Testing large-x asymptotics in nucleon and pion PDFs

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Probing large-x structure from QCD scattering data

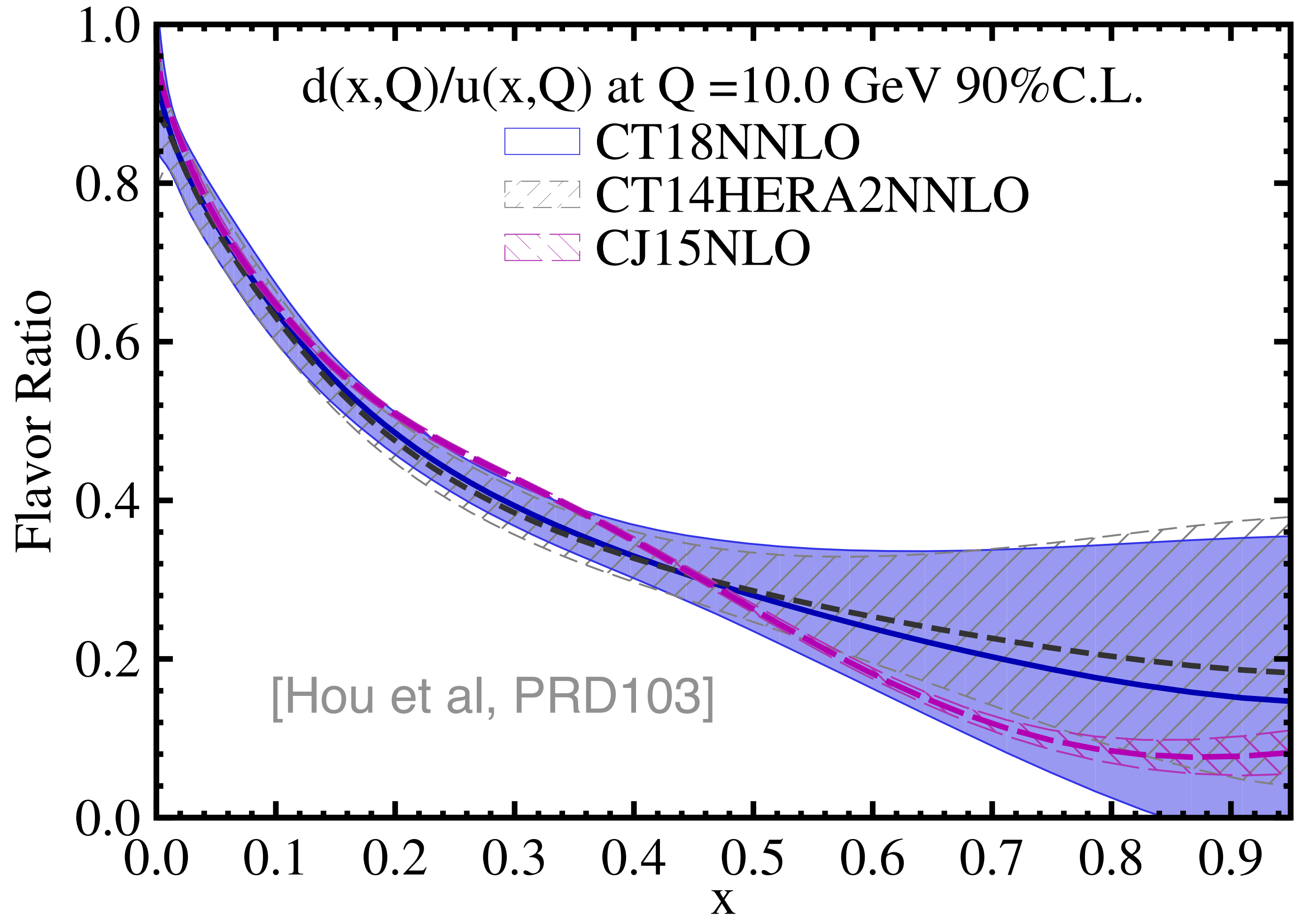
Plenary talk by C. Keppel

*Expect large improvements in our understanding of the large x valence regime in the **next 1-2 years**!*

Our goal is to understand **how to interpret** primordial PDFs based on scattering data.

⇨ continous efforts to understand the large-x regime — CT18 for ratio d/u

⇨ *e.g.* role of parametrization, detailed uncertainties — at NNLO in QCD



Talk by P. Nadolsky

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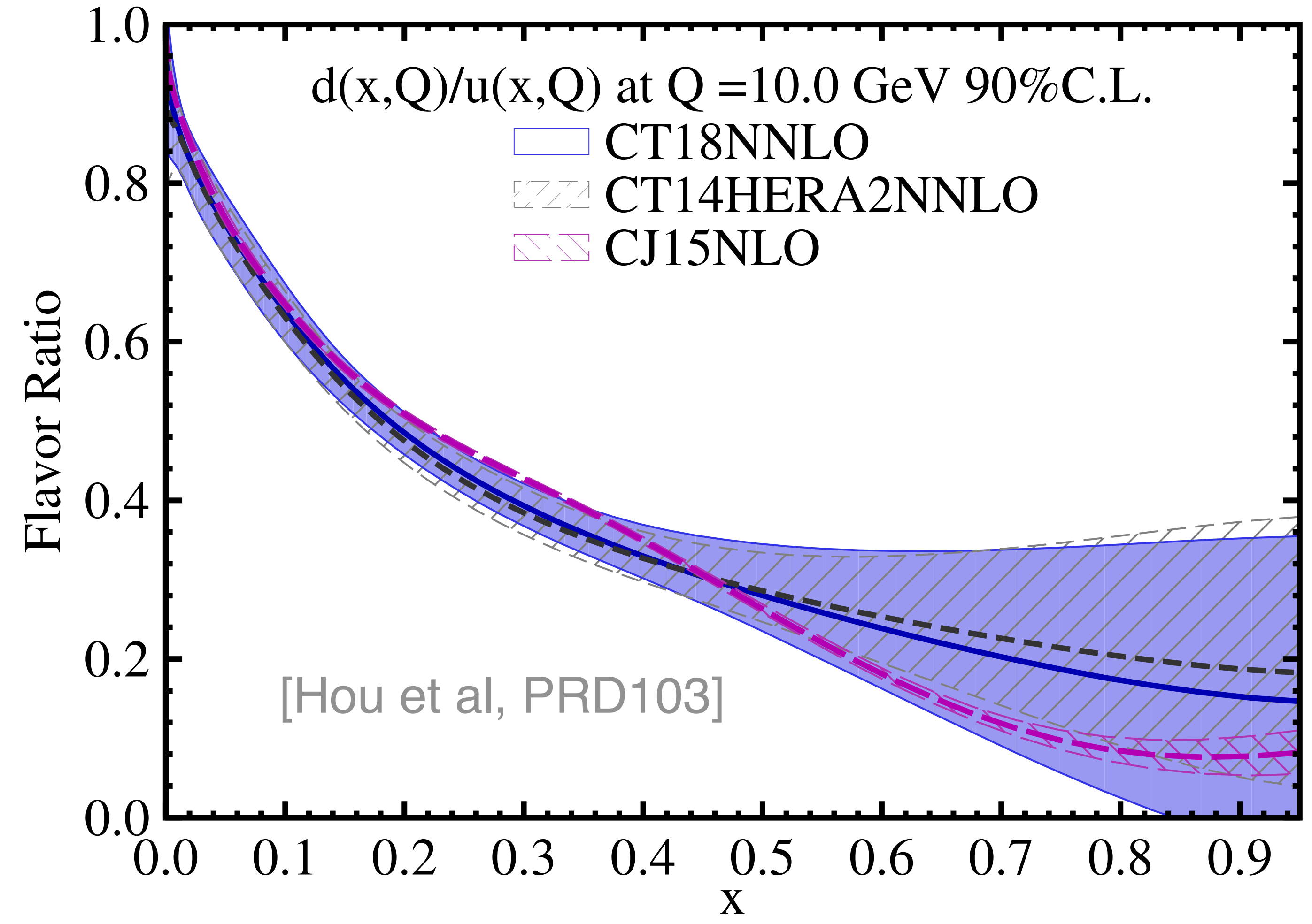


Talk by P. Nadolsky

⇨ PDF at large x reflect the valence region (x~.5) and threshold limits (x→1)

⇨ ideal for probing the nonperturbative structure of hadrons

- early QCD pictures of hadron dynamics → have evolved into pQCD
- early models for hadron dynamics → have paved the way for nonperturbative approaches

Probing large-x structure from QCD scattering data

Plenary talk by C. Keppel → *Expect large improvements in our understanding of the large x valence regime in the **next 1-2 years**!*

Large-x PDFs of common interest across the energy spectrum.

⇨ We explore what we can learn about **quark counting rules** — the classical prediction of QCD theory.

⇨ We explore the functional mimicry of PDFs — the rôle of the parametrization.

⇨ We use the CT18NNLO global fit of proton PDFs.

⇨ we discuss implications for the pion PDFs.

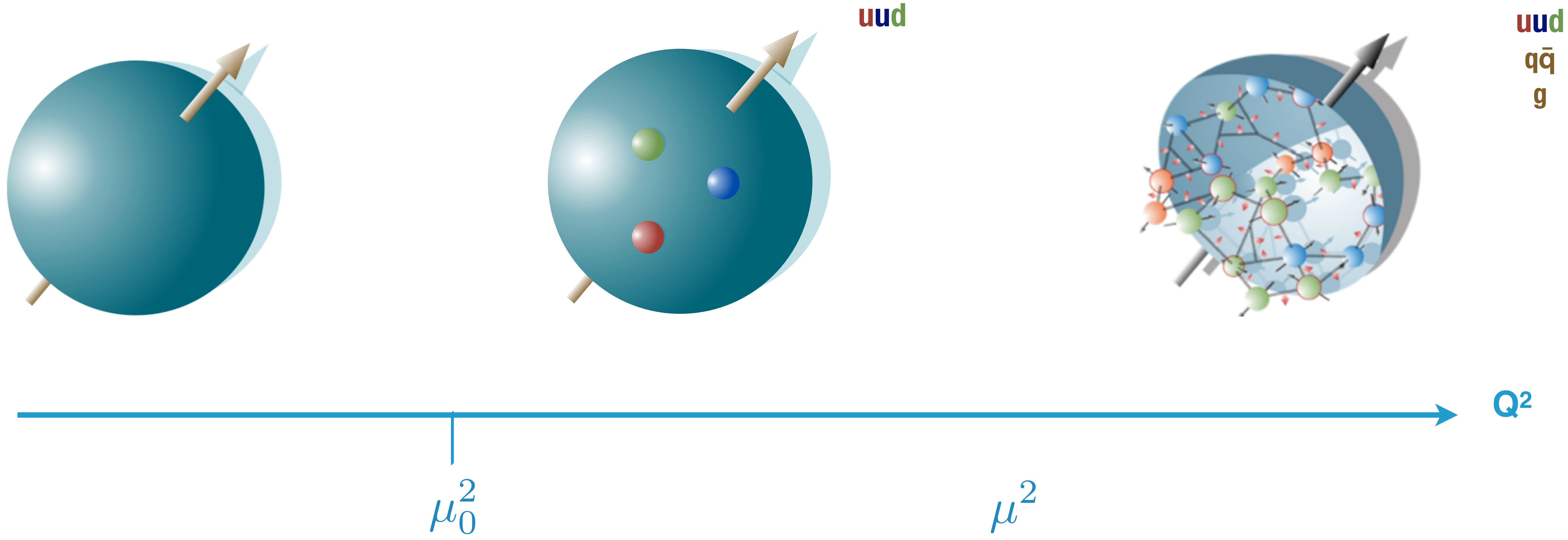
⇨ We revisit the interpretation of the “data vs. physical manifestations”.

