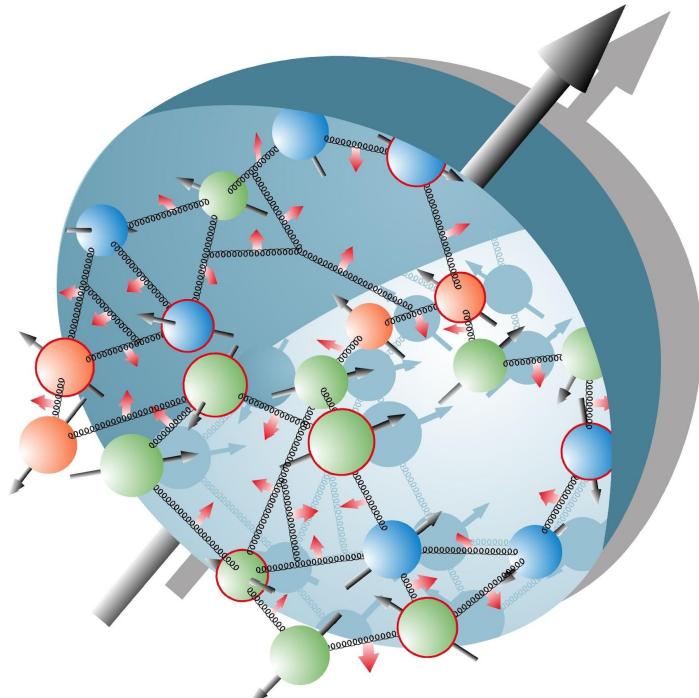


# SIVERS EXTRACTION WITH NEURAL NETWORK



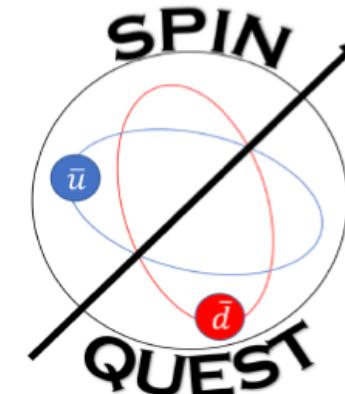
Ishara Fernando, Nicholas Newton, Devin Seay & Dustin Keller

*University of Virginia (UVA) Spin Physics Group*

@ **DIS-2021**  
April 13, 2021



**UNIVERSITY  
of  
VIRGINIA**

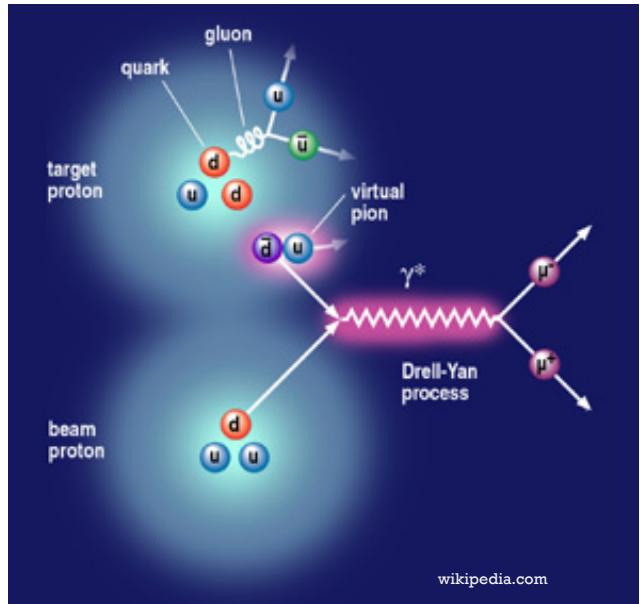


 **Fermilab**

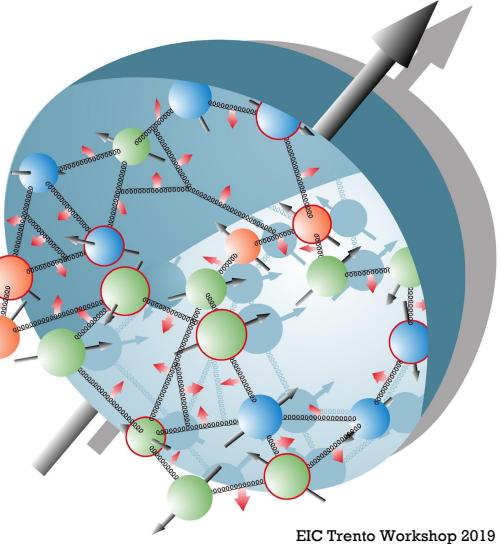
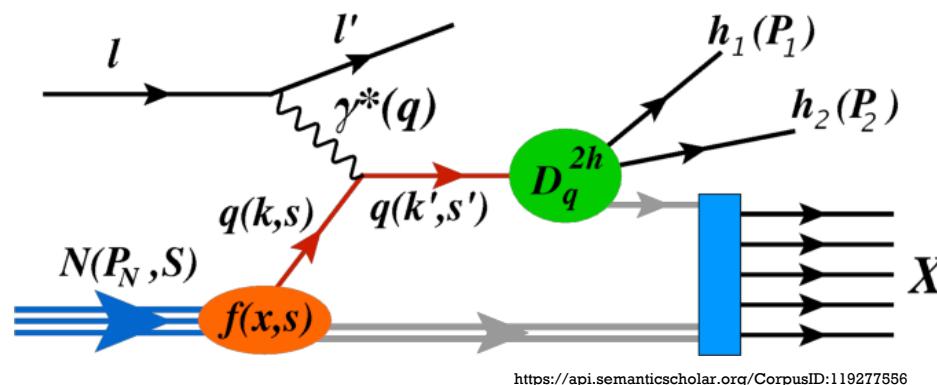
# PROBING TMDs

Transverse  
Momentum  
Dependent  
Parton Distribution  
Functions  
(TMD-PDFs)

Drell-Yan (DY)

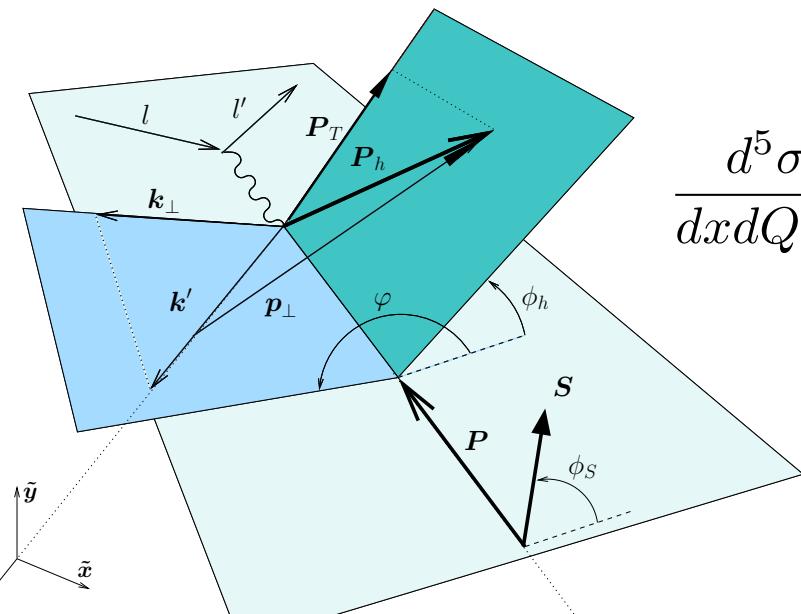


Semi Inclusive Deep Inelastic Scattering (SIDIS)



