QCD Resummation in the Lattice Calculation of PDFs

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Abstract

We study the threshold resummation in the large-momentum effective theory (LaMET) and pseudo distribution approaches to calculate parton distribution functions (PDFs) from lattice QCD, and apply it to the lattice data for the pion valence quark PDF. Our analysis shows that with the current data quality and highest pion momentum $P^z = 2.4$ GeV, the effect of threshold resummation is negligible for the lowest moments and large-x behavior of the PDF.

Introduction

Many present lattice QCD approaches to calculate the parton distribution functions (PDFs) rely on a factorization formula or effective theory expansion to extract them from certain Euclidean matrix elements in boosted hadron states. In the threshold region, the perturbative matching in the factorization or expansion formula includes large logarithms that need to be resummed, which could affect the prediction of the large-x behavior of PDFs.

The pion valence PDFs at large x, which is characterized by a power law behavior $(1-x)^\beta$, has been under debate among various approaches. In the asymptotic limit, the Brodsky-Farrar quark counting rules predicted that $\beta \sim 2$, but recent analyses by JAM and **xFitter** suggest that $\beta \sim 1$. With the inclusion of threshold resummation in the Drell-Yan cross section formula, it was found that $\beta \sim 2$. Therefore, it will be important if lattice QCD can offer insight on this property.

References

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[2] X. Gao, L. Jin, C. Kallidonis, N. Karthik, S. Mukherjee, P. Petreczky, C. Shugert, S. Syritsyn, and Y. Zhao, "Valence parton distribution of the pion from lattice QCD: Approaching the continuum limit," *Phys. Rev. D*, vol. 102, no. 9, p. 094513.

Origin of threshold logarithms

Both the quasi-PDF (qPDF) in LaMET and pseudo-PDF (pPDF) can be related to a 3D distribution defined as

$$\tilde{q}(x,\vec{b}_\perp,P^z) = \frac{1}{2P^0}\int\frac{dz}{2\pi}e^{iz(xP^z)} \times \langle P|\bar{\psi}(b)W(b,0)\gamma^t\psi(0)|P\rangle\,, \quad (1)$$

where $b^\mu = (0,\vec{b}_\perp,z)$, and $W(b,0)$ is a straight Wilson line connecting 0 and b^μ.

The qPDF:

$$\tilde{q}(x,P^z) = \lim_{b_\perp\to 0}\tilde{q}(x,\vec{b}_\perp,P^z)\,. \quad (2)$$

The pPDF:

$$\mathcal{P}(x,b_\perp^2) = \lim_{P^z\to\infty}\tilde{q}(x,\vec{b}_\perp,P^z)\,. \quad (3)$$

The leading soft divergence in the 3D distribution $\tilde{q}(x,\vec{b}_\perp,P^z)$ comes from the diagram in Fig. 1.

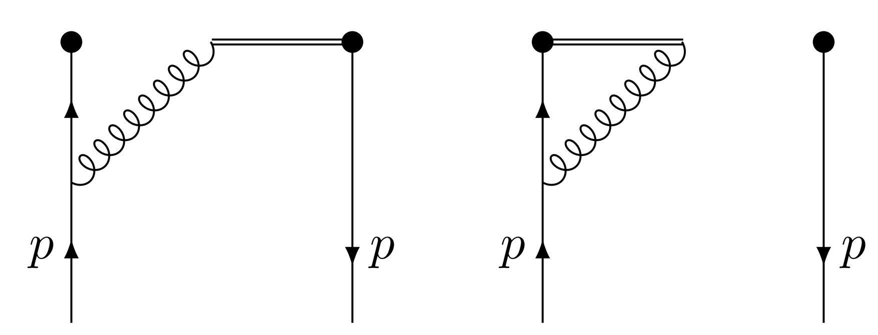

Figure 1: One-loop Feynman diagrams (and conjugates) that contribute to the leading soft divergence of the 3D distribution.