PDFs in nonperturbative QCD

- **at hadronic scale μ_0^2 < 1GeV²**
  - ⇨ prefactorization picture
  - ⇨ nonperturbative dynamics
  - ⇨ model's degrees of freedom

Phenomenological PDFs

- **at factorization scale μ^2 > 1GeV²**
  - ⇨ quasi-free partonic degrees of freedom
  - ⇨ defined in the MSbar scheme
  - ⇨ leading-power approximation to full dynamics

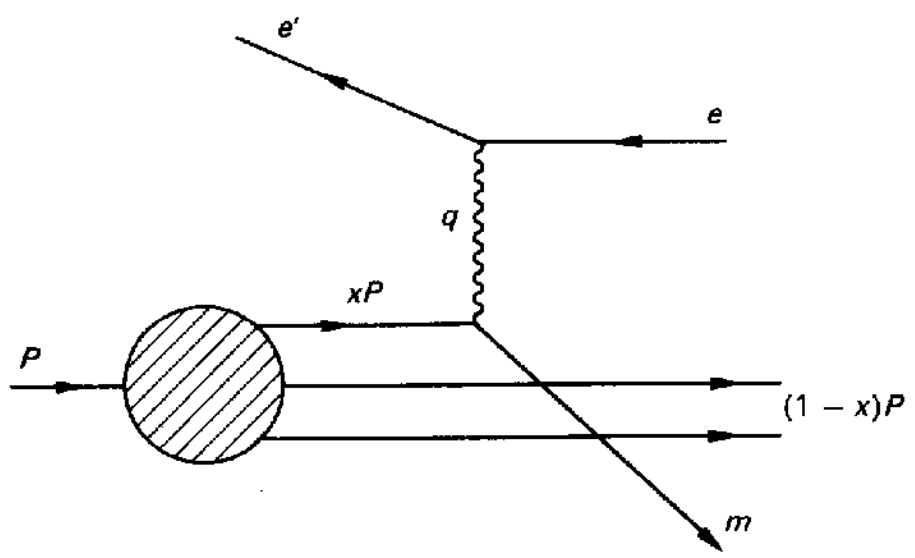


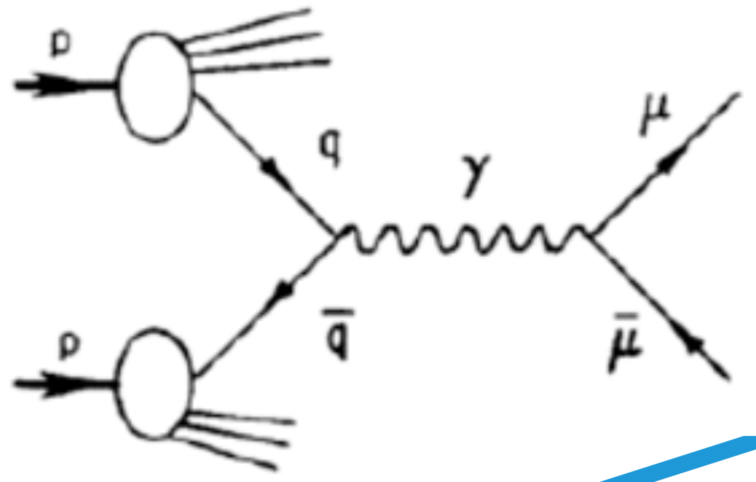
PDFs in nonperturbative QCD

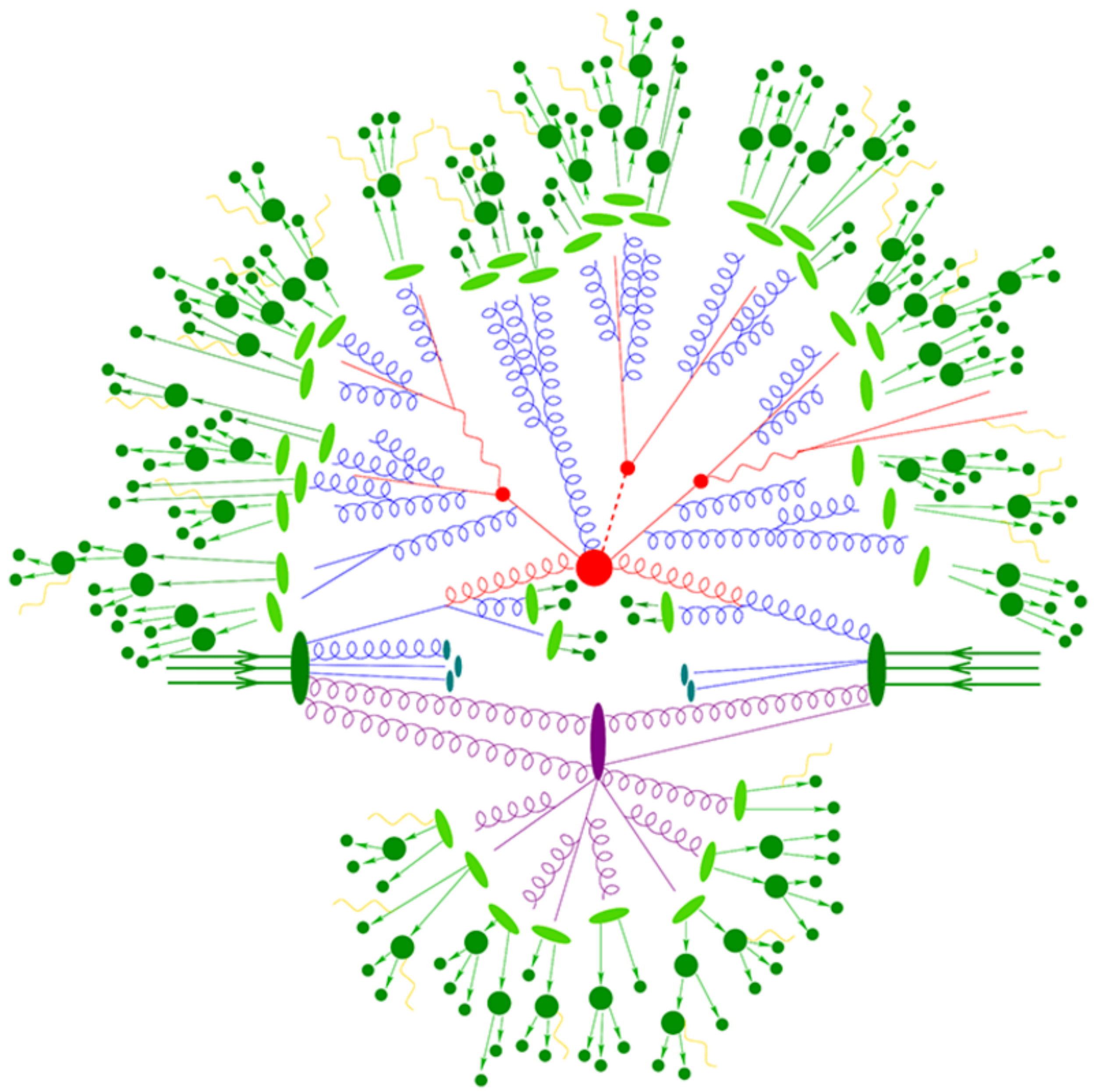
Phenomenological PDFs

How to relate the x dependence of the perturbative and nonperturbative pictures?

⇨ we can learn about nonperturbative dynamics by comparing predictions to data for the simplest scattering processes (DIS and DY)







⇨ pheno PDFs are determined from analyzing many processes with complex scattering dynamics

μ_0^2 $\mu^2 \sim Q^2$

Testing Quark Counting Rules

1. Structure functions

Threshold limit in DIS ⇨ x →1
Proved for exclusive and inclusive *processes*

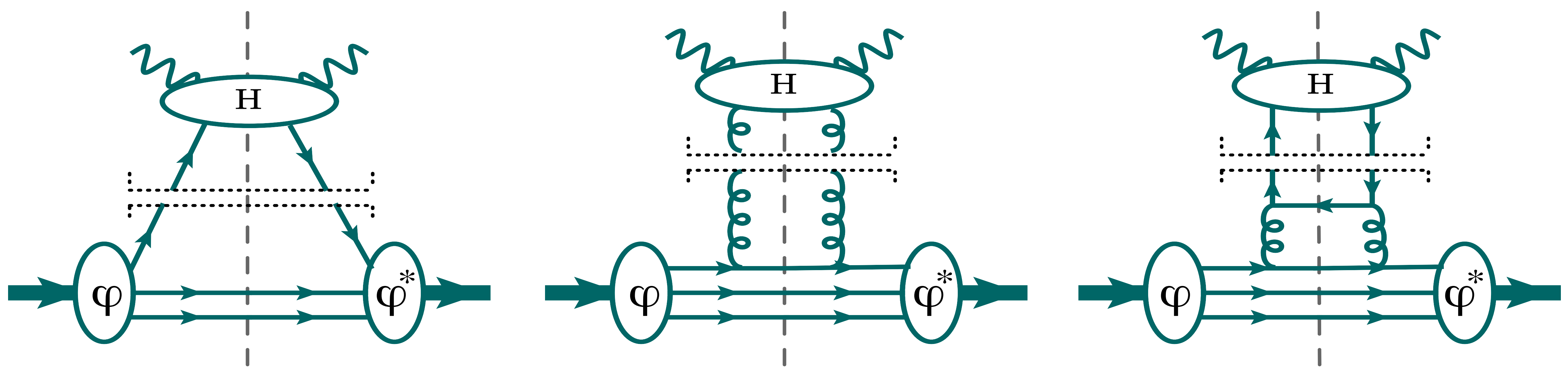
Brodsky and Farrar, PRL31 and PRD11
Ezawa, Nuovo Cim. A23
Berger and Brodsky, PRL42
Soper, PRD15

$$F_2(x_B) \xrightarrow[x_B \to 1]{} (1-x_B)^{2p-1+2|\lambda_q-\lambda_A|}$$

p=#spectators

λ_q & λ_A =helicities of active quark and target

2. Unpolarized PDFs



⇦ extended from SF — without λ

$$f(x) \xrightarrow[x_B \to 1]{} (1-x_B)^{n=3} \qquad (1-x_B)^{n>4} \qquad (1-x_B)^{n>5}$$

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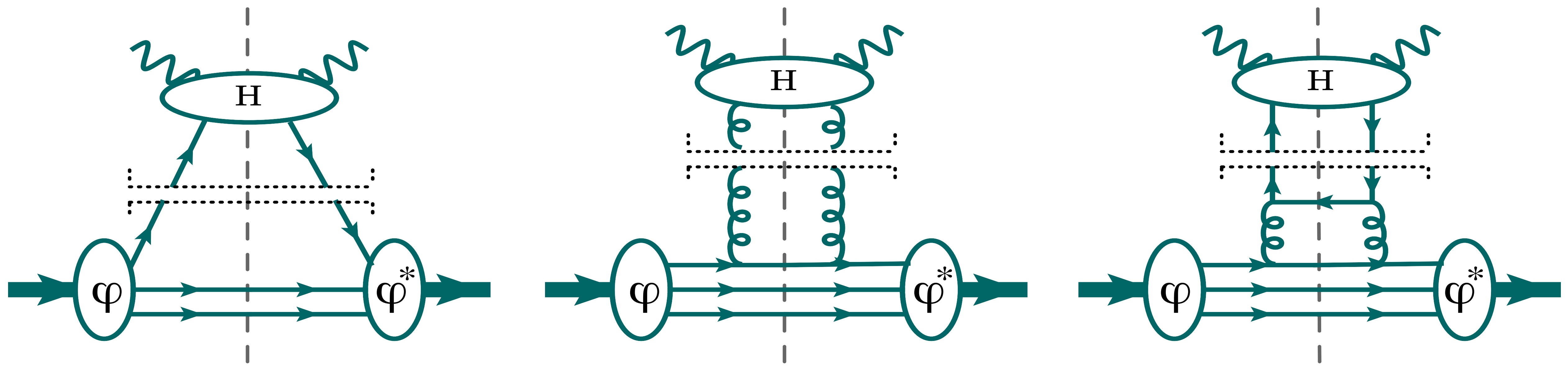

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Complementary testing to first principles

Scale dependent

Do we observe QCRs in data described by pQCD?

