(Towards a better estimate of) Perturbative uncertainties in the Drell-Yan spectrum at low q_T

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RGEs in QCD

- Generic RGE in QCD:

$R = (\alpha_S, PDF, TMD)$

$$\frac{d \ln R}{d \ln \mu} = \Gamma(\alpha_S(\mu))$$

Γ = appropriate anomalous dimension

$$\Gamma(\alpha_S(\mu)) = \sum_{n=0}^{k} \Gamma_n \alpha_S^{n+1}(\mu)$$

α_S

k = highest-order known in β-expansion

$$\frac{d \ln \alpha_S}{d \ln \mu} = \beta(\alpha_S(\mu)) = \sum_{n=0}^{k} \beta_n \alpha_S^{n+1}(\mu)$$

- Knowledge of β_k allows to resum N^{k}LL tower of $\ln(\mu/\mu_0)$
- Question: is it possible to find an analytic expression for each tower?

Analytic solutions

Closed-form analytic solutions ($\lambda = \beta_0 \alpha_{S0} \ln(\mu/\mu_0)$)

LL: *exact* beyond LL: *approximate* (rely on PT, RGE not satisfied)

$g_1(\lambda)$

$$\alpha_S^{LL}(\mu) = \alpha_{S0}\left(\frac{1}{1-\lambda}\right)$$

$g_2(\lambda)$

$$\alpha_S^{NLL}(\mu) = \alpha_{S0}\left(\frac{1}{1-\lambda}\right) - \alpha_{S0}^2\left(\frac{\beta_1 \ln(1-\lambda)}{(1-\lambda)^2}\right)$$

$g_3(\lambda)$

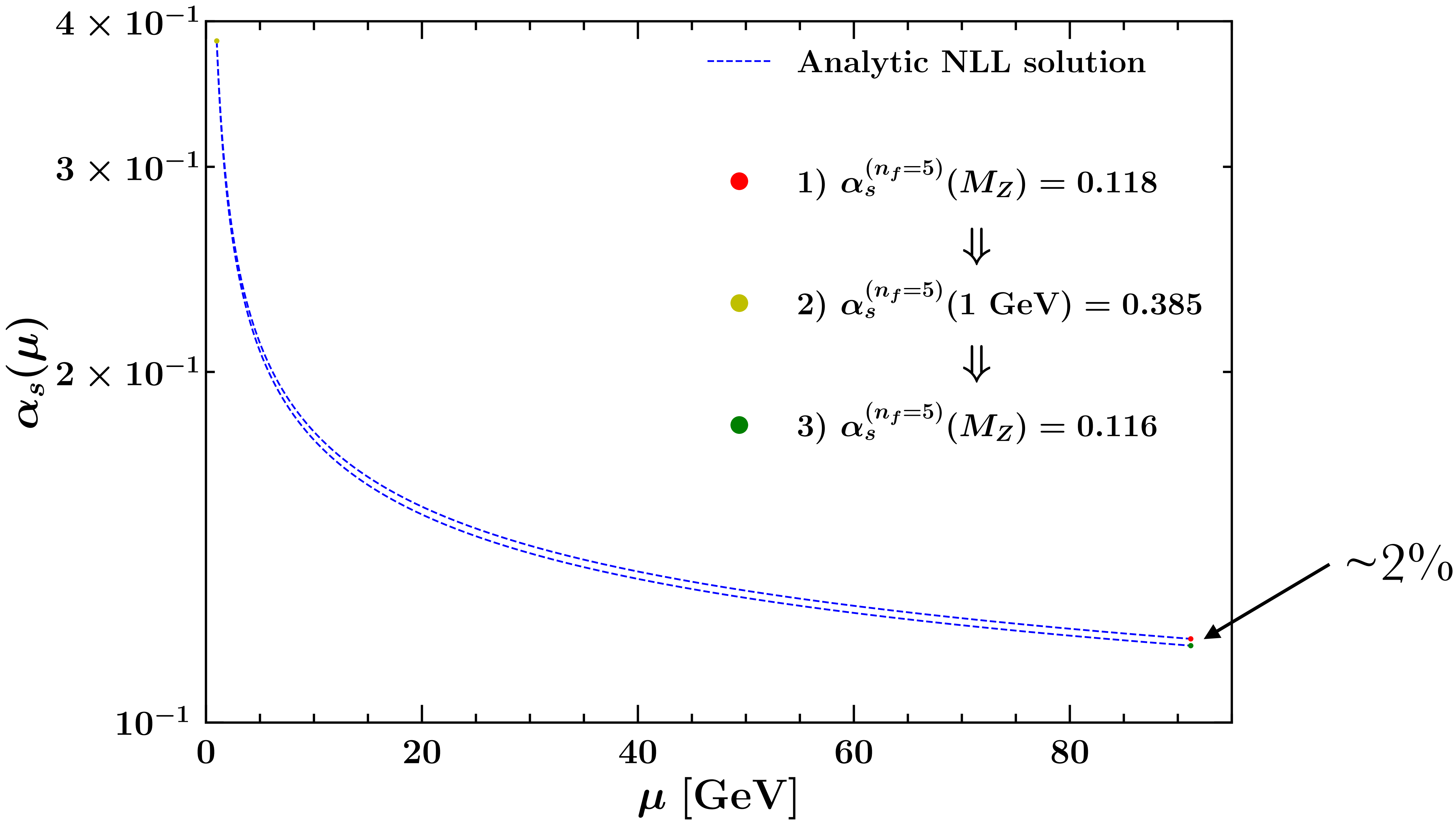
$$\alpha_S^{NLL}(\mu) = \alpha_{S0}\left(\frac{1}{1-\lambda}\right) - \alpha_{S0}^2\left(\frac{\beta_1 \ln(1-\lambda)}{(1-\lambda)^2}\right) + \alpha_{S0}^3\left(\frac{\beta_2\lambda - \beta_1^2(\lambda + \ln(1-\lambda) - \ln^2(1-\lambda)}{(1-\lambda)^3}\right)$$