In the limit of the $x\to 1$,

$$\tilde{q}^{(1)}_{\rm cs}(x,p^z) \overset{x\to 1^-}{\Longrightarrow} -\frac{g^2C_F}{8\pi^2}\frac{1}{\epsilon}\Big(\frac{\mu^2}{p_z^2}\Big)^\epsilon\frac{1+(1-x)^{-2\epsilon}}{1-x}\,, \quad (4)$$

$$\mathcal{P}^{(1)}_{\rm cs}(x,b_\perp^2) \overset{x\to 1^-}{\Longrightarrow} \frac{g^2C_F}{4\pi^2}\Gamma(-\epsilon)(b_\perp^2\mu^2)^\epsilon\frac{(1-x)^{2\epsilon}}{(1-x)}\,. \quad (5)$$

The different signs of ϵ in the exponents in Eqs. (4) and (5) lead to different dynamical behaviors of the thershold logarithms, as the qPDF approaches the infrared (IR) region while the pDF approaches an ultraviolet (UV) fixed point. In the pPDF case, the emitted gluon remains off-shell with virtual mass $-k_\perp^2 \sim 1/b_\perp^2$ in the limit of $x\to 1$, so the gluon is in the UV region when $b_\perp^2$ is small. Moreover, since the limit $p^z\to\infty$ has been taken first, only collinear and soft emissions with $k_\perp \to 0$ are allowed, and the emission of a gluon with finite $k_\perp$ is suppressed, which explains the suppression factor $(1-x)^{2\epsilon}$. On the other hand, in the qPDF case, since $k_\perp$ is integrated over, the limit $x\to 1$ includes contributions from both hard and soft transverse momentum modes, with the latter being sensitive to IR physics.

Threshold resummation at next-to-leading-logarithmic accuracy

Let us work on the short-distance OPE of the spatial correlator $\tilde{h}_{\gamma^t}(\lambda, z^2\mu^2)$ that defines the qPDF and pPDF,

$$\tilde{h}_{\gamma^t}(\lambda,z^2\mu^2) = \sum_{N=0}^{\infty}\frac{(-i\lambda)^N}{N!}C_N(z_0^2\mu^2)a_N(\mu)\,, \quad (6)$$

where $z_0 = ze^{\gamma_E}/2$, and $a_N(\mu)$ is the Mellin moment of the PDF. At large N, the Wilson coefficients C_N include large threshold logarithms of $\alpha_s\ln^2 N$ and $\alpha_s \ln N$, which can be resummed as

$$\ln C_N^{\rm NLL} = \int dx\frac{x^{N-1}-1}{1-x}\Big[\int_{\mu^2}^{\frac{(1-x)^a}{z_0^2}}\frac{dk^2}{k^2}A(\alpha_s(k^2)) + B(\alpha_s((1-x)^a/z_0^2))\Big]\,, \quad (7)$$

where $a=-2$ corresponds to a UV fixed point. At next-to-leading-logarithmic (NLL) accuracy,

$$\ln C_N^{\rm NLL}(\alpha_s(\mu),z_0^2\mu^2) = -\frac{\pi^2}{3}a_sC_F + \ln N' g_1(\tau,L) + g_2(\tau,L), \quad (8)$$

where $\tau = \beta_0 a_s \ln N'$, $L = \ln(z_0^2\mu^2)$. The functions g_1 and g_2 can be found in Ref. [1]. We can also include the resummation of $\alpha_s L$ with DGLAP evolution,

$$C_N^{\rm evo}(z_0^2\mu^2) = C_N(\alpha_s(z_0^{-1}),1)\left(\frac{\alpha_s(z_0^{-1})}{\alpha_s(\mu)}\right)^{\frac{\gamma_N^{(0)}}{\beta_0}}\,, \quad (9)$$

where $\gamma_N^{(0)}$ is the anomalous dimension of C_N and η_0 is the leading coefficient in the QCD β function.