Narrowing the question: Do we find clear evidence or tentative hints?

Agreement of model with data — here a $(1-x)^{A_2}$ behavior

uncertainties needed for a faithful conclusion.

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Evidence of polynomial form:

there is more than one possible solution to the choice of functional form.

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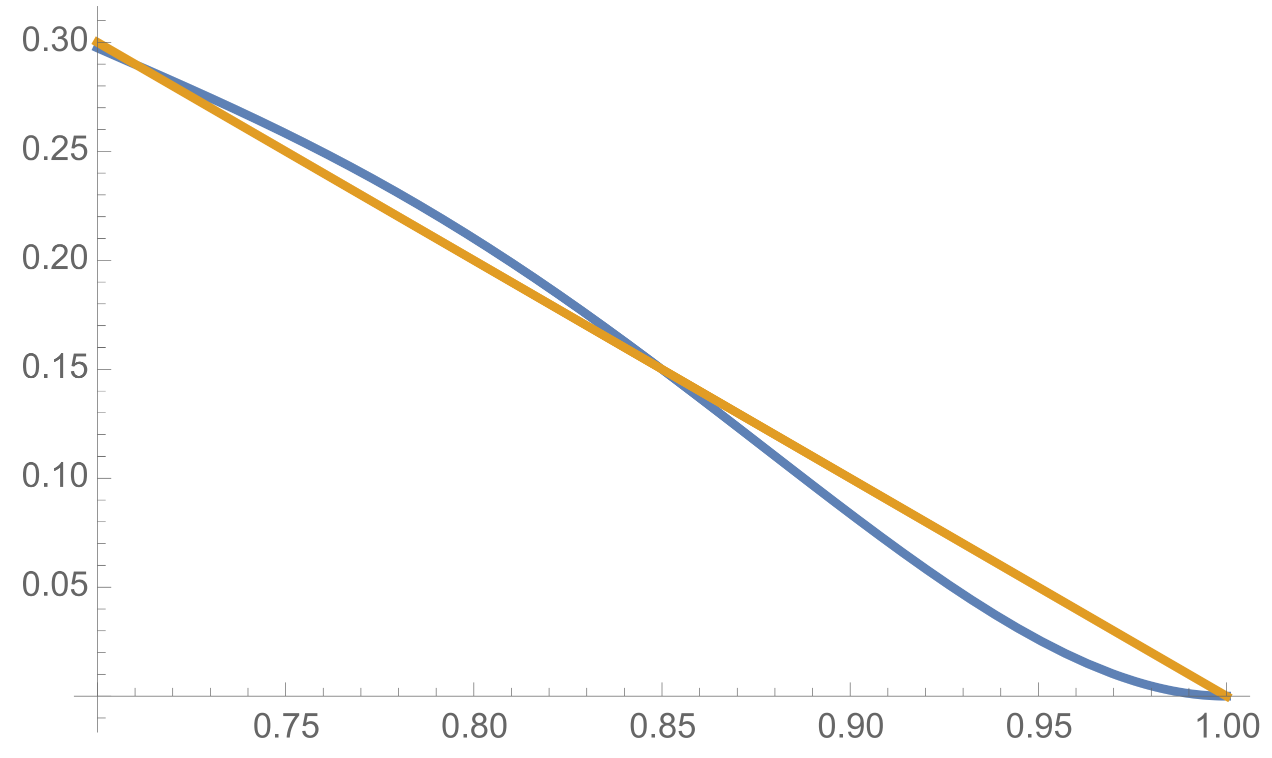
Testing specific x-shapes:

1. Global/local degree of polynomial

   What is large x? Pheno: $x \in [.6, \sim .9]$, theory: $x \rightarrow 1$

   A sum of (1-x)-power contributions can *globally* fit to a lower power $\sum_{n=2}^{4} \alpha_n (1-x)^n = (1-x)$



2. Polynomial mimicry

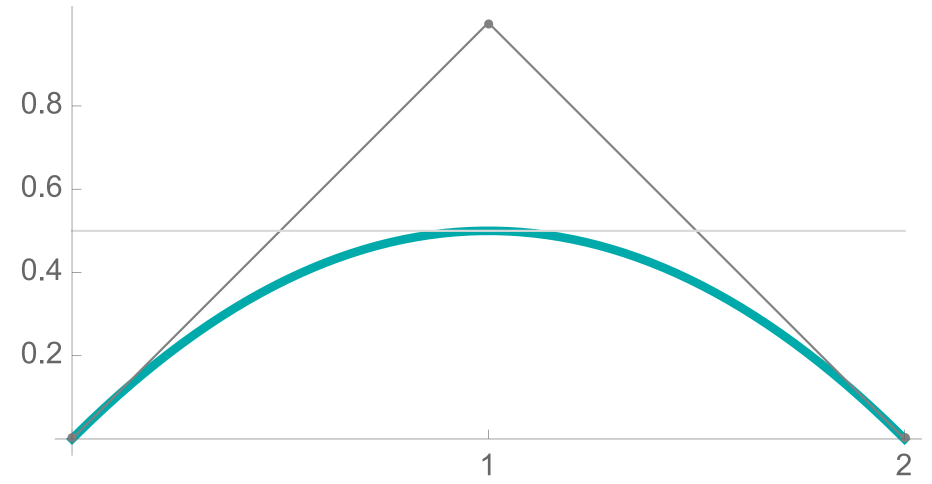
   Mathematical equivalence of polynomials of different orders.

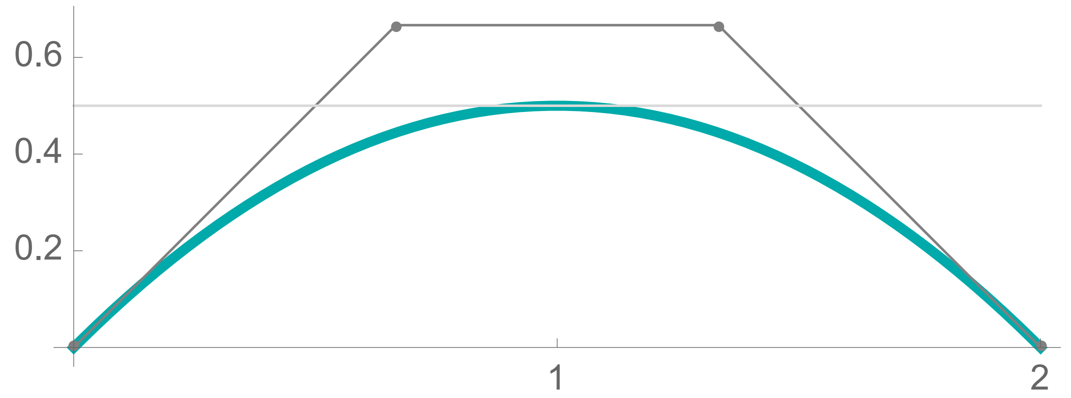
Bézier curves

[AC & Nadolsky, PRD 103 (2021)]

2. Bézier curves give an example of mathematical equivalence of polynomials of different orders

- defined using Bernstein basis polynomials:





$$f(x) = \alpha(1-x)^2 + 2\beta(1-x)x + \gamma x^2$$

$$f(x) = \alpha(1-x)^3 + 3\beta'(1-x)^2 x + 3\beta''(1-x)x^2 + \gamma x^3 \qquad \beta', \beta'' \equiv F[\alpha, \beta, \gamma]$$

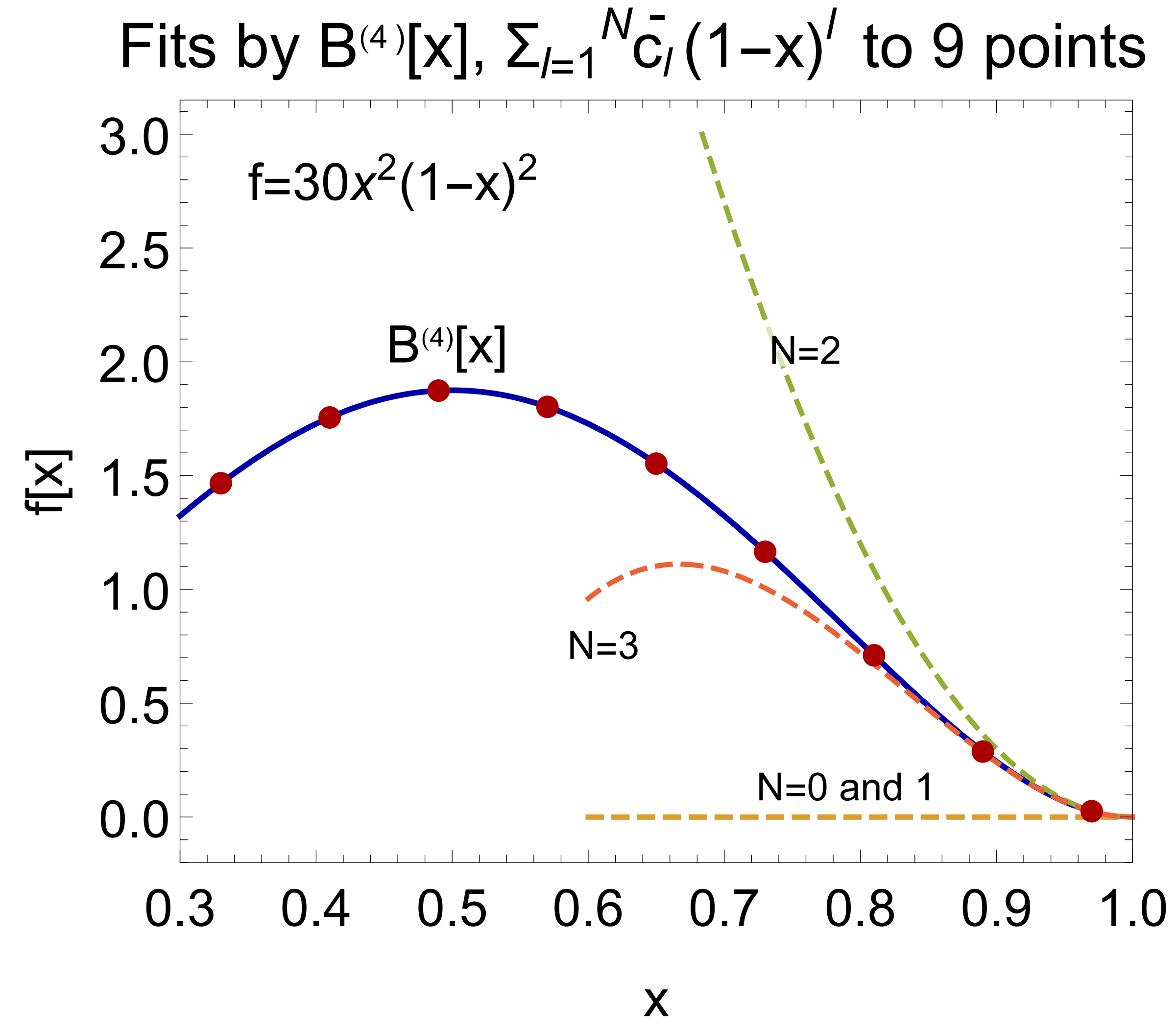
- can be used to interpolate discrete data points

