Transverse-momentum-dependent parton distribution functions for spin-1 hadrons

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XXVIII International Workshop on Deep-Inelastic Scattering (DIS) and Related Subjects, 12-16 April 2021, Stony Brook, NY.

Reference: S. Kumano and Qin-Tao Song, Phys. Rev. D 103 (2021) 014025.

Spin structure of spin-1 hadrons

The TMDs in spin-1 hadrons

- Spinless (unpolarized) part: same as pion meson
- Vector-polarized part: same as proton
- Tensor-polarized part: NEW $\dashrightarrow$ Leading-twist quark TMDs: firstly studied by Profs. Bacchetta and Mulders.

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In this talk we will discuss:

a complete decomposition of quark correlation function and higher-twist TMDs for spin-1 hadrons

J. P. Ralston and D. E. Soper, NPB152, 109 (1979).
R. D. Tangerman and P. J. Mulders, PRD 51, 3357 (1995).
P. J. Mulders and R. D. Tangerman, NPB461, 197 (1996).
A. Bacchetta and P. Mulders, PRD 62, 114004 (2000).
Daniel Boer, Sabrina Cotogno and Tom van Daal *et al*, JHEP 10, 013 (2016)

Quark correlation function of spin-1 hadrons

$$\Phi^{[c]}_{ij}(k,P,S,T) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi} \times \langle P,S,T \,|\, \bar{\psi}_j(0)\, W^{[c]}(0,\xi)\psi_i(\xi) \,|\, P,\, S,T\rangle$$

A. Bacchetta and P. Mulders, PRD 62, 114004 (2000).
S. Kumano and Qin-Tao Song, PRD 103 (2021) 014025.

S is the spin vector and T is spin tensor (5 parameters).

Terms of A_{13} –A_{20} are given by Bacchetta and Mulders.
The new terms of B_{21} –B_{52} terms are dependent on the lightcone vector n due to the gauge link.

$$\begin{aligned}
\Phi(k,P,T\,|n) = {} & \frac{A_{13}}{M}T_{kk} + \frac{A_{14}}{M^2}T_{kk}\not{P} + \frac{A_{15}}{M^2}T_{kk}\not{k} + \frac{A_{16}}{M^3}\sigma_{Pk}T_{kk} + A_{17}T^{k\nu}\gamma_\nu + \frac{A_{18}}{M}\sigma_{\nu P}T^{k\nu} + \frac{A_{19}}{M}\sigma_{\nu k}T^{k\nu} \\
& + \frac{A_{20}}{M^2}\varepsilon^{\mu\nu Pk}\gamma_\mu\gamma_5 T_{\nu k} + \frac{B_{21}M}{P\cdot n}T_{kn} + \frac{B_{22}M^3}{(P\cdot n)^2}T_{nn} + \frac{B_{23}}{P\cdot nM}\varepsilon^{\mu kPn}T_{\mu k}(i\gamma_5) + \frac{B_{24}M}{(P\cdot n)^2}\varepsilon^{\mu kPn}T_{\mu n}(i\gamma_5) + \frac{B_{25}}{P\cdot n}\not{n}T_{kk} \\
& + \frac{B_{26}M^2}{(P\cdot n)^2}\not{n}T_{kn} + \frac{B_{27}M^4}{(P\cdot n)^3}\not{n}T_{nn} + \frac{B_{28}}{P\cdot n}\not{P}T_{kn} + \frac{B_{29}M^2}{(P\cdot n)^2}\not{P}T_{nn} + \frac{B_{30}}{P\cdot n}\not{k}T_{kn} + \frac{B_{31}M^2}{(P\cdot n)^2}\not{k}T_{nn} + \frac{B_{32}M^2}{P\cdot n}\gamma_\mu T^{\mu n} \\
& + \frac{B_{33}}{P\cdot n}\varepsilon^{\mu\nu Pk}\gamma_\mu\gamma_5 T_{\nu n} + \frac{B_{34}}{P\cdot n}\varepsilon^{\mu\nu Pn}\gamma_\mu\gamma_5 T_{\nu k} + \frac{B_{35}M^2}{(P\cdot n)^2}\varepsilon^{\mu\nu Pn}\gamma_\mu\gamma_5 T_{\nu n} + \frac{B_{36}}{P\cdot nM^2}\varepsilon^{\mu kPn}\gamma_\mu\gamma_5 T_{kk} \\
& + \frac{B_{37}}{(P\cdot n)^2}\varepsilon^{\mu kPn}\gamma_\mu\gamma_5 T_{kn} + \frac{B_{38}M^2}{(P\cdot n)^3}\varepsilon^{\mu kPn}\gamma_\mu\gamma_5 T_{nn} + \frac{B_{39}}{(P\cdot n)^2}\not{n}\gamma_5 T_{\mu k}\varepsilon^{\mu kPn} + \frac{B_{40}M^2}{(P\cdot n)^3}\not{n}\gamma_5 T_{\mu n}\varepsilon^{\mu kPn} \\
& + \frac{B_{41}}{P\cdot nM}\sigma_{Pk}T_{kn} + \frac{B_{42}M}{(P\cdot n)^2}\sigma_{Pk}T_{nn} + \frac{B_{43}}{P\cdot nM}\sigma_{Pn}T_{kk} + \frac{B_{44}M}{(P\cdot n)^2}\sigma_{Pn}T_{kn} + \frac{B_{45}M^3}{(P\cdot n)^3}\sigma_{Pn}T_{nn} + \frac{B_{46}}{P\cdot nM}\sigma_{kn}T_{kk} \\
& + \frac{B_{47}M}{(P\cdot n)^2}\sigma_{kn}T_{kn} + \frac{B_{48}M^3}{(P\cdot n)^3}\sigma_{kn}T_{nn} + \frac{B_{49}M}{P\cdot n}\sigma_{\mu n}T^{\mu k} + \frac{B_{50}M^3}{(P\cdot n)^2}\sigma_{\mu n}T^{\mu n} + \frac{B_{51}M}{P\cdot n}\sigma_{\mu P}T^{\mu n} + \frac{B_{52}M}{P\cdot n}\sigma_{\mu k}T^{\mu n},
\end{aligned}$$