Electron-positron annihilation

# Semi Inclusive Deep Inelastic Scattering (SIDIS) Process



arXiv:hep-ph/0501196

$$\begin{aligned} f_{q/p\uparrow}(x, \mathbf{k}_\perp) &= f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p\uparrow}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \\ &= f_{q/p}(x, k_\perp) - \frac{k_\perp}{m_p} f_{1T}^{\perp q}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \end{aligned}$$

$$\frac{d^5 \sigma^{lp \rightarrow lhX}}{dx dQ^2 dz d^2 p_{hT}} \propto \sum_q e_q^2 \int d^2 \mathbf{k}_\perp \mathcal{K}(x, p_{hT}, Q^2) f_q(x, \mathbf{k}_\perp) D_{h/q}(z, p_\perp) + \mathcal{O}(\mathbf{k}_\perp/Q)$$

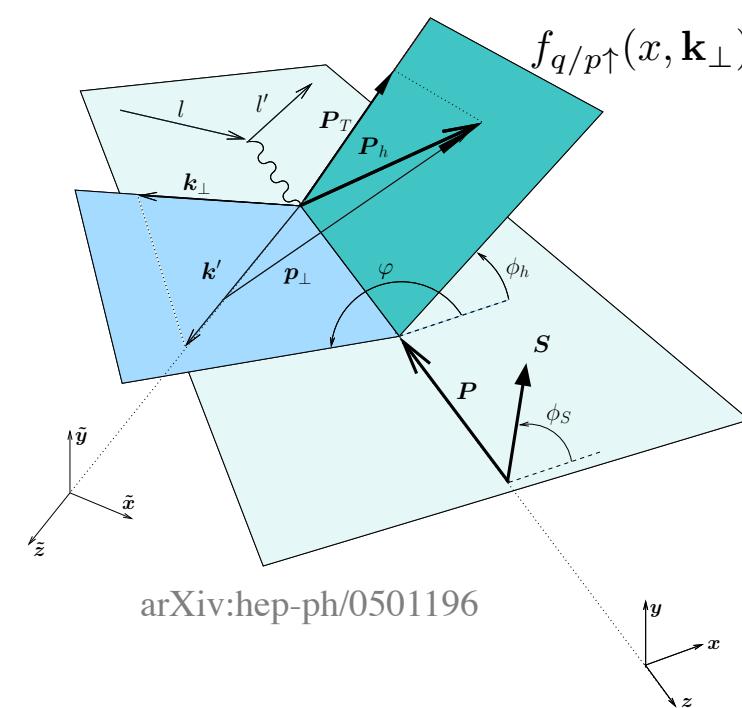
Fragmentation Functions

Leading Twist TMDs

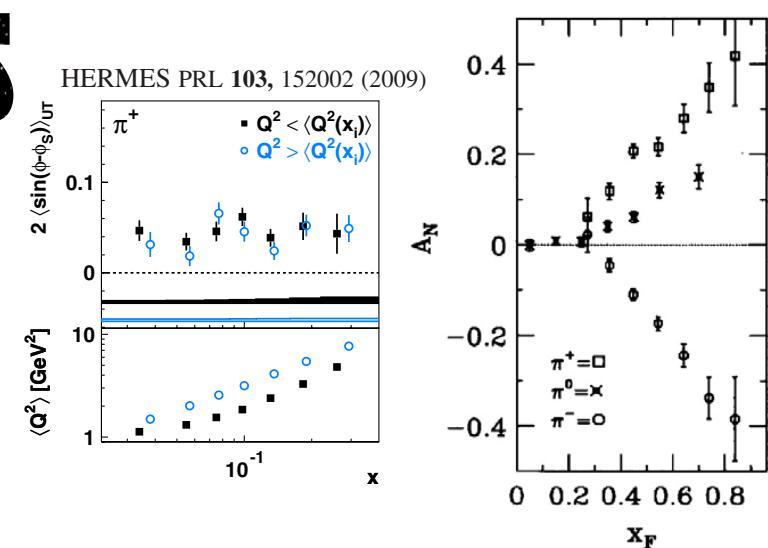
Nucleon Spin    Quark Spin

Nucleon Polarization	Quark Polarization		
	Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_1 = \bullet$		$h_1^\perp = \bullet - \bullet$ Boer-Mulders
L		$g_{1L} = \bullet \rightarrow - \bullet \rightarrow$ Helicity	$h_{1L}^\perp = \bullet \rightarrow - \bullet \rightarrow$
T	$f_{1T}^\perp = \bullet - \bullet$ Sivers	$g_{1T}^\perp = \bullet \uparrow - \bullet \downarrow$	$h_{1T}^\perp = \bullet \uparrow - \bullet \uparrow$ Transversity

# SIVERS ASYMMETRY FROM SIDIS



$$\begin{aligned} f_{q/p\uparrow}(x, \mathbf{k}_\perp) &= f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p\uparrow}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \\ &= f_{q/p}(x, k_\perp) - \frac{k_\perp}{m_p} f_{1T}^{\perp q}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \end{aligned}$$



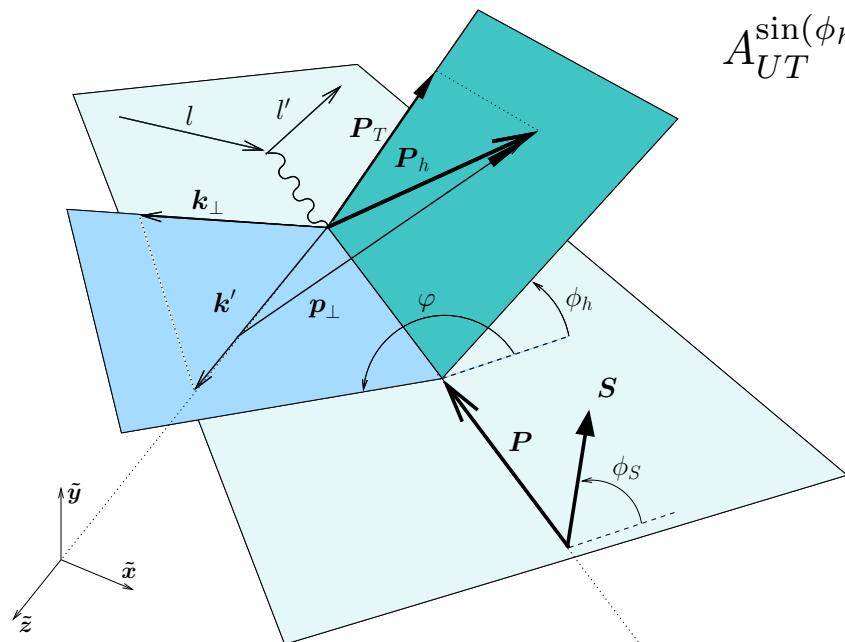
Asymmetry in  $pp^\uparrow \rightarrow \pi X$  pion production from E704

