T-odd quark-gluon–quark correlation function in the light-front quark-diquark model

DIS 2021 Presentation

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In Collaboration with

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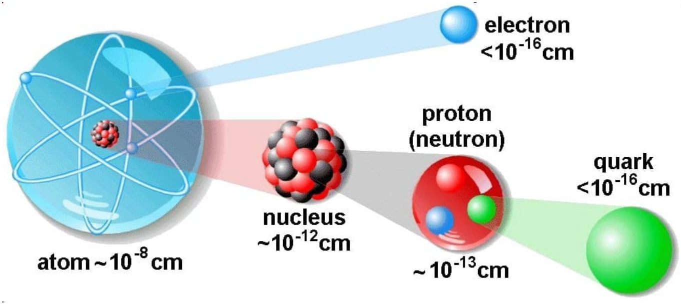
Introduction



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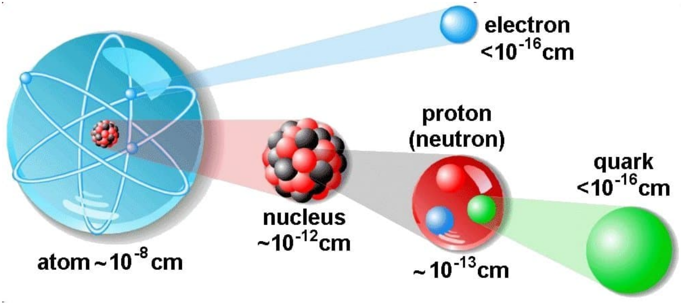

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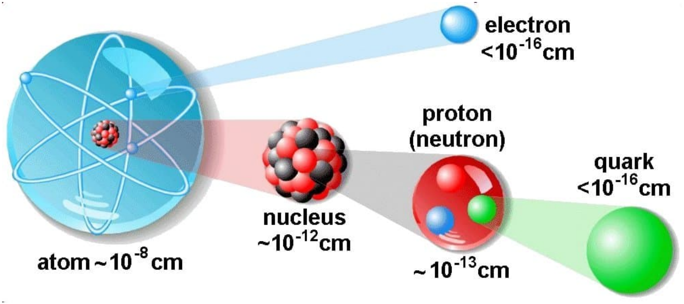

- The theory of the strong interaction which provides the fundamental description of hadronic structure and dynamics in terms of their elementary quarks and gluons degrees of freedom is Quantum Chromodynamics (QCD).
- The foremost problem of hadron physics is to unravel the internal structure of hadron.

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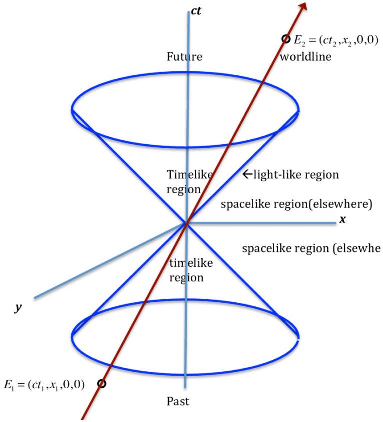


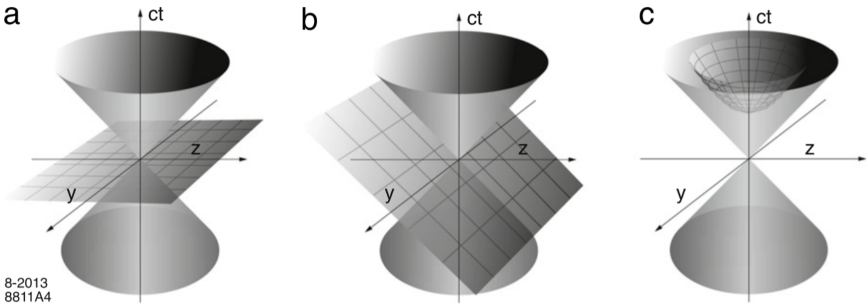
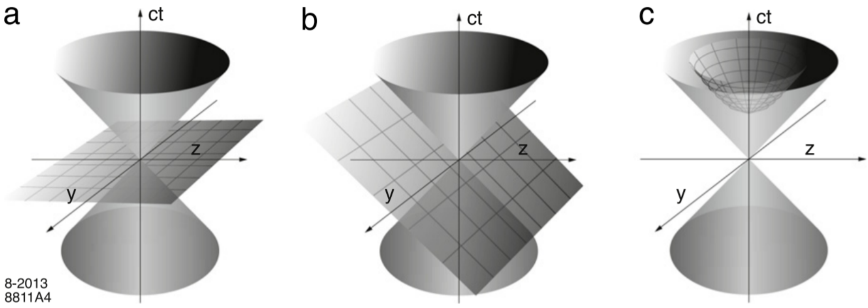


Figure 1: (a) the instant form, (b) the front form, (c) the point form.



