Study of the Potential Transverse Momentum and Potential Angular Momentum within the Scalar Diquark Model

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March 8th, 2021

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Introduction and Outline [1]

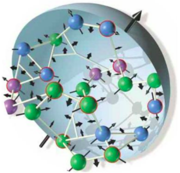

Figure – Sketch of the inside of a proton

In the proton

- $\frac{1}{2} = \langle\langle S_z^q \rangle\rangle + \langle\langle S_z^G \rangle\rangle + \langle\langle L_z \rangle\rangle$
- Experiences have shown $\frac{1}{2} \neq \langle\langle S_z^q \rangle\rangle$ (eg., $\langle\langle S_z^q \rangle\rangle_{COMPASS} \sim 0.3$)
- Have to deal with a relativistic quantum system

How would we get an understanding for L_z ?

Outline :

- *Nucleon pin decomposition* ($k_{\perp,pot}$ and L^z_{pot} definitions)
- *The Sivers Shift*
- *Proposed lensing mechanism* (Intuitive link between $k_{\perp,pot}$ and L^z_{pot})
- *Study in the SDM*

1. Presentation based on the article : D.-A. Amor-Quiroz, M. Burkardt WF et C. Lorcé. 2020. arXiv : 2010.13451 [hep-ph]

The Ji and JM Decomposition [2] [3]

$$\text{Ji} : J = L_q + S_q + J_g$$

- $L_q \sim \vec{r} \times i\vec{D}$
- Kinetic decomposition (corresponds to the classical notion $r \times p_{kin}$)
- L_q, S_q, J_g are gauge invariant, can be measured
- Local definition of the derivative

$$\text{Jaffe-Manohar} : J = \mathcal{L}_q + S_q + \mathcal{L}_g + S_g$$

- $\mathcal{L}_q \sim \vec{r} \times i\vec{\partial}$
- Canonical decomposition in the light-cone gauge (corresponds to the Noether theorem operators)
- Total decomposition into both quark and gluon spin and OAM
- Only $\mathcal{S}_q$ and $(\mathcal{L}_q + \mathcal{L}^g_{JM} + S^g_{JM})$ are gauge invariant
- Generators of rotation
- Non-local definition of the derivative
- Gauge invariance can be restored provided one replaces ∂ by a non-local definition of the gauge covariant derivative

2. Elliot Leader et Cédric Lorcé. In : *Physics Reports* 541.3 (2014).
3. M. Wakamatsu. In : *Physical Review D* 81.11 (2010).

Potential Transverse Momentum and Angular Momentum

Using the Ji and JM definitions, we investigate the following quantities [4] :

$$k_{pot} \triangleq \langle k_\perp^{JM} \rangle - \langle k_\perp^{Ji} \rangle$$
$$L^z_{pot} \triangleq \langle \mathcal{L}^z_q \rangle - \langle L^z_q \rangle \quad (1)$$

In the light-cone gauge $A^+ = 0$. These can be shown to be :

$$k_{\perp,pot} = -\, e_q \langle \int \mathrm{d}^2 r_\perp \bar{\psi}(r_\perp)\gamma^+ A_{\perp,phys}(r_\perp)\psi(r_\perp)\rangle$$
$$L^z_{pot} = -\, e_q \langle \int \mathrm{d}^2 r_\perp \bar{\psi}(r_\perp)\gamma^+ (r \times A_{\perp,phys})\psi(r_\perp)\rangle \quad (2)$$

- $L^z_{pot} = 0$ at one-loop [5]
- Lattice QCD shows a "significantly enhanced" $\langle \mathcal{L}^z_q \rangle$ compared to $\langle L^z_q \rangle$ [6]
- Can we understand or estimate one quantity from the other ?

4. we use light-cone variables $[x^+, x^-, x_\perp = (x_1, x_2)]$, $x^\pm = \frac{1}{\sqrt{2}}(A^0 \pm A^3)$
5. X. Ji et al. In : *Phys Rev D.* 93.5, 054013 (2016).
6. M. Engelhardt et al. *Quark orbital angular momentum in the proton evaluated using a direct derivative method.* 2019. arXiv : 1901.00843 [hep-lat].

