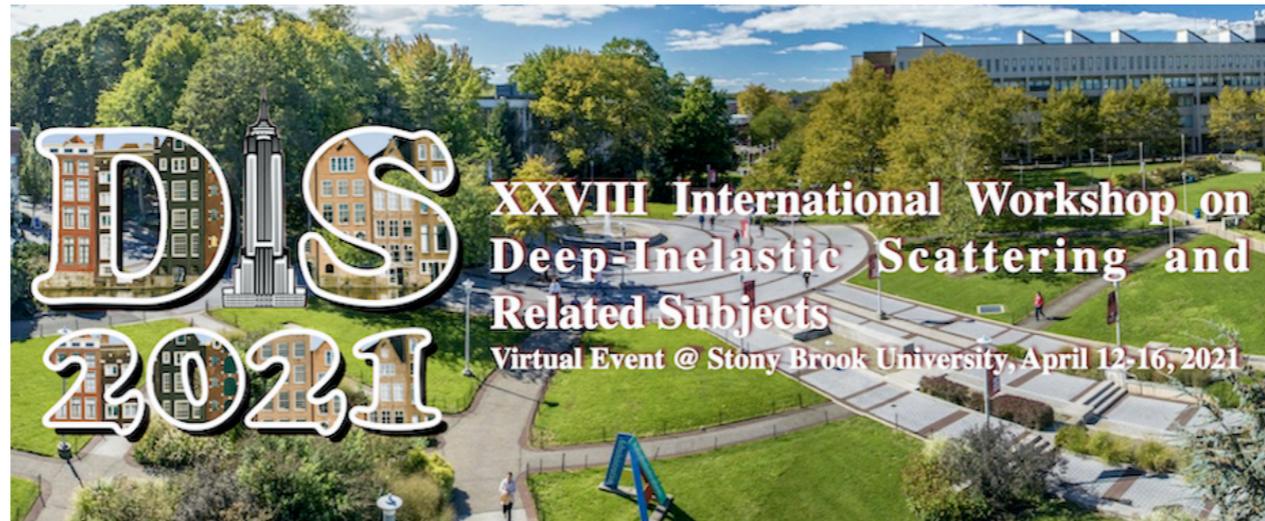


Λ^\uparrow Polarization in e^+e^- collisions



Leonard Gamberg



w/ Zhong-Bo Kang, Ding Yu Shao, John Terry, Fanyi Zhao
arXiv:2102.05553



U.S. DEPARTMENT OF
ENERGY

Office of
Science



Outline

- Motivation to study on Λ^\uparrow physics long standing challenge describe via QCD factz.
- Review “outsized” role of Lambda in studying TSSAs look @ data
- Twist -2 TMD fact. description in terms of PFF. $D_{1T}^\perp(z, p_\perp, Q^2)$
 - ★ Thrust observable $\Lambda(\text{Thrust}) + X \quad e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust}) X$
 - ★ Back to back hadrons $h + \Lambda \quad e^+e^- \rightarrow \Lambda^\uparrow h X$
- Inclusive process $e^+e^- \rightarrow \Lambda^\uparrow X$ possible & interesting to process to study
- Twist -3 fact. description in terms of $D_T(z, Q^2)$
 - * Change of ref frame COM of e^+e^- pair $e^+e^- \rightarrow \Lambda^\uparrow X$
Test of naive time reversal in QCD

Dilemma

VOLUME 41, NUMBER 25

PHYSICAL REVIEW LETTERS

18 DECEMBER 1978

Transverse Quark Polarization in Large- p_T Reactions, e^+e^- Jets, and Leptoproduction: A Test of Quantum Chromodynamics

G. L. Kane

Physics Department, University of Michigan, Ann Arbor, Michigan 48109

and

J. Pumplin and W. Repko

Physics Department, Michigan State University, East Lansing, Michigan 48823

(Received 5 July 1978)

We point out that the polarization P of a scattered or produced quark is calculable perturbatively in quantum chromodynamics for $e^+e^- \rightarrow q\bar{q}$, large- p_T hadron reactions, and large- Q^2 leptoproduction, and is infrared finite. The quantum-chromodynamics prediction is that $P=0$ in the scaling limit. Experimental tests are or will soon be possible in $pp \rightarrow \Lambda X$ [where presently $P(\Lambda) \simeq 25\%$ for $p_T > 2$ GeV/c] and in $e^+e^- \rightarrow$ quark jets.

In this note we have pointed out that the asymmetry off a polarized target, and the transverse polarization of a produced quark in $e^+e^- \rightarrow q\bar{q}$, or in $qq \rightarrow qq$ at large p_T , or in leptoproduction, should all be calculable perturbatively in QCD. The result is zero for $m_q = 0$ and is numerically small if we calculate m_q/\sqrt{s} corrections for light quarks. We discuss how to test the predictions.

At least for the cases when P is small, tests should be available soon in large- p_T production [where currently $P(\Lambda) = 25\%$ for $p_T \gtrsim 2$ GeV/c], and e^+e^- reactions. While fragmentation effects could dilute polarizations, they cannot (by parity considerations) induce polarization. Consequently, observation of significant polarizations in the above reactions would contradict either QCD or its applicability.

TMD factorization says otherwise:

Mulders Tangerman, NPB1996

Boer, Jakob, Mulders NPB1997, 2000

Anselmino Boer, D'Alesio, Murgia. PRD 2001, 2002

Boer, Kang, Vogelsang, Yuan, PRL 2010

Measurement of Lambda-polarization through weak decay $\Lambda^0 \rightarrow p \pi^-$

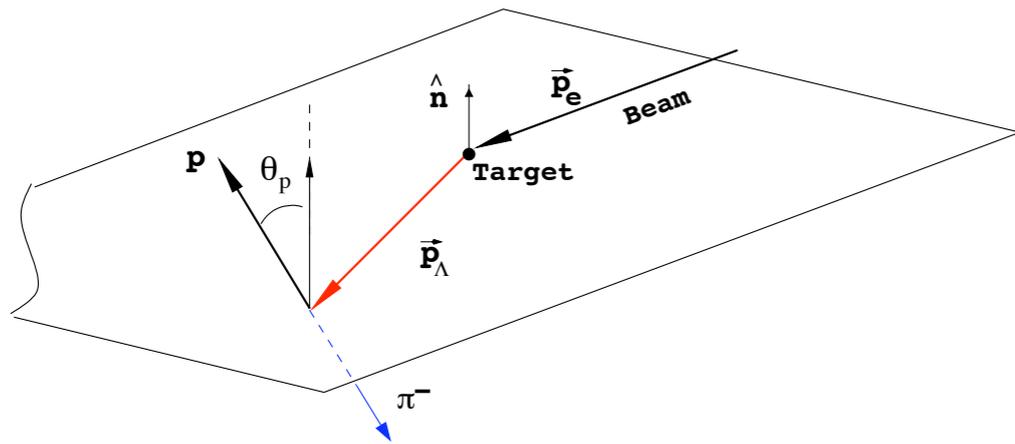


FIG. 1: Schematic diagram of inclusive Λ production and decay. The angle θ_p of the decay proton with respect to the normal \hat{n} to the production plane is defined in the Λ rest frame.

- Proton preferentially emitted along Λ -spin
- In Λ rest frame: pol. decay distribution

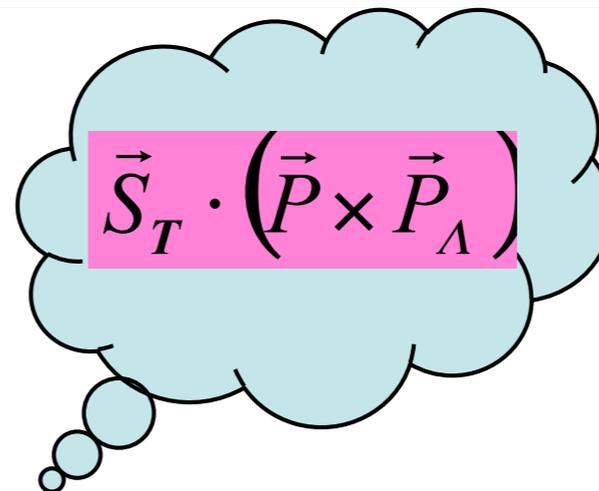
$$\left(\frac{dN}{d\Omega_p}\right)_{\text{pol}} = \left(\frac{dN}{d\Omega_p}\right)_{\text{unpol}} (1 + \alpha P_n^\Lambda \cos(\theta_p))$$

P^Λ : Transverse Lambda Polarization

$$P_\perp^\Lambda(z_a, j_\perp) = \frac{d\Delta\sigma}{dz_\Lambda d^2j_\perp} \bigg/ \frac{d\sigma}{dz_\Lambda d^2j_\perp}$$

QCD is Parity conserving so any final state hadron must be polarised perpendicular to the production plane

The Status of Transverse Spin Physics . . .



What does Exp Say ...

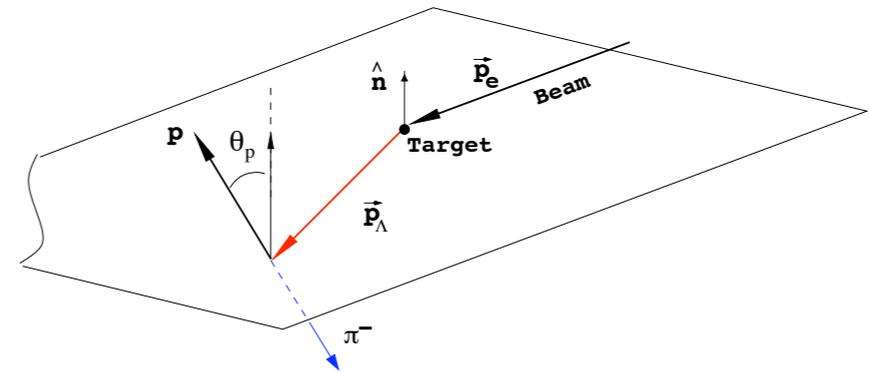
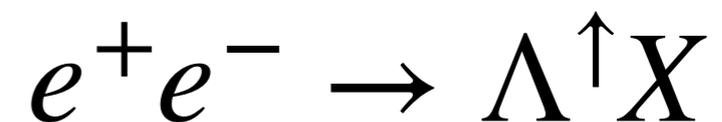
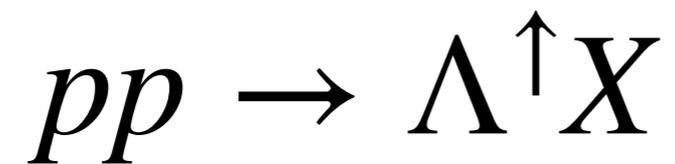
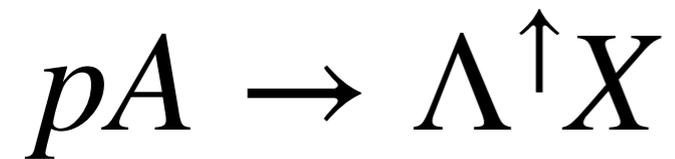
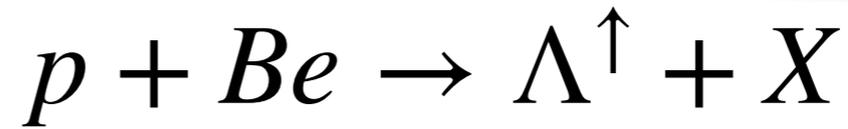


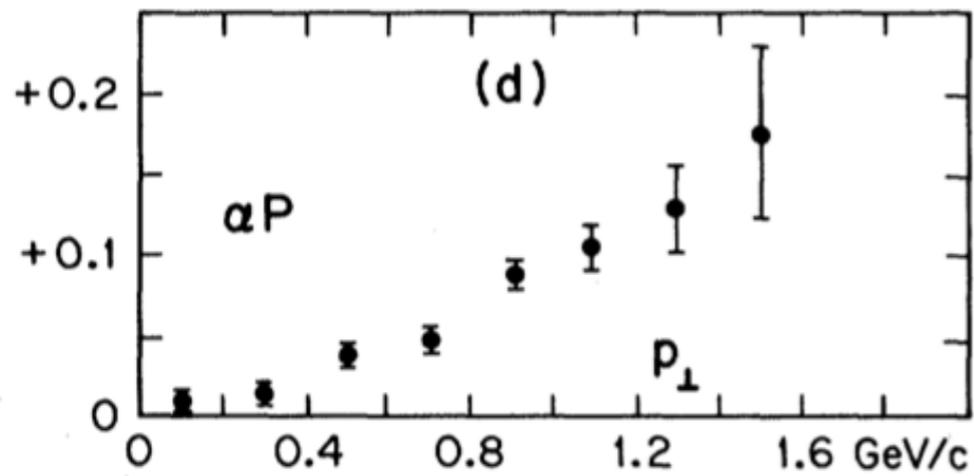
FIG. 1: Schematic diagram of inclusive Λ production and decay. The angle θ_p of the decay proton with respect to the normal \hat{n} to the production plane is defined in the Λ rest frame.

Transverse Λ polarisation a long history

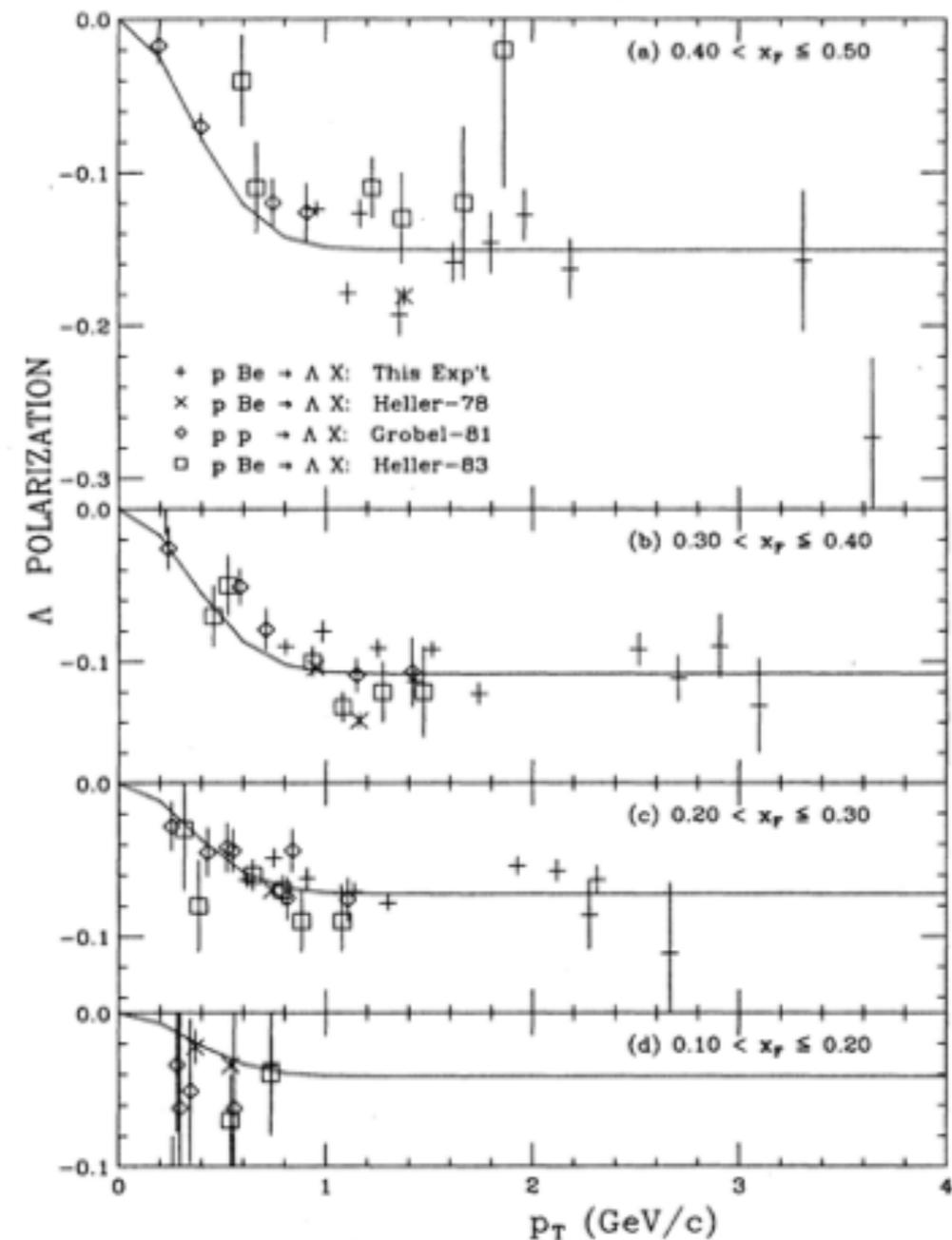
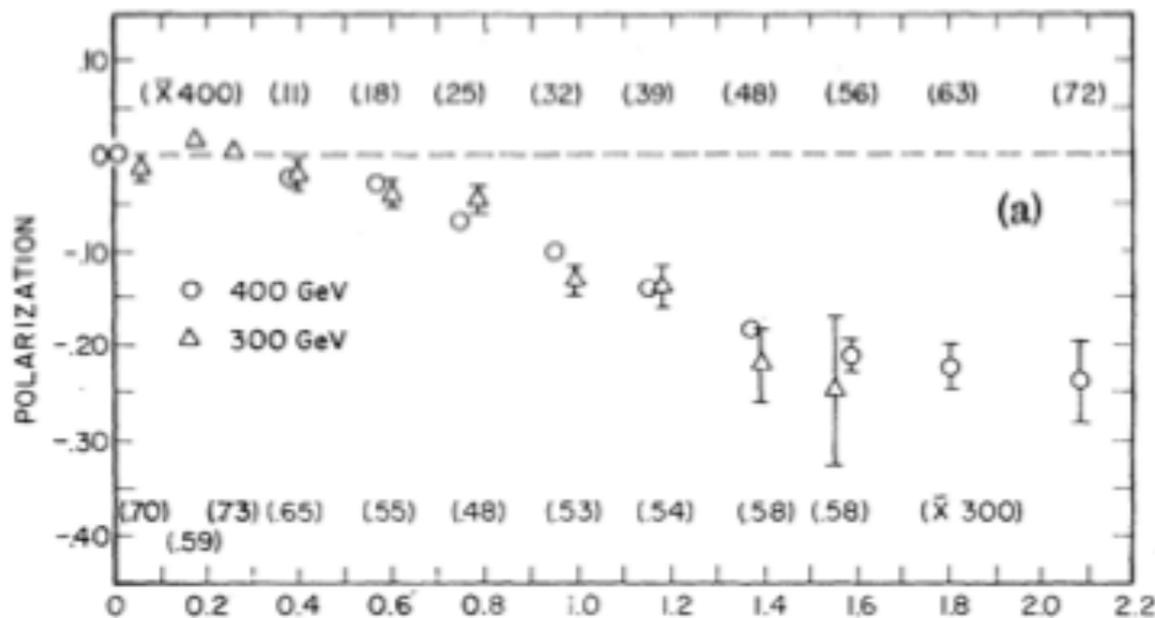