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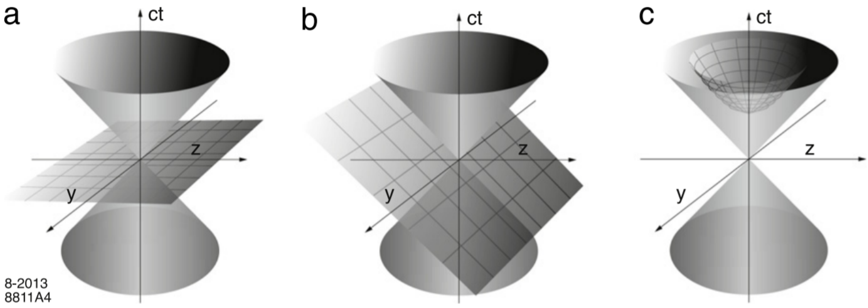

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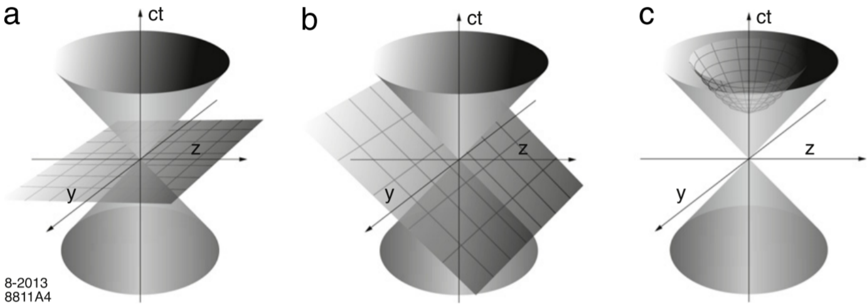

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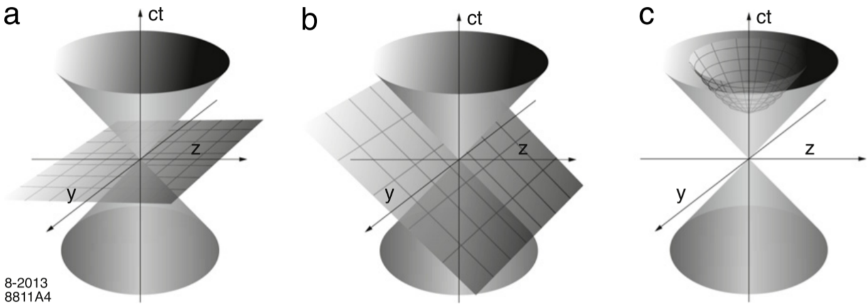

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  - Dispersion Relation (for ON shell particles)

    $$k^- = \frac{(k\perp)^2 + m^2}{k^+}$$

    $\sim$ no square root factor.

Light-Front Coordinates

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- Similarly we can define the longitudinal momentum $k^+ = k^0 + k^3$ and light-front energy $k^- = k^0 - k^3$.

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- The distribution of the constituents in momentum space can be measured through deep inelastic knock-out scattering.
- The longitudinal momentum distribution of partons in a hadron is described by **Parton distribution functions (PDFs)**.
- The distribution of a partons in the transverse plane is described by **Generalized parton distributions (GPDs)**. They unify the spatial picture produced by form factors with the momentum picture produced by PDF's.

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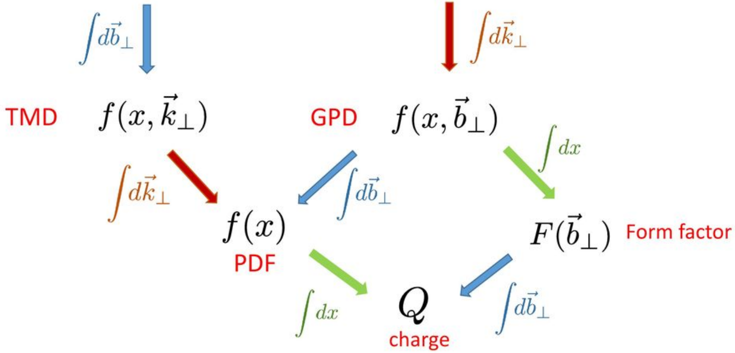
- The **TMD's** give details of transverse momentum distributions of partons inside the hadrons.
- **Wigner distributions** unify the position & momentum distributions and provide subtle details for partons inside the hadron.

Relation between Wigner, TMD's, GPD's and PDF's

Belitsky, Ji, Yuan (2003);

$$W(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int \frac{dz^- d^2z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P-\frac{\Delta}{2}|\bar{q}(-z/2)\gamma^+ q(z/2)|P+\frac{\Delta}{2}\rangle$$



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- Target polarization 4-vector and quark momentum is parametrized as

$$S = \left(\frac{\lambda_h P^+}{M}, \mathbf{S}_T, \frac{-\lambda_h M}{2P^+} \right)$$

$$p = \left(xP^+, \mathbf{p}_T, \frac{p^2 + p_T^2}{2xP^+} \right)$$

TMDs

- TMDs contain information on both the longitudinal and transverse momentum of partons in the hadron.
- They describe the probability to find a parton with longitudinal momentum fraction x and transverse momentum with respect to the direction of the parent hadron momentum in a hadron.
- TMDs are also of particular importance because they give rise to single spin asymmetries (SSAs).
- They can be measured in a variety of reactions in lepton-proton and proton-proton collisions as SIDIS [1, 2] and DY production [3] where a final-state particle is observed with a transverse momentum.