Transverse Momentum Distributions

The leading-twist quark TMD correlator and corresponding Wilson line are defined as [7] [8] :

$$\phi^{[\gamma^+]}(P, x, k_\perp, S; \gamma) = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3} e^{-iz\cdot k}\langle P, S|\psi(-\tfrac{z}{2})\gamma^+\mathcal{W}_\gamma(-\tfrac{z}{2}, \frac{z}{2})\psi(\tfrac{z}{2})|P, S\rangle_{z^+=0} \quad (3)$$

$$\mathcal{W}_\gamma(z_1, z_2) = \mathcal{P}\exp\left(-ig\int_\gamma \mathrm{d}z\cdot A(z)\right) \quad (4)$$

The Wilson line

- makes the correlator color-gauge invariant
- encodes Final or Initial State Interactions (FSI, ISI)

Figure – Different path correspond to different processes

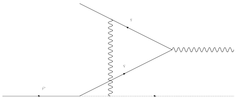

Figure – Example of an Initial State Interaction

- is process dependent
- breaks the naive time reversal symmetry

7. D. Boer, P.J. Mulders et F. Pijlman. In : *Nuclear Physics B* 667.1-2 (2003).
8. J.C. et al. Collins. In : *Transversity 2005* (2006). DOI : 10.1142/9789812773272_0025.

The Sivers Shift

Generically : $\phi(P,x,k_\perp,S;\gamma) = f_1(x,k_\perp) - \frac{\epsilon_\perp^{ij} S_\perp^i k_\perp^i}{M} f_{1T}(x,k_\perp)$
where f_1 is the unpolarized parton distributions and f_{1T} is the Sivers function
After some algebra :

$$\begin{aligned} k_{\perp,pot} = \langle k_\perp \rangle_{JM} &= \int \mathrm{d}x \int \mathrm{d}^2 k_\perp \; k_\perp \phi^{[\gamma^+]}(P,x,\vec{k}_\perp,S) \\ &= -\epsilon_\perp^{ij} S_\perp^k \int \mathrm{d}x \int \mathrm{d}^2 k_\perp \frac{k_\perp^2}{M} f_{1T}(x,k_\perp) \end{aligned} \tag{5}$$

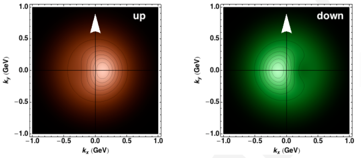

Figure – The up and down quark density distortion in transverse-momentum space, obtained by studies of the Sivers function

- One of two leading twist function that are odd under time reversal for a spin-1/2 target
- Can lead to a non-zero transverse momentum
- In SIDIS (Drell-Yan), the Sivers function encodes the presence of Final (Initial) State interactions through gluon exchanges

How can a nonzero torque arise ?

Burkardt[9] made the case for the following "lensing mechanism" :

- The distribution of unpolarized quarks in a transversely polarized nucleon is shifted due to the Sivers shift before it fragments
- The attracting interactions bend the observed hadrons in the direction opposite to the struck quark
- The lensing parameter would formally be written "SSA = GPD x L(x)"

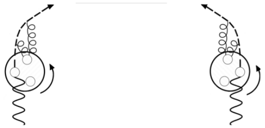

Figure – sketch of the proposed lensing mechanism

Pasquini, Rodini and Bacchetta showed that under restrictive conditions, such a lensing mechanism could be exhibited in the case of the pion[10].

9. Matthias Burkardt. In : *Nuclear Physics A* 735.1-2 (2004).
10. B. Pasquini, S. Rodini et A. Bacchetta. In : *Physical Review D* 100.5 (2019).

