

# Production of three isolated photons in the high-energy factorization approach

V. A. Saleev<sup>1,2</sup>

<sup>1</sup> Samara National Research University

<sup>2</sup> Joint Institute for Nuclear Research

12-16 April 2021  
Stony Brook, NY

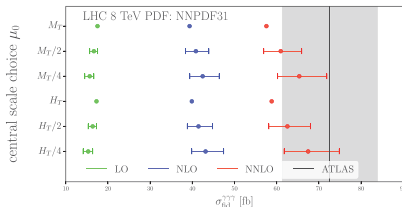
**Talk at XXVIII International Workshop on Deep-Inelastic  
Scattering and Related Subjects**

## Outline

- 1 Introduction
- 2 Parton Reggeization Approach
- 3 UnPDFs with exact normalization
- 4 ReggeQCD and KaTie
- 5 Three-photon production at the LHC
- 6 Conclusions

## Introduction

NNLO predictions in CPM of QCD  
 versus  
 ATLAS, 7 TeV,  $\sigma = 72_{-6.5}^{+6.5}(\text{stat})_{-9.2}^{+9.2}(\text{syst})$  fb



**Figure 1.** Predictions for the fiducial cross-section in LO (green), NLO (blue) and NNLO (red) QCD versus ATLAS data (black). Shown are predictions for six scale choices. The error bars on the theory predictions reflect scale variation only. For two of the scales only the central predictions are shown.

*H.A.Chawdhry, M.L.Czaron, A.Mitov and  
 R.Poncelet, NNLO QCD corrections to  
 three-photon production at the LHC,  
 JHEP 02 (2020), 057*

**Table 2**

Predictions for fiducial  $pp \rightarrow \gamma\gamma\gamma + X$  cross sections at different centre-of-mass energies; the numbers in brackets are integration errors, while at NNLO they also include systematic uncertainties from  $r_{\text{cut}}$  dependence, see Ref. [50]; the percentages correspond to scale uncertainties;  $K_{\text{NLO}} = \sigma_{\text{NLO}}/\sigma_{\text{LO}}$ ,  $K_{\text{NNLO}} = \sigma_{\text{NNLO}}/\sigma_{\text{NLO}}$ .

$\sqrt{s}$ [TeV]	$\sigma_{\text{LO}}$ [fb]	$\sigma_{\text{NLO}}$ [fb]	$\sigma_{\text{NNLO}}$ [fb]	$K_{\text{NLO}}$	$K_{\text{NNLO}}$
7	$13.8237(14)_{-7.0\%}^{+6.0\%}$	$37.6084(35)_{-7.5\%}^{+9.7\%}$	$57.84(20)_{-8.3\%}^{+10.7\%}$	2.72	1.54
8	$15.3023(15)_{-8.0\%}^{+6.9\%}$	$43.1076(22)_{-7.6\%}^{+9.9\%}$	$67.42(20)_{-8.5\%}^{+11.0\%}$	2.82	1.56
13	$21.8814(22)_{-11.2\%}^{+10.4\%}$	$69.6330(60)_{-11.7\%}^{+10.3\%}$	$114.60(43)_{-9.1\%}^{+11.9\%}$	3.18	1.65
14	$23.0839(23)_{-11.7\%}^{+10.9\%}$	$74.7875(82)_{-10.0\%}^{+10.4\%}$	$123.83(24)_{-9.2\%}^{+12.0\%}$	3.24	1.66
27	$36.9540(37)_{-16.1\%}^{+16.0\%}$	$138.797(13)_{-14.8\%}^{+12.2\%}$	$245.91(48)_{-9.9\%}^{+13.2\%}$	3.76	1.77
100	$92.3779(92)_{-24.0\%}^{+26.6\%}$	$442.310(39)_{-23.0\%}^{+21.7\%}$	$878.9(24)_{-13.5\%}^{+15.0\%}$	4.79	1.99

*S.Kallweit, V.Sotnikov and  
 M.Wiesemann, Triphoton production at  
 hadron colliders in NNLO QCD, Phys.  
 Lett. B 812 (2021), 136013*

## Parton Reggeization Approach (PRA)

Details of the LO PRA are presented in Refs.