One of the first transverse spin effects at **Fermilab** (1976):

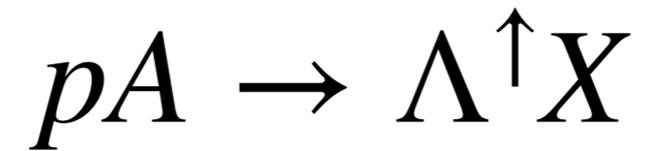
PRD 89 Lundberg



Bunce PRL 76
Heller PRL 78



Proton-Nuclei cont ...



V. Fanti et al.: NA 48 450 GeV proton energy

Eur. Phys. J. C 6, 265–269 (1999) **CERN SPS**

Lundberg et al PRD40 (1989) 400 GeV

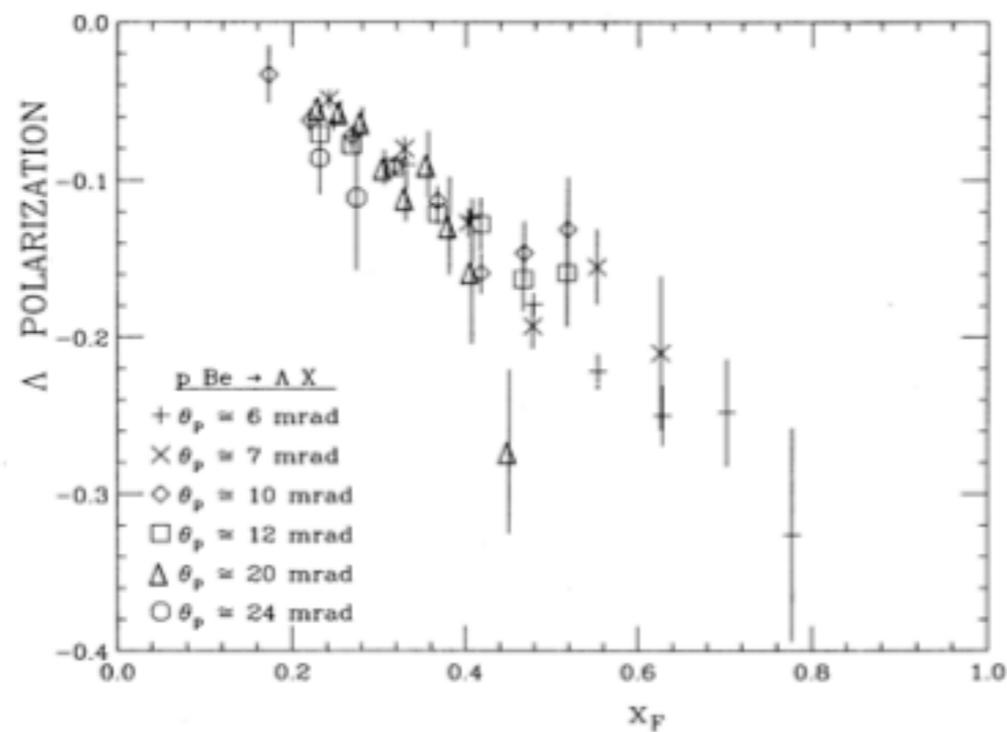
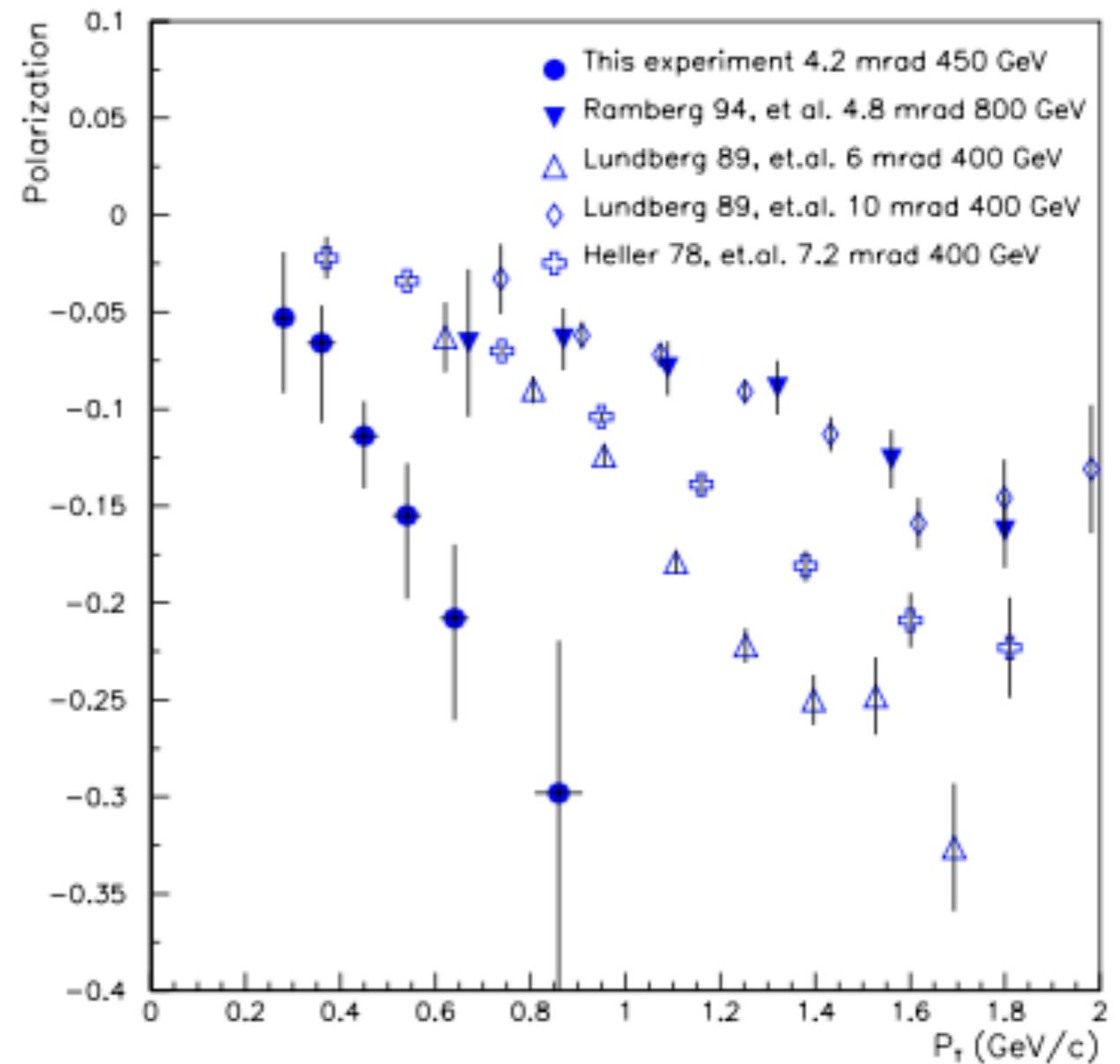
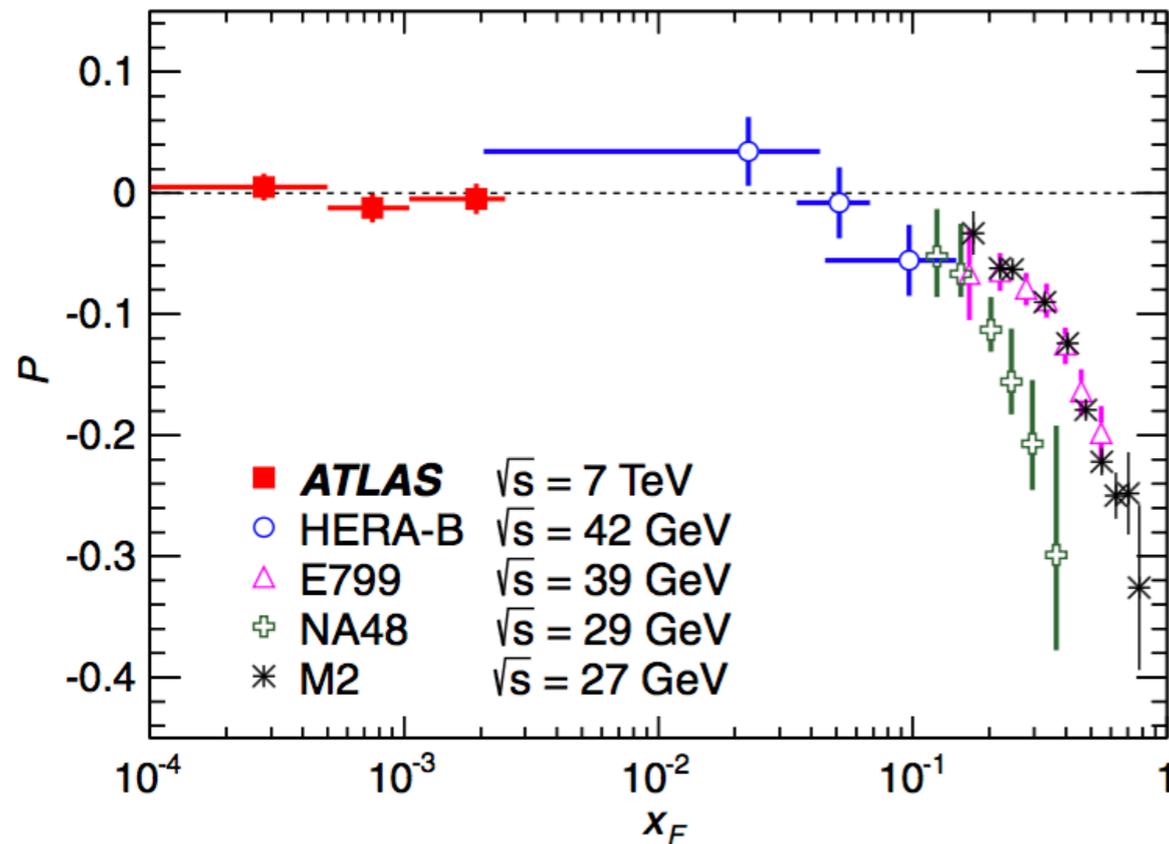


FIG. 4. The Λ polarization is shown as a function of x_F for all production angles. Over this range of production angles and within experimental uncertainties, the polarization is angle (or p_T) independent.



What about LHC?

Is it feasible at a high energy collider?



Recent ATLAS measurement
at $\sqrt{S} = 7$ TeV

PRD 91, 032004 (2015)

**Small Polarisation at mid rapidity but
Such expts. demonstrate
feasibility to study Λ^\uparrow @ hi energy**

What does Exp Say ...

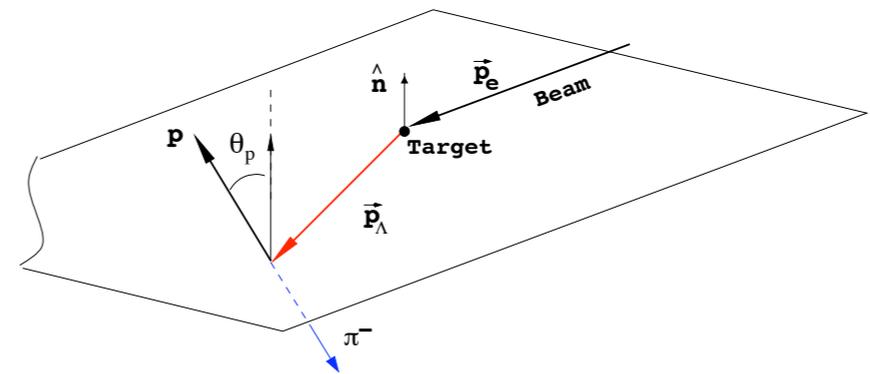
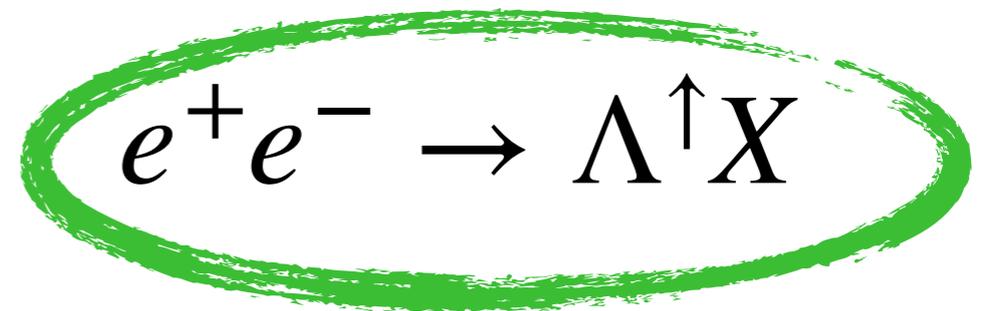
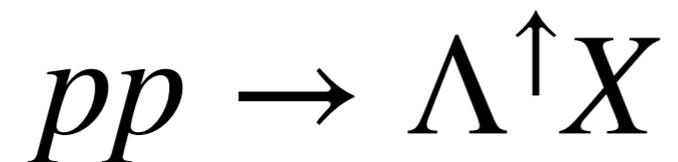
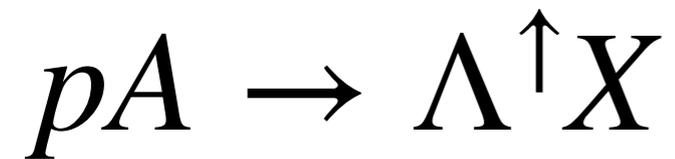
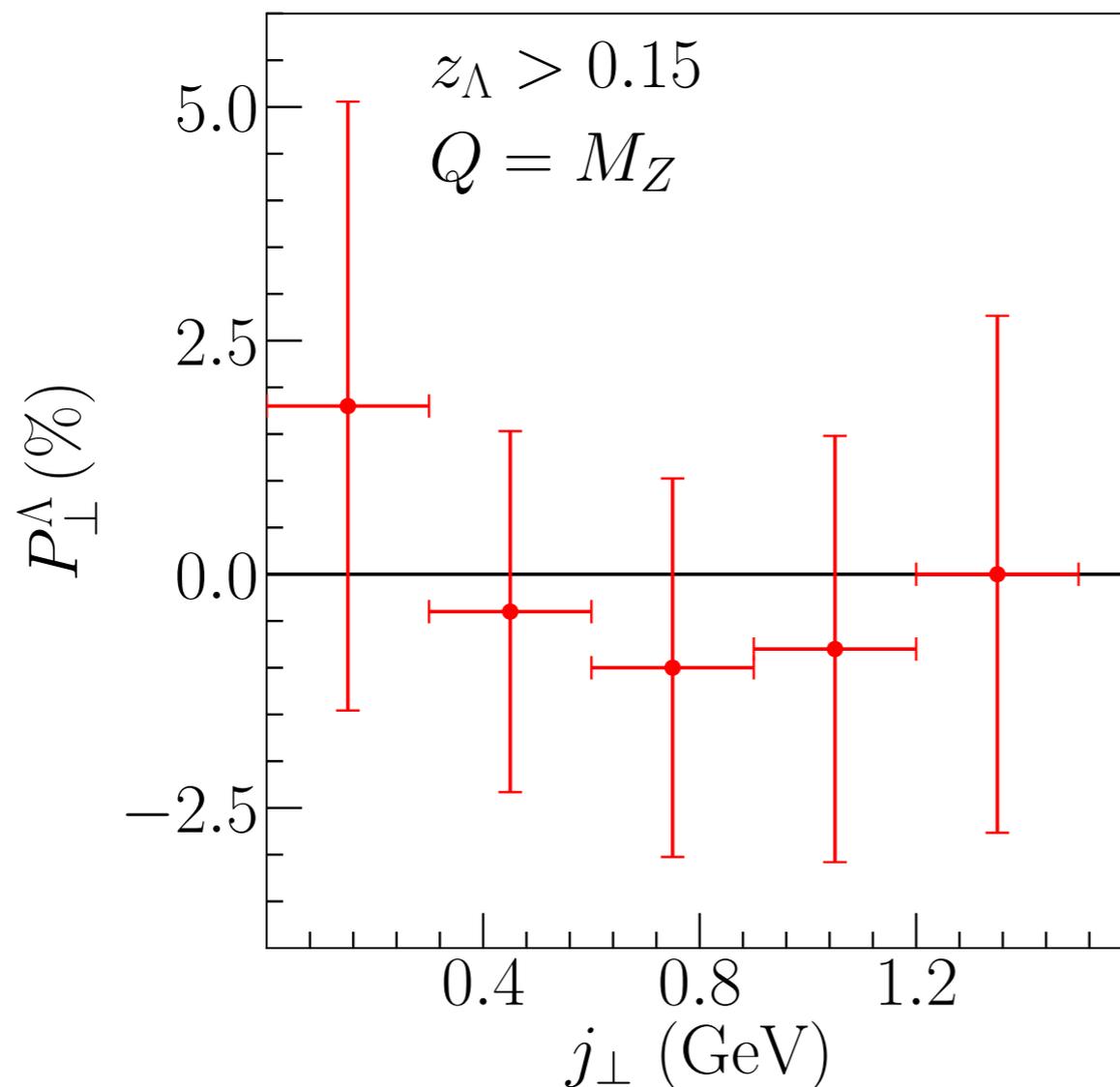