## Single Spin Asymmetry (Sivers Asymmetry)

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, p_{hT}) = \frac{d\sigma^{l\uparrow p \rightarrow hlX} - d\sigma^{l\downarrow p \rightarrow lhX}}{d\sigma^{l\uparrow p \rightarrow hlX} + d\sigma^{l\downarrow p \rightarrow lhX}} \equiv \frac{d\sigma \uparrow - d\sigma \downarrow}{d\sigma \uparrow + d\sigma \downarrow}$$

$$\begin{aligned} A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, p_{hT}) &= \frac{[z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle] \langle k_S^2 \rangle^2}{[z^2 \langle k_S^2 \rangle + \langle p_\perp^2 \rangle] \langle k_\perp^2 \rangle^2} \exp \left[ -\frac{p_{hT}^2 z^2 (\langle k_S^2 \rangle - \langle k_\perp^2 \rangle)}{(z^2 \langle k_S^2 \rangle + \langle p_\perp^2 \rangle) (z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle)} \right] \\ &\times \frac{\sqrt{2e} z p_{hT}}{M_1} \frac{\sum_q \mathcal{N}_q e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)} \end{aligned}$$

# SIVERS ASYMMETRY FROM SIDIS



arXiv:hep-ph/0501196

$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, p_{hT}) = \frac{[z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle] \langle k_S^2 \rangle^2}{[z^2 \langle k_S^2 \rangle + \langle p_\perp^2 \rangle] \langle k_\perp^2 \rangle^2} \exp \left[ -\frac{p_{hT}^2 z^2 (\langle k_S^2 \rangle - \langle k_\perp^2 \rangle)}{(z^2 \langle k_S^2 \rangle + \langle p_\perp^2 \rangle) (z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle)} \right]$$

$$\times \frac{\sqrt{2e} z p_{hT}}{M_1} \frac{\sum_q \mathcal{N}_q e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)}$$

$$\langle k_S^2 \rangle = \frac{M_1^2 \langle k_\perp^2 \rangle}{M_1^2 + \langle k_\perp^2 \rangle} \quad \langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2 \quad \langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$D_{h/q}(z)$  is the fragmentation function for a quark with flavor  $q$  in a hadron  $h$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-p_\perp^2 / \langle p_\perp^2 \rangle}$$

$\langle p_\perp^2 \rangle$  is the width of the unpolarized TMD-PDFs

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, p_{hT}) = \mathcal{A}_0(z, p_{hT}, M_1) \left( \frac{\sum_q \mathcal{N}_q e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)} \right)$$

$$\mathcal{N}_{\bar{q}}(x) = N_{\bar{q}}$$

# FITTING METHODOLOGY

Inputs:

- Unpolarized PDFs : LHAPDF6 (CTEQ61)
- Fragmentation Functions:

- Pi+: NNFF10\_Pip\_nlo
- Pi- : NNFF10\_Pim\_nlo
- Pi0: NNFF10\_Pisum\_nlo
- K+: NNFF10\_Kap\_nlo
- K- : NNFF10\_Kam\_nlo

V. Bertone *et. al* arXiv:1706.07049

Data Sets (on consideration):

- HERMES\_p\_2009 (from Luciano Pappalardo)
- COMPASS\_d\_2009 (from Bakur Parsamyan )
- COMPASS\_p\_2015 (from Bakur Parsamyan )
- HERMES\_p\_2020 (from Luciano Pappalardo)

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, p_{hT}) = \frac{[z^2\langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle]\langle k_S^2 \rangle^2}{[z^2\langle k_S^2 \rangle + \langle p_\perp^2 \rangle]\langle k_\perp^2 \rangle^2} \exp \left[ -\frac{p_{hT}^2 z^2 (\langle k_S^2 \rangle - \langle k_\perp^2 \rangle)}{(z^2\langle k_S^2 \rangle + \langle p_\perp^2 \rangle)(z^2\langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle)} \right]$$
$$\times \frac{\sqrt{2e} z p_{hT}}{M_1} \frac{\sum_q \mathcal{N}_q e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)}$$

Fit parameters (13):

$M_1$

$N_u, \alpha_u, \beta_u, N_{\bar{u}}$

$N_d, \alpha_d, \beta_d, N_{\bar{d}}$

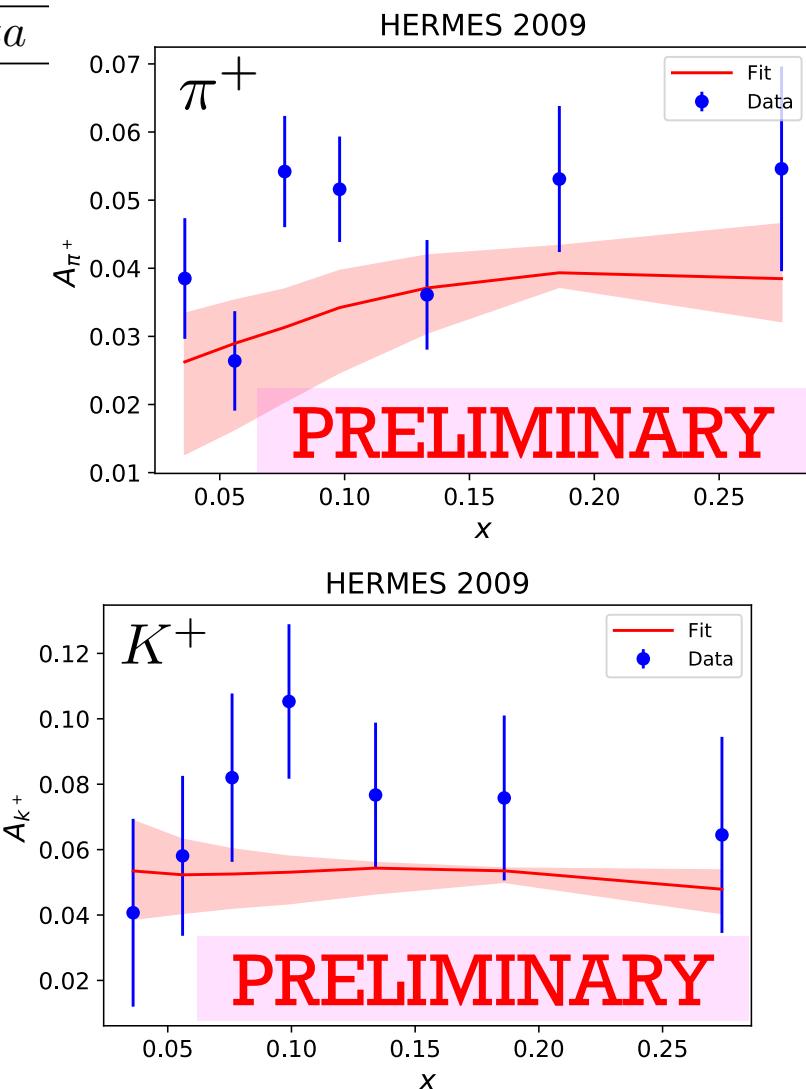
$N_s, \alpha_s, \beta_s, N_{\bar{s}}$

Fitting routines:

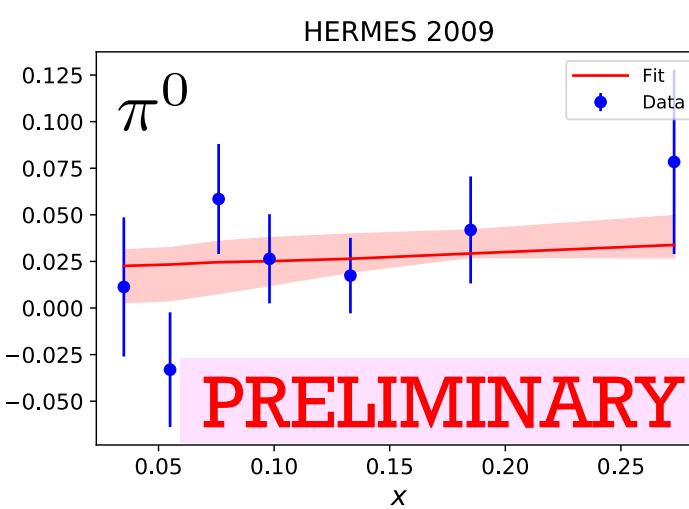
- “iminuit” (python supported version of MINUIT)
- Python `scipy.optimize.curve_fit`
- Using a Neural Network approach

# FITS TO HERMES(2009) [PRELIMINARY]

Hadron	Dependence	n data	$\chi^2/n data$
$\pi^+$	$x$	7	2.53
$\pi^+$	$z$	7	1.02
$\pi^+$	$p_{hT}$	7	5.23
$\pi^-$	$x$	7	1.94
$\pi^-$	$z$	7	2.45
$\pi^-$	$p_{hT}$	7	1.61
$\pi^0$	$x$	7	0.85
$\pi^0$	$z$	7	1.11
$\pi^0$	$p_{hT}$	7	2.00
$K^+$	$x$	7	1.22
$K^+$	$z$	7	2.97
$K^+$	$p_{hT}$	7	2.65
$K^-$	$x$	7	0.49
$K^-$	$z$	7	0.52
$K^-$	$p_{hT}$	7	0.96
Total		105	1.84

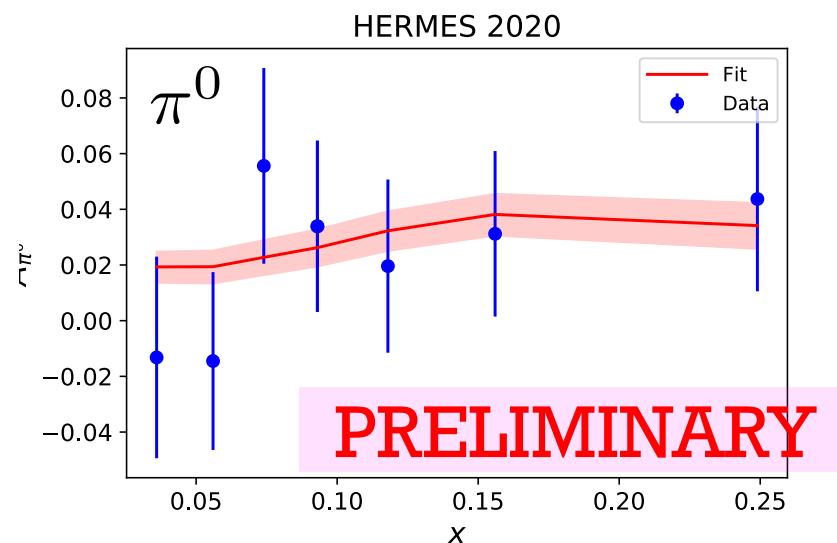
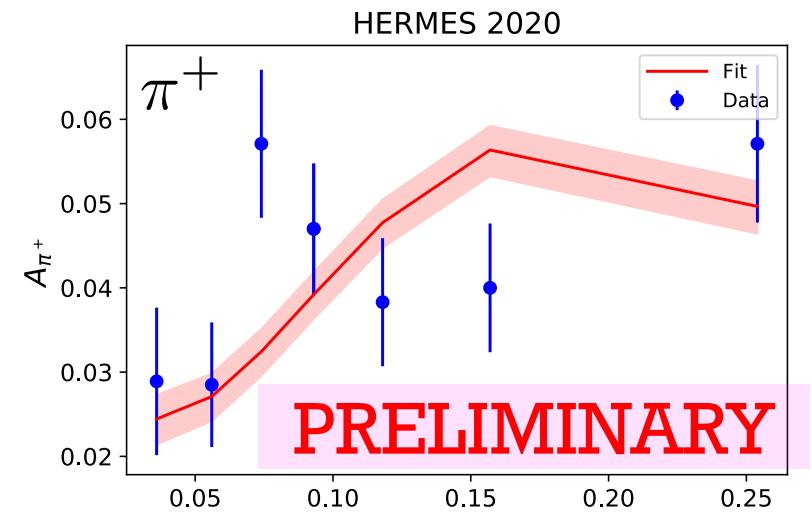


Parameter	Value
$M_1$	$1.303 \pm 0.010$
$N_u$	$0.169 \pm 0.002$
$\alpha_u$	$0.645 \pm 0.125$
$\beta_u$	$3.122 \pm 2.661$
$N_{\bar{u}}$	$0.007 \pm 0.003$
$N_d$	$-0.434 \pm 0.005$
$\alpha_d$	$1.777 \pm 0.909$
$\beta_d$	$7.788 \pm 2.144$
$N_{\bar{d}}$	$-0.142 \pm 0.048$
$N_s$	$0.563 \pm 0.073$
$\alpha_s$	$(6.84 \pm 10.00) \times 10^{-5}$
$\beta_s$	$(5.987 \pm 8.77) \times 10^{-10}$
$N_{\bar{s}}$	$-0.122 \pm 0.504$