- *Nefedov M.A., Saleev V.A., Shipilova A.V. Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach. Phys. Rev. D. 2013. V. 87. P. 094030.*
- *Karpishkov A.V., Nefedov M.A. and Saleev V.A.,  $B B\bar{B}$  angular correlations at the LHC in parton Reggeization approach merged with higher-order matrix elements// Phys. Rev. D. 2017. V. 96. P. 096019.*
- *M. A. Nefedov and V. A. Saleev, High-Energy Factorization for Drell-Yan process in  $pp$  and  $p\bar{p}$  collisions with new Unintegrated PDFs, Phys. Rev. D **102** (2020), 114018*

Developments of PRA in NLO can be found here

- M. A. Nefedov, Towards stability of NLO corrections in High-Energy Factorization via Modified Multi-Regge Kinematics approximation, JHEP **08** (2020), 055
- M. A. Nefedov, Computing one-loop corrections to effective vertices with two scales in the EFT for Multi-Regge processes in QCD,” Nucl. Phys. B **946** (2019), 114715
- M. Nefedov and V. Saleev, On the one-loop calculations with Reggeized quarks,” Mod. Phys. Lett. A **32** (2017) no.40, 1750207

# Parton Reggeization Approach (PRA)

## I. High-energy factorization

$$d\sigma = \sum_{i,j} \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \Phi_i(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \Phi_j(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{ij}^{\text{PRA}},$$

where  $t_{1,2} = -\mathbf{q}_{T1,2}^2$ ,  $i, j = R, Q, \bar{Q}$

$\Phi_{R/Q}(x, t, \mu^2)$  are gluon/quark unintegrated parton distribution functions (uPDFs)

$d\hat{\sigma}_{ij}^{\text{PRA}}$  is partonic cross section which is written via off-shell squared matrix elements  $\overline{|M^{\text{PRA}}|^2}$  of PRA, and

$$\lim_{t_1, t_2 \rightarrow 0} \int \frac{d\phi_1}{2\pi} \int \frac{d\phi_2}{2\pi} \overline{|M^{\text{PRA}}|^2} = \overline{|M^{\text{CPM}}|^2}$$

## Parton Reggeization Approach

### II. Off-shell amplitudes as multi-Regge limit of auxiliary $(n + 2)$ QCD amplitudes

The PRA hard-scattering amplitude is *gauge-invariant* because the initial-state off-shell partons are treated as Reggeized partons  $(R, Q, \bar{Q})$  in a sense of gauge-invariant EFT for QCD processes in Multi-Regge Kinematics(MRK), introduced by L.N. Lipatov.

#### Feynman's rules of EFT:

- L. N. Lipatov, Gauge invariant effective action for high-energy processes in QCD, Nucl. Phys. B **452**, 369 (1995).
- L. N. Lipatov and M. I. Vyazovsky, QuasimultiRegge processes with a quark exchange in the  $t$  channel, Nucl. Phys. B **597**, 399 (2001).
- M. A. Nefedov, ReggeQCD model-file for FeynArts.

## Parton Reggeization Approach

### III. Modified KMR unintegrated PDFs with exact normalization at arbitrary $x$

$$\Phi_i(x, t, \mu^2) = \frac{d}{dt} [T_i(t, \mu^2, \mathbf{x}) F_i(x, t)],$$

where  $T_i(t, \mu^2, \mathbf{x})$  is usually referred to as *Sudakov formfactor*, satisfying the boundary conditions  $T_i(t = 0, \mu^2, \mathbf{x}) = 0$  and  $T_i(t = \mu^2, \mu^2, \mathbf{x}) = 1$ .

$$\Phi_i(x, t, \mu_Y^2) = \frac{\alpha_s(t)}{2\pi} \frac{T_i(t, \mu^2, \mathbf{x})}{t} \sum_{j=q, \bar{q}, g} \int_x^1 dz P_{ij}(z) F_j\left(\frac{x}{z}, t\right) \theta(\Delta(t, \mu_Y^2) - z).$$