The Scalar Diquark Model

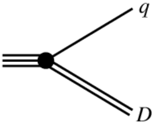

Figure – sketch of the scalar diquark model

- The nucleon splits into a quark and scalar diquark structure
- Lorentz covariance is maintained
- Both the quark and diquark are charged ($e_q = -e_S$)

Without loss of generality, calculations are :

- computed in the LC gauge ($A^+ = 0$), SIDIS link
- Regularized through dimensional regularization (dimension $D = 4 - 2\varepsilon$)

Potential momentum (1)

It can be shown that the transverse momentum following Ji and JM's decomposition write [11] :

$$
\begin{aligned}
\langle \vec{k}_\perp \rangle_{Ji} &= \frac{i}{4P^+} \langle P, S | \bar{\psi}(0) \overleftrightarrow{D}_\perp \psi(0) | P, S \rangle \\
\langle \vec{k}_\perp \rangle_{JM} &= \frac{i}{4P^+} \langle P, S | \bar{\psi}(0) \overleftrightarrow{D}_{pure,\perp} \psi(0) | P, S \rangle \\
k_{pot} &= \frac{i}{4P^+} \langle P, S | \bar{\psi}(0) A_{phys,\perp} \psi(0) | P, S \rangle
\end{aligned}
\tag{6}
$$

From symmetry :

$$
\begin{aligned}
\langle \vec{k}_\perp \rangle_{Ji} &= - \langle \vec{k}_\perp \rangle_{Ji} \\
\langle \vec{k}_\perp \rangle_{JM,DIS} &= - \langle \vec{k}_\perp \rangle_{JM,DY}
\end{aligned}
\tag{7}
$$

- Only the JM $k_\perp$ contributes.
- No asymmetry can arise without a gluon or photon exchange.

11. $\langle X \rangle = \frac{\langle P,S|X|P,S \rangle}{\langle P,S|P,S \rangle}$

Potential momentum (2)

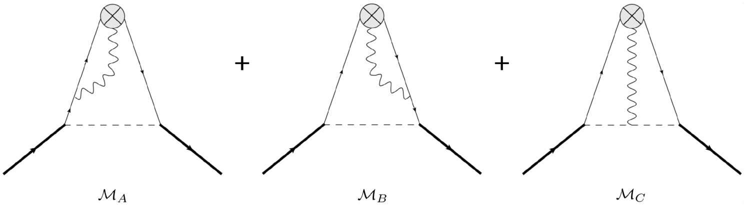

Figure – Diagramms contributing to $k_\perp^q$

Calculating these diagramms will give the potential OAM, keeping in mind :

- $\langle k_\perp^q \rangle_{JI}^{A+B} = \langle k_\perp^q \rangle_{JM}^{A+B} = 0$ (conservation of momentum)
- $\langle k_\perp^q \rangle_{JI}^{C} = 0$ (PT symmetry)
- $\langle k_\perp^q \rangle + \langle k_\perp^S \rangle = 0$ (Burkardt sum rule)

Potential momentum (3)

All in all, the calculation yields [12] :

$$k_{pot}^{q,i} = -\,\epsilon_{\perp}^{ij}\, s_{\perp}^{j}\,\frac{\pi}{6}(3m_q + M)\frac{\lambda^2 e_q e_S}{(4\pi)^2(4\pi\varepsilon)^2} + \mathcal{O}(1/\varepsilon) \tag{8}$$

Remarks :

- $k_{\perp,pot}^{q} \neq 0$! There is a Sivers shift in the SDM
- Crosscheck with known Sivers function [13].
- The Burkardt sum rule holds $\sum_{q,S}\langle k_{\perp}^{q,S}\rangle = 0$
- If the same mechanism produces the transverse momentum and the angular momentum, we need to look at two loops.

12. m_q and M are the masses of the struck quark and proton respectively. λ is the field coupling, e_q and e_S are the charges of the quark and diquark repectively.

13. S. MeiSSner, A. Metz et K. Goeke. "Relations between generalized and transverse momentum dependent parton distributions". In : *Physical Review D* 76.3 (2007).

Potential Angular Momentum

Looking now at L^z_{pot}, we calculate :

$$L^z_{pot} = \frac{-g\epsilon^{ij}_{\perp}}{2P^+}\left[-i\nabla^i_{\Delta_\perp}\langle p+\Delta, S'|\bar{\psi}(0)\gamma^+ A^j_{phys,\perp}\psi(0)|p,S\rangle\right]_{\Delta=0} \tag{9}$$
$$= \mathcal{O}(1/\varepsilon)$$