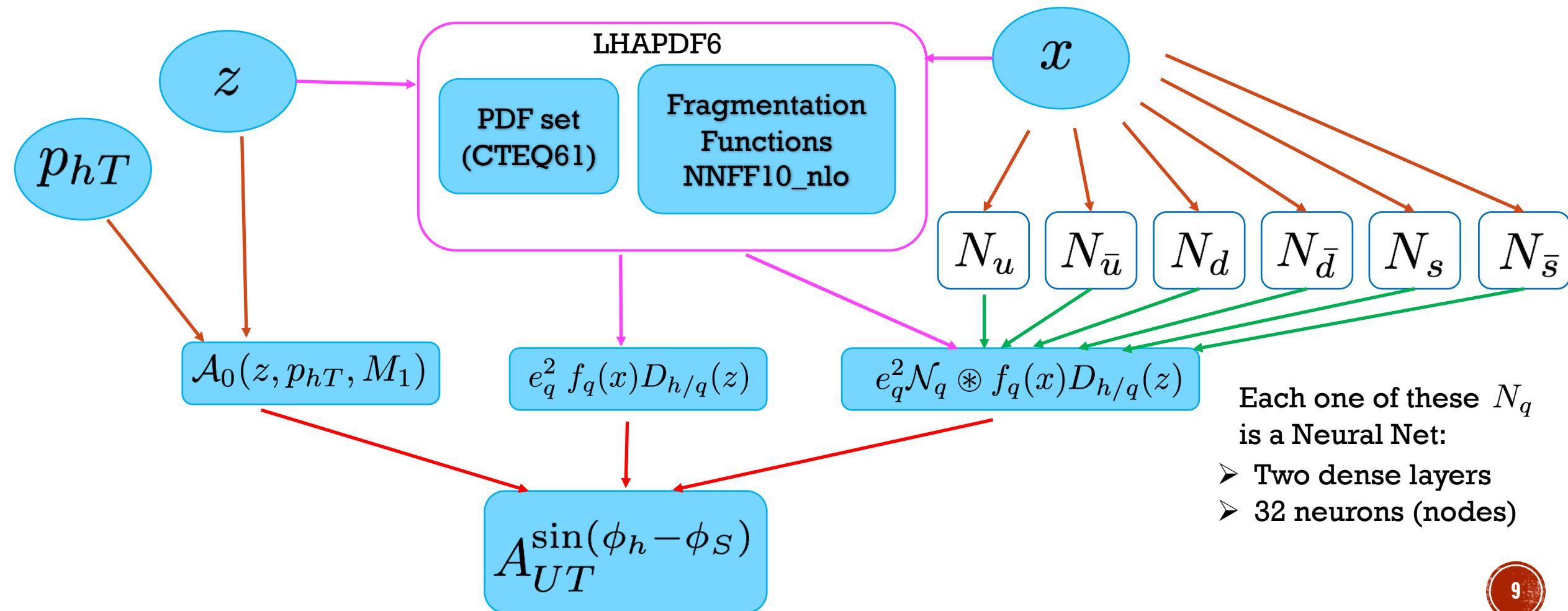
# FITS TO HERMES(2020) [PRELIMINARY]

Hadron	Dependence	ndata	$\chi^2/ndata$
$\pi^+$	$x$	8	2.12
$\pi^+$	$z$	11	1.49
$\pi^+$	$p_{hT}$	8	1.14
$\pi^-$	$x$	8	1.81
$\pi^-$	$z$	11	1.16
$\pi^-$	$p_{hT}$	8	1.20
$\pi^0$	$x$	8	0.40
$\pi^0$	$z$	11	0.95
$\pi^0$	$p_{hT}$	8	0.50
$K^+$	$x$	8	0.48
$K^+$	$z$	11	6.31
$K^+$	$p_{hT}$	8	1.26
$K^-$	$x$	8	0.26
$K^-$	$z$	10	0.93
$K^-$	$p_{hT}$	8	0.79
Total		134	1.477



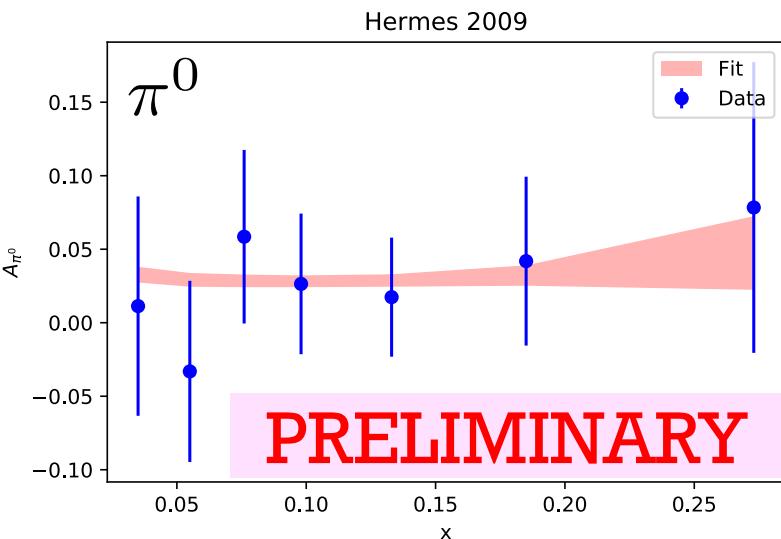
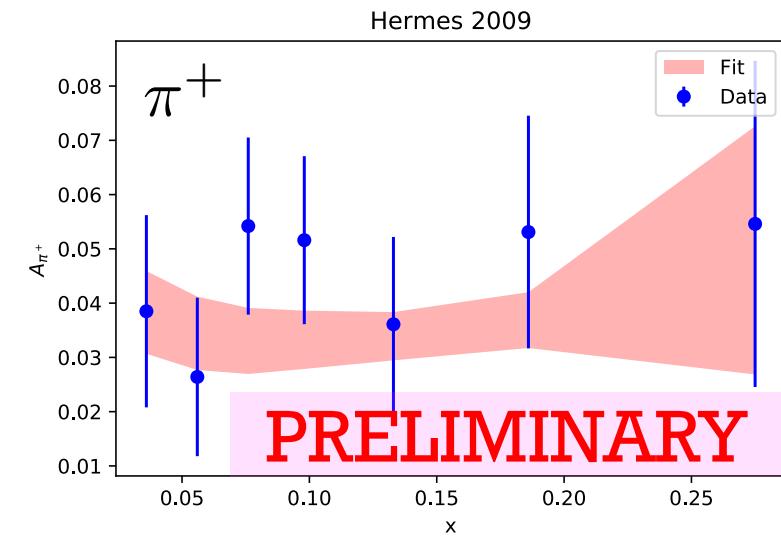
# NEURAL NETWORK APPROACH

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, p_{hT}) = \mathcal{A}_0(z, p_{hT}, M_1) \left( \frac{\sum_q \mathcal{N}_q e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)} \right)$$