FIG. 1: Schematic diagram of inclusive Λ production and decay. The angle θ_p of the decay proton with respect to the normal \hat{n} to the production plane is defined in the Λ rest frame.

Simplest and cleanest process : $e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust}) X$

OPAL at LEP at Z-pole [Eur.Phys.J C2, 49 (1998)]

Longitudinal Polarization, small/zero Transverse Polarization w/ errors



QCD is Parity Conserving TSSAs Scattering plane transverse to spin
Naively "T-odd"

$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times \mathbf{P}_{\perp}) \otimes (\text{"T-odd" QCD-phases})$$

Spin orbit

Simplest and cleanest process Λ^\uparrow in e^+e^-

Belle data: Transverse Polarization

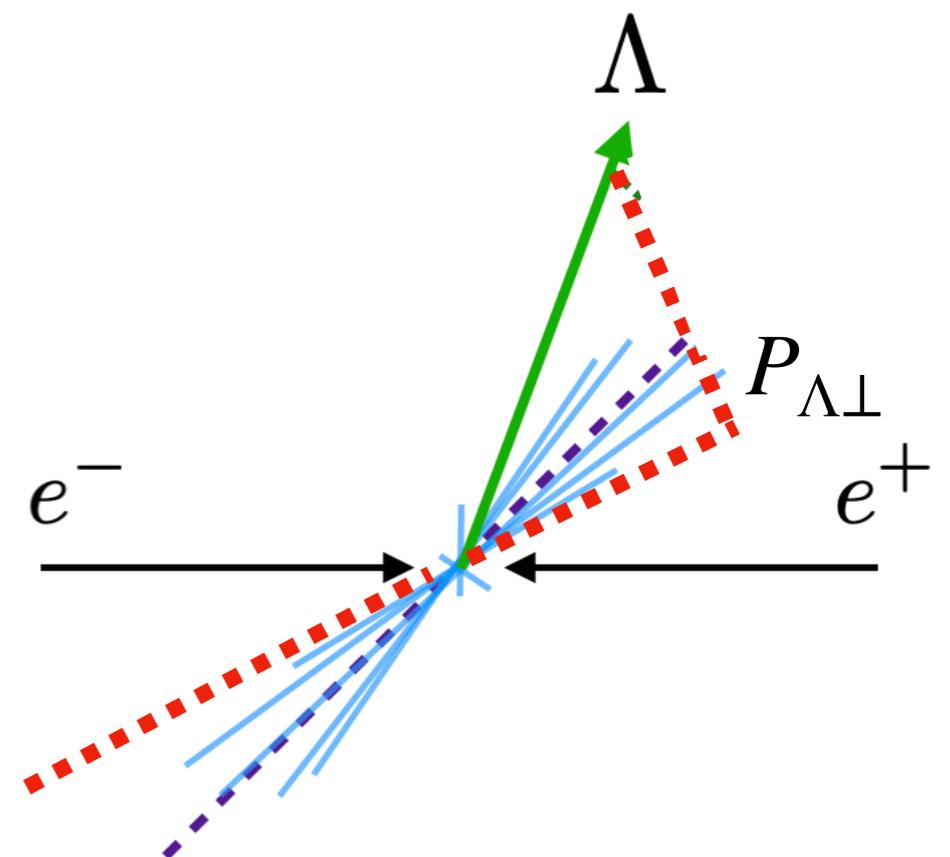
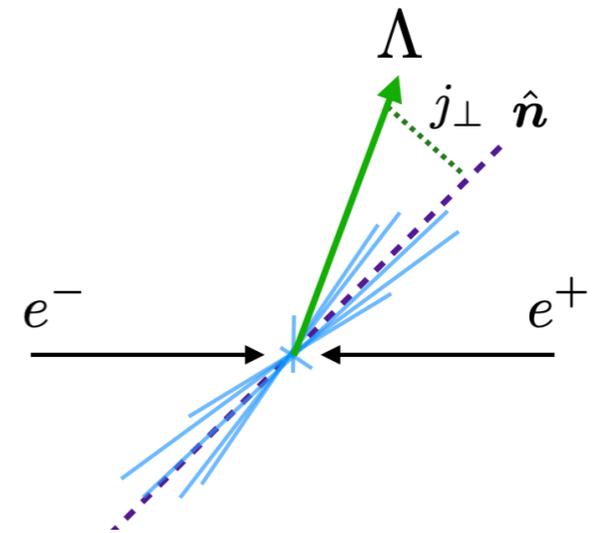
Y. Guan, et al. PRL 122 (2019) → talk by Anselm here @ Jets Workshop

⇒ significant transverse polarization

$$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust}) X$$

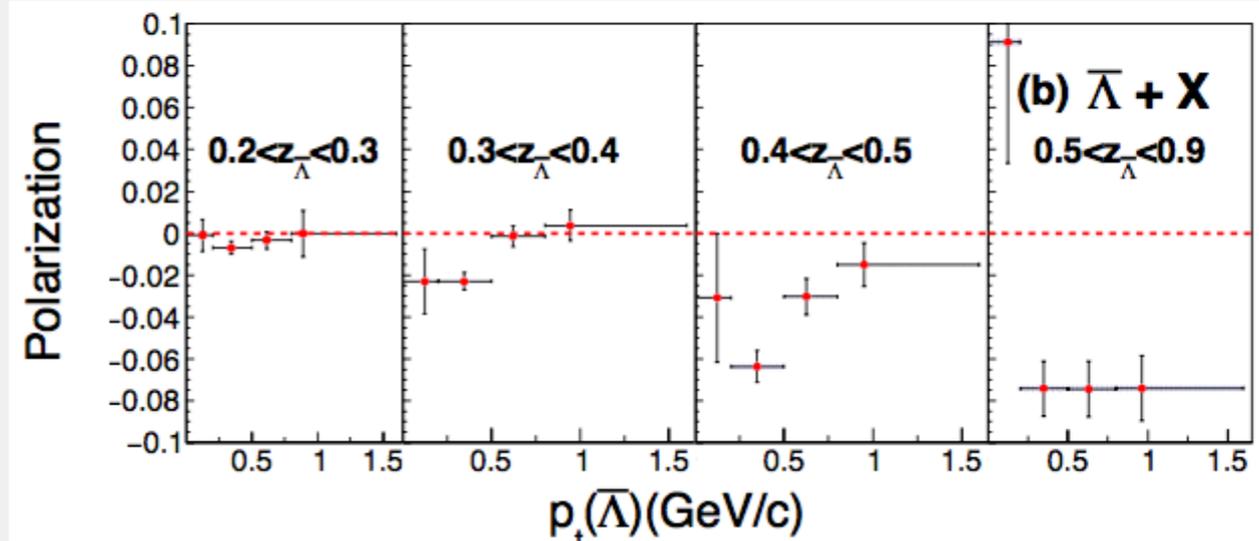
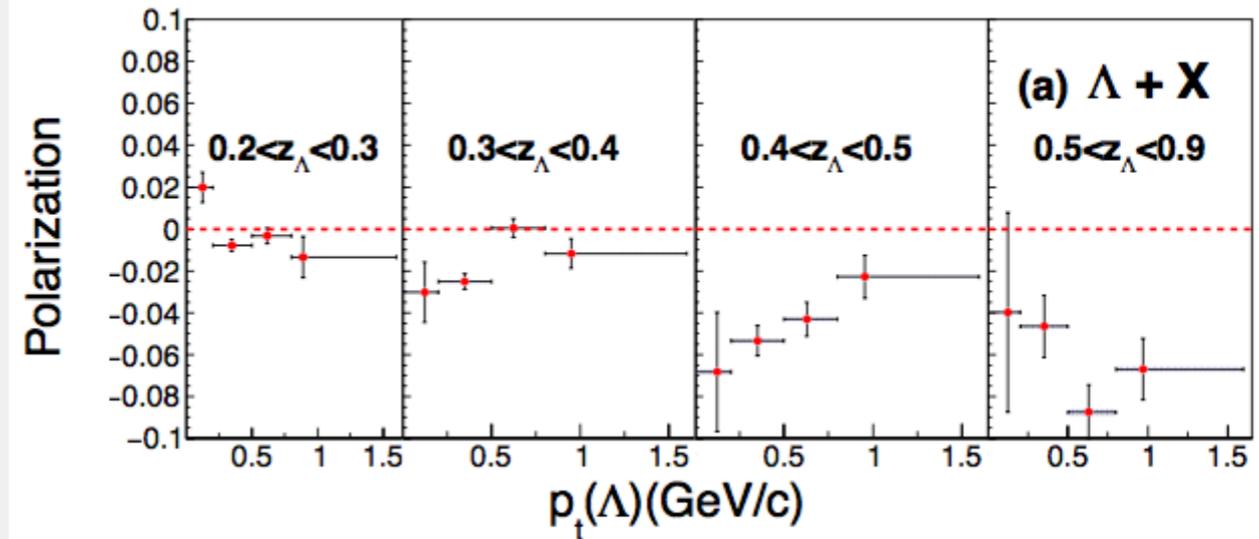
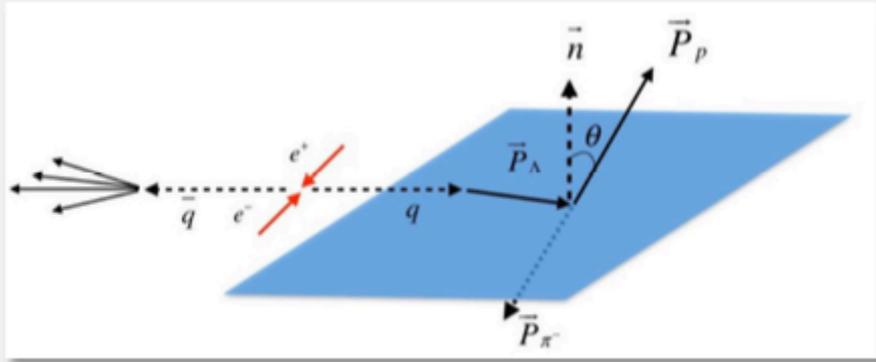
$$e^+e^- \rightarrow \Lambda^\uparrow h X,$$

Measured w.r.t. thrust axis &
back to back hadrons="bTOb"



FIRST OBSERVATION BY BELLE

Thrust



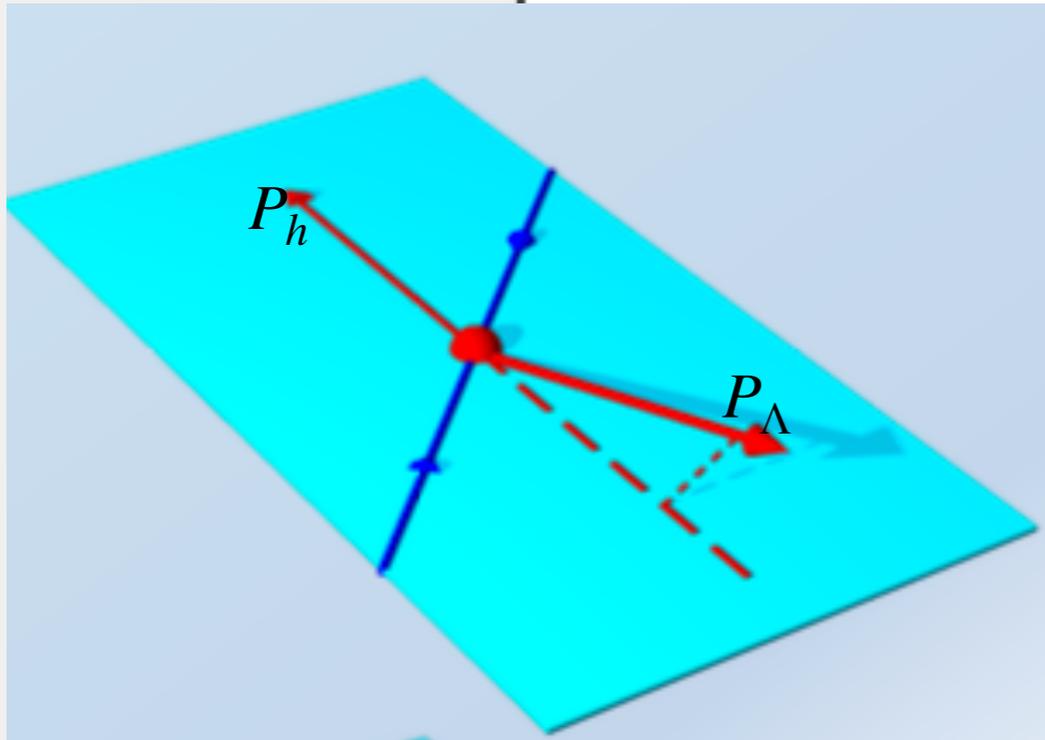
PHYSICAL REVIEW LETTERS 122, 042001 (2019)

