New Approach for Calculation of Valence Quark Distributions in the Nucleon

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Valence quarks in the nucleon and nucleus

with Chris Leon and Joseph Maerovitz

Valence quarks play a unique role in QCD dynamics of the nucleon

- They define baryonic number of the nucleon:
- They represent effective "three fermion" system with complex interaction among themselves and with nucleon environment
- Because of their conserved number the concept of mean-field interaction can be introduce to discuss their interaction with the nucleon environment
- Quantum mechanically, this becomes a problem of fermions in the strong external field (see e.g. Migdal Fermions and Bosons in the Strong Field)
 - Short range interaction among three valence quarks responsible to the generation of high x distribution of PDFs

Valence quarks in the nucleon at medium to high x: 0.1 < x < 1



- Treating the height of the peak $h(x_p,Q^2)$ and position of the peak x_p as physical observables:



"New Approaches" in modeling valence quark dynamics

- In general the peaking property of bound Fermi-system is a hallmark for mean-field dynamics
- Our assumption is that the peaking feature of valence quark distributions is due to interaction of valence quarks in the strong mean field generated by "residual nucleon system"
- We introduce concept of "residual nucleon system" as as composite part of the light-front wave function of valence quarks











Main assumptions of the model

Dynamics: The main assumption is that the mean field, two- and three- quark short-range correlations define the dynamics of the valence quarks in the range of $0.1 \le x \le 1$.



Valence Quarks in the Nucleon:

- The model assumes an existence of almost massless valence three-quark cluster V in the nucleon.
- The cluster is compact with the transverse separation between any qq, $b_{qq} \leq 0.3$ Fm.
- Valence quark system defines the baryonic number but not necessarily the total isospin of the nucleon. It can have total total isospin, $I_V = 1/2$ or 3/2 each of them corresponding to the different excitations or masses of the residual nucleon system. (For the lowest mass of the recoil system one expects the 3q system to have the same isospin and its projection that the considered nucleon has.)



Residual Structure:



- Introducing residual structure of the nucleon with the spectrum of mass, m_R -(spectral function formalism in the description of the nucleon structure)
- The model assumes a certain universality of the residual structure, R, entering in all three mechanisms of generation of valence quark distribution.
- This universality is reflected in the fact that one can fix its main properties within mean field and apply it in the calculation of 2q- and 3q- correlation contributions.
- -The mass spectrum of the residual system is continuous and effectively depends on whether u- or d- valence quarks are probed. $m_R(u/d) = \alpha_{u/d} \cdot m_R(I_V = \frac{1}{2}, I_V^3 = \frac{1}{2}) + \beta_{u/d} \cdot m_R(I_V = \frac{1}{2}, I_V^3 = -\frac{1}{2}) + \gamma(u/d) \sum_{v=1}^{\frac{1}{2}} m_R(I_V = \frac{3}{2}, I_V^3) + \cdots$ For proton: $m_R(u) < m_R(d)$ - QCD evolution will increase: $m_R(Q^2)$



Mean-Field Model of Valence Quark Distributions

- The valence 3q system occupies a region of 0.6Fm and is described by mutually coupled three-dimensional harmonic oscillators, thus satisfying confinement condition.

- Valence quarks are almost massless with the invariant energy of 3q system contributing to the nucleon mass.

- The residual system generates the mean field and occupies a volume less or equal to the nucleon volume.



Reference Frame, Kinematics and Structure Function

$$p_{N}^{\mu} = (p_{N}^{+}, \frac{m_{N}^{2}}{p_{N}^{+}}, \mathbf{o}_{\perp}), \quad q^{\mu} = (0, \frac{2p \cdot q}{p_{N}^{+}}, \mathbf{q}_{\perp}), \quad Q^{2} = -q^{2} = |\mathbf{q}_{\perp}|^{2}, \qquad p_{N}^{+} \gg m_{N}, k_{i}^{-}, k_{i,\perp},$$