# NN FITS TO HERMES(2009) [PRELIMINARY]

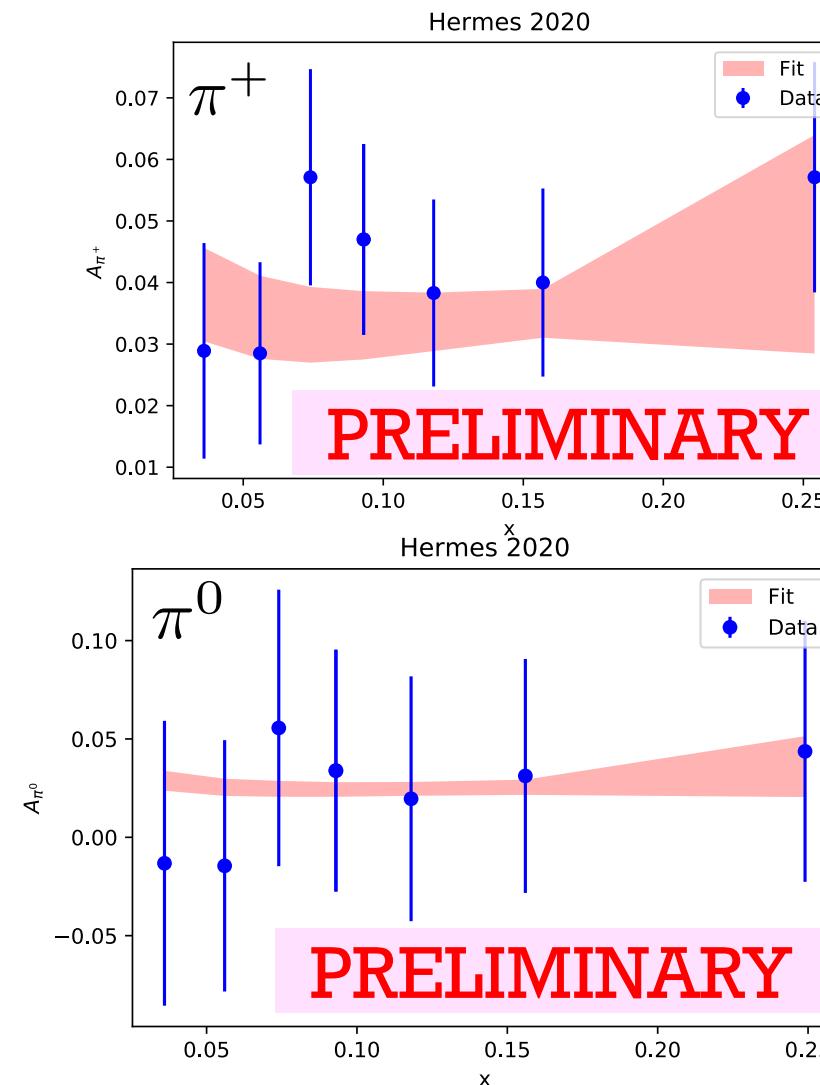
Hadron	Dependence	ndata	$\chi^2/ndata$
$\pi^+$	$x$	7	2.29
$\pi^+$	$z$	7	1.01
$\pi^+$	$p_{hT}$	7	3.40
$\pi^-$	$x$	7	3.13
$\pi^-$	$z$	7	0.52
$\pi^-$	$p_{hT}$	7	1.96
$\pi^0$	$x$	7	0.90
$\pi^0$	$z$	7	1.13
$\pi^0$	$p_{hT}$	7	1.61
$K^+$	$x$	7	1.78
$K^+$	$z$	7	3.69
$K^+$	$p_{hT}$	7	1.29
$K^-$	$x$	7	0.52
$K^-$	$z$	7	0.57
$K^-$	$p_{hT}$	7	0.73
Total		105	1.64



# NN PREDICTIONS FOR HERMES(2020) [PRELIMINARY]

Trained using HERMES 2009 data set

Hadron	Dependence	n data	$\chi^2/n data$
$\pi^+$	$x$	8	2.23
$\pi^+$	$z$	11	1.63
$\pi^+$	$p_{hT}$	8	2.07
$\pi^-$	$x$	8	2.82
$\pi^-$	$z$	11	0.57
$\pi^-$	$p_{hT}$	8	1.44
$\pi^0$	$x$	8	0.50
$\pi^0$	$z$	11	0.97
$\pi^0$	$p_{hT}$	8	0.73
$K^+$	$x$	8	1.45
$K^+$	$z$	11	7.99
$K^+$	$p_{hT}$	8	2.45
$K^-$	$x$	8	0.54
$K^-$	$z$	10	1.11
$K^-$	$p_{hT}$	8	2.93
Total		134	2.02