The interpolation through Bézier curves is unique if the polynomial degree= (# points-1):
there's a closed-form solution to the problem,

$$\mathcal{B}^{(n)}(x) = \sum_{l=0}^{n} c_l \; B_{n,l}(x) \qquad \text{with the Bernstein pol.} \qquad B_{n,l}(x) \equiv \binom{l}{n} x^l (1-x)^{n-l}.$$

The interpolation through Bézier curves allows an expansion in (1-x) about x=1:

$$u_\pi(x \to 1) = \sum_{i=0}^{n} \bar{c}_i \, (1-x)^i$$

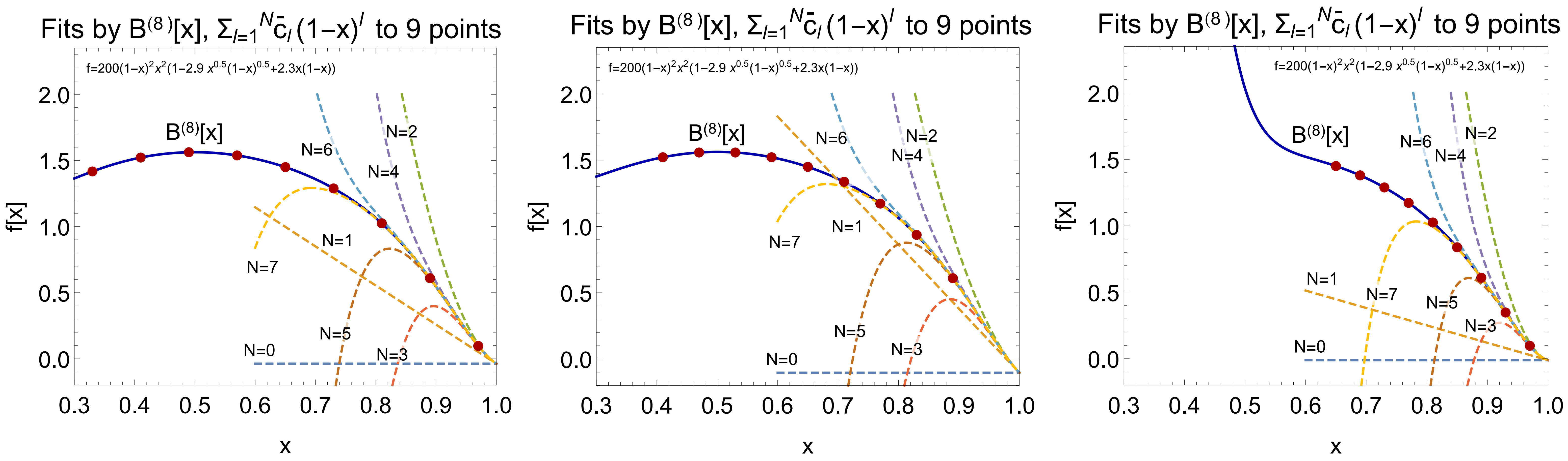


Pinning down the large-x behaviour?

Take a realistic functional form, here $f=200(1-x)^2x^2(1-2.9\,x^{0.5}(1-x)^{0.5}+2.3x(1-x))$

- Sample 9 points with $x < 0.975$ from this function and interpolate by a Bezier curve. [This interpolation is exact.]
- The lowest coefficients of the monomial expansion (N=1, 2, 3) are spurious and depend on the range and spacing of the sampled data.

⇨ The lowest powers of the expansion cannot be meaningfully reconstructed.



Can we still find hints for QCRs?

Data on F_2 described by pQCD

Effective exponent for x →1

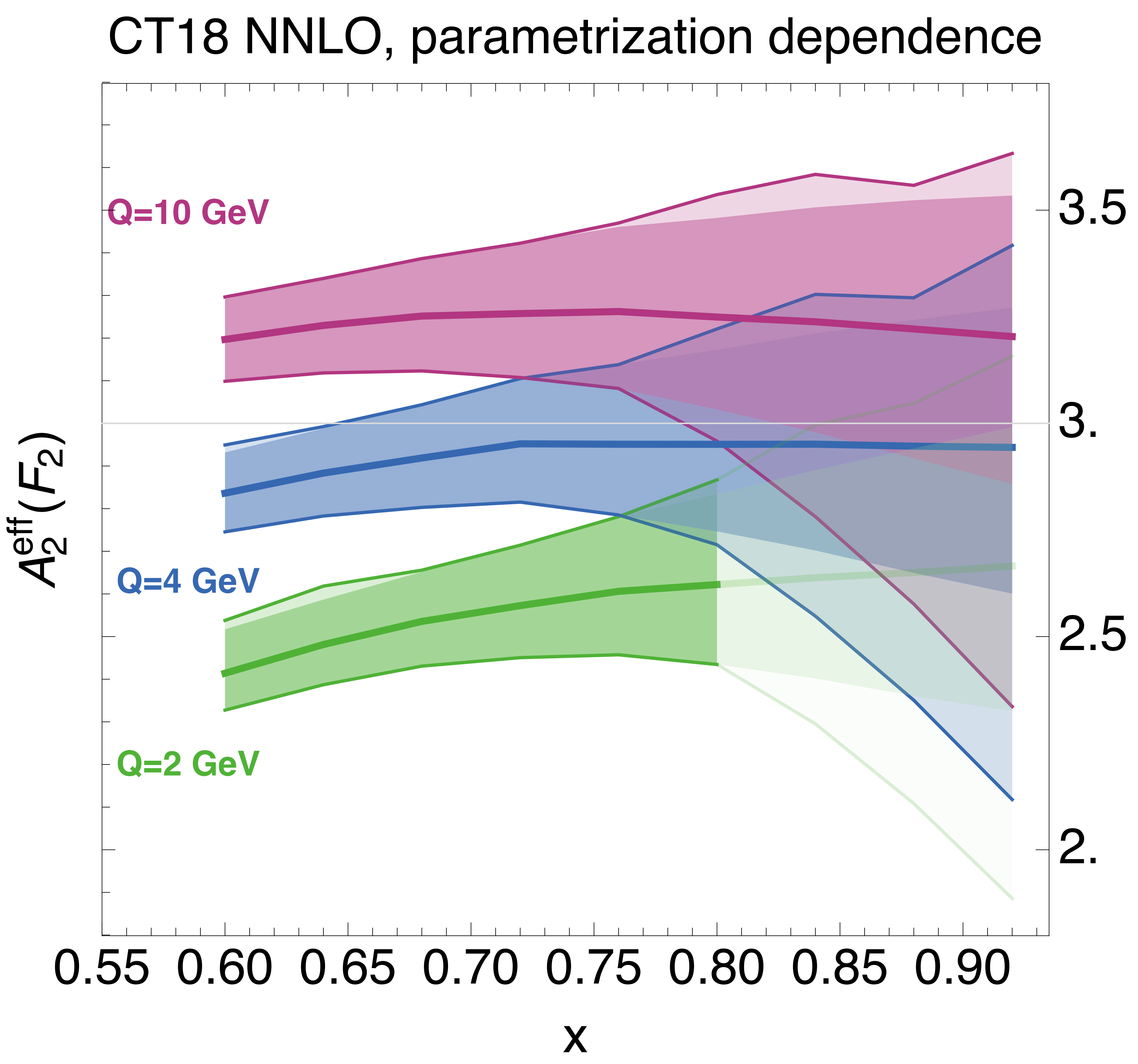
$$\boxed{A_2^{\rm eff}(F_2) \equiv \frac{\partial \ln(F_2(x,Q))}{\partial \ln(1-x)}}$$

see Ball et al, EPJC76 as well

CT18 [Hou et al, PRD103] and all main global fits assume

$$f_{a/A}(x,Q_0^2) = x^{A_{1,a}}(1-x)^{A_{2,a}} \times \Phi_a(x)$$

$\Phi_{a,n}$ a Bézier curve for CT18
≡smooth pol. of degree n



Structure Function follows QCRs within uncertainties.

- Non-negligible running with Q^2
- Bjorken regime defined for particular W^2 and Q^2 regions ($Q^2 \to\infty$ and $W^2 > m_p^2+(1-x)/x\, Q^2$)

Two sources of uncertainties showed:
1. Hessian errors from fit output
2. Parametrization choice