$$\alpha_S(\mu) = \alpha_{S0}\, g_1(\lambda) + \alpha_{S0}^2\, g_2(\lambda) + \alpha_{S0}^3\, g_3(\lambda) + \cdots$$

(purposely reminiscent of threshold/q_T resummation)

Perturbative hysteresis: α_s

$$\alpha_S^{NLL}(\mu) = \alpha_{S0}\, g_1(\lambda) + \alpha_{S0}^2\, g_2(\lambda)$$



Resummation scale

Closed-form analytic solutions ($\lambda = \beta_0 \alpha_{S0} \ln(\mu/\mu_0)$)

LL: *exact* beyond LL: *approximate* (rely on PT, RGE not satisfied)

$g_1(\lambda)$ $g_2(\lambda)$ $g_3(\lambda)$

$$\alpha_S^{LL}(\mu) = \alpha_{S0}\left(\frac{1}{1-\lambda}\right)$$

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$$\alpha_S(\mu) = \alpha_{S0}\, g_1(\lambda) + \alpha_{S0}^2\, g_2(\lambda) + \alpha_{S0}^3\, g_3(\lambda) + \cdots$$

(purposely reminiscent of threshold/q_T resummation)

- Introduce a "resummation scale" $\mu_{res} = \kappa\mu$

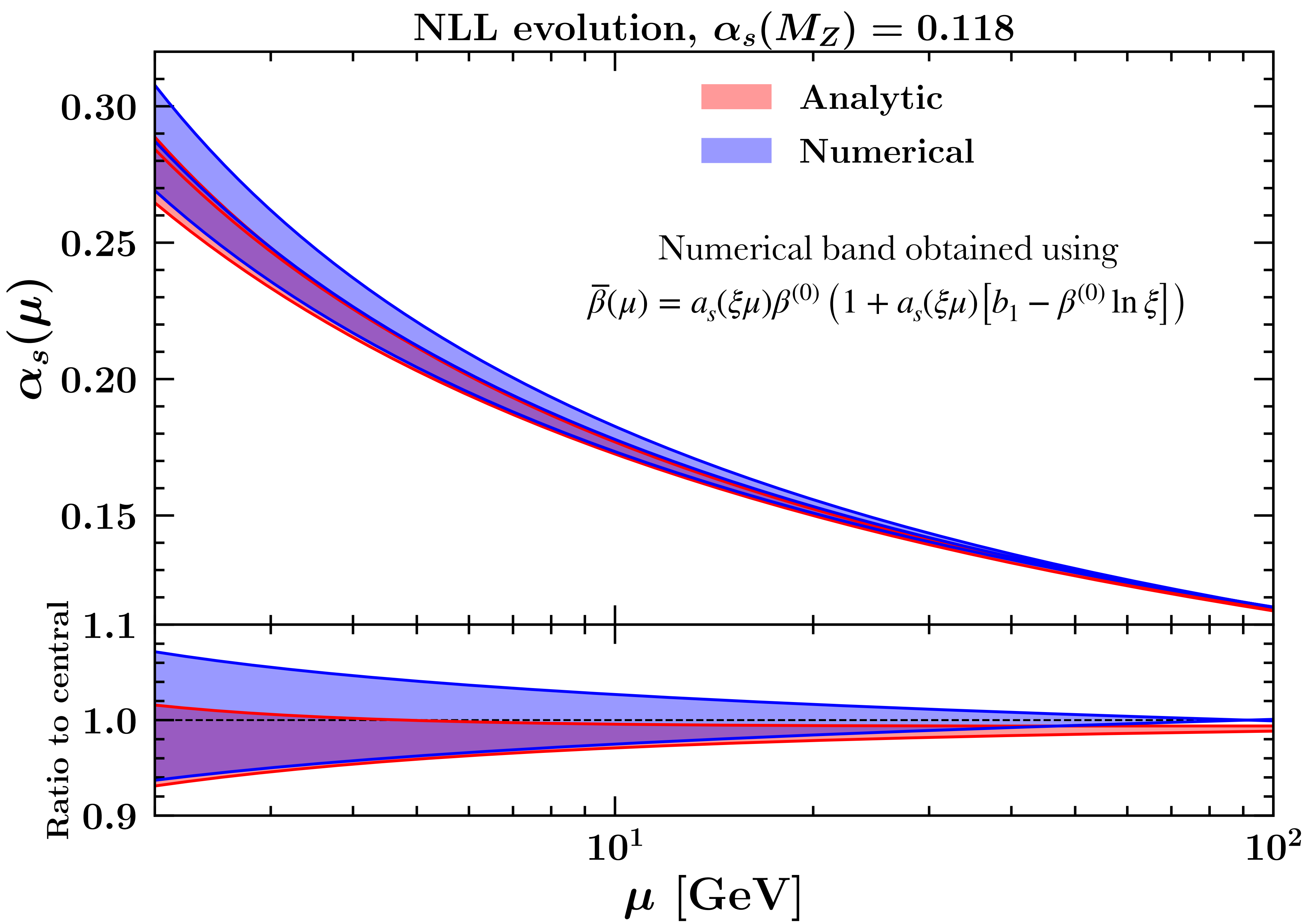
$$\lambda = \overbrace{\boxed{\beta_0 \alpha_{S0} \ln(\mu_{res}/\mu_0)}}^{\bar{\lambda}} - \beta_0 \alpha_{S0} \ln\kappa \longrightarrow g_1(\bar{\lambda}),\ g_2(\bar{\lambda}, \ln\kappa),\ g_3(\bar{\lambda}, \ln\kappa),\ \cdots$$