## Parton Reggeization Approach

Modified KMR unintegrated PDFs with exact normalization at arbitrary  $x$

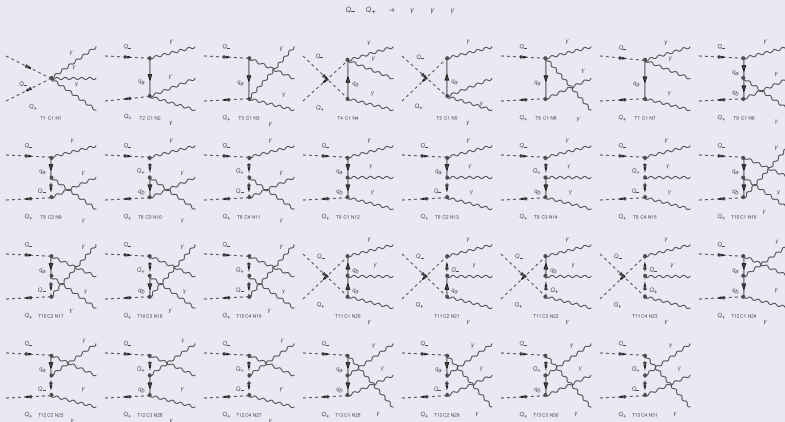
$$\begin{aligned}
 T_i(t, \mu^2, \mathbf{x}) &= \exp \left[ - \int_t^{\mu^2} \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} (\tau_i(t', \mu^2) + \Delta\tau_i(t', \mu^2, \mathbf{x})) \right] \\
 \tau_i(t, \mu^2) &= \sum_j \int_0^1 dz z P_{ji}(z) \theta(\Delta(t, \mu^2) - z), \\
 \Delta\tau_i(t, \mu^2, \mathbf{x}) &= \sum_j \int_0^1 dz \theta(z - \Delta(t, \mu^2)) \left[ z P_{ji}(z) - \frac{F_j(\frac{x}{z}, t)}{F_i(x, t)} P_{ij}(z) \theta(z - x) \right].
 \end{aligned}$$

For details, see Ref.

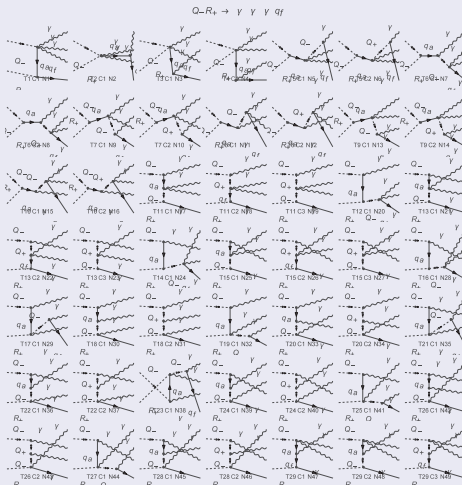
- *M. A. Nefedov and V. A. Saleev, High-Energy Factorization for Drell-Yan process in  $pp$  and  $p\bar{p}$  collisions with new Unintegrated PDFs, Phys. Rev. D **102** (2020), 114018*



## LO contribution to three-photon production

Feynman diagrams for  $Q\bar{Q} \rightarrow \gamma\gamma\gamma$  obtained with ReggeQCD,  $N_{tot} = 28$ 

## Most important real NLO correction to three-photon production

Feynman diagrams for  $QR \rightarrow \gamma\gamma\gamma q$  obtained with ReggeQCD,  $N_{tot} = 238$ 

## MC event generator KaTie

As it has been shown in Refs.

- *Nefedov M.A., Saleev V.A., Shipilova A.V. Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach. Phys. Rev. D. 2013. V. 87. P. 094030.*
- *Karpishkov A.V., Nefedov M.A. and Saleev V.A.,  $B B\bar{B}$  angular correlations at the LHC in parton Reggeization approach merged with higher-order matrix elements// Phys. Rev. D. 2017. V. 96. P. 096019.*
- *Kutak K., Maciula R., Serino M., Szczurek A. and van Hameren A., Four-jet production in single- and double-parton scattering within high-energy factorization// JHEP. 2016. V. 1604. P. 175,*

at the level of tree diagrams, analytical formalism based on Lipatov's EFT fully coincide with numerically generated off-shell amplitudes using MC event generator KaTie

- *van Hameren A., KaTie: For parton-level event generation with  $kT$ -dependent initial states. Comput.Phys.Commun. 2018. V. 224. P. 371.*