$$F_{2}(x, Q^{2}) \equiv \sum_{i} e_{i}^{2} x f_{i}(x, Q^{2}), = \frac{MQ^{2}}{2x(p_{N}^{+})^{2}} W_{N}^{++}$$

$$W_{N}^{\mu\nu} = \frac{1}{4\pi M} \int \sum_{X} \sum_{s_{X}} J^{\mu,\dagger}(p_{X}, s_{X}, p_{N}, s_{N}) J^{\nu}(p_{X}, s_{X}, p_{N}, s_{N}) (2\pi)^{4} \delta^{4}(q + p_{N} - p_{X}) \delta(p_{X}^{2} - M_{X}^{2}) \frac{d^{4}p_{X}}{(2\pi)^{3}} \frac{1}{2(2\pi)^{3}}$$



$$A^{\mu} = \sum_{h_{V},h_{1}} \frac{1}{k_{V}^{+}} \frac{1}{k_{1}^{+}} \frac{\bar{u}(k_{1}',h_{1}')(ie_{1}\gamma^{\mu})u(k_{1},h_{1})}{\mathcal{D}_{1}} \frac{\prod_{i=1}^{3} \bar{u}(k_{i},h_{i})\Gamma^{V \to 3q} \chi_{V} \bar{\chi_{V}} \bar{\chi_{R}} \Gamma^{B \to VR} u(p_{N},h_{N})}{\mathcal{D}_{2}}$$

Calculation of the scattering amplitude



$$A^{\mu} = \sum_{h_{1},h_{V}} \frac{1}{x_{V}} \frac{1}{\beta_{1}} \frac{\bar{u}(k_{1}',h_{1}')(ie_{1}\gamma^{\mu})u(k_{1},h_{1})}{m_{V}^{2} - \sum_{i=1}^{3} \frac{k_{i,\perp}^{2} + m_{i}^{2}}{\beta_{i}}} \frac{\prod_{i=1}^{3} \bar{u}(k_{i},h_{i})\Gamma^{V \to 3q} \chi_{V} \bar{\chi}_{V} \bar{\chi}_{R} \Gamma^{B \to VR} u(p_{N},h_{N})}{M^{2} - \frac{k_{V,\perp}^{2} + m_{V}^{2}}{x_{V}} - \frac{k_{R,\perp}^{2} + m_{R}^{2}}{x_{R}}}$$
$$\psi_{VR}(x_{V},\mathbf{k}_{R,\perp},x_{R},\mathbf{k}_{V,\perp}) = \frac{\bar{\chi}_{V} \bar{\chi}_{R} \Gamma^{B \to VR} u(p_{N},h_{N})}{m_{N}^{2} - \frac{k_{V,\perp}^{2} + m_{V}^{2}}{x_{V}} - \frac{k_{R,\perp}^{2} + m_{R}^{2}}{x_{R}}}}{\psi_{3q}(\{\beta_{i},\mathbf{k}_{i,\perp},h_{i}\}_{i=1}^{3})} = \frac{\prod_{i=1}^{3} \bar{u}(k_{i},h_{i})\Gamma^{V \to 3q} \chi_{V}}{m_{V}^{2} - \sum_{i=1}^{3} \frac{k_{i,\perp}^{2} + m_{I}^{2}}{\beta_{i}}}}$$

where $\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3$ denotes the LC momenta and helicities of the three valence quarks in the wave function.

$$A^{\mu} = \sum_{h_1, h_V} \bar{u}(k_1, h_1) (ie_1 \gamma^{\mu}) u(k_1, h_1) \frac{\psi_{VR}(x_V, \mathbf{k}_{R, \perp}, x_R, \mathbf{k}_{V, \perp})}{x_V} \frac{\psi_{3q}(\{\beta_i, \mathbf{k}_{i, \perp}, h_i\}_{i=1}^3)}{\beta_1}$$



$$F_2(x,Q^2) \equiv \sum_i e_i^2 x f_i(x,Q^2) = \frac{MQ^2}{2x(p_N^+)^2} W_N^{++}$$