Twist-2 TMDs

$$\Phi^{[\gamma^+]}(x, k_T, T) = f_{1LL}(x, k_T^2) S_{LL} - f_{1LT}(x, k_T^2) \frac{S_{LT} \cdot k_T}{M} + f_{1TT}(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2},$$

time reversal-even ones

γ^μ terms

$$\Phi^{[\gamma^+\gamma_5]}(x, k_T, T) = g_{1LT}(x, k_T^2) \frac{S_{LT\mu}\epsilon_T^{\mu\nu} k_{T\nu}}{M} + g_{1TT}(x, k_T^2) \frac{S_{TT\mu\rho} k_T^\rho \epsilon_T^{\mu\nu} k_{T\nu}}{M^2},$$

$$\Phi^{[\sigma^{i+}]}(x, k_T, T) = h_{1LL}^\perp(x, k_T^2) \frac{S_{LL} k_T^i}{M} + h'_{1LT}(x, k_T^2) S_{LT}^i - h_{1LT}^\perp(x, k_T^2) \frac{k_T^i S_{LT} \cdot k_T}{M^2} - h'_{1TT}(x, k_T^2) \frac{S_{TT}^{ij} k_{Tj}}{M} + h_{1TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \frac{k_T^i}{M}$$

time reversal-odd ones

$\gamma^\mu\gamma_5$ and $\sigma^{\mu\nu}$ terms

10 leading-twist TMDs

A. Bacchetta and P. Mulders, PRD 62, 114004 (2000).

Twist-3 TMDs: time reversal-even ones

$$\Phi^{[\gamma^i]}(x,k_T,T) = \frac{M}{P^+}\left[f_{LL}^{\perp}(x,k_T^2)S_{LL}\frac{k_T^i}{M} + f'_{LT}(x,k_T^2)S_{LT}^i - f_{LT}^{\perp}(x,k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2} - f'_{TT}(x,k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M} + f_{TT}^{\perp}(x,k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\frac{k_T^i}{M}\right].$$

$$\Phi^{[\mathbf{1}]}(x,k_T,T) = \frac{M}{P^+}\left[e_{LL}(x,k_T^2)S_{LL} - e_{LT}^{\perp}(x,k_T^2)\frac{S_{LT}\cdot k_T}{M} + e_{TT}^{\perp}(x,k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\right],$$

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+}\left[e_{LT}(x,k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu}k_{T\nu}}{M} - e_{TT}(x,k_T^2)\frac{S_{TT\mu\rho}k_T^{\rho}\varepsilon_T^{\mu\nu}k_{T\nu}}{M^2}\right].$$

Only dependent on the new terms (B_{21} –B_{52}) in the correlation function.

