

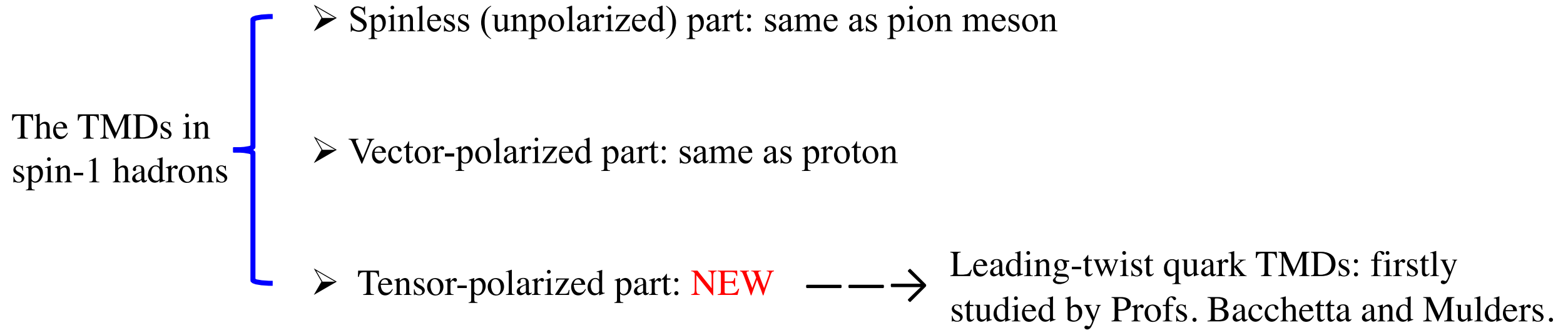
Transverse-momentum-dependent parton distribution functions for spin-1 hadrons

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Reference: S. Kumano and Qin-Tao Song, Phys. Rev. D 103 (2021) 014025.

Spin structure of spin-1 hadrons



In this talk we will discuss:

a complete decomposition of quark correlation function and higher-twist TMDs for spin-1 hadrons

J. P. Ralston and D. E. Soper, NPB152, 109 (1979).

R. D. Tangerman and P. J. Mulders, PRD 51, 3357 (1995).

P. J. Mulders and R. D. Tangerman, NPB461, 197 (1996).

A. Bacchetta and P. Mulders, PRD 62, 114004 (2000).

Daniel Boer, Sabrina Cotogno and Tom van Daal *et al*, JHEP 10, 013 (2016)

Quark correlation function of spin-1 hadrons

$$\Phi_{ij}^{[c]}(k, P, S, T) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \times \langle P, S, T \left| \bar{\psi}_j(0) W^{[c]}(0, \xi) \psi_i(\xi) \right| P, S, T \rangle$$

A. Bacchetta and P. Mulders, PRD 62, 114004 (2000).

S. Kumano and Qin-Tao Song, PRD 103 (2021) 014025.

Terms of A_{13} – A_{20} are given by Bacchetta and Mulders.

The new terms of B_{21} – B_{52} terms are dependent on the lightcone vector n due to the gauge link.

S is the spin vector and T is spin tensor (5 parameters).