Calculation

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Most generalized form of quark-gluon-quark correlator is given by [4]

$$\Phi^{\alpha}_{Aij}(p, p - p_1; P, S) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4} \frac{\mathrm{d}^4\eta}{(2\pi)^4} e^{ip\cdot\xi} e^{ip_1\cdot(\eta-\xi)} \times \left\langle P, S \left| \bar{\psi}_j(0) g A^{\alpha}(\eta) \psi_i(\xi) \right| P, S \right\rangle .$$

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where,

- $gA^{\alpha}(\eta)$ is the operator corresponding to gluon field.
- p & p_1 are parton's momenta.

We have started our work with this form of quark-gluon-quark correlator [5]

$$(\tilde{\Phi}_A^{[\pm]\alpha})_{ij}(x, p_T) \equiv \int \frac{d^2\xi_T \, d\xi^-}{(2\pi)^3} e^{ip\xi} \times \langle P, S|\bar{\psi}_j(0) g \int_{\pm\infty}^{\xi^-} d\eta^- \, \mathcal{L}^{[\pm]}(0, \eta^-) F^{+\alpha}(\eta) \times \mathcal{L}^{\xi_T, \xi^+}(\eta^-, \xi^-) \psi_i(\xi)|P, S\rangle_c \Big|_{\substack{\eta^+=\xi^+=0 \\ \eta_T=\xi_T \\ p^+=xP^+}},$$

where,

- $F^{\mu\nu}$ is the antisymmetric field strength tensor of the gluon,
- $\mathcal{L}^{[\pm]}$ and $\mathcal{L}\xi_T, \xi^+$ are the gauge-links ensuring the gauge-invariance of the definition.
- The sign $\pm$ in the superscript or subscript indicates that the gauge-link between the quark and the gluon is future/past-pointing.

Above correlator can be rewritten as [5]

$$(\tilde{\Phi}_A^{[\pm]\alpha})_{ij}(x, p_T) = ig \int \frac{d^2\xi_T \, d\xi^- \, d\eta^-}{(2\pi)^4} \int dx' \, \frac{e^{ix'P^+\eta^-}}{(x' \mp i\epsilon)}$$
$$\times e^{i[(x-x')P^+ \cdot \xi^- - \boldsymbol{p}_T \cdot \boldsymbol{\xi}_T]} \langle P, S | \bar{\psi}_j(0) \mathcal{L}^{[\pm]}(0, \eta^-) F^{+\alpha}(\eta)$$
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- Ignore all gauge links in the correlator.
- We choose to work in quark-diquark model by considering axial-vector diquark.

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- Twist-3 T-odd TMD's are $\tilde{e}_L, \tilde{e}_T, \tilde{e}_T^{\perp}, \tilde{f}_T$ (or $\tilde{f}'_T$), $\tilde{f}_T^{\perp}, \tilde{f}_L^{\perp}, \tilde{g}^{\perp}, \tilde{h}$.
- Real part of Left hand side term appearing in the above correlator corresponds to T-odd(or T-even) TMD's if imaginary (or real) part is considered.

$$\frac{1}{(x' \mp i\epsilon)} = \mathrm{P}\left(\frac{1}{x'}\right) \pm i\delta(x').$$

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Above expression is proportional to

$$\begin{aligned}(F^{+\alpha})[&\psi^{\pm}_{-+}(x,p_T)\psi^{\pm}_{++}(x,p_T) + \psi^{\pm}_{-0}(x,p_T)\psi^{\pm}_{+0}(x,p_T)\\ &+\psi^{\pm}_{--}(x,p_T)\psi^{\pm}_{+-}(x,p_T) + \psi^{\pm}_{++}(x,p_T)\psi^{\pm}_{-+}(x,p_T)\\ &+\psi^{\pm}_{+0}(x,p_T)\psi^{\pm}_{-0}(x,p_T) + \psi^{\pm}_{+-}(x,p_T)\psi^{\pm}_{--}(x,p_T)]\end{aligned} \tag{1}$$

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After fixing the helicity of nucleon using Light Front Wave Functions

$$\begin{aligned}(F^{+\alpha})[&\psi^{+}_{-+}(x,p_T)\psi^{+}_{++}(x,p_T) + \psi^{+}_{-0}(x,p_T)\psi^{+}_{+0}(x,p_T) \\ &+\psi^{+}_{++}(x,p_T)\psi^{+}_{-+}(x,p_T) + \psi^{+}_{+0}(x,p_T)\psi^{+}_{-0}(x,p_T)]\end{aligned} \tag{2}$$

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thank you!