➡ κ generates subleading corrections to analytic solution

➡ allows estimate of missing higher orders

Scale variation

$$\alpha_S^{NLL}(\mu) = \alpha_{S0}\, g_1(\overline{\lambda}) + \alpha_{S0}^2\, g_2(\overline{\lambda}, \ln \kappa)$$



- Analytic solution strongly asymmetric: central scale outside variation band
- Numerical band shrinking to zero at Z mass, where coupling is exactly known

PDFs

k = highest-order known in γ-expansion

$$\frac{d\ln f}{d\ln\mu} = \gamma(\alpha_S(\mu)) = \sum_{n=0}^{k} \gamma_n \alpha_S^{n+1}(\mu)$$

- We repeat the procedure outlined for α_S
- At NLL:

$$f^{NLL}(\mu) = \underbrace{\left[1 + \frac{1}{\beta_0}\left(\gamma_1 - \beta_1\gamma_0\right)\left(\alpha_S^{LL}(\mu) - \alpha_{S0}\right)\right]}_{g_0^{NLL}(\overline{\lambda}, \ln\kappa)} \underbrace{\left(\frac{\alpha_S^{NLL}(\mu)}{\alpha_{S0}}\right)^{\frac{\gamma_0}{\beta_0}}}_{g_1(\overline{\lambda}), g_2(\overline{\lambda}, \ln\kappa), \ldots} f(\mu_0)$$

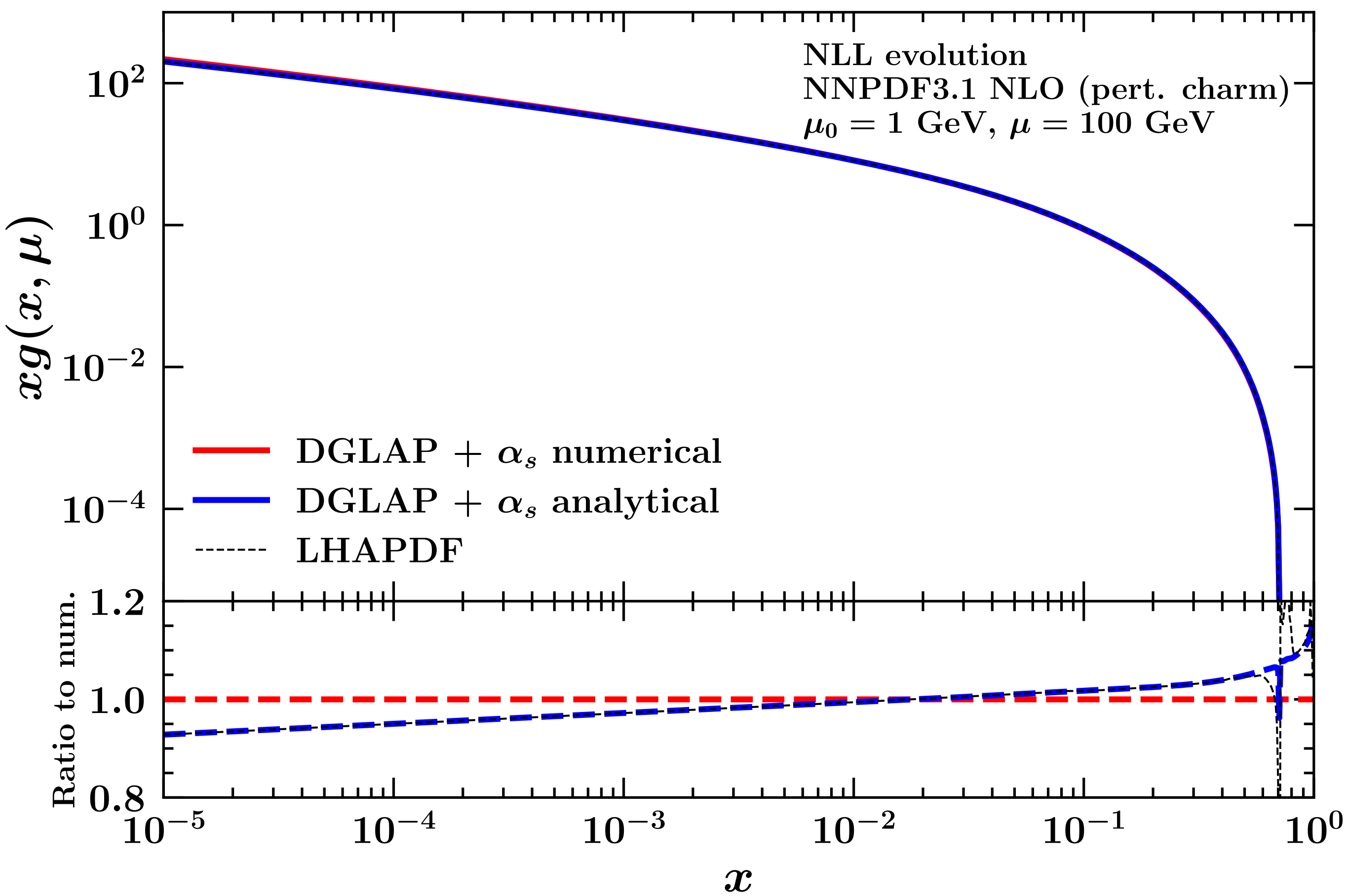
- g-functions and "Resummation scale"