From Anselm's INT talk

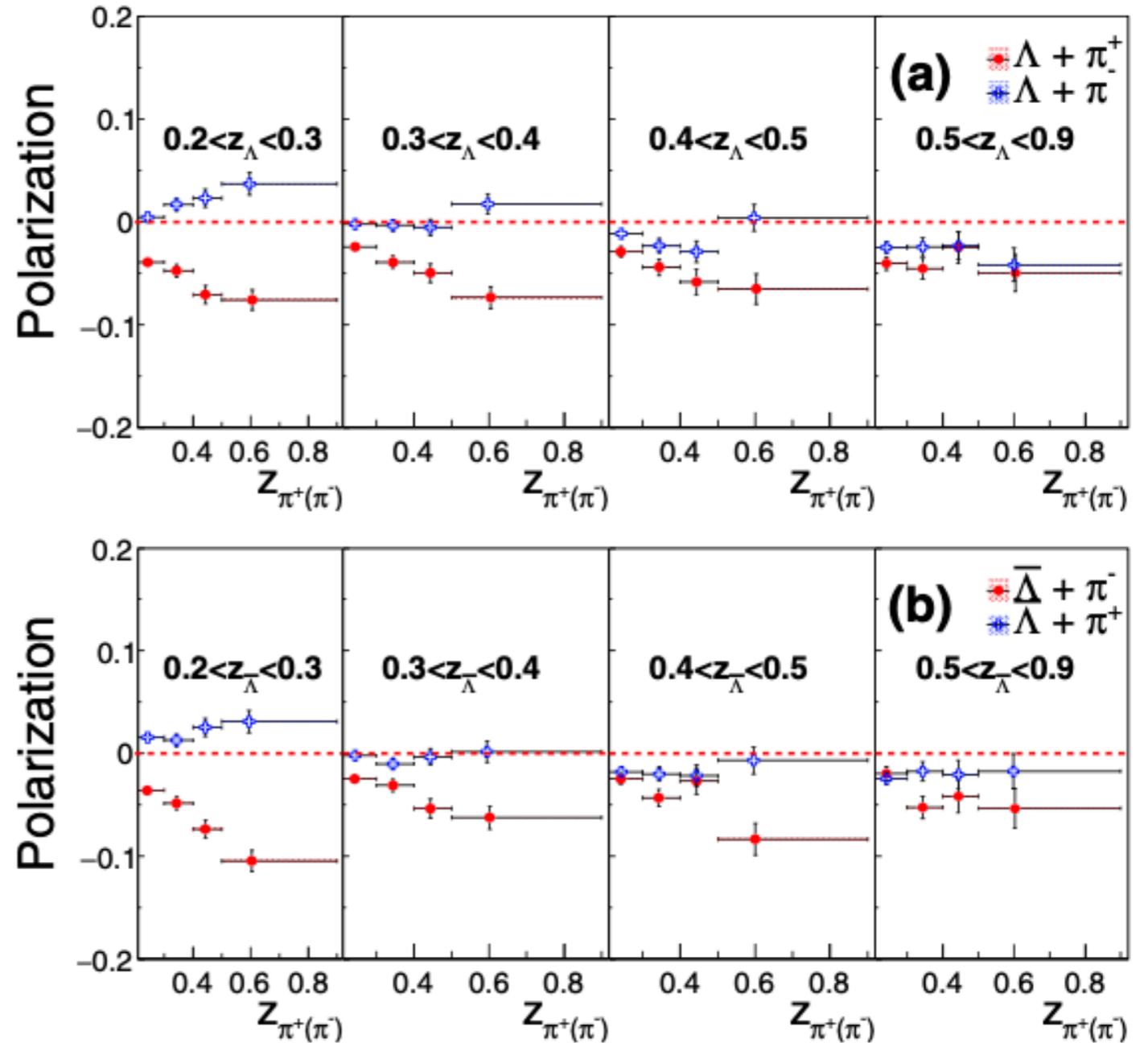
The P_t is measured as the transverse momentum of Λ relative to the **thrust axis**

$$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust}) X$$

FIRST OBSERVATION BY BELLE bTOB



From Anselm's INT talk



$$e^+e^- \rightarrow \Lambda^\uparrow h X$$

Back to back hadrons
integrated over p_\perp NOT SMALL

Question for global analysis & to test Universality Belle BeS BaBar + EIC

$$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust}) X$$

$$e^+e^- \rightarrow \Lambda^\uparrow h X,$$

Questions/issues: is “mechanism” the same ??

- TMD factorization “2” two scale fact. Theorems ?

*TMD factorization formalism ?? for thrust axis measurement

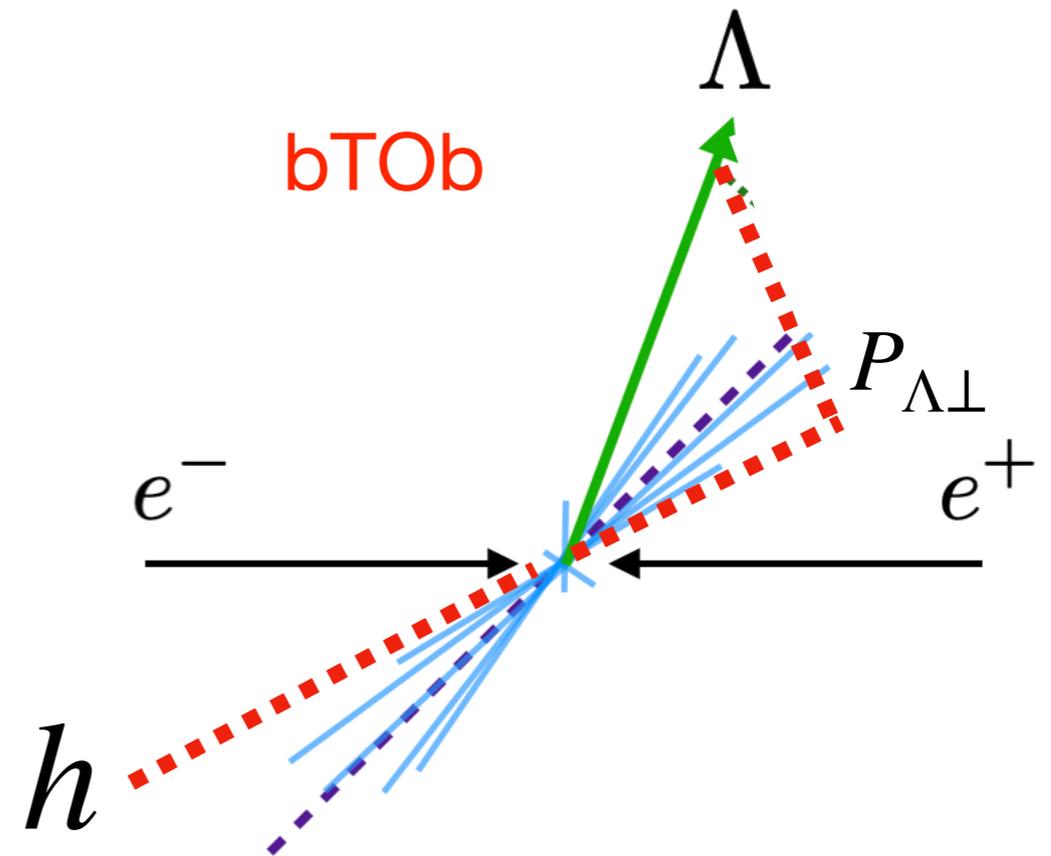
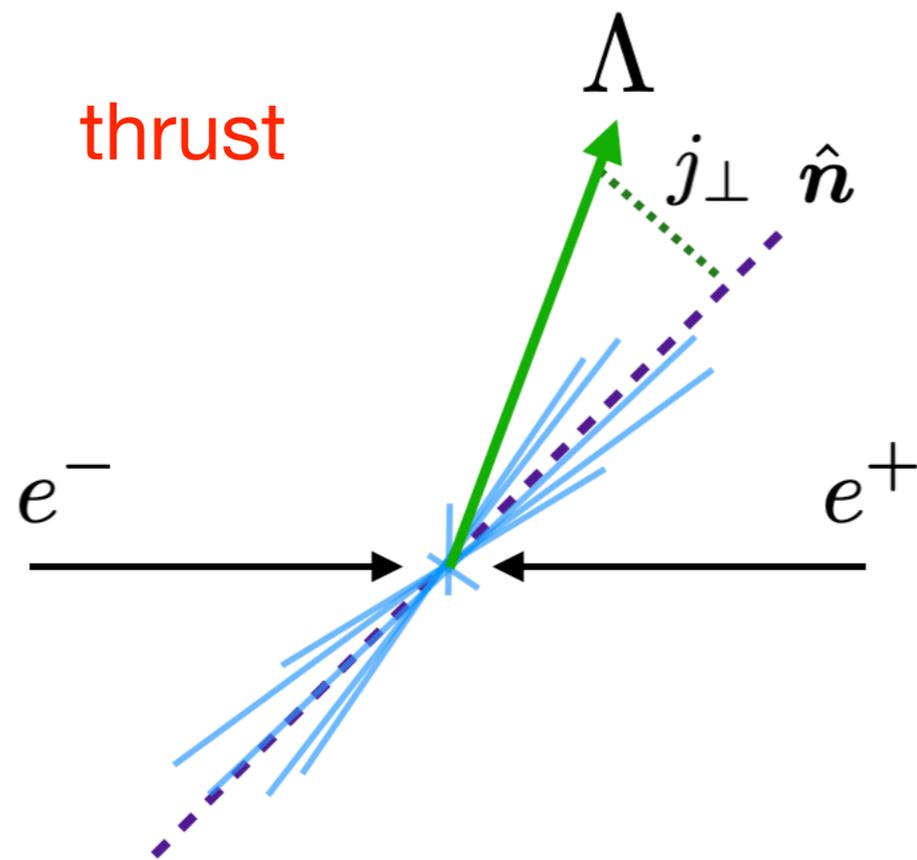
-

$$\Lambda_{QCD} \lesssim p_\perp \ll Q$$

$$\Lambda_{QCD} \lesssim j_\perp \ll Q$$

Global analysis test Universality Belle BeS BaBar + EIC

Is it same PFF function in bTOB hadron \mathcal{E} hadron + thrust measurements ?



$$T = \frac{\sum_i |\mathbf{p}_i \cdot \hat{n}|}{\sum_i |\mathbf{p}_i|}$$

The thrust axis defined by vector, \hat{n} which maximizes the thrust variable T

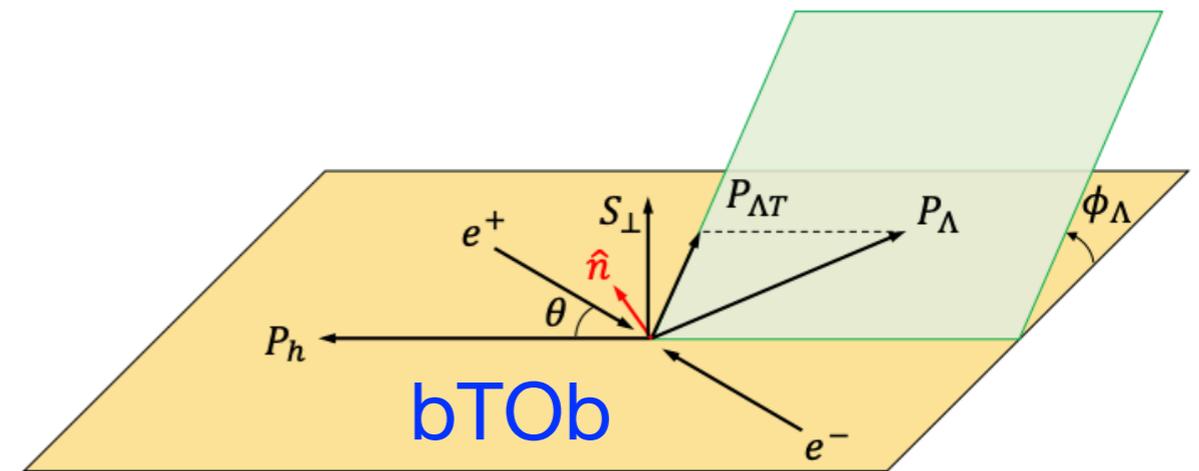
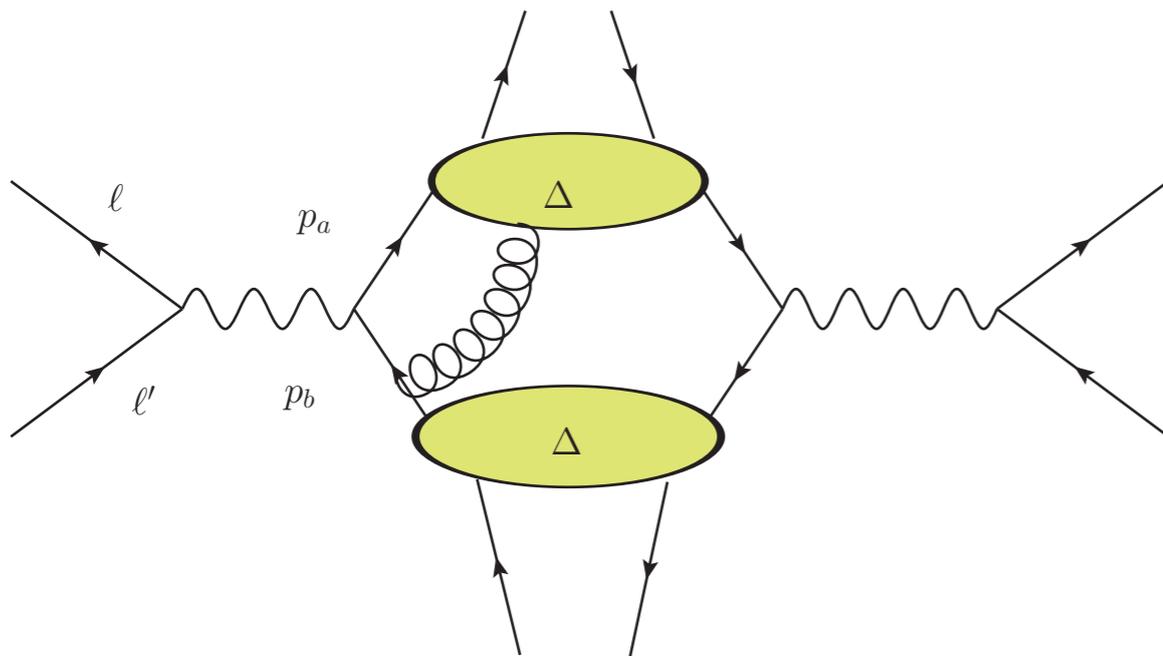
- What about “T”-odd universality can we test it with all data?

Explain non trivial P_{Λ^\uparrow} via TMD FFs
 polarization fragmentation function PFF unsurpressed

TMD framework for **bTOb** production of
 $\Lambda + h$ chiral even, naively T-odd fragmentation function, universal

Parton Model factorization *Mulders & Tangerman 1996, Boer Jakob Mulders 1996*

Boer & Mulders 1997



$$\hat{D}_{\Lambda/q}(z_\Lambda, \mathbf{p}_\perp, \mathbf{S}_\perp, Q) = \frac{1}{2} \left[D_{\Lambda/q}(z_\Lambda, p_{\Lambda\perp}, Q) + \frac{1}{z_\Lambda M_\Lambda} D_{1T,\Lambda/q}^\perp(z_\Lambda, p_\perp, Q) \epsilon_{\perp\rho\sigma} p_\perp^\rho S_\perp^\sigma \right]$$

Explain via TMD fact.