## Three-photon production at the LHC

### Setup of ATLAS measurements at 8 TeV

- For photons transverse momenta:  $p_{T1} > 27$  GeV,  $p_{T2} > 22$  GeV,  $p_{T3} > 15$  GeV
- For pseudorapidity all photons:  $|\eta_\gamma| < 2.37$ , excluding the range  $1.37 < |\eta_\gamma| < 1.56$
- Minimum three-photon invariant mass:  $M_{3\gamma} > 50$  GeV
- Photon-photon separation conditions:  $\Delta R_{ij} > R_{\gamma\gamma} = 0.45$ , where  $\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$
- Photon-parton separation conditions:  $\Delta R_{ij} > R_0 = 0.40$

Fragmentation contribution is included using the Frixione condition for  $QR \rightarrow q\gamma\gamma\gamma$  subprocesses

$$E_T^{iso}(\Delta R_{q\gamma}) = p_{Tq} < E_T^{max} \frac{1 - \cos(\Delta R_{q\gamma})}{1 - \cos(R_0)}, \text{ where } E_T^{max} = 10 \text{ GeV}$$

## Three-photon production at the LHC: total cross section

### Details of numerical calculations

- All final calculations have been done using MC event generator KaTie.
- LO contribution of subprocess  $Q\bar{Q} \rightarrow \gamma\gamma\gamma$  for crosscheck were calculated both with KaTie and using direct integration of analytical squared amplitudes obtained with the help of Feynman rules of Lipatov EFT (with ReggeQCD model file in FeynArts and FeynCalc).
- To extract double counting between LO ( $Q\bar{Q} \rightarrow \gamma\gamma\gamma$ ) and NLO ( $QR \rightarrow q\gamma\gamma\gamma$ ) diagrams with emission of additional quark which is strongly separated in rapidity of three-photon system we apply approach proposed early in PRA, see discussion in [\*].
- NLO contribution from subprocess  $Q\bar{Q} \rightarrow g\gamma\gamma\gamma$  is negligibly small, see discussion in [\*].

[\*] *M. Nefedov and V. Saleev, Diphoton production at the Tevatron and the LHC in the NLO approximation of the parton Reggeization approach, Phys. Rev. D* **92** (2015) no.9, 094033

## Three-photon production at the LHC: total cross section

The scale dependence,  $\mu_F = \mu_R = \mu$

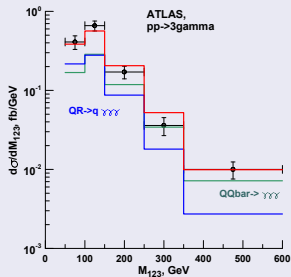
- $\mu \sim M_{3\gamma} = \sqrt{(k_{1\gamma} + k_{2\gamma} + k_{3\gamma})^2}$  - invariant mass of three-photon system,
- $\mu \sim k_{T,3\gamma} = k_{T,1\gamma} + k_{T,2\gamma} + k_{T,3\gamma}$  - sum of transverse momentum moduli,
- $\mu \sim E_{T,3\gamma} = \sqrt{(\mathbf{k}_{T1} + \mathbf{k}_{T2} + \mathbf{k}_{T3})^2 + M_{3\gamma}^2}$  - transverse energy of three-photon system.

Hard scale, $\mu$	$\sigma_{\text{LO}}$ [fb]	$\sigma_{\text{NLO}}$ [fb]
$M_{3\gamma}/2$	$31.07^{+8.87}_{-6.76}$	$69.22^{+4.05}_{-1.07}$
$p_{T,3\gamma}/2$	$29.72^{+9.22}_{-6.72}$	$69.76^{+4.29}_{-1.85}$
$E_{T,3\gamma}/2$	$32.50^{+9.80}_{-2.65}$	$71.00^{+4.93}_{-2.65}$

$\sqrt{s}$ [TeV]	$\sigma_{\text{LO}}$	$\sigma_{\text{NLO}}$	$K_{\text{NLO}}$	$\sigma_{\text{NNLO}}^{\text{CPM}}$
8	$32.50^{+9.80}_{-7.46}$	$71.00^{+4.93}_{-2.65}$	2.18	$67.42^{+7.41}_{-5.73}$
13	$53.91^{+18.14}_{-14.11}$	$126.79^{+10.43}_{-7.30}$	2.35	$114^{+13.64}_{-10.54}$
27	$115.25^{+45.09}_{-34.45}$	$298.54^{+30.71}_{-25.55}$	2.59	$245.91^{+32.46}_{-24.34}$