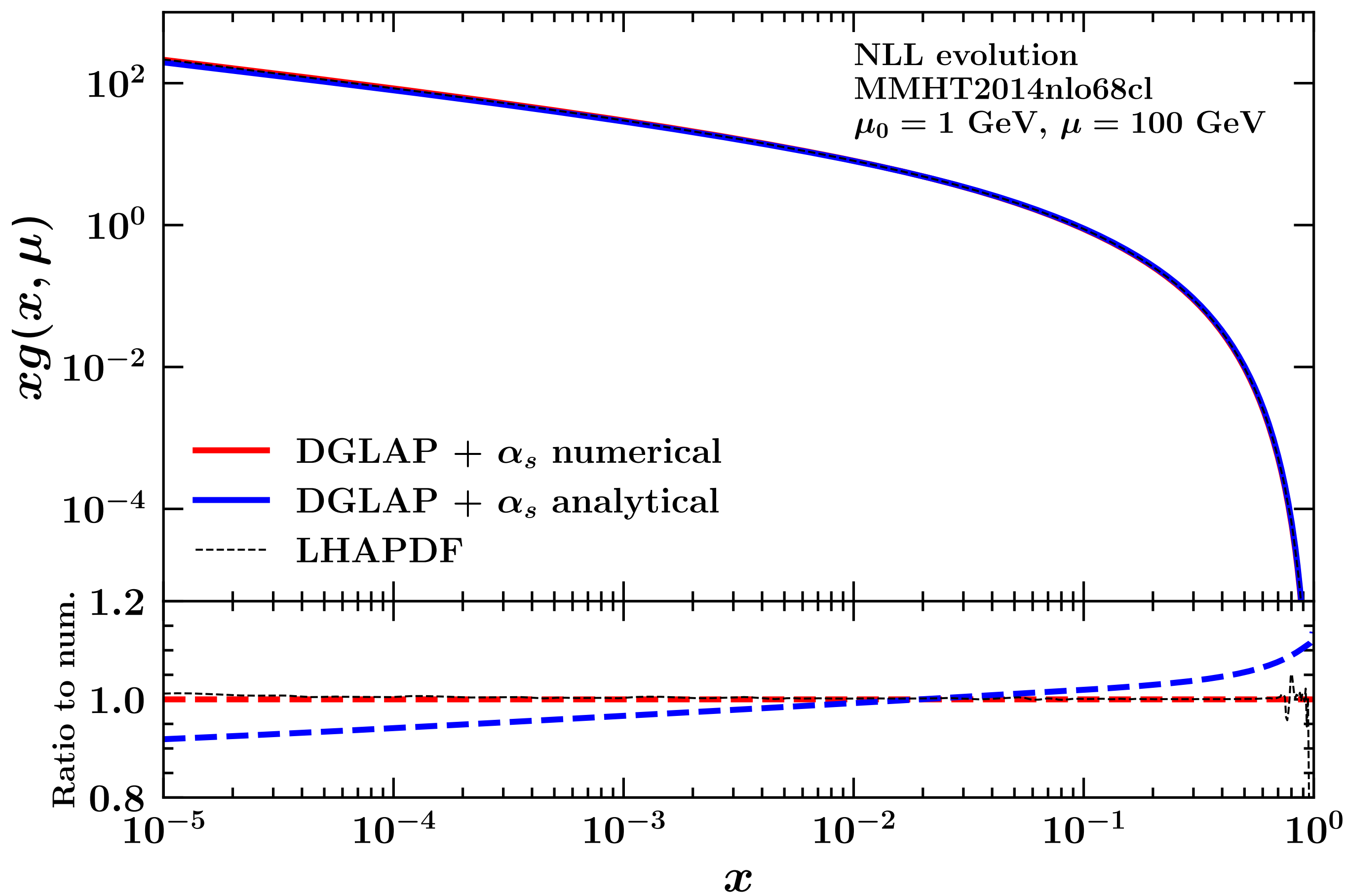
$$f^{\mathrm{N}^k\mathrm{LL}}(\mu) = g_0^{\mathrm{N}^k\mathrm{LL}}(\overline{\lambda})\exp\left[\sum_{l=0}^{k} a_s^l(\mu_0) g_{l+1}(\overline{\lambda})\right] f(\mu_0)\,,$$