TMD PDFs (x, k_T)

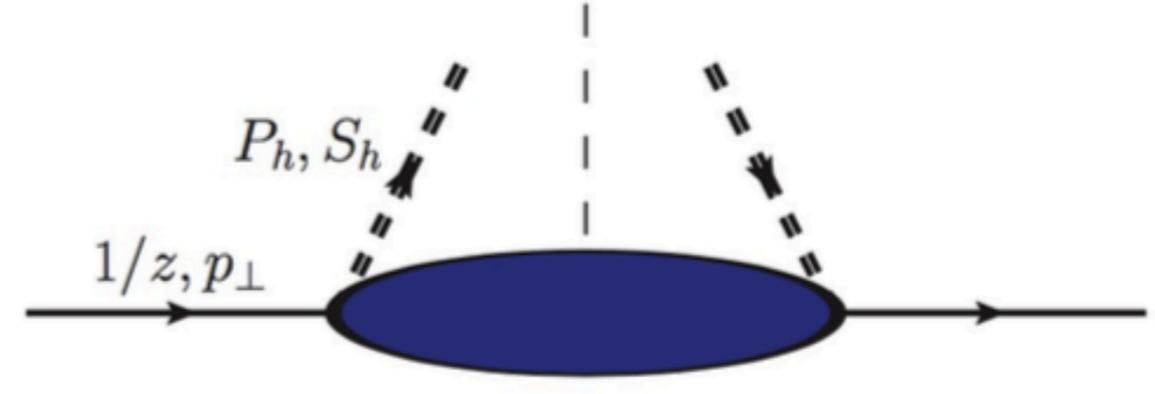
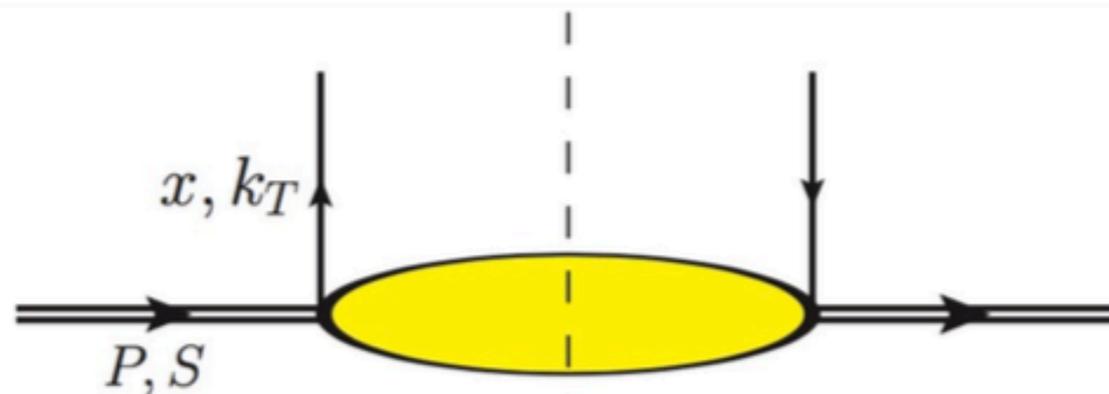
q pol. \ H pol.	U	L	T
U	f_1		h_1^\perp
L		g_{1L}	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_{1T} h_{1T}^\perp

(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

TMD FFs (z, p_\perp)

q pol. \ H pol.	U	L	T
U	D_1		H_1^\perp
L		G_{1L}	H_{1L}^\perp
T	D_{1T}^\perp	G_{1T}	H_{1T} H_{1T}^\perp

(Boer, Jakob, Mulders (1997))



Courtesy of Daniel Pitonyak

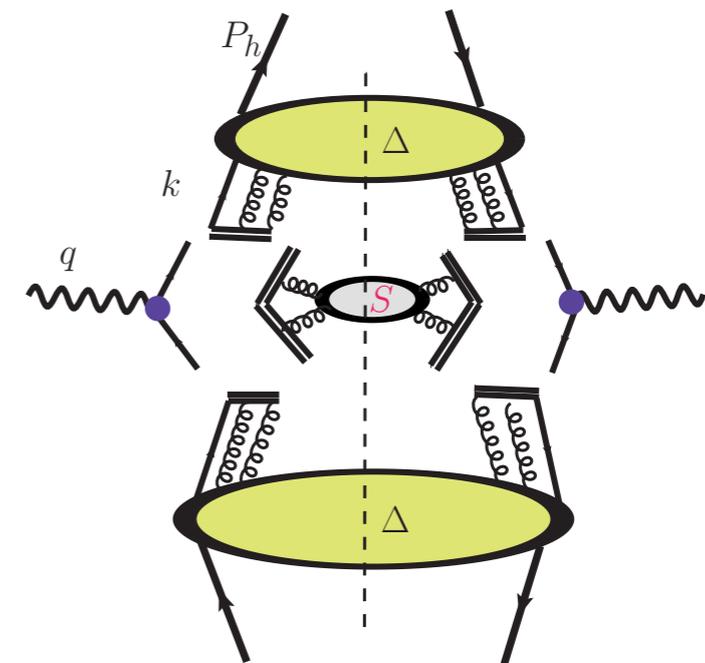
bTOb beyond leading order TMD Factorization

QCD factorization Collins Soper 1982 NPB,
Collins Foundations of PQCD Cambridge Press 2011

Collins Soper (81,82), Collins, Soper, Sterman (85),
Boer (01) (09) (13), Ji, Ma, Yuan (04,05,06),
Collins-Cambridge University Press (11), Aybat Rogers PRD (11),
Aybat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11),
Bacchetta, Prokudin (13), Sun, Yuan (13), Echevarria, Idilbi, Scimemi
JHEP 2012, Collins Rogers 2015
SCET: Bauer, Fleming Pirjol Rothstein, Stewart PRD 2002, Chiu,
Jain, Neill, Rothstein JHEP 2013, Rothstein & Stewart JHEP 2016,
...

JCC Soft factor further “repartitioned”
This is done to

- 1) cancel LC divergences in “unsubtracted” TMDs
- 2) separate “right & left” movers i.e. full factorization
- 3) remove double counting of momentum regions



$$\tilde{D}_{H/j}^{\text{sub}}(z_A, b_T; \mu, \zeta) = \lim_{\substack{y_A \rightarrow +\infty \\ y_B \rightarrow -\infty}} \underbrace{\tilde{D}_{H/j}^{\text{unsub}}(z_A, b_T; \mu, y_A - y_B)} \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B) \tilde{S}(b_T; y_n, y_B)}}} \times UV_{\text{renorm}}$$

$$\updownarrow$$

$$\tilde{D}_{H/j}^{\text{unsub}}(z_A, b_T; \mu, y_P - y_B) = \frac{1}{z_A} \int \frac{db^+}{2\pi} e^{-ik_A^- b^+} \langle 0 | \gamma^- \mathcal{U}_{[0,b]} \psi(b) | X P_A \rangle \langle P_A X | \bar{\psi}(0) | 0 \rangle |_{b^- = 0}$$

Use both data sets to study universality of T-odd fragmentation?

What is prediction of TMD Factorization

Universality of T-odd Collins function: $H_{1,\pi/q}^{\perp(1)}(z, b, Q)$

Metz PLB2002,

Boer Mulders Pijlman NPB2003

Collins Metz PRL 2004,

Gamberg, Mukerjee, Mulders PRD2007,

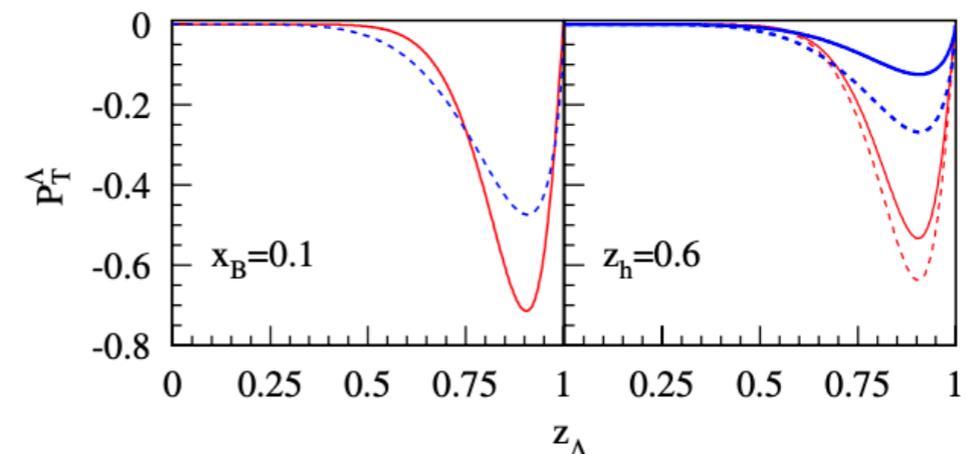
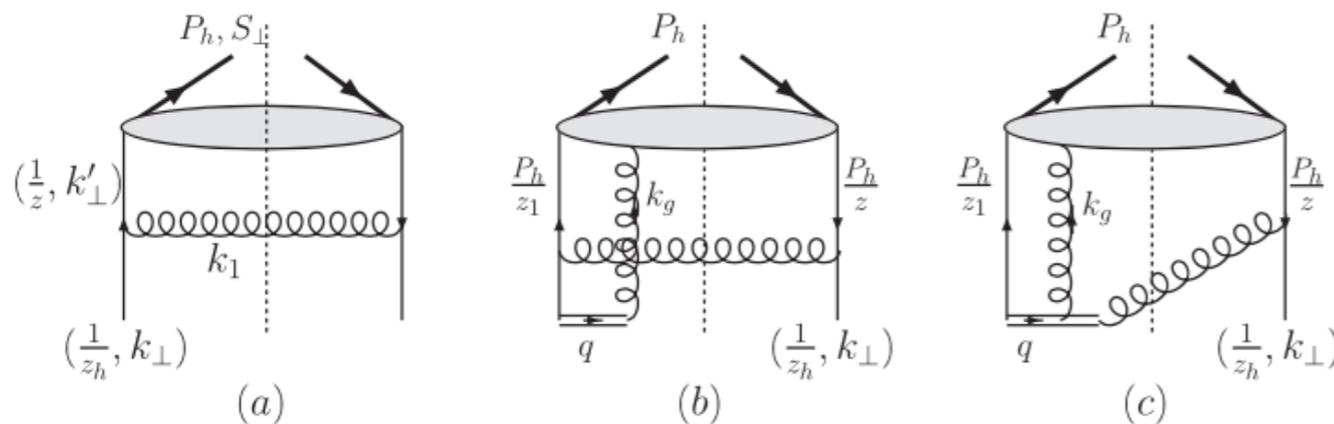
Meissner Metz PRL 2009,

Gamberg Mukherjee, Mulders PRD 2008

Universality of T-odd PFF prediction from pQCD - $D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, b, Q)$

phase from FSI but not gluonic/fermionic pole

Boer, Kang, Vogelsang, Yuan PRL 2010



Λ Belle data fall into 2 classes

$$e^+e^- \rightarrow \Lambda^\uparrow h X \quad \& \quad \Lambda^\uparrow(\text{Thrust}) X$$

$$D_{1T,\Lambda/q}^\perp(z_\Lambda, p_\perp, Q)$$

? Is it true that the PFF is the same TMD in both process?

Recent extractions address this

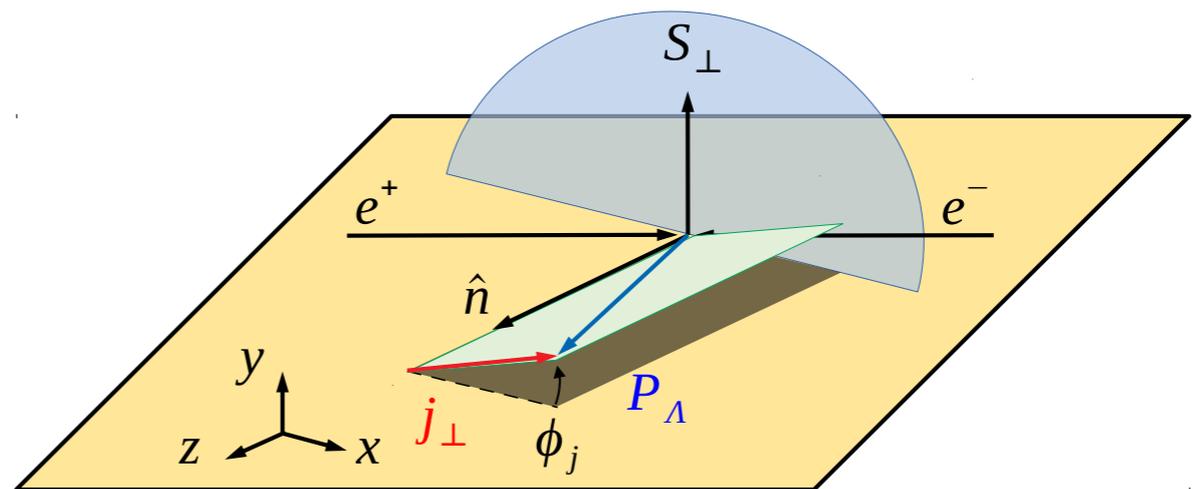
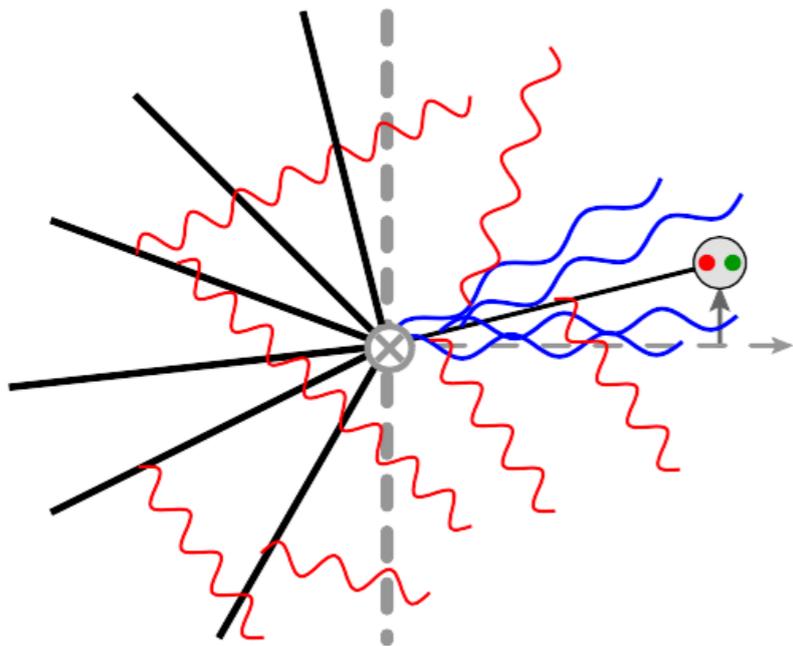
- 1) D'Alesio & Murgia Zaccheddu PRD2020 **bTOb + Thrust assumed same factz.here**
- 2) Callos, Kang, Terry PRD2020 **bTOb only**

Other pheno studies

- *Anselmino, Kishore, Mukherjee PRD 2019
single inclusive case and the role of the PFFs
twist-2 in place of twist-3 ?
- *Earlier Anselmino Boer, D'Alesio, Murgia. PRD 2001, 2002
TMD factorization applied to inclusive process ?

? Same PFF ? in $e^+e^- \rightarrow \Lambda^\uparrow$ (Thrust)

In TMD factorization framework for production of Λ (Thrust) we have non-global observable “right hemisphere” only
 ? chiral even, naively T-odd fragmentation function, universal ?