Application to lattice QCD

Through a semi-model-independent approach, we can demonstrate that the NLL threshold resummation tends to increase the value of β [1]. We also apply the resummed Wilson coefficients to analyze the lattice matrix elements for the pion valence PDF. By constructing the ratios of the matrix elements at different pion momenta, we can fit them to the resummed OPE formula as

$$\frac{\tilde{h}_{\gamma^t}(z,P^z)}{\tilde{h}_{\gamma^t}(z,P_0^z)} = \frac{\sum_N C_N(z^2\mu^2)\frac{(-izP^z)^N}{N!}\langle x^N\rangle}{\sum_N C_N(z^2\mu^2)\frac{(-izP_0^z)^N}{N!}\langle x^N\rangle}\,. \quad (10)$$

The lattice data are obtained from Ref. [2] at spacings $a = 0.04, 0.06$ fm with valence pion mass $m_\pi = 300$ MeV. The largest pion momenta are $P^z = 2.15$ GeV and 2.42 GeV for the $a = 0.04$ and 0.06 fm lattices, respectively. Due to the requirement of small z, the largest $\lambda = zP^z$ is limited, so we can only reliably extract up to the fourth moment $\langle x^4\rangle$, as shown in Fig. 2.

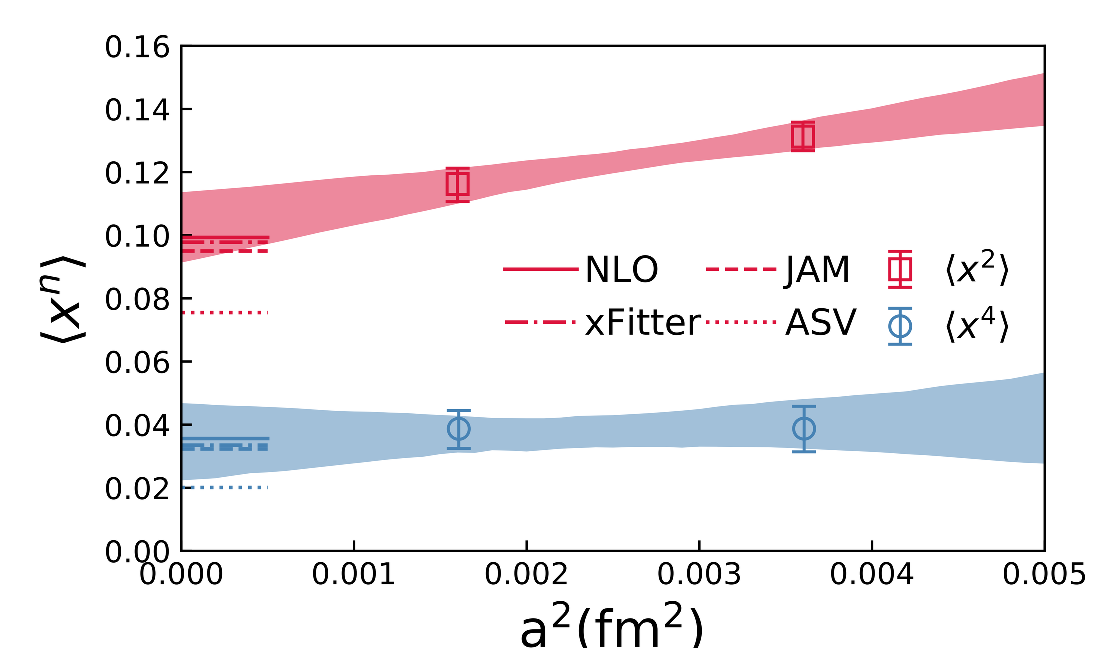

Figure 2: Results of $\langle x^2\rangle$ (red band)and $\langle x^4\rangle$ (blue band) in this work, compared with NLO fit, as well as global fits.

Compared to the fit with next-to-leading order (NLO) OPE formula, the effect of threshold resummation is negligible for $\langle x^2\rangle$ and $\langle x^4\rangle$. We also parameterized the PDF with a simple model $\sim x^\alpha(1-x)^\beta$ and fit it to the lattice data, but found that the NLL resummation barely changes β.

The main reason is because our lattice data are not sensitive the higher moments or large-x PDF. Therefore, to better constrain β, we will need to reach larger P^z and higher statistics, then the threshold resummation will also become important.