 $f_q(x_B) = \sum_{h_i} \int \delta(1 - \sum_{i=1}^3 x_i - x_R) \frac{dx_R}{x_R} \prod_{i=1}^3 \frac{dx_i}{x_i} [d^2 \mathbf{k}_{\perp}] 16\pi^3 \delta^{(2)} (\sum_{i=1}^3 \mathbf{k}_{i,\perp} + \mathbf{k}_{R,\perp}) \frac{d^2 \mathbf{k}_{R,\perp}}{16\pi^3} \prod_{i=1}^3 \frac{d^2 \mathbf{k}_{i,\perp}}{16\pi^3} \times \delta(x_1 - x_B) |\psi_{3q}(\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3)|^2 |\psi_V(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp})|^2.$

Modeling Wave Functions

Wave function of 3q valence system: Relativistic coupled Harmonic Oscillator

$$\psi_{3q}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1}^3) = 16\pi^3 m_N A_V \exp\left[-\frac{B_V}{2}(k_{12,cm}^2 + k_{23,cm}^2 + k_{31,cm}^2)\right]\sqrt{x_2 x_3},\tag{1}$$

where A_V and B_V are parameters and $x_i, k_{i,\perp}$, $(i \neq j = 1, 2, 3)$ are LC momentum fractions and transverse momenta of each valence quark in the reference considered frame. The $k_{ij,cm}^2$ s, $(i \neq j = 1, 2, 3)$ in the exponent of the wave function represent relative three momenta in the CM system of i, j pairs defined as follows:

$$k_{ij,cm}^{2} = \frac{(s_{ij} - (m_i - m_j)^2)(s_{ij} - (m_i + m_j)^2)}{4s_{ij}},$$
(2)

where the invariant energy of the i, j pair is:

$$s_{ij} = (k_i + k_j)^+ (k_i + k_j)^- - (\mathbf{k}_{i,\perp} + \mathbf{k}_{j,\perp})^2 = (x_i + x_j) \left(\frac{k_{i,\perp}^2 + m_i^2}{x_i} + \frac{k_{j,\perp}^2 + m_j^2}{x_j} \right) - (\mathbf{k}_{i,\perp} + \mathbf{k}_{j,\perp})^2.$$
(3)

$$\psi_{3q}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1}^3) = 16\pi^3 m_N A_V \exp\left[-\frac{B_V}{8} \left(\sum_{i=1}^3 x_V \frac{\tilde{k}_{i,\perp}^2 + m^2}{x_i} - 9m^2\right)\right]$$



 $\tilde{\mathbf{k}}_{i,\perp} = \mathbf{k}_{i,\perp} - \frac{x_i}{x_V} \mathbf{k}_{V,\perp}, \quad (i = 1, 2, 3)$

$$\sum_{i=1}^{3} \tilde{k}_{i,\perp} = 0.$$

Modeling Wave Functions

Wave function of V-R system: Model in a Gaussian form

 $\psi_R(x_R, \mathbf{p}_{R,\perp}) = \sqrt{16\pi^3 m_N} A_R e^{-B_R p_R^2} \sqrt{x_R}$

 x_R – light-cone momentum fraction of the recoil system

p_R – relative momentum between CMs of V and R system

- Considering a non-relativistic approximation for recoil system pR <mR

 $p_{R,z} \approx (x_R m_N - m_R).$

$$f_q(x_B) = \sum_{h_i} \int \delta(1 - \sum_{i=1}^3 x_i - x_R) \frac{dx_R}{x_R} \prod_{i=1}^3 \frac{dx_i}{x_i} [d^2 \mathbf{k}_{\perp}] 16\pi^3 \delta^{(2)} (\sum_{i=1}^3 \mathbf{k}_{i,\perp} + \mathbf{k}_{R,\perp}) \frac{d^2 \mathbf{k}_{R,\perp}}{16\pi^3} \prod_{i=1}^3 \frac{d^2 \mathbf{k}_{i,\perp}}{16\pi^3} \times \delta(x_1 - x_B) |\psi_{3q}(\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3)|^2 |\psi_V(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp})|^2.$$