$$\begin{aligned} \Phi(k, P, T | n) = & \frac{A_{13}}{M} T_{kk} + \frac{A_{14}}{M^2} T_{kk} \not{P} + \frac{A_{15}}{M^2} T_{kk} \not{k} + \frac{A_{16}}{M^3} \sigma_{Pk} T_{kk} + A_{17} T^{k\nu} \gamma_\nu + \frac{A_{18}}{M} \sigma_{\nu P} T^{k\nu} + \frac{A_{19}}{M} \sigma_{\nu k} T^{k\nu} \\ & + \frac{A_{20}}{M^2} \varepsilon^{\mu\nu Pk} \gamma_\mu \gamma_5 T_{\nu k} + \frac{B_{21} M}{P \cdot n} T_{kn} + \frac{B_{22} M^3}{(P \cdot n)^2} T_{nn} + \frac{B_{23}}{P \cdot n M} \varepsilon^{\mu k P n} T_{\mu k} (i\gamma_5) + \frac{B_{24} M}{(P \cdot n)^2} \varepsilon^{\mu k P n} T_{\mu n} (i\gamma_5) + \frac{B_{25}}{P \cdot n} \not{n} T_{kk} \\ & + \frac{B_{26} M^2}{(P \cdot n)^2} \not{n} T_{kn} + \frac{B_{27} M^4}{(P \cdot n)^3} \not{n} T_{nn} + \frac{B_{28}}{P \cdot n} \not{P} T_{kn} + \frac{B_{29} M^2}{(P \cdot n)^2} \not{P} T_{nn} + \frac{B_{30}}{P \cdot n} \not{k} T_{kn} + \frac{B_{31} M^2}{(P \cdot n)^2} \not{k} T_{nn} + \frac{B_{32} M^2}{P \cdot n} \gamma_\mu T^{\mu n} \\ & + \frac{B_{33}}{P \cdot n} \varepsilon^{\mu\nu Pk} \gamma_\mu \gamma_5 T_{\nu n} + \frac{B_{34}}{P \cdot n} \varepsilon^{\mu\nu Pn} \gamma_\mu \gamma_5 T_{\nu k} + \frac{B_{35} M^2}{(P \cdot n)^2} \varepsilon^{\mu\nu Pn} \gamma_\mu \gamma_5 T_{\nu n} + \frac{B_{36}}{P \cdot n M^2} \varepsilon^{\mu k P n} \gamma_\mu \gamma_5 T_{kk} \\ & + \frac{B_{37}}{(P \cdot n)^2} \varepsilon^{\mu k P n} \gamma_\mu \gamma_5 T_{kn} + \frac{B_{38} M^2}{(P \cdot n)^3} \varepsilon^{\mu k P n} \gamma_\mu \gamma_5 T_{nn} + \frac{B_{39}}{(P \cdot n)^2} \not{n} \gamma_5 T_{\mu k} \varepsilon^{\mu k P n} + \frac{B_{40} M^2}{(P \cdot n)^3} \not{n} \gamma_5 T_{\mu n} \varepsilon^{\mu k P n} \\ & + \frac{B_{41}}{P \cdot n M} \sigma_{Pk} T_{kn} + \frac{B_{42} M}{(P \cdot n)^2} \sigma_{Pk} T_{nn} + \frac{B_{43}}{P \cdot n M} \sigma_{Pn} T_{kk} + \frac{B_{44} M}{(P \cdot n)^2} \sigma_{Pn} T_{kn} + \frac{B_{45} M^3}{(P \cdot n)^3} \sigma_{Pn} T_{nn} + \frac{B_{46}}{P \cdot n M} \sigma_{kn} T_{kk} \\ & + \frac{B_{47} M}{(P \cdot n)^2} \sigma_{kn} T_{kn} + \frac{B_{48} M^3}{(P \cdot n)^3} \sigma_{kn} T_{nn} + \frac{B_{49} M}{P \cdot n} \sigma_{\mu n} T^{\mu k} + \frac{B_{50} M^3}{(P \cdot n)^2} \sigma_{\mu n} T^{\mu n} + \frac{B_{51} M}{P \cdot n} \sigma_{\mu P} T^{\mu n} + \frac{B_{52} M}{P \cdot n} \sigma_{\mu k} T^{\mu n}, \end{aligned}$$

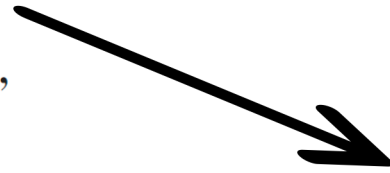
Twist-2 TMDs

$$\Phi^{[\gamma^+]}(x, k_T, T) = f_{1LL}(x, k_T^2) S_{LL} - f_{1LT}(x, k_T^2) \frac{S_{LT} \cdot k_T}{M} \\ + f_{1TT}(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2},$$



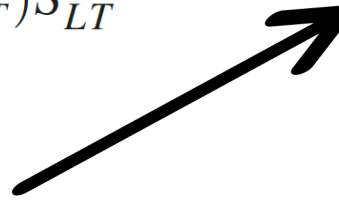
time reversal-even ones
 γ^μ terms

$$\Phi^{[\gamma^+ \gamma_5]}(x, k_T, T) = g_{1LT}(x, k_T^2) \frac{S_{LT\mu} \epsilon_T^{\mu\nu} k_{T\nu}}{M} \\ + g_{1TT}(x, k_T^2) \frac{S_{TT\mu\rho} k_T^\rho \epsilon_T^{\mu\nu} k_{T\nu}}{M^2},$$



time reversal-odd ones
 $\gamma^\mu \gamma_5$ and $\sigma^{\mu\nu}$ terms

$$\Phi^{[\sigma^{i+}]}(x, k_T, T) = h_{1LL}^\perp(x, k_T^2) \frac{S_{LL} k_T^i}{M} + h'_{1LT}(x, k_T^2) S_{LT}^i \\ - h_{1LT}^\perp(x, k_T^2) \frac{k_T^i S_{LT} \cdot k_T}{M^2} \\ - h'_{1TT}(x, k_T^2) \frac{S_{TT}^{ij} k_{Tj}}{M} \\ + h_{1TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \frac{k_T^i}{M}$$

