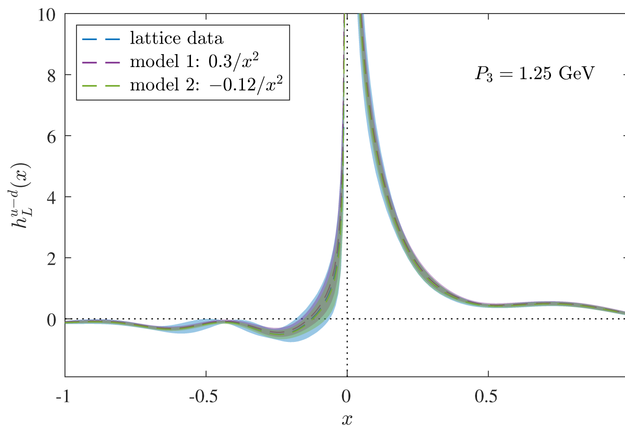


influence only for $|x| \lesssim 0.05$

Another check:
Standard procedure: apply a cut such that $\tilde{q}(x > x_c) = 0$.

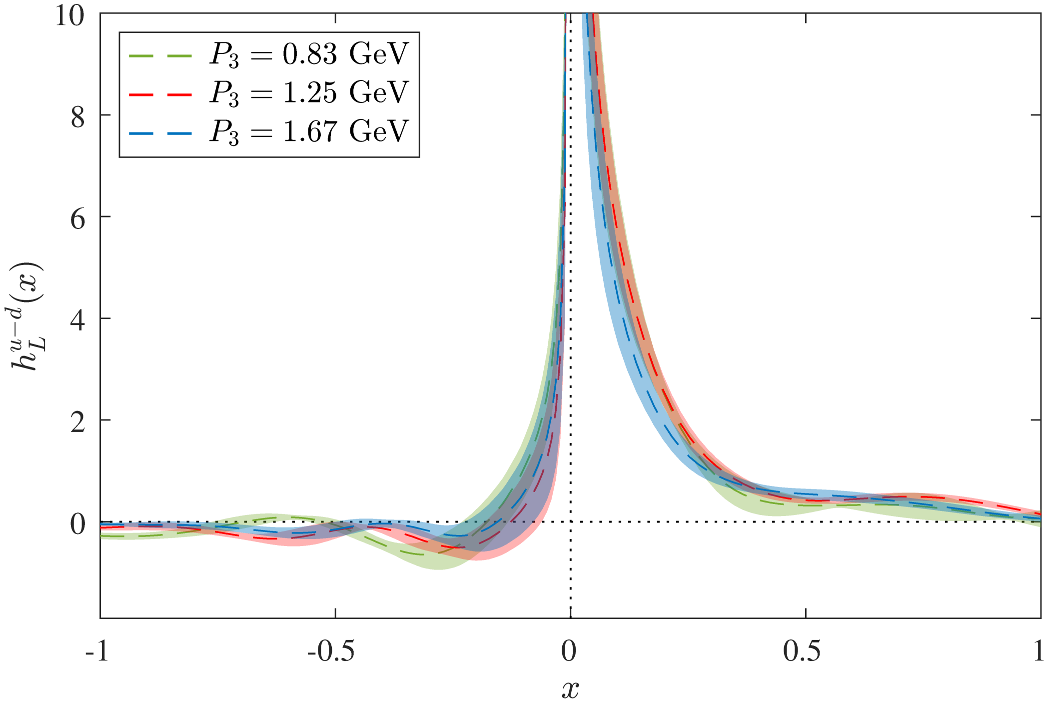
Check whether the large-x behavior of $\tilde{q}$ plays any role by employing PT-inspired models of the form: $\tilde{h}_L(x) = c/(x^2+b)$ with c, b chosen to match lattice data in some region of x.