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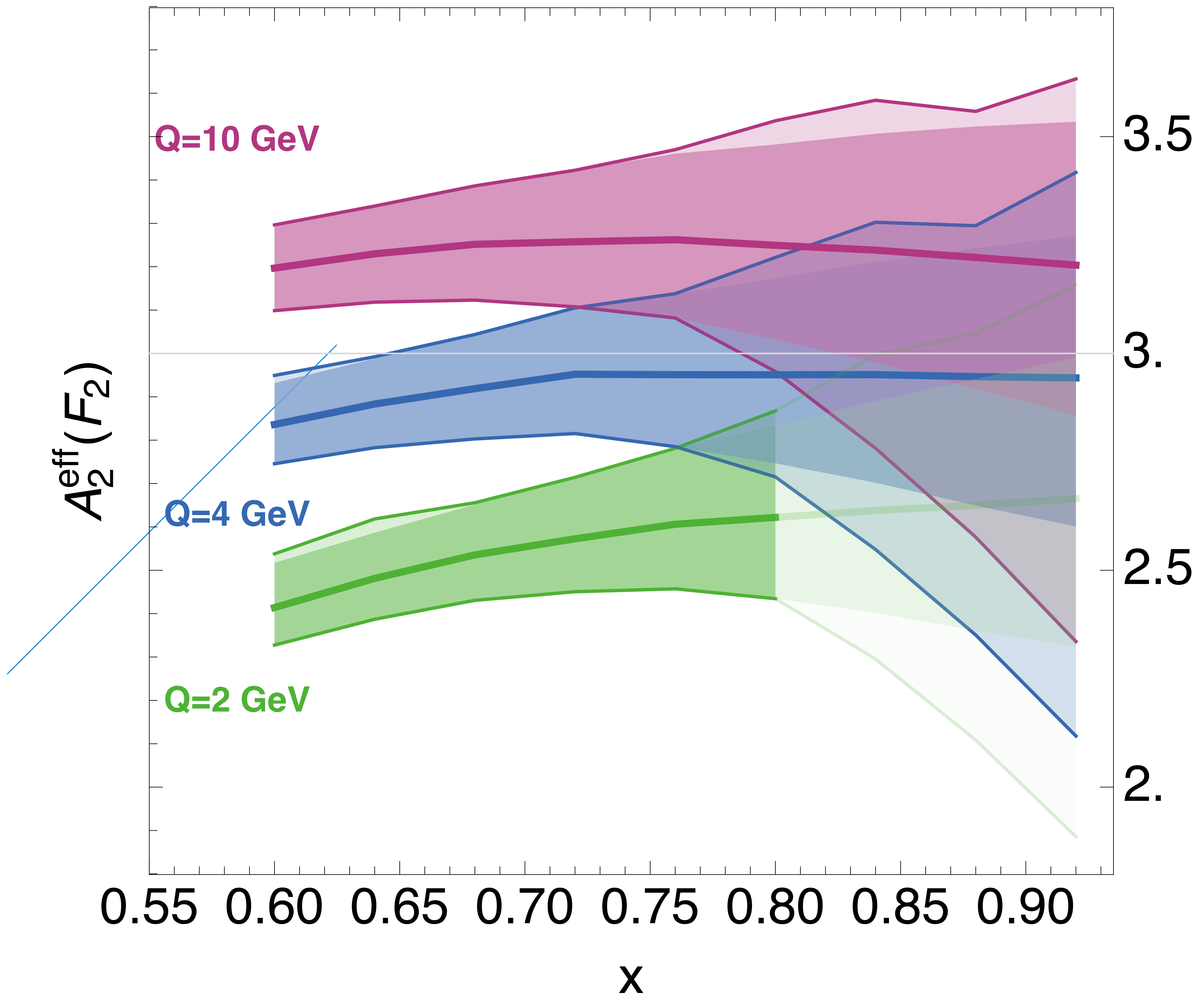
CT18 NNLO, parametrization dependence



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NNLO Pheno PDFs

Talk by P. Nadolsky

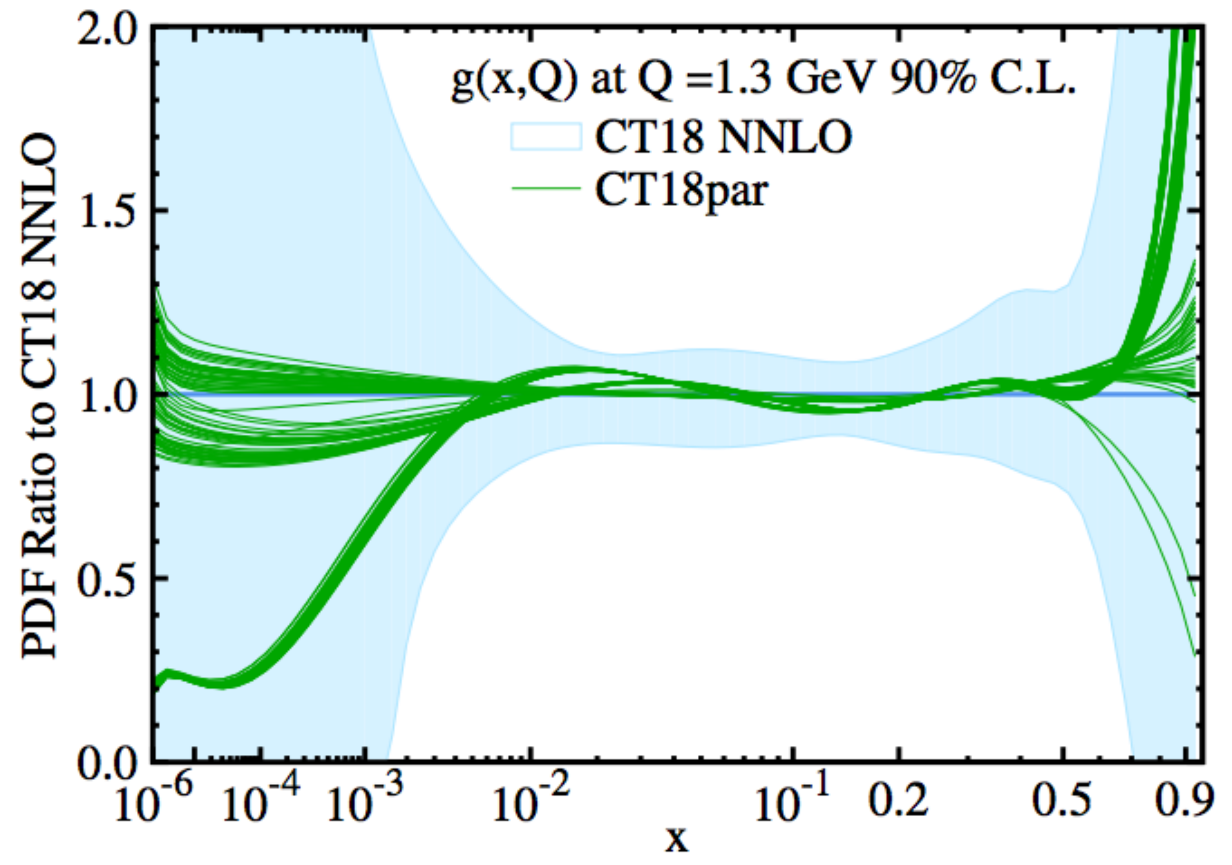
Data do not constrain the x> 0.8 region
⇨ experiments compatible with multiple parametric forms at large x

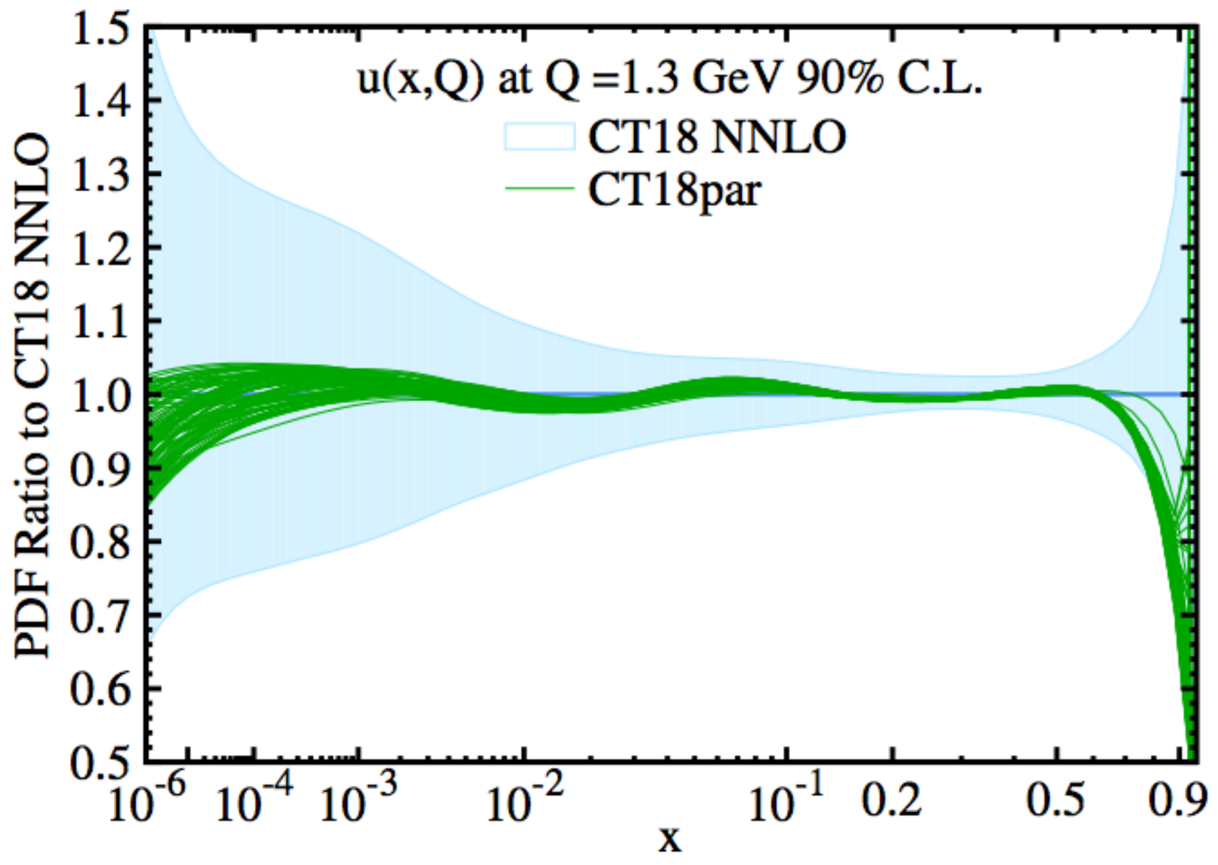
Q_0=1.3 GeV is the starting evolution scale

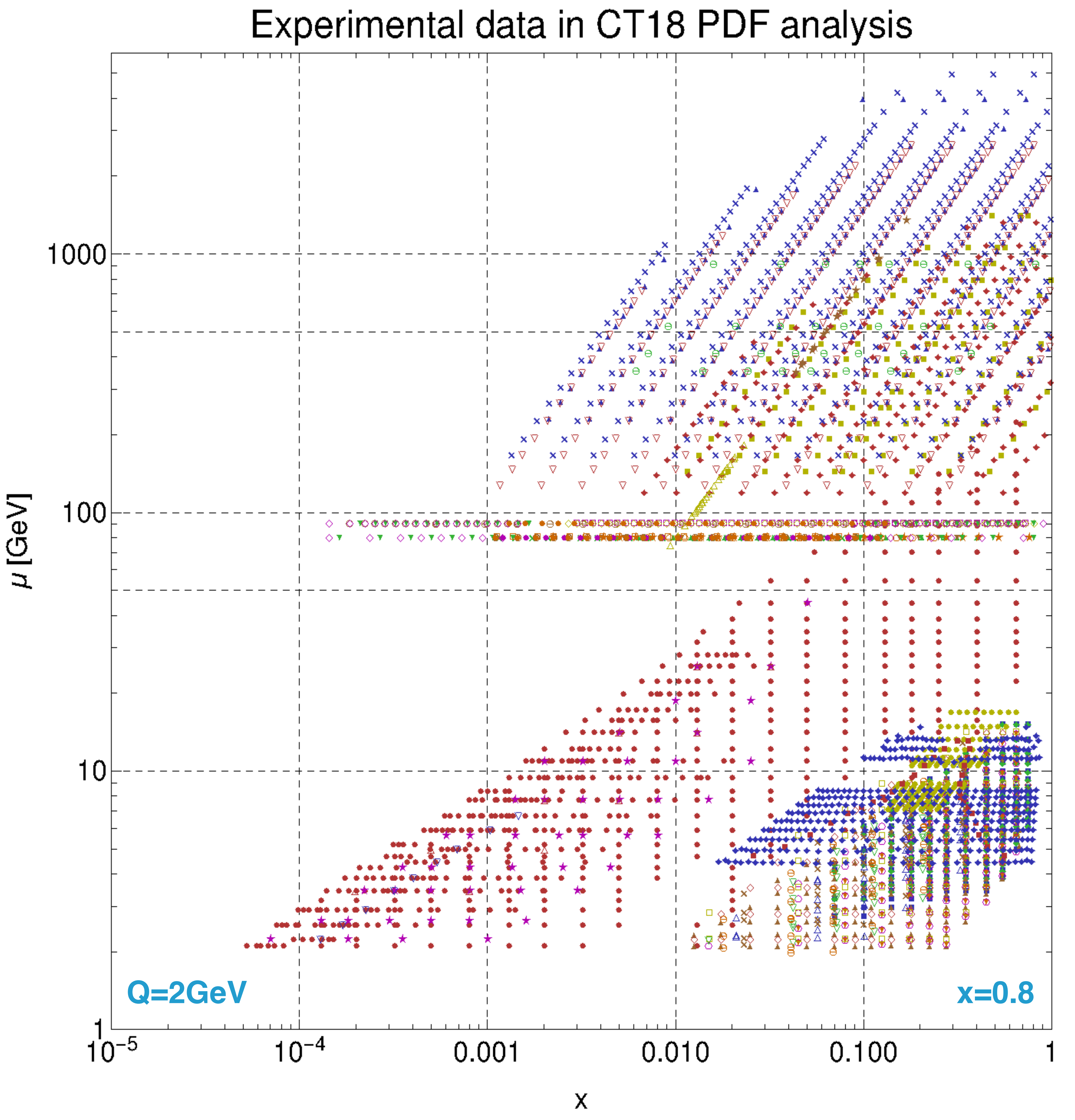
1. Hessian error propagation from fit output
2. Choice of functional form:
   - there is more than one representation with equally good agreement
   - dominant error for valence quarks