S. Kumano and Qin-Tao Song, PRD 103 (2021) 014025.

Twist-3 TMDs: time reversal-odd ones

$$\Phi^{[\sigma^{-+}]}(x,k_T,T)=\frac{M}{P^+}\left[h_{LL}(x,k_T^2)S_{LL}-h_{LT}(x,k_T^2)\frac{S_{LT}\cdot k_T}{M}+h_{TT}(x,k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\right],$$

$$\Phi^{[\sigma^{ij}]}(x,k_T,T)=\frac{M}{P^+}\left[h_{LT}^{\perp}(x,k_T^2)\frac{S_{LT}^i k_T^j-S_{LT}^j k_T^i}{M}-h_{TT}^{\perp}(x,k_T^2)\frac{S_{TT}^{il}k_{Tl}k_T^j-S_{TT}^{jl}k_{Tl}k_T^i}{M^2}\right].$$

$$\Phi^{[\gamma^i\gamma_5]}(x,k_T,T)=\frac{M}{P^+}\left[-g_{LL}^{\perp}(x,k_T^2)S_{LL}\frac{\varepsilon_T^{ij}k_{Tj}}{M}-g_{LT}'(x,k_T^2)\varepsilon_T^{ij}S_{LTj}+g_{LT}^{\perp}(x,k_T^2)\frac{\varepsilon_T^{ij}k_{Tj}S_{LT}\cdot k_T}{M^2}\right.$$
$$\left.+g_{TT}'(x,k_T^2)\frac{\varepsilon_T^{ij}S_{TTjl}k_T^l}{M}-g_{TT}^{\perp}(x,k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\frac{\varepsilon_T^{ij}k_{Tj}}{M}\right].$$

Only dependent on the new terms (B_{21} –B_{52}) in the correlation function.

There are 20 twist-3 TMDs.

Twist-4 TMDs

$$\Phi^{[\gamma^-]} = \frac{M^2}{P^{+2}}\left[f_{3LL}(x,k_T^2)S_{LL} - f_{3LT}(x,k_T^2)\frac{S_{LT}\cdot k_T}{M} + f_{3TT}(x,k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\right],$$

$$\Phi^{[\gamma^-\gamma_5]} = \frac{M^2}{P^{+2}}\left[g_{3LT}(x,k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu}k_{T\nu}}{M} + g_{3TT}(x,k_T^2)\frac{S_{TT\mu\rho}k_T^{\rho}\varepsilon_T^{\mu\nu}k_{T\nu}}{M^2}\right],$$

$$\Phi^{[\sigma^{i-}]} = \frac{M^2}{P^{+2}}\left[h_{3LL}^{\perp}(x,k_T^2)S_{LL}\frac{k_T^i}{M} + h'_{3LT}(x,k_T^2)S_{LT}^i - h_{3LT}^{\perp}(x,k_T^2)\frac{k_T^iS_{LT}\cdot k_T}{M^2} - h'_{3TT}(x,k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M}\right.$$
$$\left. + h_{3TT}^{\perp}(x,k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\frac{k_T^i}{M}\right].$$

There are 10 twist-4 TMDs.

Sum rules for time-reversal-odd TMDs

$$\int d^2k_T h_{1LT}(x,k_T^2) = 0,$$

$$\int d^2k_T g_{LT}(x,k_T^2) = 0,$$

$$\int d^2k_T h_{LL}(x,k_T^2) = 0,$$

$$\int d^2k_T h_{3LT}(x,k_T^2) = 0.$$

Summary

(1) A complete decomposition of quark correlation function for spin-1 hadrons.

(2) Twist-3 and Twist-4 TMDs are defined for spin-1 hadrons.

(3) Sum rules are obtained for time-reversal-odd TMDs

Thank you very much