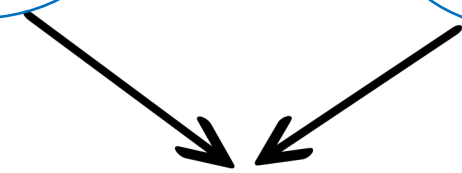


10 leading-twist TMDs

Twist-3 TMDs: time reversal-even ones

$$\Phi^{[\gamma^i]}(x, k_T, T) = \frac{M}{P^+} \left[f_{LL}^\perp(x, k_T^2) S_{LL} \frac{k_T^i}{M} + f'_{LT}(x, k_T^2) S_{LT}^i - f_{LT}^\perp(x, k_T^2) \frac{k_T^i S_{LT} \cdot k_T}{M^2} - f'_{TT}(x, k_T^2) \frac{S_{TT}^{ij} k_{Tj}}{M} \right. \\ \left. + f_{TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \frac{k_T^i}{M} \right].$$

$$\Phi^{[1]}(x, k_T, T) = \frac{M}{P^+} \left[e_{LL}(x, k_T^2) S_{LL} - e_{LT}^\perp(x, k_T^2) \frac{S_{LT} \cdot k_T}{M} + e_{TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \right],$$

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+} \left[e_{LT}(x, k_T^2) \frac{S_{LT\mu} \epsilon_T^{\mu\nu} k_{T\nu}}{M} - e_{TT}(x, k_T^2) \frac{S_{TT\mu\rho} k_T^\rho \epsilon_T^{\mu\nu} k_{T\nu}}{M^2} \right].$$


Only dependent on the new terms ($B_{21} - B_{52}$) in the correlation function.

Twist-3 TMDs: time reversal-odd ones

$$\Phi^{[\sigma^{-+}]}(x, k_T, T) = \frac{M}{P^+} \left[h_{LL}(x, k_T^2) S_{LL} - h_{LT}(x, k_T^2) \frac{S_{LT} \cdot k_T}{M} + h_{TT}(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \right],$$

$$\Phi^{[\sigma^{ij}]}(x, k_T, T) = \frac{M}{P^+} \left[h_{LT}^\perp(x, k_T^2) \frac{S_{LT}^i k_T^j - S_{LT}^j k_T^i}{M} - h_{TT}^\perp(x, k_T^2) \frac{S_{TT}^{il} k_{Tl} k_T^j - S_{TT}^{jl} k_{Tl} k_T^i}{M^2} \right].$$

$$\begin{aligned} \Phi^{[\gamma^i \gamma_5]}(x, k_T, T) = \frac{M}{P^+} & \left[-g_{LL}^\perp(x, k_T^2) S_{LL} \frac{\epsilon_T^{ij} k_{Tj}}{M} - g_{LT}^\perp(x, k_T^2) \epsilon_T^{ij} S_{LTj} + g_{LT}^\perp(x, k_T^2) \frac{\epsilon_T^{ij} k_{Tj} S_{LT} \cdot k_T}{M^2} \right. \\ & \left. + g_{TT}^\perp(x, k_T^2) \frac{\epsilon_T^{ij} S_{TTjl} k_T^l}{M} - g_{TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \frac{\epsilon_T^{ij} k_{Tj}}{M} \right]. \end{aligned}$$



Only dependent on the new terms (B_{21} – B_{52}) in the correlation function.

There are 20 twist-3 TMDs.

Twist-4 TMDs

$$\Phi^{[\gamma^-]} = \frac{M^2}{P^{+2}} \left[f_{3LL}(x, k_T^2) S_{LL} - f_{3LT}(x, k_T^2) \frac{S_{LT} \cdot k_T}{M} + f_{3TT}(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \right],$$

$$\Phi^{[\gamma^- \gamma_5]} = \frac{M^2}{P^{+2}} \left[g_{3LT}(x, k_T^2) \frac{S_{LT\mu} \epsilon_T^{\mu\nu} k_{T\nu}}{M} + g_{3TT}(x, k_T^2) \frac{S_{TT\mu\rho} k_T^\rho \epsilon_T^{\mu\nu} k_{T\nu}}{M^2} \right],$$

$$\begin{aligned} \Phi^{[\sigma^{i-}]} = \frac{M^2}{P^{+2}} & \left[h_{3LL}^\perp(x, k_T^2) S_{LL} \frac{k_T^i}{M} + h'_{3LT}(x, k_T^2) S_{LT}^i - h_{3LT}^\perp(x, k_T^2) \frac{k_T^i S_{LT} \cdot k_T}{M^2} - h'_{3TT}(x, k_T^2) \frac{S_{TT}^{ij} k_{Tj}}{M} \right. \\ & \left. + h_{3TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \frac{k_T^i}{M} \right]. \end{aligned}$$

There are 10 twist-4 TMDs.

Sum rules for time-reversal-odd TMDs

$$\int d^2 k_T h_{1LT}(x, k_T^2) = 0,$$

$$\int d^2 k_T g_{LT}(x, k_T^2) = 0,$$

$$\int d^2 k_T h_{LL}(x, k_T^2) = 0,$$

$$\int d^2 k_T h_{3LT}(x, k_T^2) = 0.$$

Summary

- (1) A complete decomposition of quark correlation function for spin-1 hadrons.
- (2) Twist-3 and Twist-4 TMDs are defined for spin-1 hadrons.
- (3) Sum rules are obtained for time-reversal-odd TMDs

Thank you very much