# SIVERS FUNCTION

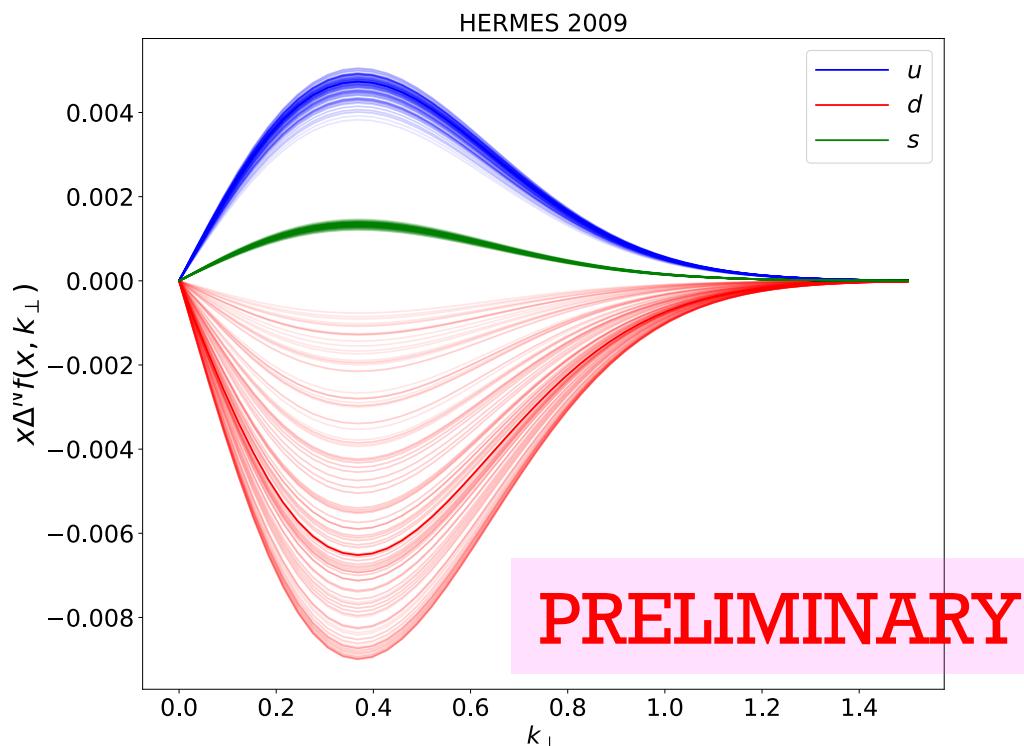
From regular fit results

$$f_{q/p}(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle},$$

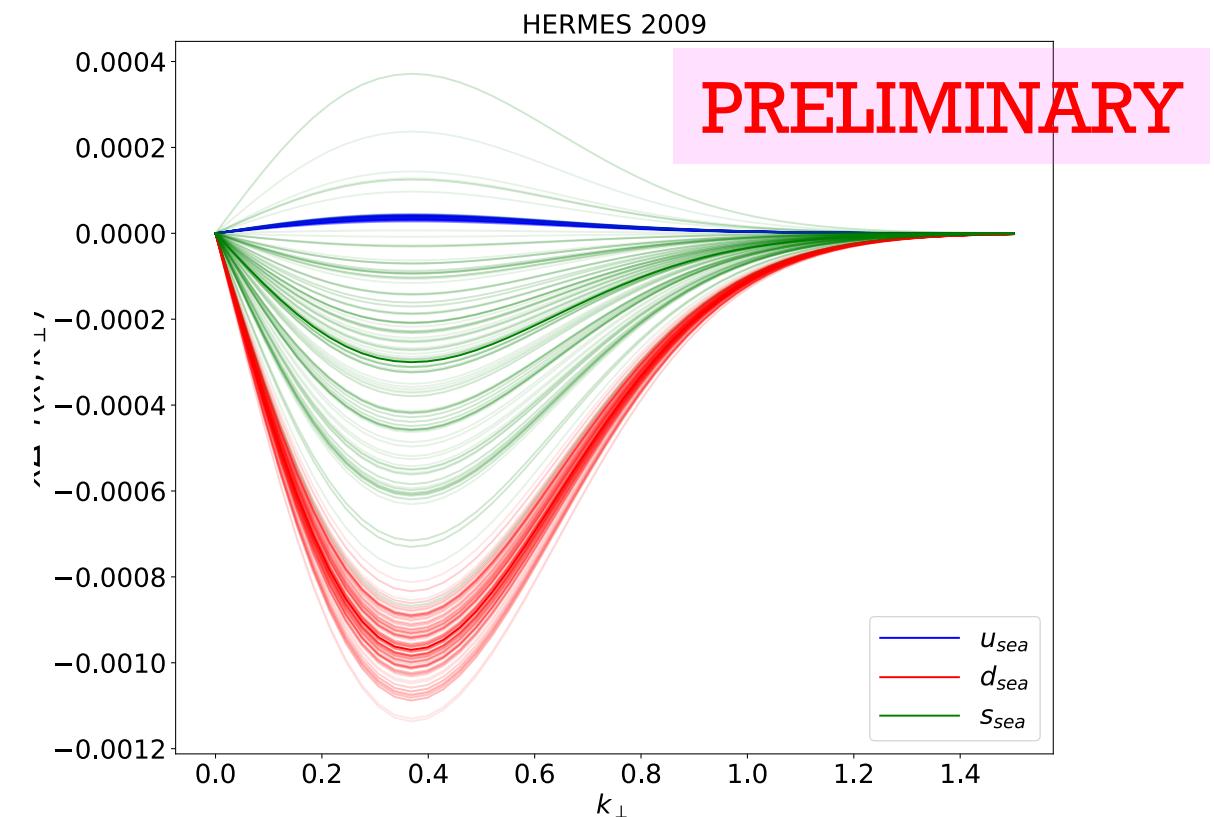
$$\Delta^N f_{q/p\uparrow}(x, k_\perp) = 2\mathcal{N}_q(x) h(k_\perp) f_{q/p}(x, k_\perp),$$

$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}},$$

$$h(k_\perp) = \sqrt{2e} \frac{k_\perp}{M_1} e^{-k_\perp^2 / M_1^2}$$



$$x \Delta^N f_{q/p\uparrow}(x, k_\perp)$$



$$x = 0.1$$

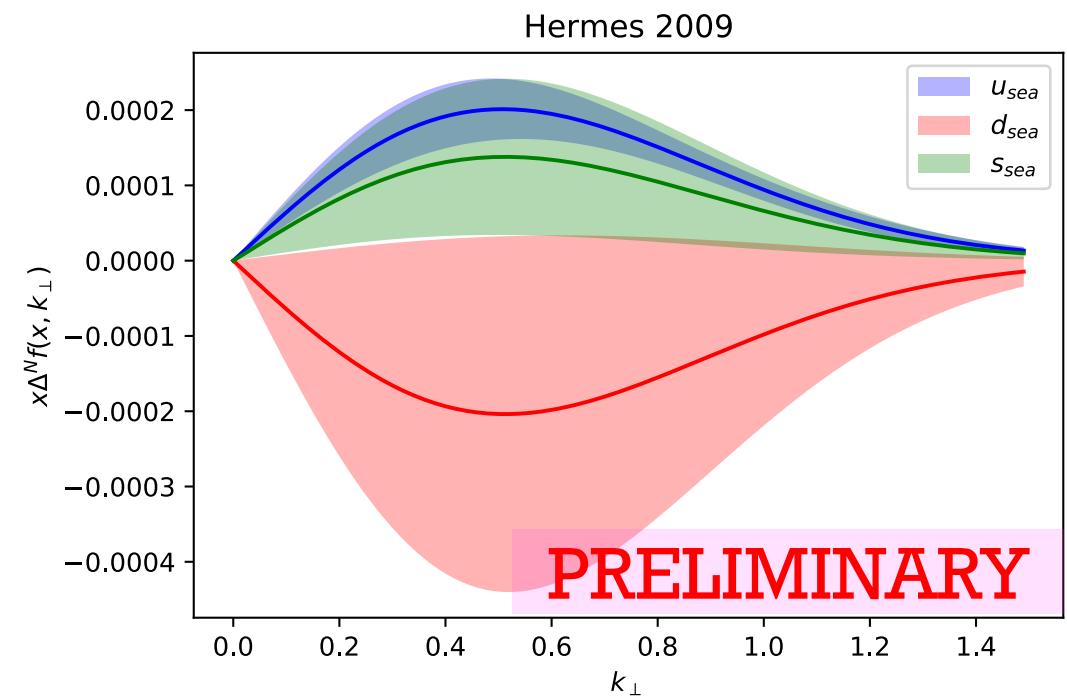
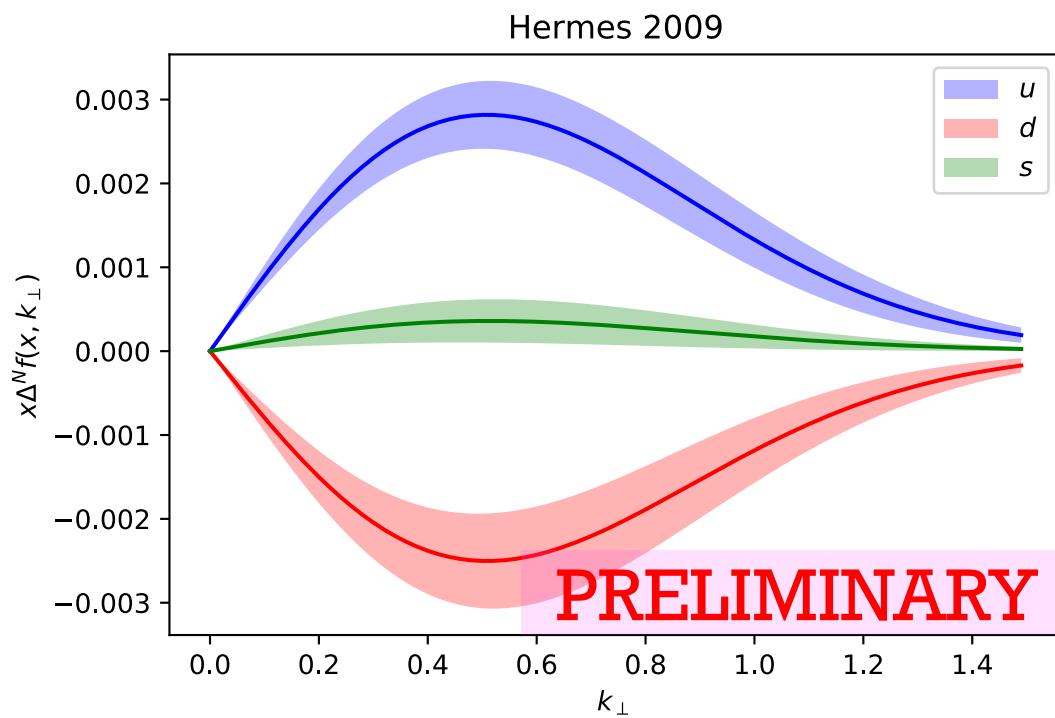
$$Q^2 = 2.4 \text{ GeV}^2/c^2$$