➡ no need for an additional resummation scale associated to DGLAP evolution

Analytic vs. numerical: gluon PDF

$$f^{NLL}(\mu) = g_0^{NLL}(\lambda)\ \exp\left(g_1(\lambda) + \alpha_s g_2(\lambda)\right)\ f(\mu_0)$$

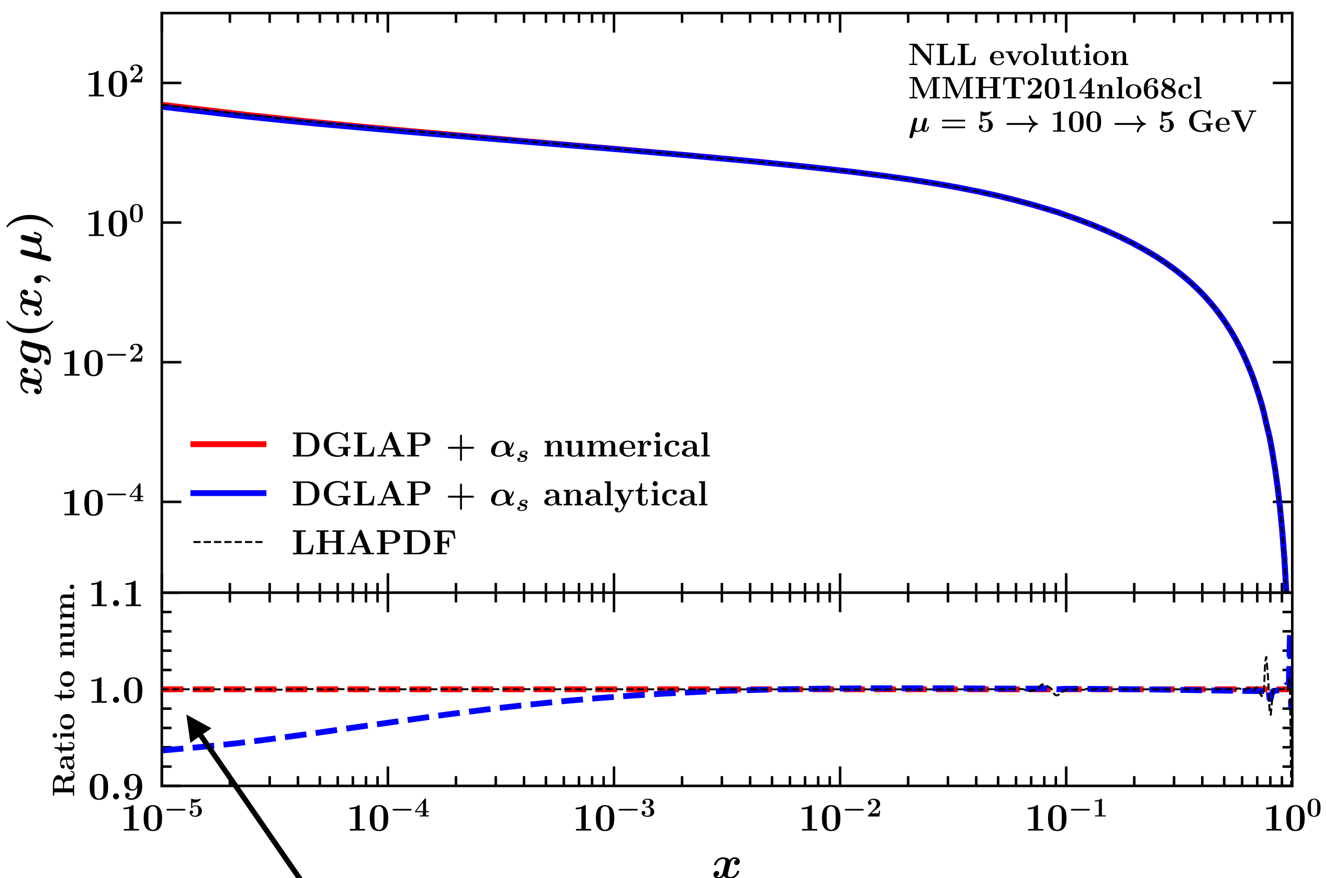




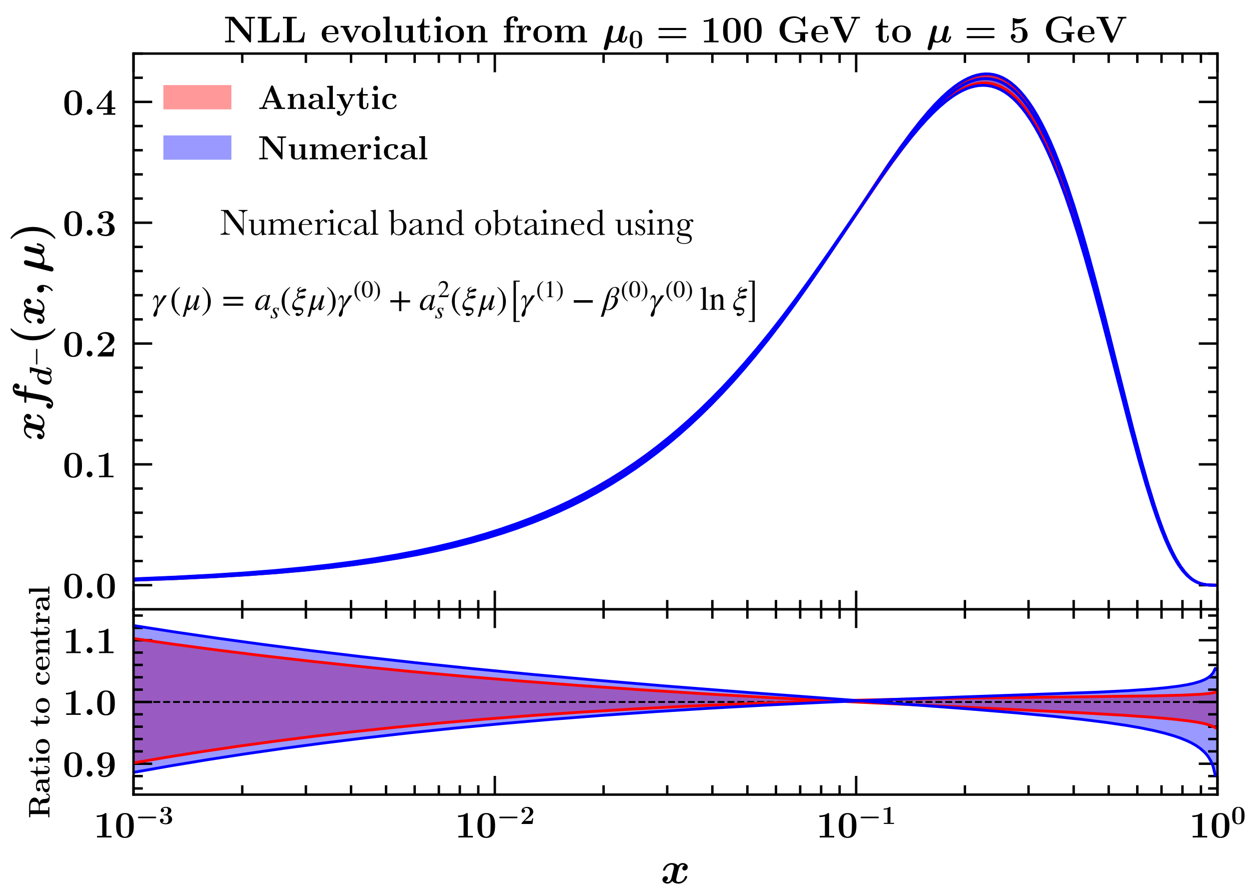
- Mismatch can exceed 10% at very low and very high values of x
- Comparison with native evolution (from LHAPDF) shows that MMHT (NNPDF) implements the numerical (analytic) evolution

Hysteresis and scale variation

$$f_{NLL}(\mu) = g_0(\bar{\lambda}, \ln\kappa)\exp\left(g_1(\bar{\lambda}) + \alpha_s g_2(\bar{\lambda}, \ln\kappa)\right) f(\mu_0)$$



perturbative hysteresis:
~6% at very low x



- Similar deviations in both numerical and analytic evolution
- Size and shape of variations consistent with previous plots

Drell-Yan q_T spectrum

- Many formalisms exist to resum large logs at small transverse-momenta
- Example of RGE-based framework: TMDs $\frac{\partial \ln F}{\partial \ln \sqrt{\zeta}} = K(\mu), \quad \frac{\partial \ln F}{\partial \ln \mu} = \gamma(\mu, \zeta)$

$$\frac{d\sigma}{dq_T} \propto H(Q) \int_0^\infty db\, b\, J_0(bq_T) \exp(S(b,Q))\, F_1(b,Q) F_2(b,Q)$$

- Common task: computing the Sudakov form factor

$$S = -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[A(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B(\alpha_S(q^2)) \right]$$

fully numerical
(α_s and Sudakov)

example: NLL integrand

$$\ln(\frac{M^2}{q^2})\left(A_1 \alpha_s^{NLL} + A_2(\alpha_s^{NLL})^2\right) + B_1 \alpha_s^{NLL}$$

fully analytic
(α_s and Sudakov)