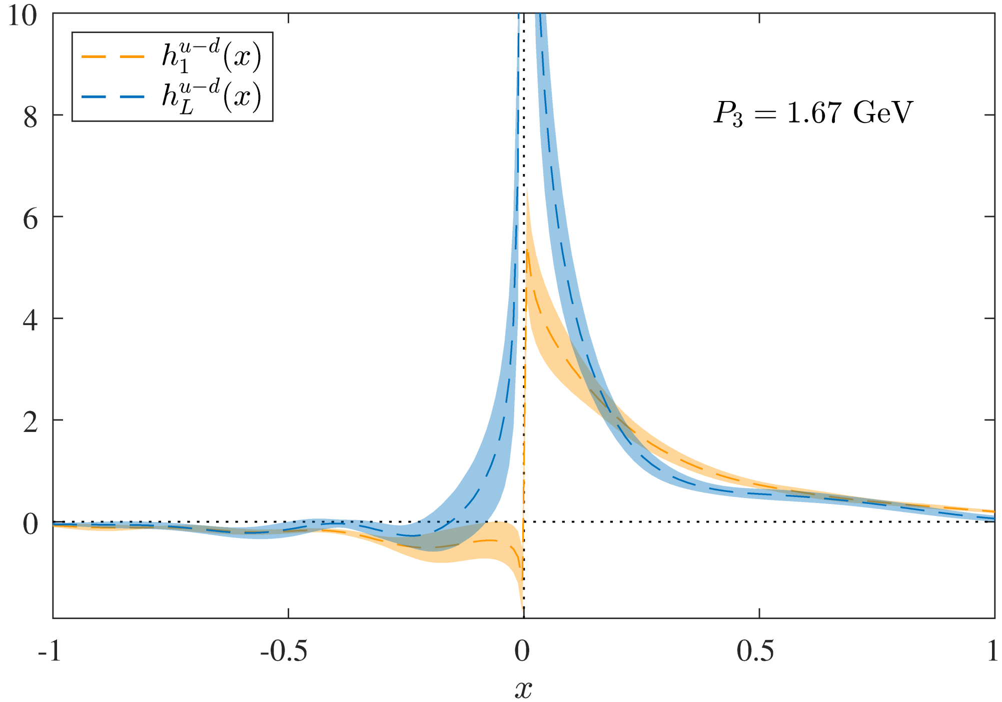


no influence of the large-x behavior of $\tilde{h}_L(x)$

Nucleon boost dependence
(after matching; all terms)
(quasi-h_L reconstructed with BG)



Twist-2 h_1 vs. twist-3 h_L
(at the largest boost)

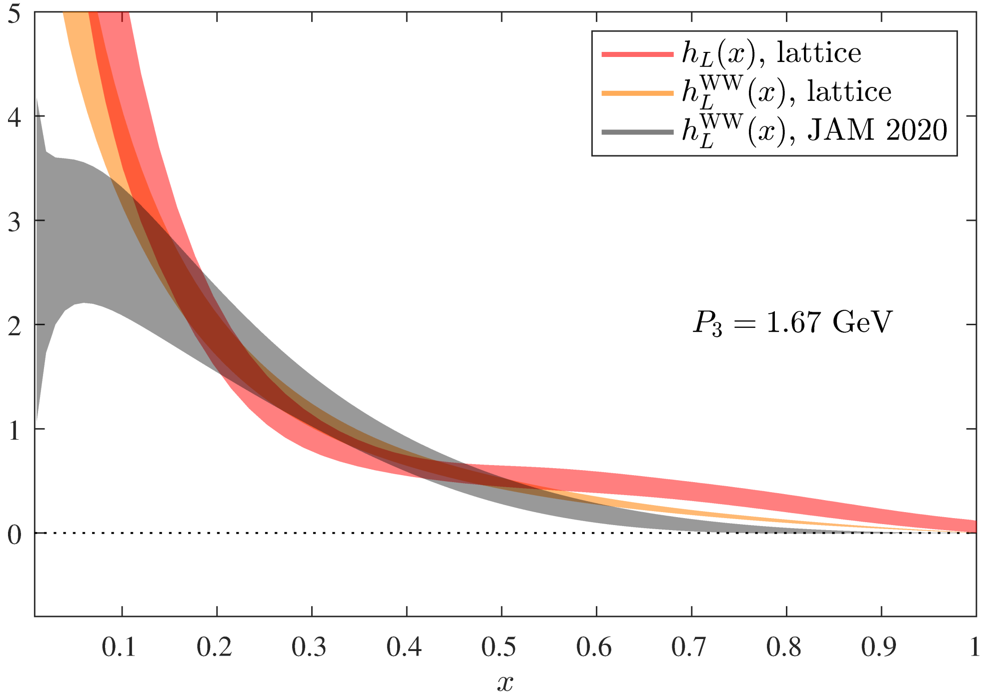


Wandzura-Wilczek approximation (h_L)

WW approximation:

twist-3 $h_L(x)$ fully determined by twist-2 $h_1(x)$:

$$h_L(x) \approx h_L^{\mathrm{WW}}(x) = 2x \int_x^1 \frac{dy}{y^2} h_1(y)$$



Similar outcome as for $g_T(x) \approx g_T^{\mathrm{WW}}(x)$

agreement between $h_L(x)$ and $h_L^{\mathrm{WW}}(x)$ for $x \lesssim 0.55$ within uncertainties

still: possible violation up to 30-40%

Conclusions and prospects

- Message of the talk:
  promising results from first exploratory quasi-distribution approach studies of twist-3 PDFs $g_T(x)$ and $h_L(x)$.
- Preliminary physics result: Wandzura-Wilczek approximation holds at $x \lesssim 0.5$, but possible violation up to 30-40% when uncertainties are taken into account.
- To make the studies more robust, lots of future work needed to assess the systematics:
  - ⋆ lattice: nucleon boost dependence, pion mass dependence, cut-off effects, FVE, excited states, ...
  - ⋆ other: mixing with quark-quark-gluon operators, role of zero modes, truncation effects, higher-twist effects, ...
- In the future, lattice results for twist-3 can have important impact on phenomenology.

Thank you for your attention!