CT18: W^2>15GeV^2 , fixed order, Q^2>4GeV^2

[Hou et al, 1912.10053]







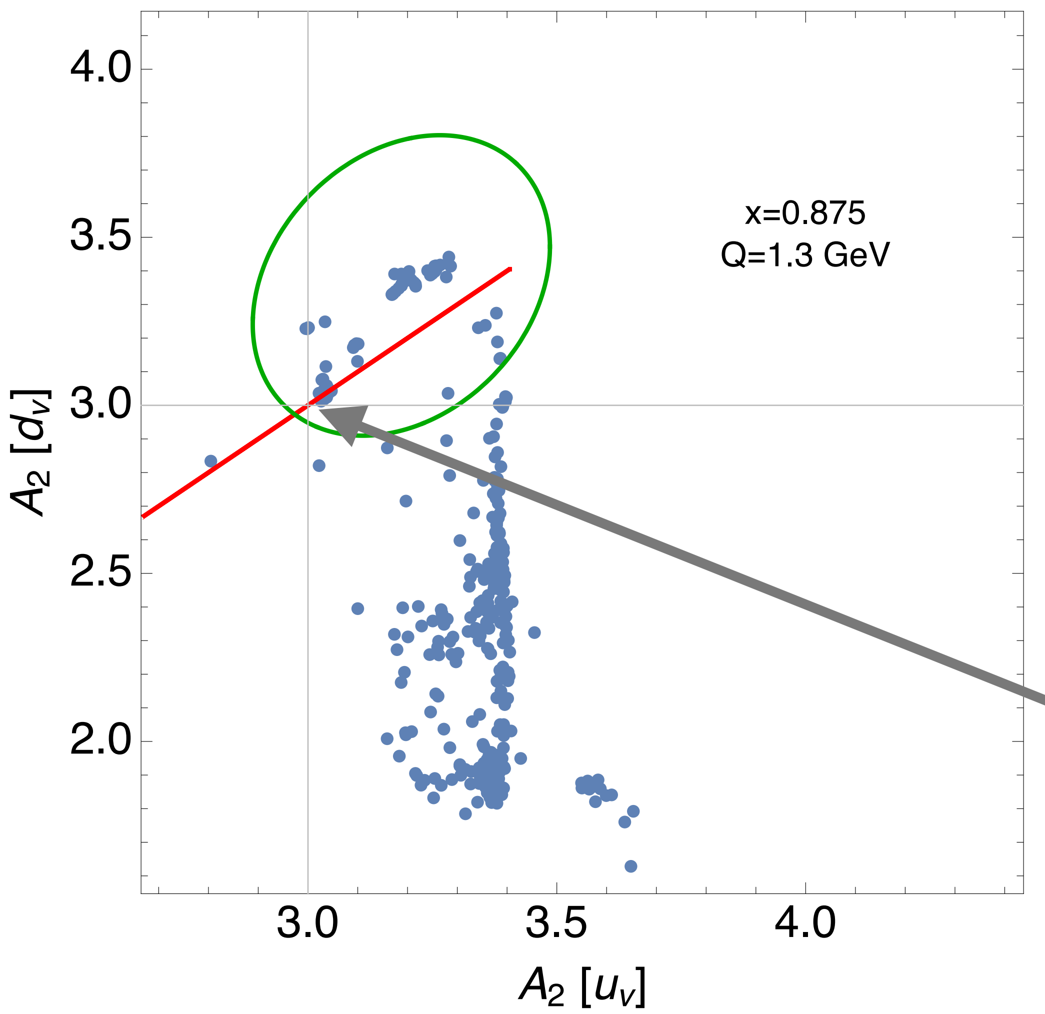
Can we see the evidence for QCRs in u_V and d_V?

Repeat the fit with N=363 functional forms

Variations of less than .5% in χ^2

Extrapolation in the region x>0.75-0.8

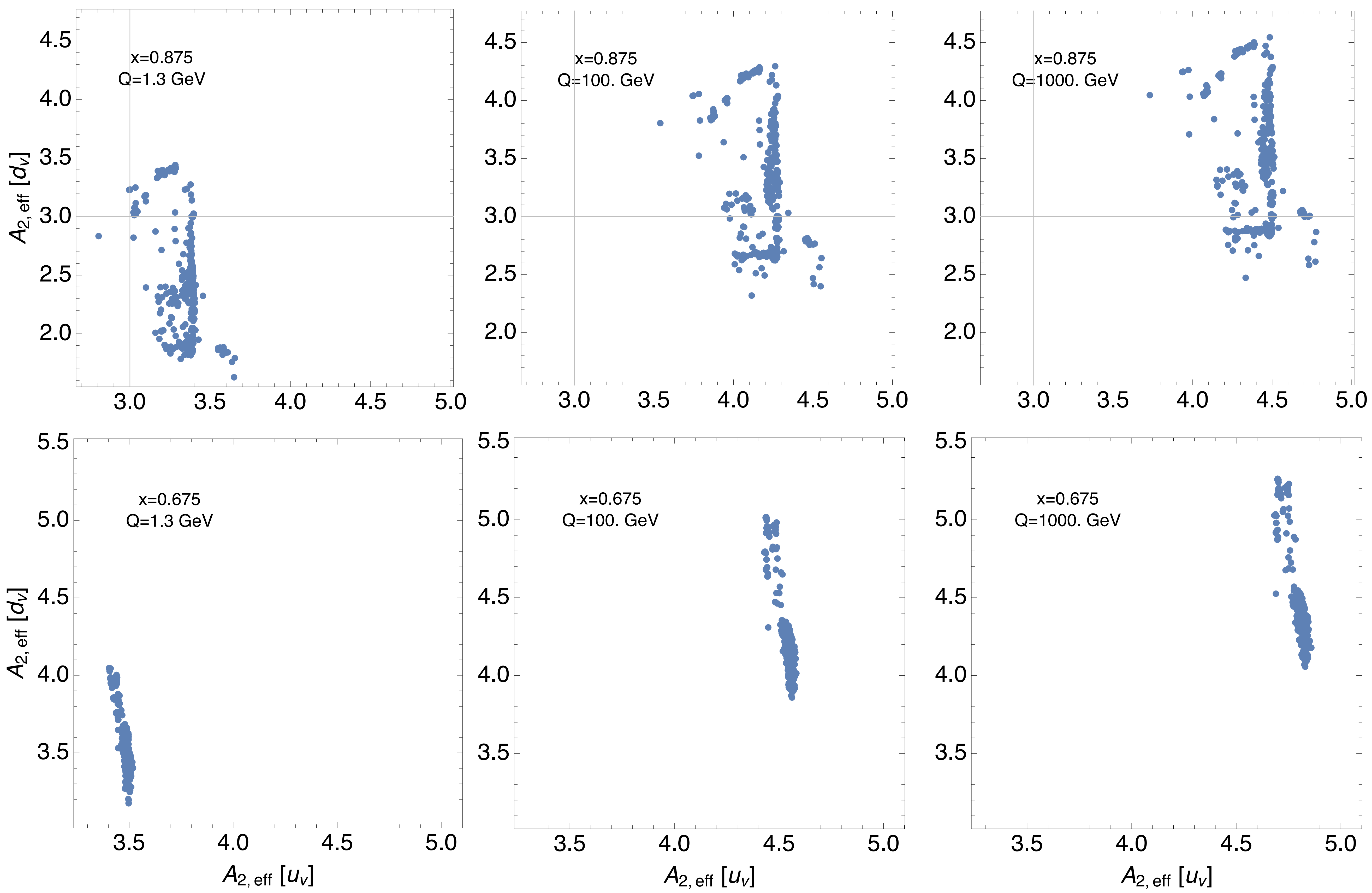
1. Hessian error propagation from tabulated A_2
2. Hessian error propagation for $A_{2,\mathrm{eff}}$ from CT18NNLO
3. **Scatter plot for the central fits for N parametrizations**



QCRs expectation

Mild running expected in pQCD

(e.g. Eur.Phys.J.C 76, Phys. Lett. B112)