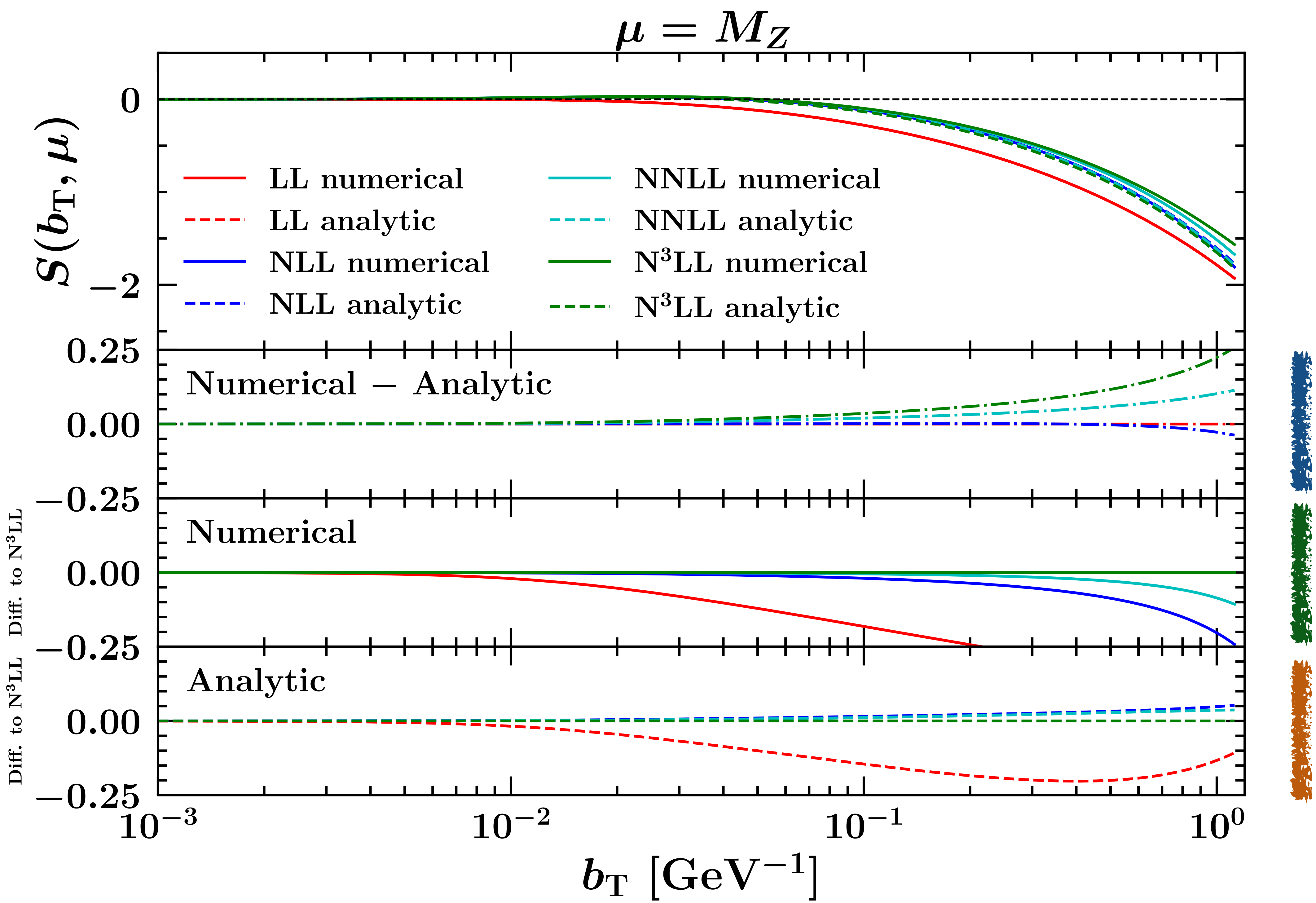
$$S = Lg_1(\alpha_s \beta_0 L) + g_2(\alpha_s \beta_0 L) + \alpha_s g_3(\alpha_s \beta_0 L) + \ldots$$

example: NLL integrand

$$\ln(\frac{M^2}{q^2})\left(A_1 \alpha_s^{NLL} + A_2(\alpha_s^{LL})^2\right) + B_1 \alpha_s^{LL}$$

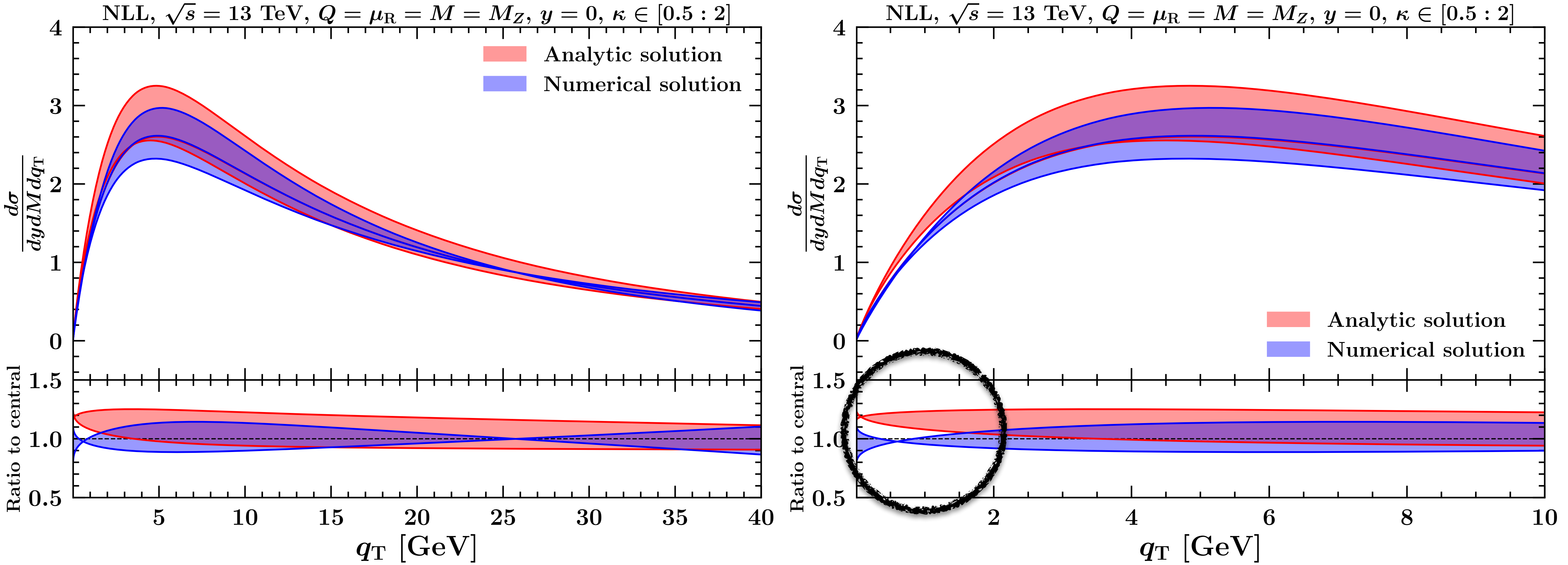
Intermediate approximations have been analysed in
Billis, Tackmann, Talbert, JHEP 03 (2020), 182