- Z.B Kang, D.Y. Shao, F. Zhao 2007.14425

$$\hat{D}_{\Lambda/q}(z_\Lambda, \mathbf{p}_\perp, \mathbf{S}_\perp, Q) = \frac{1}{2} \left[D_{\Lambda/q}(z_\Lambda, p_{\Lambda\perp}, Q) + \frac{1}{z_\Lambda M_\Lambda} \left(D_{1T,\Lambda/q}^\perp(z_\Lambda, p_\perp, Q) \epsilon_{\perp\rho\sigma} p_\perp^\rho S_\perp^\sigma \right) \right]$$

TMD factorization & Thrust observable

Recent work

- M. Boglione & A. Simonelli, 2007.13674
- Z.B Kang, D.Y. Shao, F. Zhao 2007.14425
- M. Boglione & A. Simonelli, 2007.13674

Z.B Kang, D.Y. Shao, F. Zhao 2007.14425 — *see talk of Dingyu*

Derive TMD factorization for unpolarized transverse momentum distribution for the single hadron production with the thrust axis in electron-positron collision

Lets Drill Down TMD factorization

Recent work

- Z.-B Kang, D.Y. Shao, F. Zhao 2007.14425

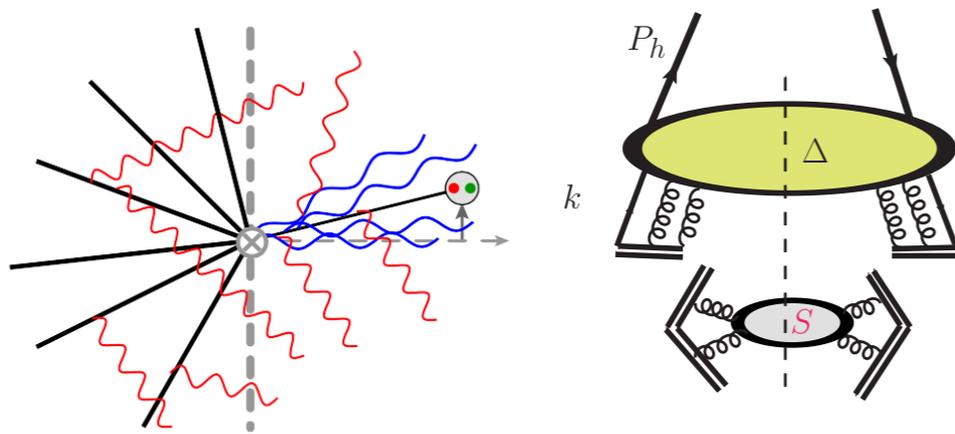
Derive TMD factorization for unpolarized TMD FF for single hadron production with the thrust axis in electron-positron collision $e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust})$ non-global observable

$$\frac{d\sigma}{dz_\Lambda d^2j_\perp} = \sigma_0 H(Q, \mu) \sum_q e_q^2 \int d^2p_\perp d^2\lambda_\perp \delta^{(2)}(\mathbf{j}_\perp - \mathbf{p}_\perp - z_\Lambda \boldsymbol{\lambda}_\perp) D_{\Lambda/q}(z_\Lambda, p_\perp, \mu, \zeta/\nu^2) S_{\text{hemi}}(\lambda_\perp, \mu, \nu)$$

Calculated to NLO and NLL

$$S_{\text{hemi}}(b, \mu, \nu) = \sqrt{S(b, \mu, \nu)}$$

$$D_{\Lambda/q}^{\text{TMD}}(z_\Lambda, b, \mu, \zeta) = D_{\Lambda/q}(z_\Lambda, b, \mu, \zeta/\nu^2) \sqrt{S(b, \mu, \nu)}$$



$$\frac{d\sigma}{dz_\Lambda d^2j_\perp} = \sigma_0^{\text{TMD}} \sum_q e_q^2 \int_0^\infty \frac{b db}{(2\pi)} J_0\left(\frac{b j_\perp}{z_\Lambda}\right) \frac{1}{z_\Lambda^2} D_{\Lambda/q}(z_\Lambda, \mu_{b_*}) e^{-S_{\text{NP}}(b, z_\Lambda, Q_0, Q) - S_{\text{pert}}(\mu_{b_*}, Q)} U_{\text{NG}}(\mu_{b_*}, Q)$$

Non-global logs resummed

Factorization theorem \exists

Becher Rahn Shao JHEP 2017

M.Dasgupta & G.Salam PLB2001

We extend TMD factorization PFF

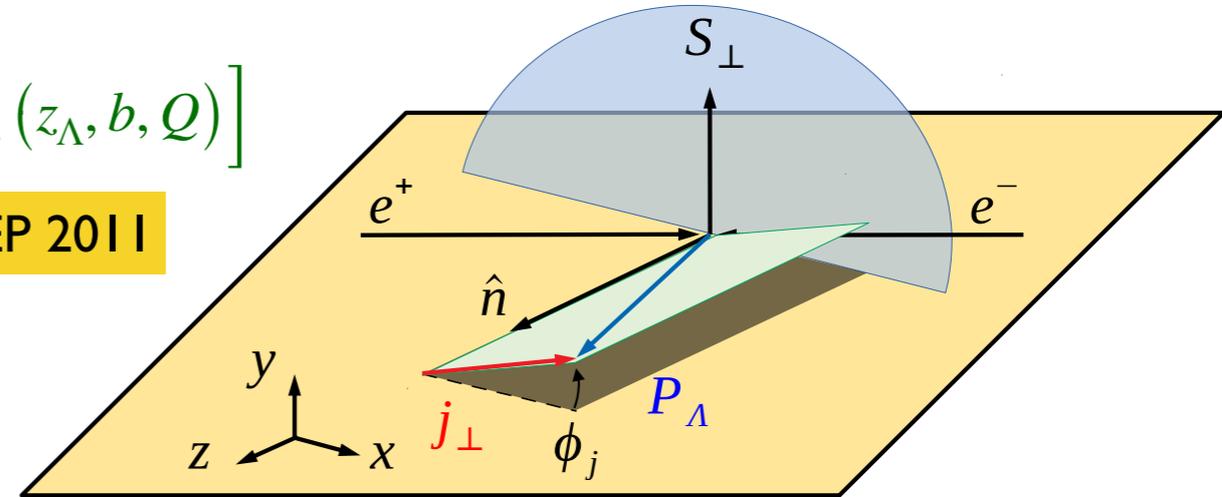
$$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust})$$

Gamberg, Kang, Shao, Terry, Zhao arXiv:2102.05553

$$\hat{D}_{\Lambda/q}(z_\Lambda, \mathbf{b}, \mathbf{S}_\perp, Q) = \frac{1}{2} \left[D_{\Lambda/q}(z_\Lambda, p_{\Lambda\perp}, Q) - i\epsilon_{\perp\rho\sigma} b^\rho S_\perp^\sigma M_\Lambda D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, b, Q) \right]$$

Boer, Gamberg, Musch, Prokudin JHEP 2011

Spin dependent
FF Obeys CSS equation



$$\begin{aligned} \frac{d\Delta\sigma}{dz_\Lambda d^2\mathbf{j}_\perp} &= \frac{d\sigma(\mathbf{S}_\perp)}{dz_\Lambda d^2\mathbf{j}_\perp} - \frac{d\sigma(-\mathbf{S}_\perp)}{dz_\Lambda d^2\mathbf{j}_\perp} \\ &= \sigma_0^{\text{TMD}} \sin(\phi_s - \phi_j) \sum_q e_q^2 \int_0^\infty \frac{b^2 db}{4\pi} J_1\left(\frac{bj_\perp}{z_\Lambda}\right) \\ &\times \frac{M_\Lambda}{z_\Lambda^4} D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, \mu b_*) e^{-S_{\text{NP}}^\perp(b, z_\Lambda, Q'_0, Q) - S_{\text{pert}}(\mu b_*, Q)} U_{\text{NG}}(\mu b_*, Q) \end{aligned}$$

UV & Rapidity subtracted TMD Universal PFF

Also, see paper of JW Qiu, T. Rogers, B. Wang *Phys.Rev.D* 101 (2020)
regarding proper definitions of weighted TMDs and talk in this workshop

Establish factorization for thrust axis factorization
carry out pheno to describe

Belle P_T and OPAL

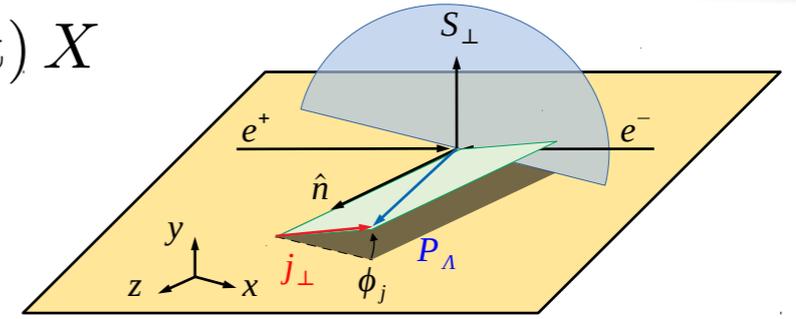
$$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust})$$



Postage stamp of input for Pheno

$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust}) X$

$$P_\perp^\Lambda(z_\Lambda, j_\perp) = \frac{d\Delta\sigma}{dz_\Lambda d^2j_\perp} \bigg/ \frac{d\sigma}{dz_\Lambda d^2j_\perp}.$$



$$S_{\text{NP}}(b, z_\Lambda, Q_0, Q) = g_h \frac{b^2}{z_\Lambda^2} + \frac{g_2}{2} \ln \frac{Q}{Q_0} \ln \frac{b}{b_*}$$

Aidala Field Gamberg Rogers PRD 2014

$$g_h = 0.042 \text{ GeV}^2 \quad g_2 = 0.84 \text{ GeV}^2$$

Implementation Issacson Sun Yuan 2014 MPA

$$U_{\text{NG}}(\mu_{b_*}, Q) = \exp \left[-C_A C_F \frac{\pi^2}{3} u^2 \frac{1 + (au)^2}{1 + (bu)^c} \right]$$

Dasgupta Salam, PLB 2001

$$\text{with } a = 0.85C_A, b = 0.86C_A, c = 1.33$$

$$u \equiv \int_{\mu_{b_*}}^Q \frac{d\mu}{\mu} \frac{\alpha_s(\mu)}{2\pi} = \frac{1}{\beta_0} \ln \left[\frac{\alpha_s(\mu_{b_*})}{\alpha_s(Q)} \right]$$

$$D_{1T,h/q}^\perp(z, p_\perp, Q'_0) = \frac{M_\Lambda}{\langle M_D^2 \rangle} D_{1T,h/q}^\perp(z, Q'_0) \frac{e^{-p_\perp^2 / \langle M_D^2 \rangle}}{\pi \langle M_D^2 \rangle}$$

$$D_{1T,h/q}^\perp(z, Q'_0) = \mathcal{N}_q(z) D_{h/q}(z, Q'_0) \quad Q'_0 = 10.58 \text{ GeV}$$

$$\mathcal{N}_q(z) = N_q z^{\alpha_q} (1-z)^{\beta_q} \frac{(\alpha_q + \beta_q - 1)^{\alpha_q + \beta_q - 1}}{(\alpha_q - 1)^{\alpha_q - 1} \beta_q^{\beta_q}}$$

$$S_{\text{NP}}^\perp(b, z, Q'_0, Q) = \frac{\langle M_D^2 \rangle}{4} \frac{b^2}{z^2} + \frac{g_2}{2} \ln \frac{Q}{Q'_0} \ln \frac{b}{b_*}$$

Parameters fit from bTOB Belle data
Callos, Kang, Terry PRD2020

Belle data fit $e^+e^- \rightarrow \Lambda^\uparrow h X$

$$D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, Q)$$

Recent extractions address this
Callos, Kang, Terry PRD2020 **bTOB only**

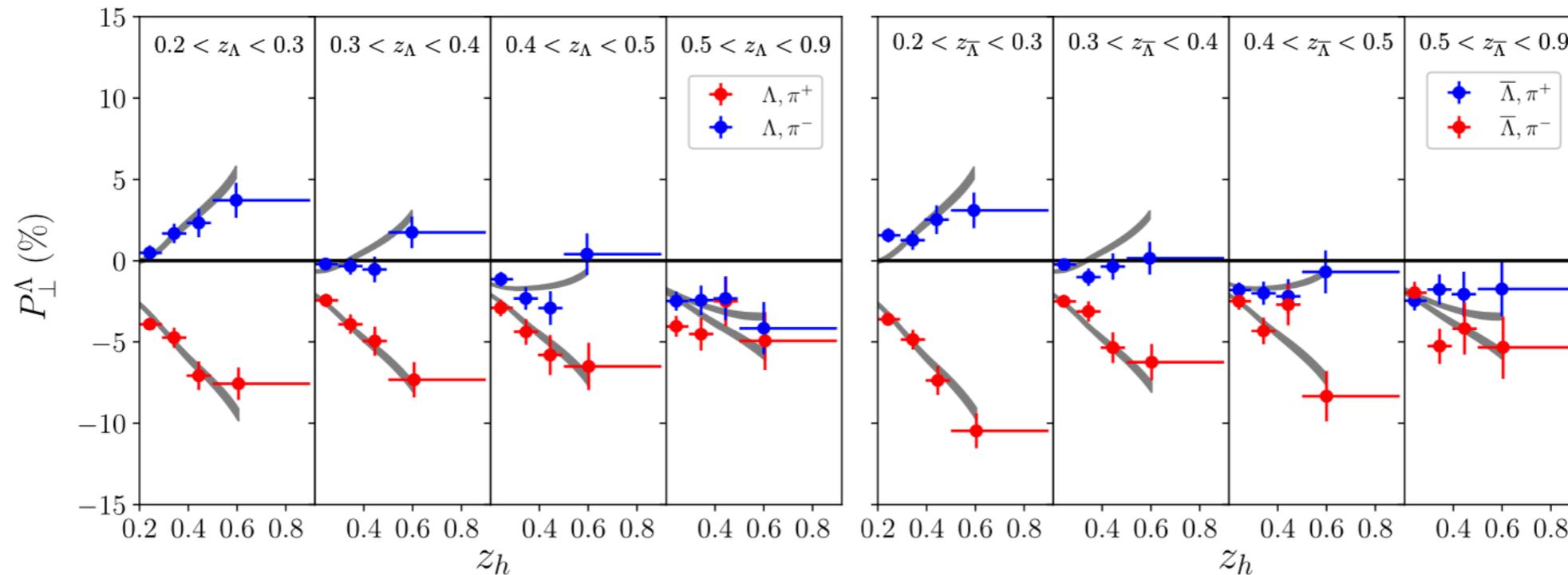


FIG. 3. The fit to the experimental data for π mesons is shown, with the gray uncertainty band displayed is generated by the replicas at 68% confidence. The left plots are for the production of $\Lambda + \pi^\pm$, while the right plots are for the production of $\bar{\Lambda} + \pi^\pm$.

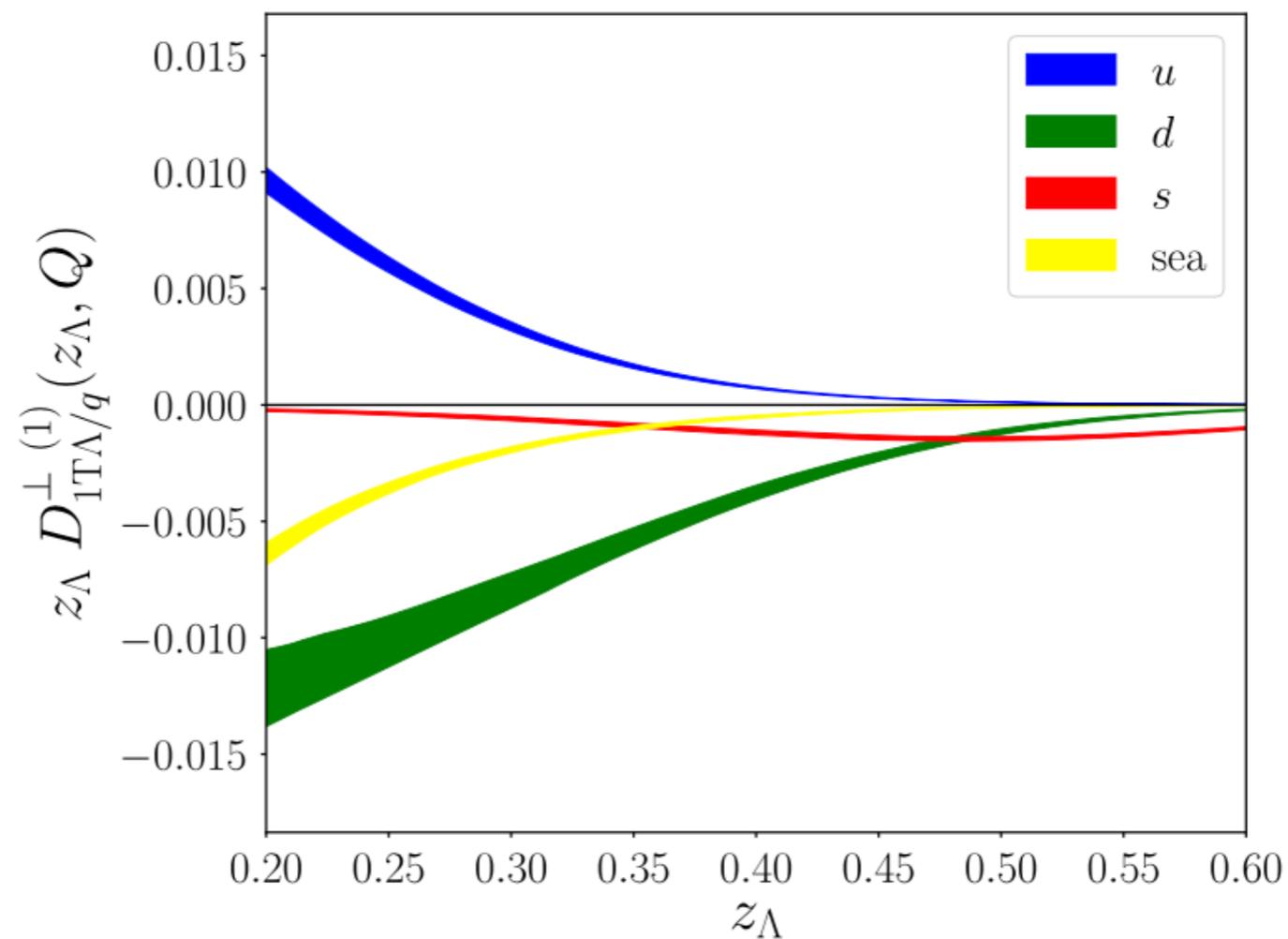
And for kaons ...