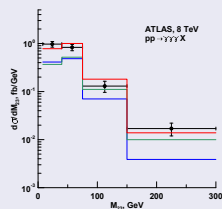
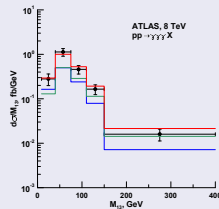
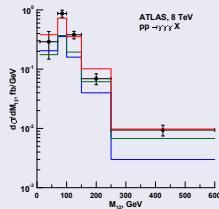
## Three-photon production at the LHC

The differential cross sections for the production of three isolated photons as functions  $M_{123}$ . The hard scale in PRA calculations are taken as invariant mass of photons,  $\mu = M_{3\gamma}$ . The green histogram corresponds LO contribution from  $Q\bar{Q} \rightarrow \gamma\gamma\gamma$  subprocess. The blue histogram corresponds NLO contribution from  $QR \rightarrow q\gamma\gamma\gamma$  subprocess. The red histogram is their sum.



## Three-photon production at the LHC

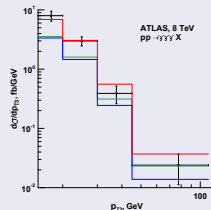
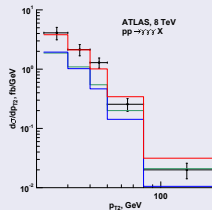
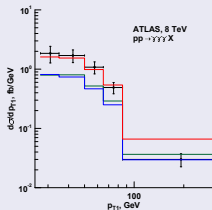
The differential cross sections for the production of three isolated photons as functions of  $M_{12}$  (left panel),  $M_{13}$  (central panel), and  $M_{23}$  (right panel).





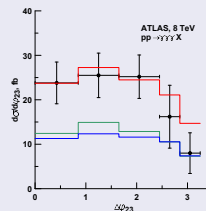
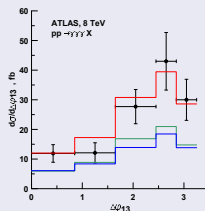
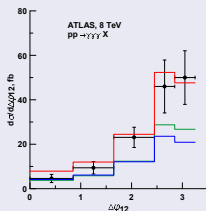
## Three-photon production at the LHC

The differential cross sections for the production of three isolated photons as functions of  $p_{T1}$  (left panel),  $p_{T2}$  (central panel) and  $p_{T3}$  (right panel).



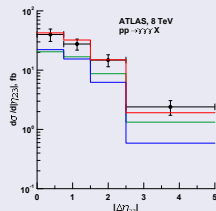
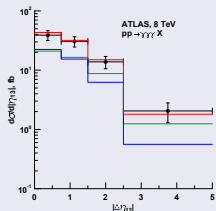
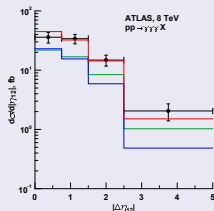
# Three-photon production at the LHC

The differential cross sections for the production of three isolated photons as functions of  $|\Delta\phi_{12}|$  (left panel),  $|\Delta\phi_{13}|$  (central panel) and  $\Delta\phi_{23}$  (right panel).



# Three-photon production at the LHC

The differential cross sections for the production of three isolated photons as functions of  $|\Delta y_{12}|$  (left panel),  $|\Delta y_{13}|$  (central panel) and  $|\Delta y_{23}|$  (right panel).



## Conclusions

- We describe cross section and spectra for three-photon production in LO PRA with real NLO corrections
- We demonstrate applicability of new KMR-type quark uPDFs for use in high-energy factorization calculations
- PRA results in LO+NLO approximation are roughly coincide with full NNLO predictions of CPM for  $\sqrt{s} = 8$  TeV
- At higher energies (13 and 27 TeV) PRA predicts larger cross sections, up to  $\sim 10$  % and  $\sim 20$  %, respectively.

**Thank you for your attention!**