Analytic vs. numerical: Sudakov



- No difference at LL (as expected)
- **Increasing** difference at high b_T (low q_T) (?)
  - Numerical Sudakov
  - Good convergence towards N3LL !!
- Analytic Sudakov
- Faster convergence towards N3LL
- (N3LL - NNLL) > (NNLL - NLL) (?)

Analytic vs. numerical: q_T spectrum



- Overlapping bands at NLL for most part of the spectrum
- Overall large variations (likely to decrease at higher orders)
- **Different behaviour of analytic/numerical scale variation**
- **Need to better understand theoretical uncertainty in the low q_T region**

NNLL and N3LL: work in progress

Conclusions

- Drell-Yan spectrum at low q_T is a perfect playground to exploit the full potentiality and the differences among q_T-resummation frameworks: one of the final goals is to agree on a common recipe for theoretical uncertainties
- We extended the “g-functions” formalism commonly used in Sudakov resummation to the evolution of the running coupling and the PDFs
- We identified in analytic vs. numerical solution of RGEs a potential source of systematic uncertainty in DY precision physics, and we quantitatively estimated the ensuing *perturbative hysteresis* effect
- Outlook: We aim at identifying an appropriate use of scale variations to estimate theoretical uncertainties in both the analytical and numerical approaches for the Drell-Yan q_T-spectrum

Backup

Solution of RGE for PDFs

$$
\begin{aligned}
f(\mu) &= \exp\left[\int_{\mu_0}^{\mu} d\ln\mu'\,\gamma(a_s(\mu'))\right] f(\mu_0) \\
&= \exp\left[\sum_{n=0}^{k}\gamma^{(n)}\int_{\mu_0}^{\mu} d\ln\mu'\,a_s^{n+1}(\mu')\right] f(\mu_0) \\
&= \prod_{n=0}^{k}\exp\left[\gamma^{(n)} I_n\right]\, f(\mu_0),
\end{aligned}
$$

$$
I_n = \int_{\mu_0}^{\mu} d\ln\mu'\,a_s^{n+1}(\mu') = \int_{a_s(\mu_0)}^{a_s(\mu)} da_s\left(\frac{a_s^n}{\overline{\beta}(a_s)}\right)
$$

$$
\begin{aligned}
I_0 &= \frac{1}{\beta^{(0)}}\int_{a_s(\mu_0)}^{a_s(\mu)}\frac{da_s}{a_s+b_1a_s^2+b_2a_s^3} = \frac{1}{\beta^{(0)}}\ln\left(\frac{a_s(\mu)}{a_s(\mu_0)}\right) - \frac{b_1}{\beta^{(0)}}(a_s(\mu)-a_s(\mu_0)) + \frac{b_1^2-b_2}{2\beta^{(0)}}(a_s^2(\mu)-a_s^2(\mu_0)) + \mathcal{O}(\alpha_s^3)\,, \\
I_1 &= \frac{1}{\beta^{(0)}}\int_{a_s(\mu_0)}^{a_s(\mu)}\frac{da_s}{1+b_1a_s+b_2a_s^2} = \frac{1}{\beta^{(0)}}(a_s(\mu)-a_s(\mu_0)) - \frac{b_1}{2\beta^{(0)}}(a_s^2(\mu)-a_s^2(\mu_0)) + \mathcal{O}(\alpha_s^3)\,, \\
I_2 &= \frac{1}{\beta^{(0)}}\int_{a_s(\mu_0)}^{a_s(\mu)}\frac{da_s\,a_s}{1+b_1a_s+b_2a_s^2} = \frac{1}{2\beta^{(0)}}(a_s^2(\mu)-a_s^2(\mu_0)) + \mathcal{O}(\alpha_s^3)\,.
\end{aligned}
\tag{20}
$$

$$
\exp\left[\gamma^{(n)} I_n\right] = 1+\gamma^{(n)}\times\mathcal{O}(\alpha_s^n)\,,\quad n\geq 1\,.
$$

$$
f(\mu) = [1+\mathcal{O}(\alpha_s)]\left(\frac{a_s(\mu)}{a_s(\mu_0)}\right)^{\frac{\gamma^{(0)}}{\beta^{(0)}}} f(\mu_0)\,.
$$

$$
\exp\left[\gamma^{(0)} I_0\right] = \exp\left[\frac{\gamma^{(0)}}{\beta^{(0)}}\ln\left(\frac{a_s(\mu)}{a_s(\mu_0)}\right)\right] + \mathcal{O}(\alpha_s) = \left(\frac{a_s(\mu)}{a_s(\mu_0)}\right)^{\frac{\gamma^{(0)}}{\beta^{(0)}}} + \mathcal{O}(\alpha_s)\,.
$$