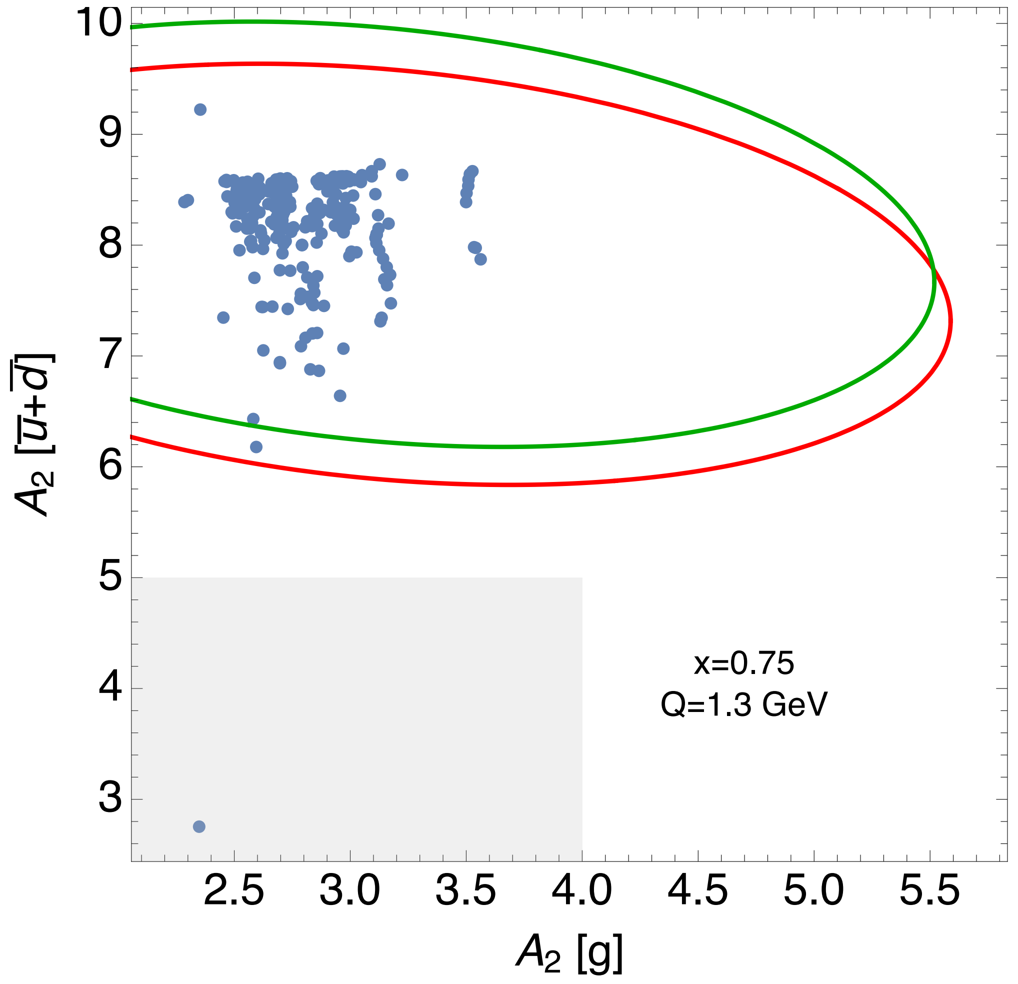
Can we see the evidence for QCRs in gluon and $\bar{u} + \bar{d}$?

Repeat the fit with N=363 functional forms

Variations of less than .5% in χ^2

Extrapolation in the region x>0.75-0.8

1. Hessian error propagation from tabulated A_2 @68%CL
2. Hessian error propagation for $A_{2,eff}$ from CT18NNLO @68%CL
3. **Scatter plot for the central fits for N parametrizations**



Running expected in pQCD

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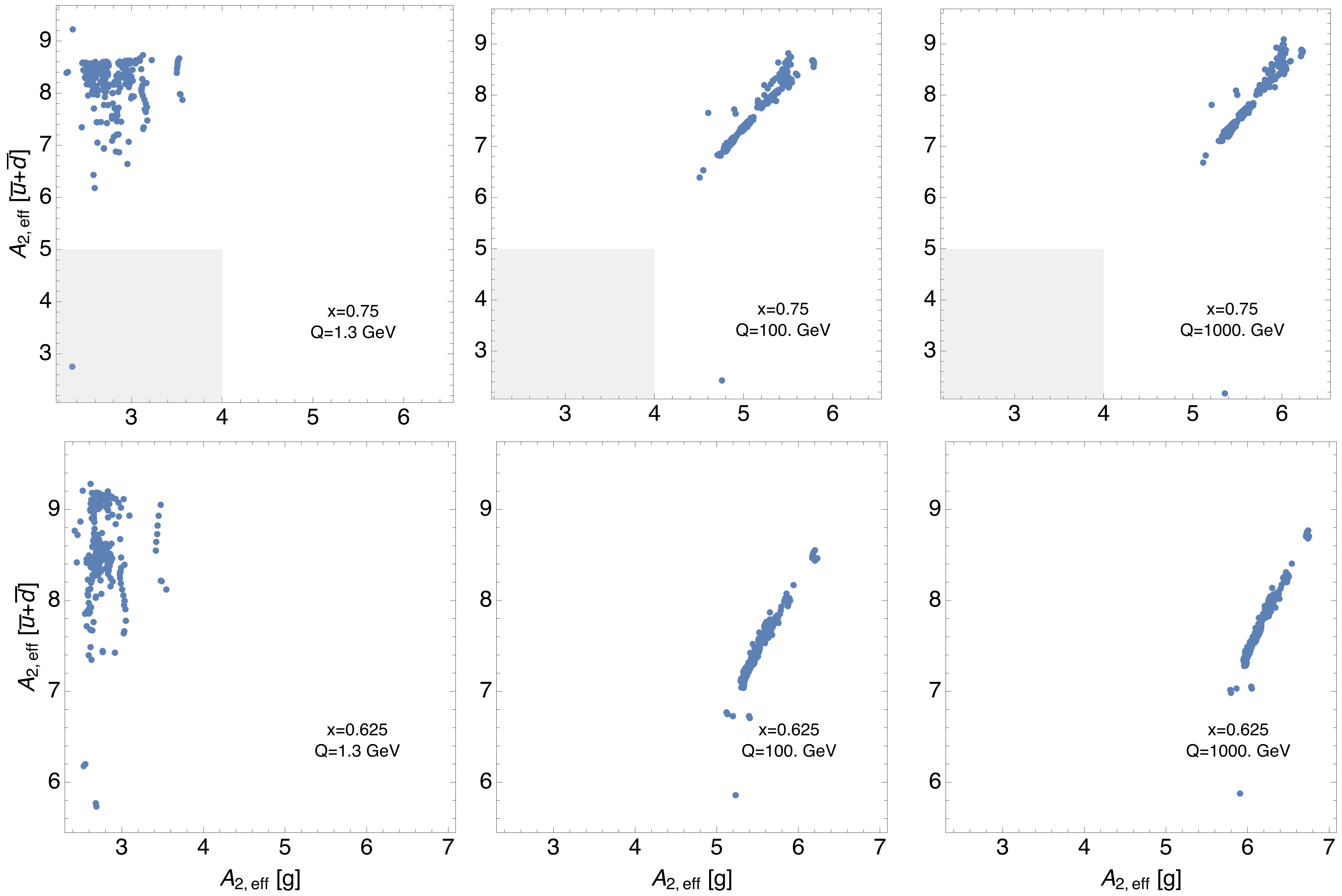
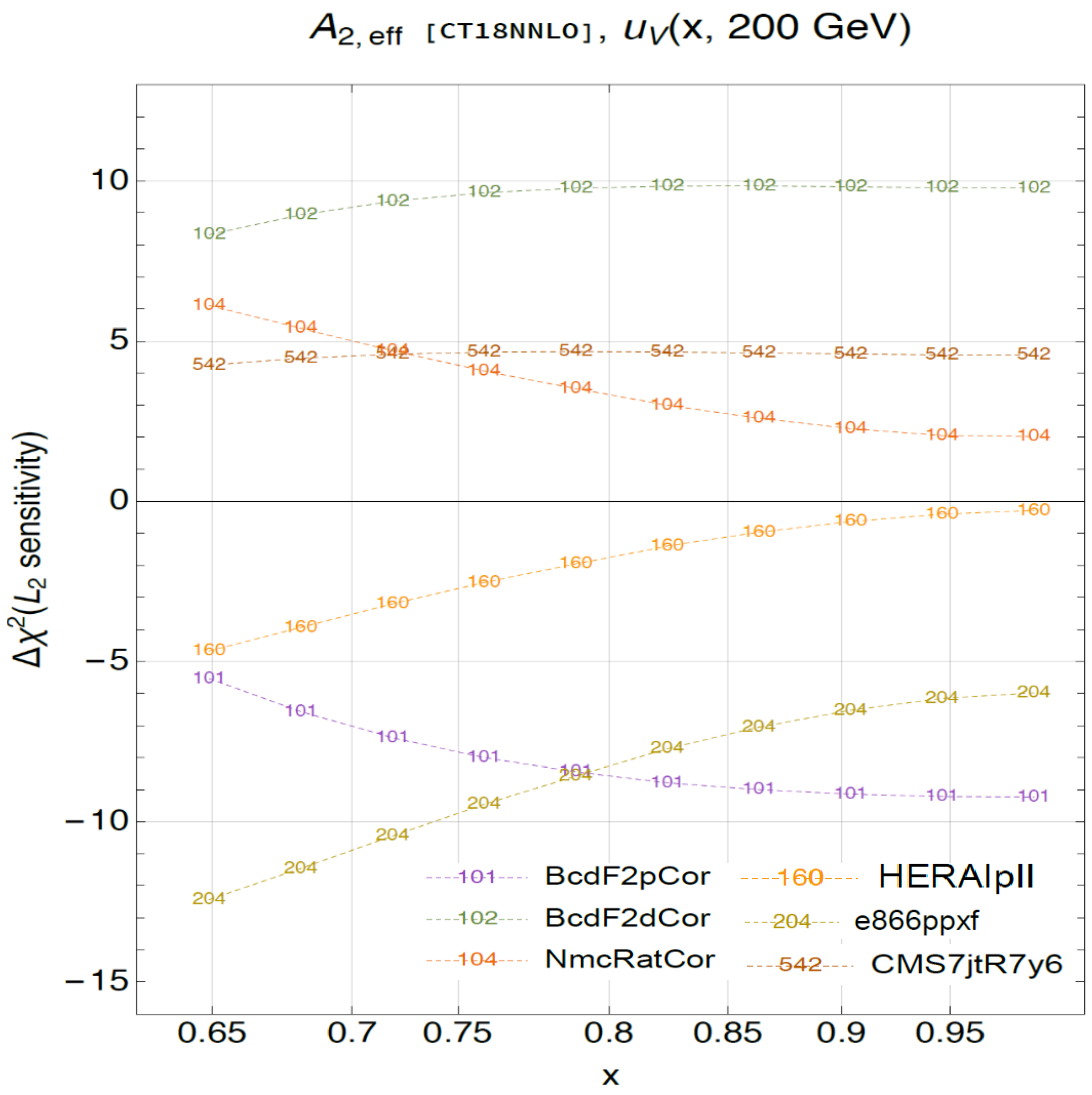
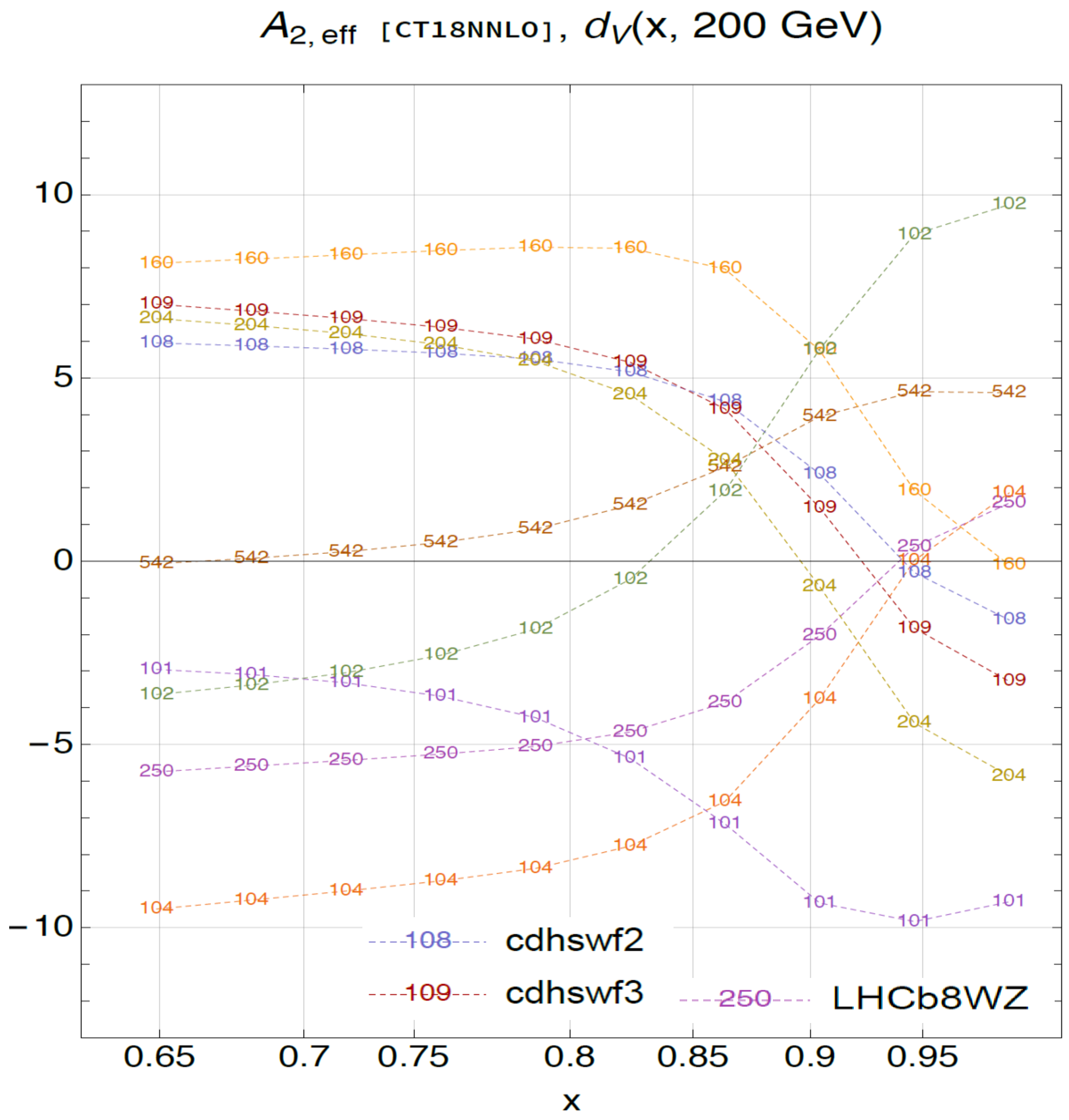