$$\begin{cases} f_q(x_B, Q^2) &= \mathcal{N} \int_0^{1-x_B} dx_2 \int_0^{1-x_B-x_2} dx_3 \exp\left[-\frac{B_V x_V}{4} \sum_{i=1}^3 \frac{m_i^2}{x_i} - B_R M_N^2 (x_V - (1 - \frac{M_R}{M_N}))^2\right] \\ &\times \frac{x_2 x_3}{x_V^3} \left(1 - e^{-a_{cm} Q_{cm}^{max^2}}\right) \left(1 - e^{-a_{rel} Q_{rel}^{max^2}}\right) \left(1 - e^{-B_R Q^2}\right) \end{cases}$$

where $a_{cm} = \frac{B_V x_V}{4} \frac{x_V}{x_3(x_1+x_2)}$ and $a_{rel} = \frac{B_V x_V}{4} \frac{x_1+x_2}{x_1x_2}$.

considering large Q^2 and $m_q \to 0$ limit

$$\left(f_q(x_B, Q^2) = \frac{\mathcal{N}}{6} \int\limits_{x_B}^1 dx_V \exp\left[-B_R m_N^2 \left(x_V - (1 - \frac{m_R}{m_N}) \right)^2 \right] \frac{(x_V - x_B)^3}{x_V^3} \right)$$
where $\mathcal{N} = \frac{16\pi^3 A_V^2 A_R^2 m_N^3}{B_R B_V^2} e^{\frac{9}{4} B_V m_q^2}$

Qualitative Features of the Model

 $f_q(x_B, Q^2) \sim (1 - x_B - \frac{m_R}{m_N})^3$ - Evaluating the integral at the maximum of exponent: In this case $h(x_B,t) = x_B f_q(x_B,Q^2) \sim x_B \left(1 - x_B - \frac{m_R}{m_N}\right)^3$ Valence and Sea Structure Functions which peaks at $x_p \approx \frac{1}{4}(1 - \frac{m_R}{m_N})$. 0.8 *x*d̄ CJ15nlo 0.7 *xd* NNPDF31nnlo kū CJ15nlo 0.6 a NNPDF31nnlo At moderate Q^2 (M_c^2) characteristic $x_p \sim 0.2$ resulting in $m_R \sim m_{\pi}$. *xt*(*x*, Q₀⁴ = 4GeV⁴) 0.0 + xd_v CJ15nlo *xd_v* NNPDF31nnlo xu_v CJ15nlo xu_v NNPDF31nnlo xd_V 68% CL - In the model: $m_R(u) < m_R(d)$: 0.2 0.1 Explains $x_p^d < x_p^u$ хđ 0.0

0.2

0.4

0.6

х

0.8

1.0

$$f_q(x_B, Q^2) = \frac{\mathcal{N}}{6} \int_{x_B}^{1} dx_V \exp\left[-B_R m_N^2 \left(x_V - (1 - \frac{m_R}{m_N})\right)^2\right] \frac{(x_V - x_B)^3}{x_V^3}$$

where $\mathcal{N} = \frac{16\pi^3 A_V^2 A_R^2 m_N^3}{B_R B_V^2} e^{\frac{9}{4} B_V m_q^2}$

Qualitative Features of the Model

- One can also evaluate the analytic behavior of $f_q(x_B, Q^2)$ at $x_B \rightarrow 1$:

For this we substitute $x_B = 1 - \epsilon$ and in the $\epsilon \to 0$ limit evaluate the integral which results in

$$f_q(x_B, Q^2) \mid_{x_B \to 1} = \frac{\mathcal{N}}{24} e^{-B_R m_R^2} \cdot (1 - x_B)^4.$$

this should be compared with $\sim (1-x)^3$ behavior following from pQCD

Numerical Estimates: choosing the parameters of the model

the model has five parameters $A_{V_{i}} B_{V_{i}} A_{R_{i}}$, B_{R} and $m_{R_{i}}$.