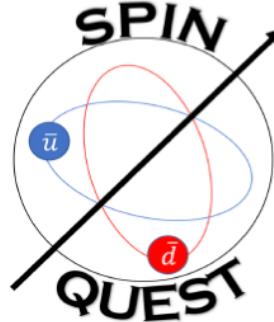
$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

# SIVERS FUNCTION FROM THE NEURAL NETWORK

$$x \Delta^N f_{q/p\uparrow}(x, k_\perp)$$



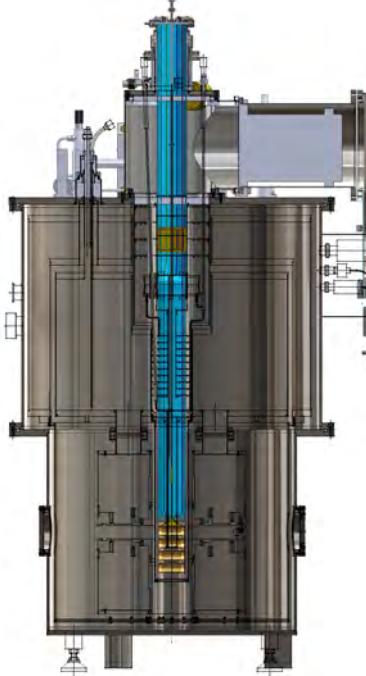


# SPIN-QUEST (E1039) EXPERIMENT AT FERMILAB

➤ First measurement of ‘sea’ quark Sivers function

$$pp \uparrow(d^\uparrow) \rightarrow \mu^+ \mu^- X, 4 < M_{\mu\mu} < 9 \text{ GeV}$$

 Fermilab



LANL-UVA  
Polarized Target

Polarized

N (target)

p (beam)

120 GeV  
proton beam

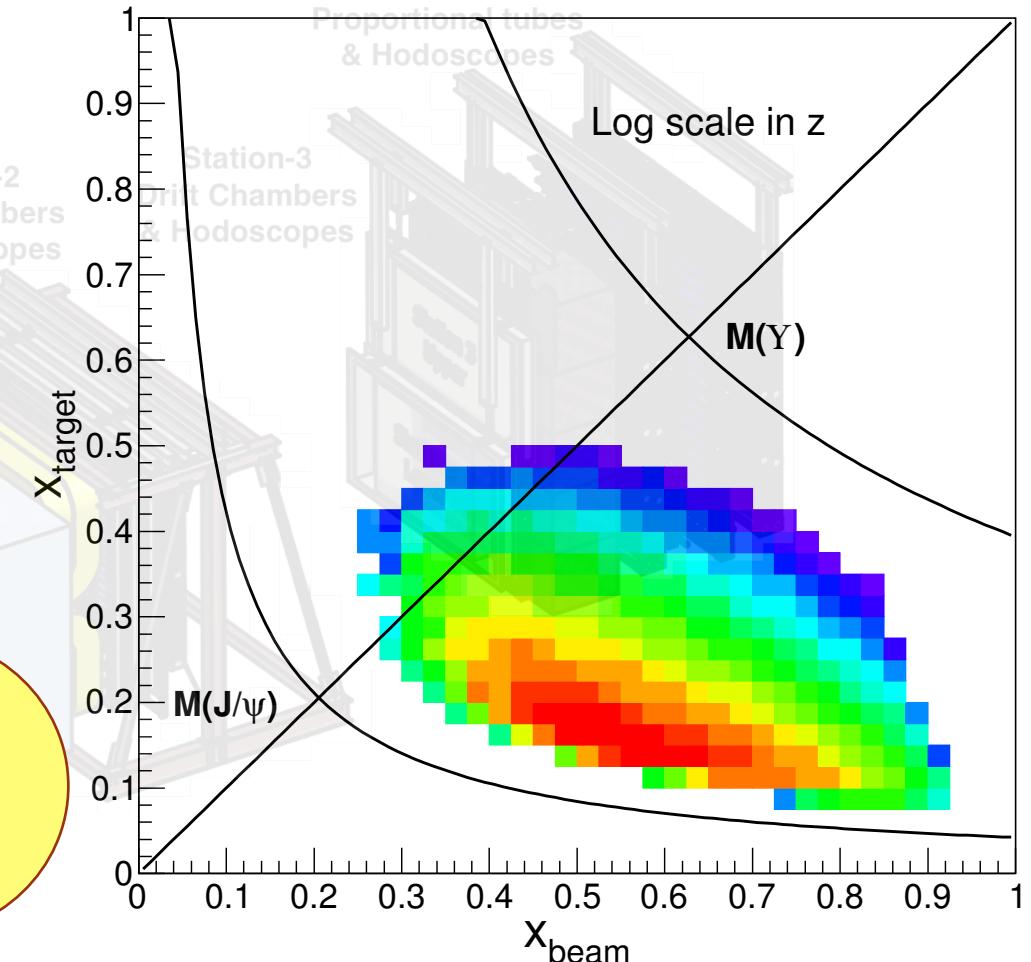
$x_2 \bar{q}$

$x_1 q$

$\gamma^*$

$\mu^+$

$\mu^-$



$$\frac{d\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9sx_1 x_2} \sum_i e_i^2 (q_i^B(x_1, Q^2) \bar{q}_i^T(x_2, Q^2) + \bar{q}_i^B(x_1, Q^2) q_i^T(x_2, Q^2))$$

# DISCUSSION & FUTURE WORK

- Fits to individual data sets can be implemented with the inclusion of s-quarks
- Performing global fits (on-going work)
- Hyperparameter search to optimize the Neural Network (NN)
- Exploring different NN architectures to handle different quark flavors
- Training with more LHAPDF sets
- Investigating towards Sivers Asymmetry extraction from Drell –Yan with/without considering the “sign-flip” of the Sivers Function.

THANK YOU!