- Both $\mathcal{L}^z$ and L^z are PT-even
- the Ji and JM definitions of OAM coïncide at two loops in the SDM to order $\mathcal{O}(1/\epsilon^2)$ unlike the transverse momentum.
- Does the two-bodied nature of the system prevent it from acquiring any Lorentz torque ? [14]

14. Cédric Lorcé. In : *The European Physical Journal C* 78.9 (2018).

Conclusions and Outlook

Summary and Conclusions :

- Both $\vec{k}^q_{pot}$ and $L^q_{z,pot}$ were computed at two-loop in the scalar diquark model to the order $\mathcal{O}(\lambda^2 e_q e_S)$
- The difference between the two decomposition appears when interactions play a role.
- We found the surprising result $L^{z,q}_{pot} = \mathcal{O}(1/\varepsilon)$, whereas $k_{\perp,pot} = \mathcal{O}(1/\varepsilon^2)$, which puts in jeopardy the inuitive proposal of a lensing mechanism

Outlook

- Deeper perturbative calculation of $L^{q,z}_{pot}$
- Continue using the SDM as a tool to challenge our physical understanding of the problems at hand
- Adress more challenging and complex models.

Additional frames

Challenges of the study of the nucleon

The proton is composed of minimum three quarks (uud) in constant interaction.

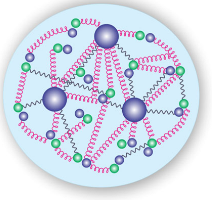

Figure – Sketch of the inside of a proton

Challenges :

- *Quantum* : size $\sim 10^{-15} m$
- *Relativistic* : High Energy, $m_{\text{nucleon}} \sim GeV$, $m_{\text{quarks u, d}} \sim MeV$
- *Particle number fluctuation*
- *Confinement* : QCD running coupling large at long distances $\rightarrow$ non-perturbative treatment
- *Gauge invariance* : Correct definition of observables is complicated

The color-gauge link

The Wilson line linking z_1 to z_2 along the path γ can be written as :

$$\mathcal{W}_\gamma(z_1, z_2) = \mathcal{P} \exp\left(-ig \int_\gamma \mathrm{d}z \cdot A(z)\right) \tag{10}$$

It has the following properties :

- makes the correlator color-gauge invariant
- encodes Final or Initial State Interactions (FSI, ISI)

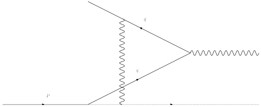

Figure – Example of an Initial State Interaction

Figure – Different path correspond to different processes

- is process dependent
- breaks the naive time reversal symmetry

The Scalar Diquark Model

The lagrangian of the model reads :

$$\mathcal{L}_{\mathsf{SDM}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\Psi}_N\left(i\partial\!\!\!/ - M\right)\Psi_N + \bar{\psi}\left(i\partial\!\!\!/ - m_q\right)\psi + \partial_\mu\phi^*\partial^\mu\phi - m_s^2\phi^*\phi$$
$$+ \lambda\left(\bar{\psi}\Psi_N\phi^* + \bar{\Psi}_N\psi\phi\right) - e_q\bar{\psi}A\!\!\!/\psi - ie_s\left(\phi^*\overleftrightarrow{\partial^\mu}\phi\right)A_\mu + e_s^2 A_\mu A^\mu\phi^*\phi \qquad (11)$$

where, the fields A, Ψ_N, ψ and ϕ represent respectively the gluon, nucleon, quark and diquark field with respective masses : 0, M, m_q, m_S.

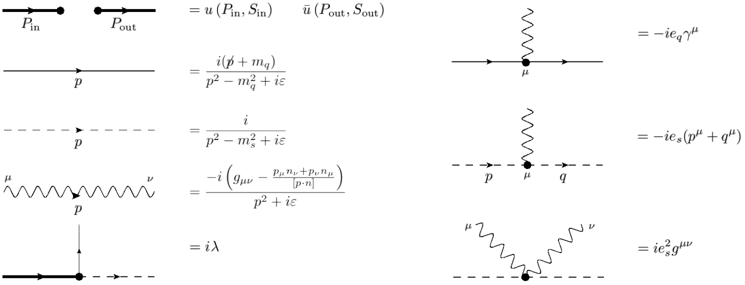

Figure – Feynaman rules for the SDM