- For valence quarks we assume that characteristic separations in the 3q system in the impact parameter space is $\langle b_{i,j}^2 \rangle \sim (0.3 \text{Fm})^2$. This allows us to evaluate $B_V = 4 \langle b_{i,j}^2 \rangle \frac{x_i}{x_V} \approx \frac{4}{3} \langle b_{i,j}^2 \rangle$.

- We assume that this parameter does not change with the QCD evolution.

- For the recoil system, because of the use of we can relate $A_R = \left(\frac{B_R}{\pi}\right)^{\frac{3}{4}}$.

- We expect the parameter B_R , which characterizes the size of the residual system to depend on the residual mass and as a result to be Q^2 dependent.

- the parameter A_V is fixed through the normalization factor, \mathcal{N} using: $\mathcal{N} = \frac{16\pi^3 A_V^2 A_R^2 m_N^3}{B_R B_V^2}$

-the remaining parameters \mathcal{N} , m_R and B_R are evaluated by fitting to empirical PDFs



Once parameters are fixed we can evaluate the strength of the missing high momentum component

- For the normalization one obtains for d-quark $N_d = 0.8$ and for u-quark $N_u = 1.54$. These results also indicates that one expects that Regge mechanism at $x < x_p$ and qq-correlations to contribute $\sim 20\%$ and $\sim 23\%$ of total normalizations for d- and u- quarks respectively.

- The evaluation of the momentum sum rule $(P_q = \int xq_V(x)dx)$ yields $P_d = 0.1$ and $P_u = 0.246$ compared to $P_d = 0.108$ and $P_u = 0.264$ of CTNN distribution. Because of lesser contribution from the Regge mechanism in this case these estimates evaluate better the contribution from qq correlations. Here for qqcorrelations one obtains for the d-quark, ~ 8% and for the u-quark is ~ 11%. Predication of d/u ratio at $xB \rightarrow 1$



$$\frac{u_V}{u_V} \mid_{x \to 1} \approx 0.21.$$

The reason of d/u fall-off is $m_R(u) < m_R(d)$





$$\mathcal{N}(t) = N_0 + N_1 t + N_2 t^2 + N_3 t^3,$$

$$B_R(t) = B_{R0} + B_{R1} t + B_{R2} t^2 + B_{R3} t^3,$$

$$m_R(t) = m_{R0} + m_{R1} t + m_{R2} t^2 + m_{R3} t^3$$

where
$$t = \log \frac{Q^2}{1 GeV^2}$$
.

$$r_R(t) \approx \sqrt{\frac{3}{2}} B_R(t) \cdot 0.197327 \text{ Fm.}$$

Q2 dependence of parameters: N, BR and mR



- Mass of the residual system increases as logQ²
- Gluon radiation of valence quarks confined in 3q system
- Mass gap is almost constant starting Q2>20 GeV2



All these observations generate predictions that can be checked at EIC

- Mass of the residual system increases as logQ²

Invariant mass of gluonic jets in target fragmentation region in ep \rightarrow e'jets will increase wit logQ2 at x=0.2 and the increase rate can be calculated

- Mass gap is almost constant starting Q2>20 GeV2

Multiplicity of target fragmented pions depends whether valence u or d quarks was struck by virtual photon

- Gluon radiation of valence quarks confined in 3q system

Angular correlation of gluon radiation in target fragmentation region with with probed x.





Conclusion and some outlook

- We developed model of valence 3q+ recoil nucleon system in which explicitly included wave function of the recoil nucleon with mass spectrum
- Fitting to phenomenological valence quark PDFs allowed to fix parameters of both
 3q mean field and residual nucleon system's wave function
- Estimated the overall possible contribution from qq and qqq correlations
- These wave functions will allow us to calculate many other observables, such as form-factors, GPDs, TMDs sensitive to x=0.2 region of valence quarks
- The project currently underway is to include the qq to qqq correlations that can contribute in generation of high x tail od PDFs



- Refinement and improvement of the model

Quarks in Nuclei