Belle data

$$e^+e^- \rightarrow \Lambda^\uparrow h X,$$

$$D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, Q)$$

Recent extractions
Callos, Kang, Terry PRD2020 **bTOb only**



Exploit Universality to describe

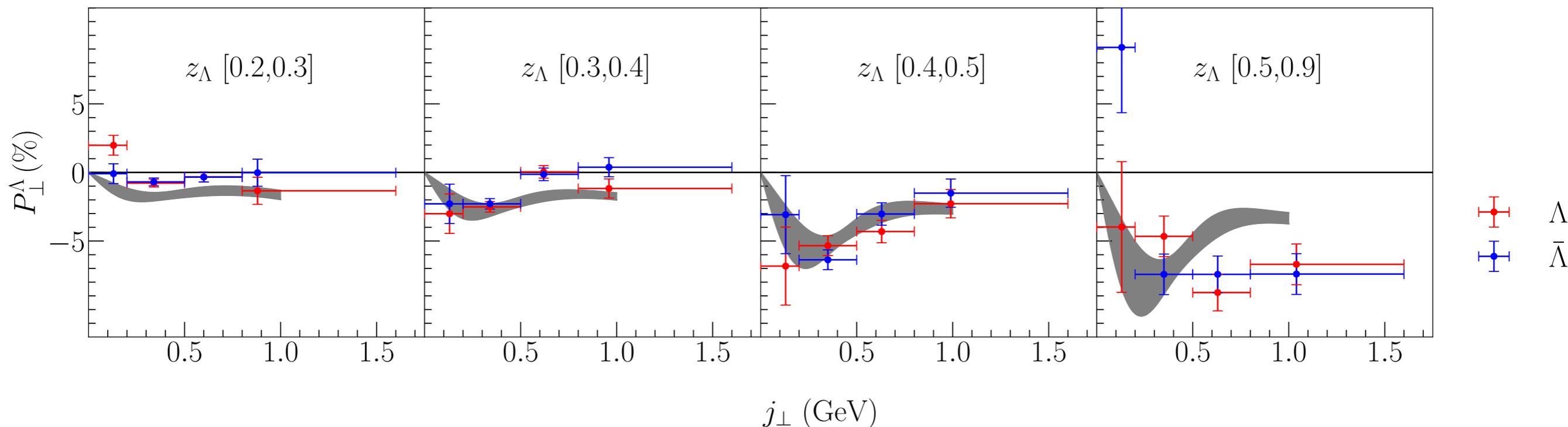
$$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust})$$



Compare theory predictions to OPAL & Belle

$$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust})$$

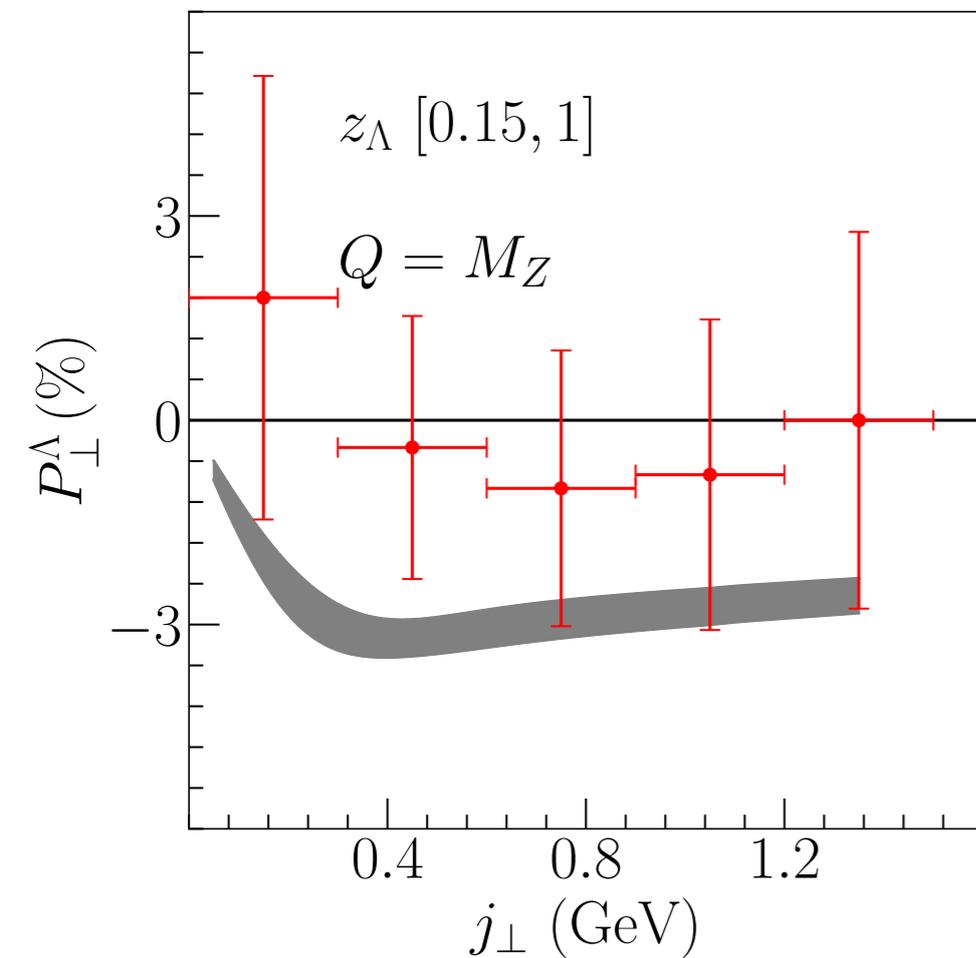
Gamberg, Kang, Shao, Terry, Zhao arXiv:2102.05553



- $P_\perp^\Lambda(z_\Lambda, j_\perp)$ for the Belle data [20]; left to right theory integrated from $0.2 < z_\Lambda < 0.3$, $0.3 < z_\Lambda < 0.4$, $0.4 < z_\Lambda < 0.5$, $0.5 < z_\Lambda < 0.6$
- The data in **red** is for Λ production while the data in **blue** is for $\bar{\Lambda}$ production
- Data plotted with total exp. uncertainty as vertical error bar & uncertainty on j_\perp horizontal error bar
- Gray band is the theoretical uncertainty which was generated from the replicas for the TMD PFF, Callos, Kang, Terry PRD2020

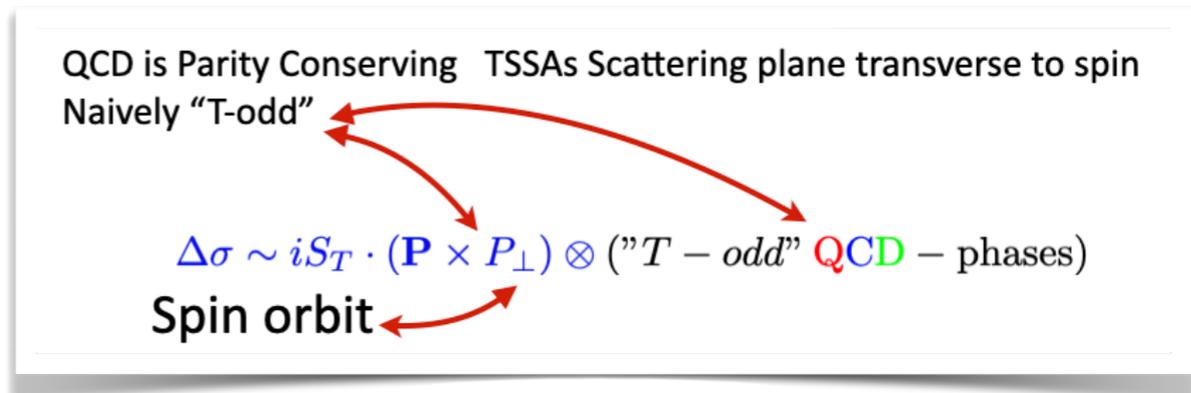
Compare theory predictions to OPAL & Belle data

$$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust})$$



Gamberg, Kang, Shao, Terry, Zhao arXiv:2102.05553

$$P_{\perp}^{\Lambda}(z_{\Lambda}, j_{\perp}) = \frac{d\Delta\sigma}{dz_{\Lambda}d^2j_{\perp}} \bigg/ \frac{d\sigma}{dz_{\Lambda}d^2j_{\perp}}.$$

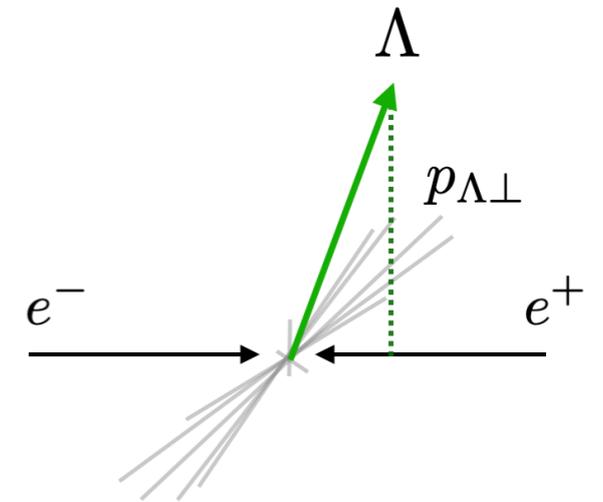


$P_{\perp}^{\Lambda}(z_{\Lambda}, j_{\perp})$ for OPAL data [19]: Theory curve is integrated over the region $0.2 < z_{\Lambda} < 0.5$.
total experimental uncertainty vertical error bar j_{\perp} horizontal error bar.
Error band, standard deviation of the replicas for TMD PFF in Callos, Kang, Terry PRD2020.

Fully inclusive process $e^+e^- \rightarrow \Lambda^\uparrow X$

\Rightarrow significant transverse polarization ?

$$e^+e^- \rightarrow \Lambda^\uparrow X$$



Measure w.r.t. COM in principle can measure at Belle ?

Questions/issues:

QCD prediction of Physics twist-3

- Twist-3 factorization one hard scale

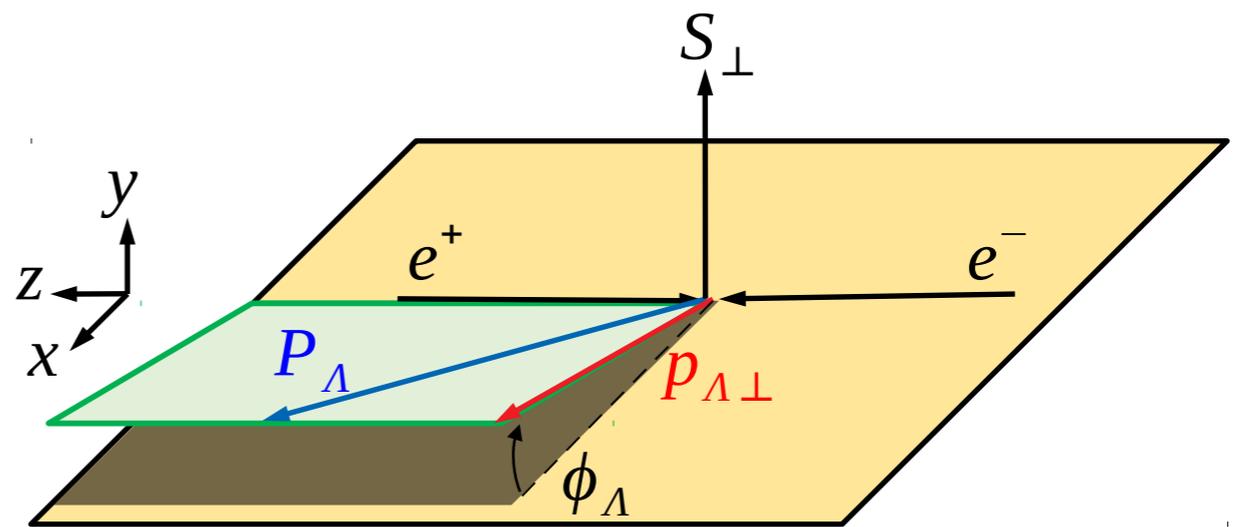
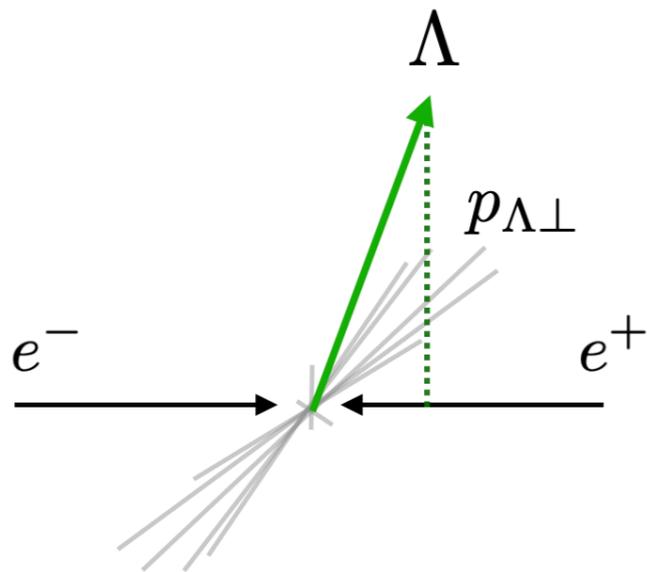
Simplest and cleanest process $e^+e^- \rightarrow \Lambda^\uparrow X$

\Rightarrow significant transverse polarization ?

$$e^+e^- \rightarrow \Lambda^\uparrow X$$

$P_{\Lambda\perp} \sim Q$ twist-3 factorization

And can be measured w.r.t. COM of e^+e^- on large scale $P_T \sim Q$



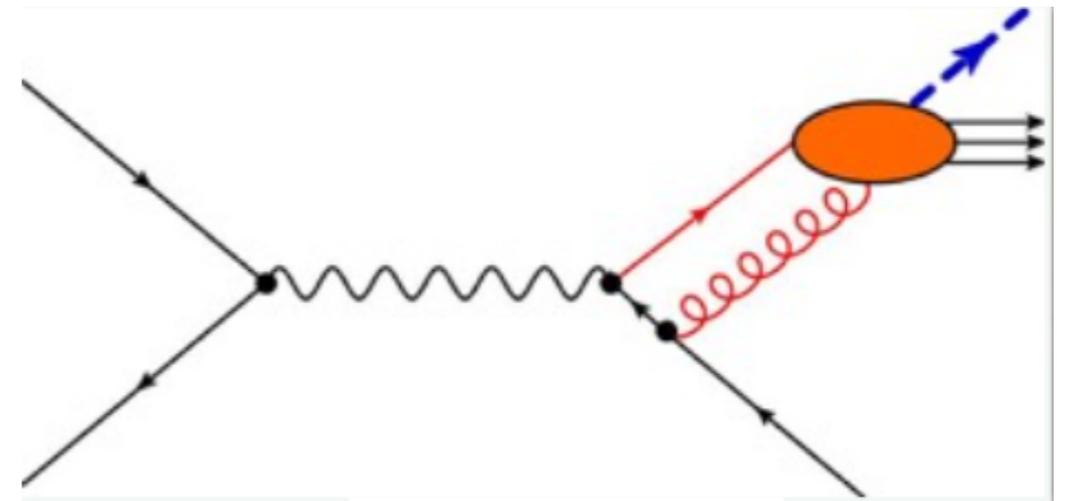
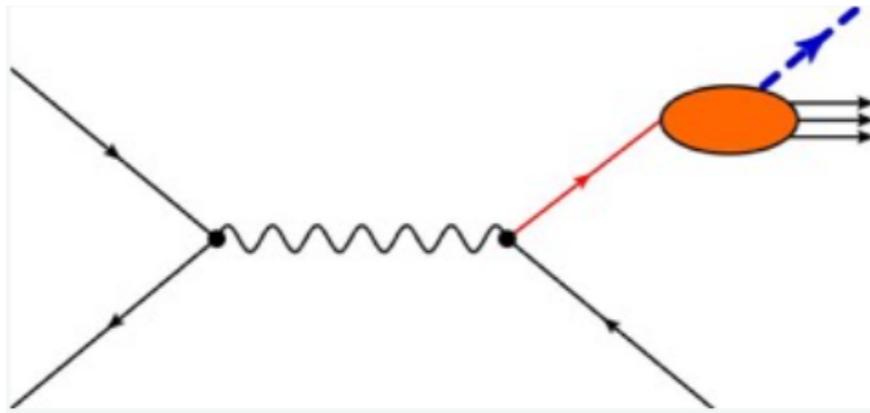
Consider Transverse $e^+e^- \rightarrow \Lambda^\uparrow X$ polarization

Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP 2019, LO & NLO

There are contributions from

'Intrinsic' & 'kinematical' twist-3 FF

'Dynamical' twist-3 FF:



Intrinsic

Kinematical

Dynamical

$$\frac{d\sigma(S_{\Lambda T})}{dz_h d\phi} = C |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[\frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{1T}^{\perp(1)\Lambda/q}(z_h) + \int_0^1 d\beta \frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta} \right]$$

Using the EOMs and LIRs CS can be expressed solely in terms of

$$D_T^{\Lambda/q}(z)$$

$$\frac{d\sigma(S_{\Lambda T})}{dz_h d\phi} = C |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[2 \frac{D_T^{\Lambda/q}(z_h)}{z_h} \right]$$

Boer, Jakob, Mulders NPB (1997)
in TMD framework at twist-3

See talk of F. Aslan on the subtleties of applying LIRs and EOMs

Twist - 3 Pheno

$$\frac{d\Delta\sigma}{dz_\Lambda d^2p_{\Lambda\perp}} = -\sin(\phi_s - \phi_\Lambda)\sigma_0^{\text{Col}} \left(\frac{8M_\Lambda}{Q}\right) \frac{p_{\Lambda\perp}}{Q} \frac{1}{z_\Lambda^3} \sum_q e_q^2 \frac{D_{T,\Lambda/q}(z_\Lambda, Q)}{z_\Lambda}$$

To describe this process, only need a parameterization for $D_{T,\Lambda/q}(z_\Lambda, Q)$

Given our lack of knowledge of this fundamental twist-3 T-odd fragmentation function we will employ the approach outlined in Gamberg, Metz, Pitonyak, Prokudin PLB 2017

Re-express the $D_{T,\Lambda/q}(z_\Lambda)$ in terms of our knowledge of $D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda)$

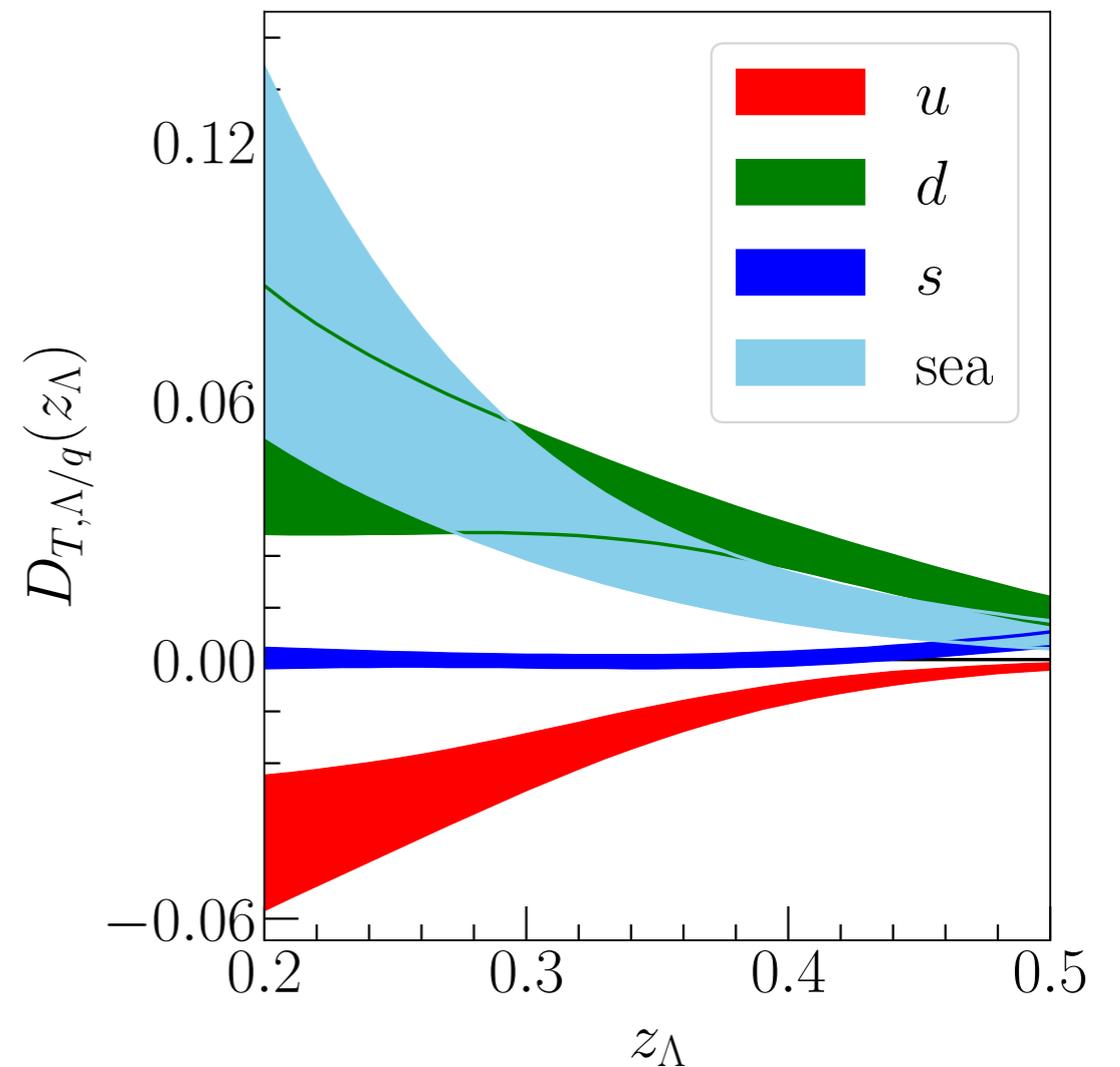
Twist - 3 Pheno

$$\frac{d\Delta\sigma}{dz_\Lambda d^2p_{\Lambda\perp}} = -\sin(\phi_s - \phi_\Lambda) \sigma_0^{\text{Col}} \left(\frac{8M_\Lambda}{Q} \right) \frac{p_{\Lambda\perp}}{Q} \frac{1}{z_\Lambda^3} \sum_q e_q^2 \frac{D_{T,\Lambda/q}(z_\Lambda, Q)}{z_\Lambda}$$

Re-express the $D_{T,\Lambda/q}(z_\Lambda)$ in terms of our knowledge of $D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda)$

$$\frac{1}{z_\Lambda} D_{T,\Lambda/q}(z_\Lambda) = - \left(1 - z_\Lambda \frac{d}{dz_\Lambda} \right) D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda) - 2 \int_0^1 d\beta \frac{\Im \left[\hat{D}_{FT}^{qg}(z_\Lambda, \beta) \right]}{(1-\beta)^2}$$

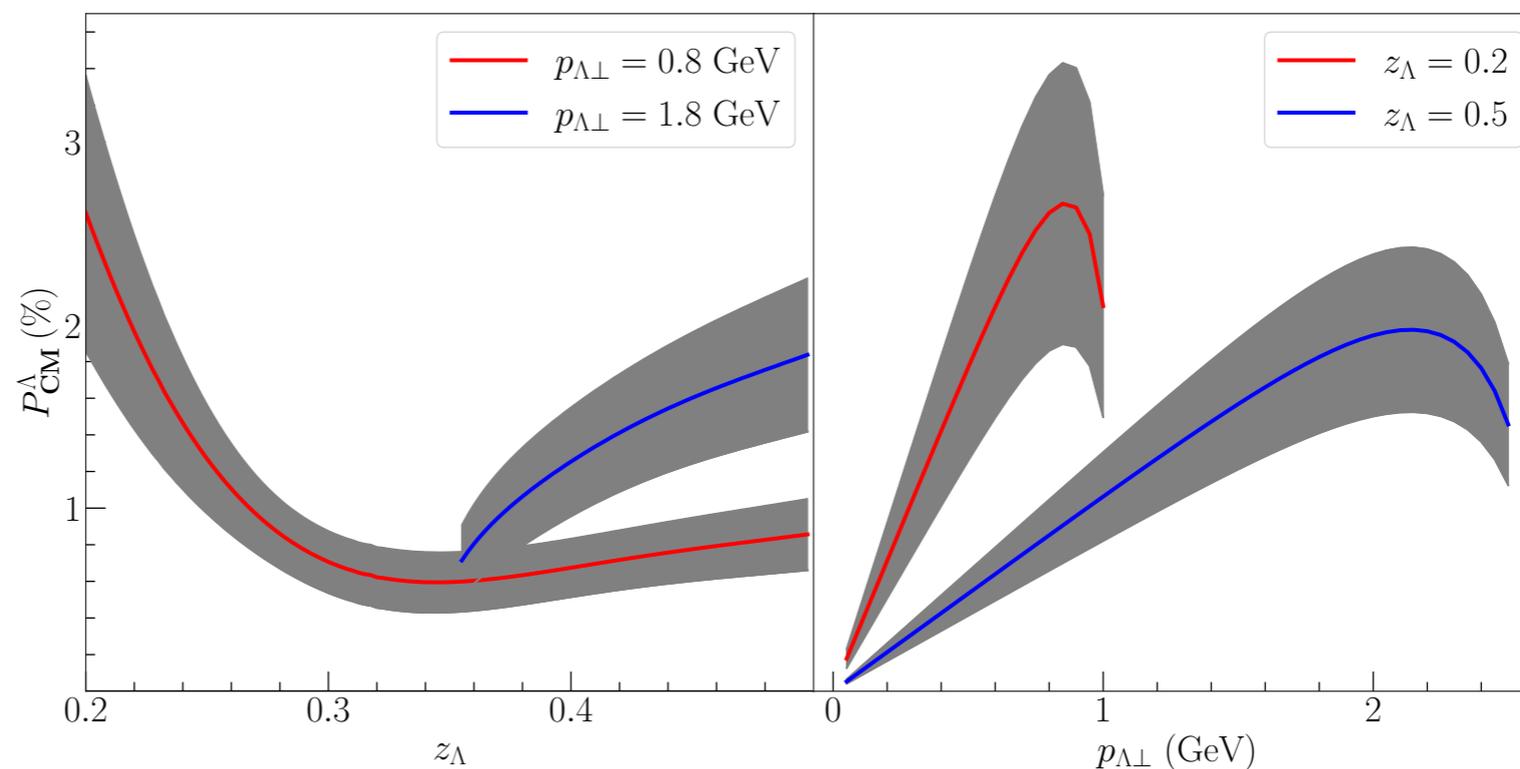
$$\frac{1}{z_\Lambda} D_{T,\Lambda/q}(z_\Lambda) \approx - \left(1 - z_\Lambda \frac{d}{dz_\Lambda} \right) D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda)$$



Prediction for Belle

Gamberg, Kang, Shao, Terry, Zhao arXiv:2102.05553

$$P_{\text{CM}}^{\Lambda}(z_{\Lambda}, p_{\Lambda\perp}) = \frac{d\Delta\sigma}{dz_{\Lambda} d^2p_{\Lambda\perp}} \bigg/ \frac{d\sigma}{dz_{\Lambda} d^2p_{\Lambda\perp}}.$$



P_{CM}^{Λ} — 3-D plot of the polarization in z_{Λ} and $p_{\Lambda\perp}$

Center: Plot of the polarization as a function of only z_{Λ} ,

Right: Plot of the polarization as a function of $p_{\Lambda\perp}$: polarization in our scheme is ~ 1 -2%

Plots are generated only using the central fit

The red and blue curves are generated using the central fit, gray band is the theoretical uncertainty

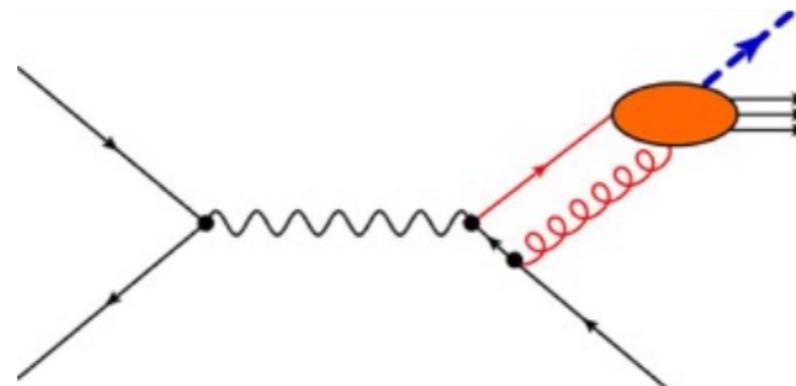
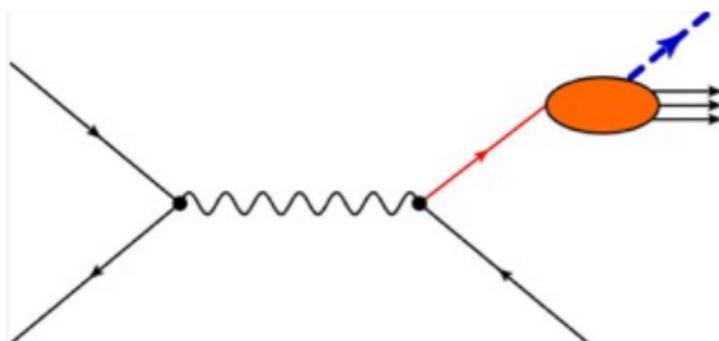
Take aways I

Comments ... $e^+e^- \rightarrow \Lambda(\text{Thrust}) X$ and $e^+e^- \rightarrow \Lambda X$

- Interesting that while these two measurements probe different distribution functions, they differ only by the definition of the measurement axis.
- That is, a measurement the polarization as a function of j_{\perp} is a useful process for probing the properties of the PFF D_{1T}^{\perp} with respect to the thrust axis
- While a measurement if polarization as a function of $p_{\Lambda\perp}$, the transverse momentum of the Λ in the lepton center-of-mass (COM) frame, is a useful process for probing the D_T function.
- Therefore the polarization in the COM frame can in principle be studied from the existing Belle data by reanalyzing the data for the inclusive $e^+e^- \rightarrow \Lambda(\text{Thrust}) X$ measurement in COM $e^+e^- \rightarrow \Lambda X$

Single-Transverse Λ^\uparrow spin asymmetry

Unique effect driven by a single fragmentation function $D_T^{\Lambda/q}(z)$ \rightarrow
 absent in DIS (1γ)



Intrinsic

$$\frac{d\sigma(S_{\Lambda T})}{dz_h d\phi} = C |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[2 \frac{D_T^{\Lambda/q}(z_h)}{z_h} \right]$$

See also Boer, Jakob, Mulders NPB (1997)

n.b. some intuition ...

Consider crossing this process to inclusive DIS for transverse polarised target

Would have the function $f_T^{q/\Lambda}(x)$, $\frac{d\sigma(S_{\Lambda T})}{dx d\phi} \sim \sin(\phi_S) \sum_q e_q^2 f_T^{q/\Lambda}(x) = 0$!!!

Constraints from time reversal on quark correlation function
 Goeke, Metz, Schlegel PLB 2006, Bacchetta et al JHEP 2007, Christ & Lee 1960

A unique test of time reversal in QCD: Non-zero intrinsic

Unique effect driven by a single fragmentation function $D_T^{\Lambda/q}(z)$ →
absent in DIS (1γ)

Single-Transverse Λ^\uparrow spin asymmetry

$$\frac{d\sigma(S_{\Lambda T})}{dz_h d\phi} = C |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[2 \frac{D_T^{\Lambda/q}(z_h)}{z_h} \right]$$

$$\frac{d\sigma(S_{\Lambda T})}{dx d\phi} \sim \sin(\phi_S) \sum_q e_q^2 f_T^{q/\Lambda}(x) = 0 \quad !!! \quad f_T^{q/\Lambda}(x)$$

Constraints from time reversal on quark correlation function
Goeke, Metz, Schlegel PLB 2006, Bacchetta et al JHEP 2007, Christ & Lee 1960

Take aways II

- Non-zero $e^+e^- \rightarrow \Lambda^{\uparrow} X$ inclusive result is an indication that there are no gluonic poles in ffs, ie time reversal is not a constraint on FFs: the simplest process is an interesting a test of time reversal in QCD, $D_T^{\Lambda/q} \neq 0$
- We are performing a test of twist-3 factorisation at NLO in $e^+e^- \rightarrow \Lambda^{\uparrow} X$
- Would be great if Belle carried out a fully inclusive measurement to directly test $D_T^{\Lambda/q} \neq 0$