Are the effective powers the same for all processes and flavors?





L_2 sensitivity shows the correlation between observable and objective function for that particular choice.

[Phys.Rev. D98 (2018)], [Phys.Rev.D 100 (2019)]

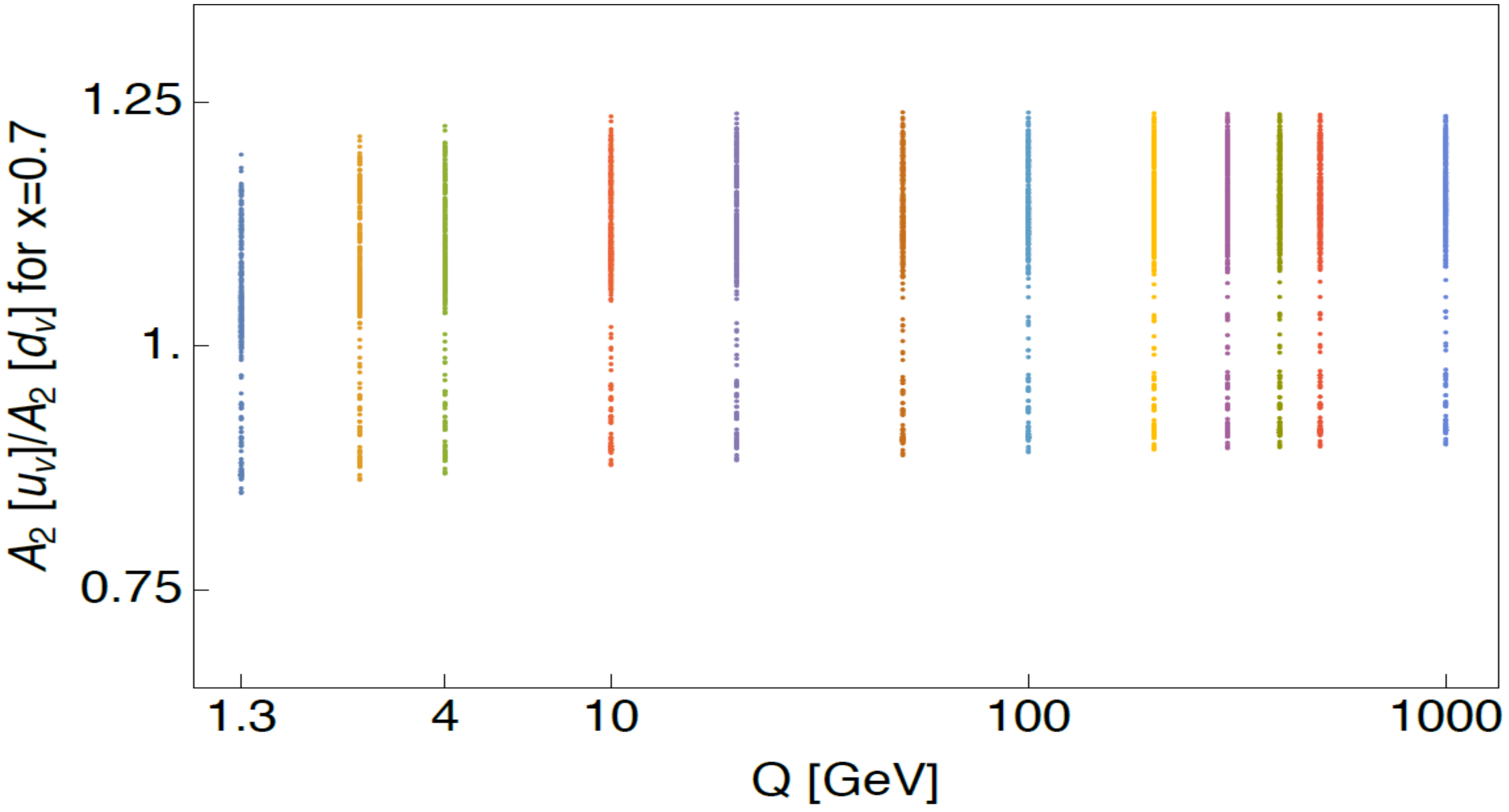
Sensitivity discussed in talk by X. Jing

⇨ **$A_2^{u_v}=A_2^{d_v}$ at x→1**

- Proton BCDMS and DY E866 favor a larger value of A_2^{eff} [u_v]
- Deuteron BCDMS favors a smaller value of A_2^{eff} [u_v]
- A_2^{eff} [u_v] largely follows A_2^{eff} [F_2]
- Tradeoff between opposite pulls of HERA and NMC at x<0.85

Large-x PDF from colliders?

High-x ZEUS data in talk by R. Aggarwal



363 replicas shown at given Q values

- **We can examine the ratio $A_2^{eff}\,[u_v]/A_2^{eff}[d_v]$ at x=0.7**
- The Q dependences of $A_2^{eff}\,[u_v]$ and $A_2^{eff}\,[d_v]$ cancel out because both obey the non-singlet DGLAP equation
- Continuity from high- to small-Q values
- The flavor separation of nonperturbative $A_2^{eff}\,[u_v]$ and $A_2^{eff}\,[d_v]$ can be deduced from high-Q data

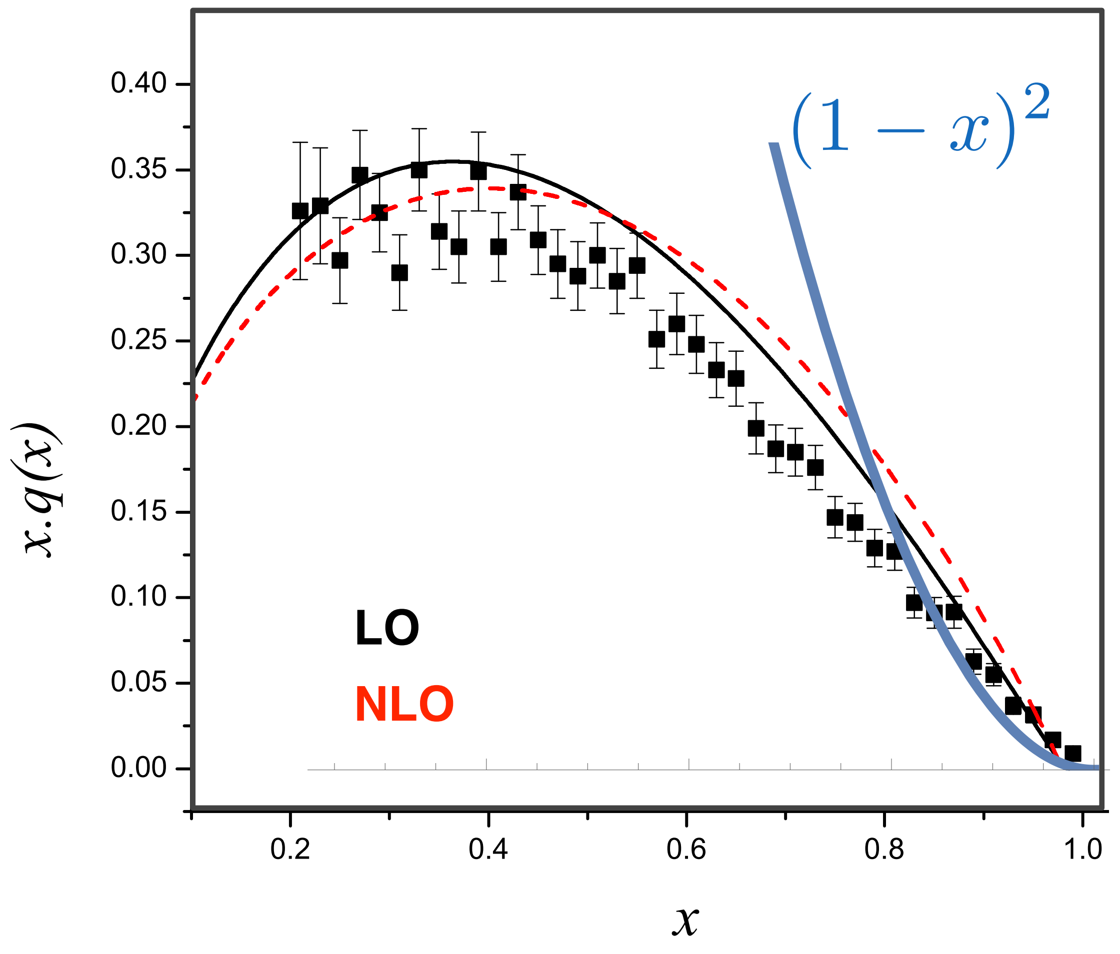
The shape of the pion PDF

Low-energy manifestations

- Emergence of Hadronic Mass: broadens the PDF at μ_0^2
- QCRs: $(1-x)^2$ tail at mid-Q^2 values

⇨ **concurring effects at a scale μ^2**

PDF broadening in NJL vs. QCRs



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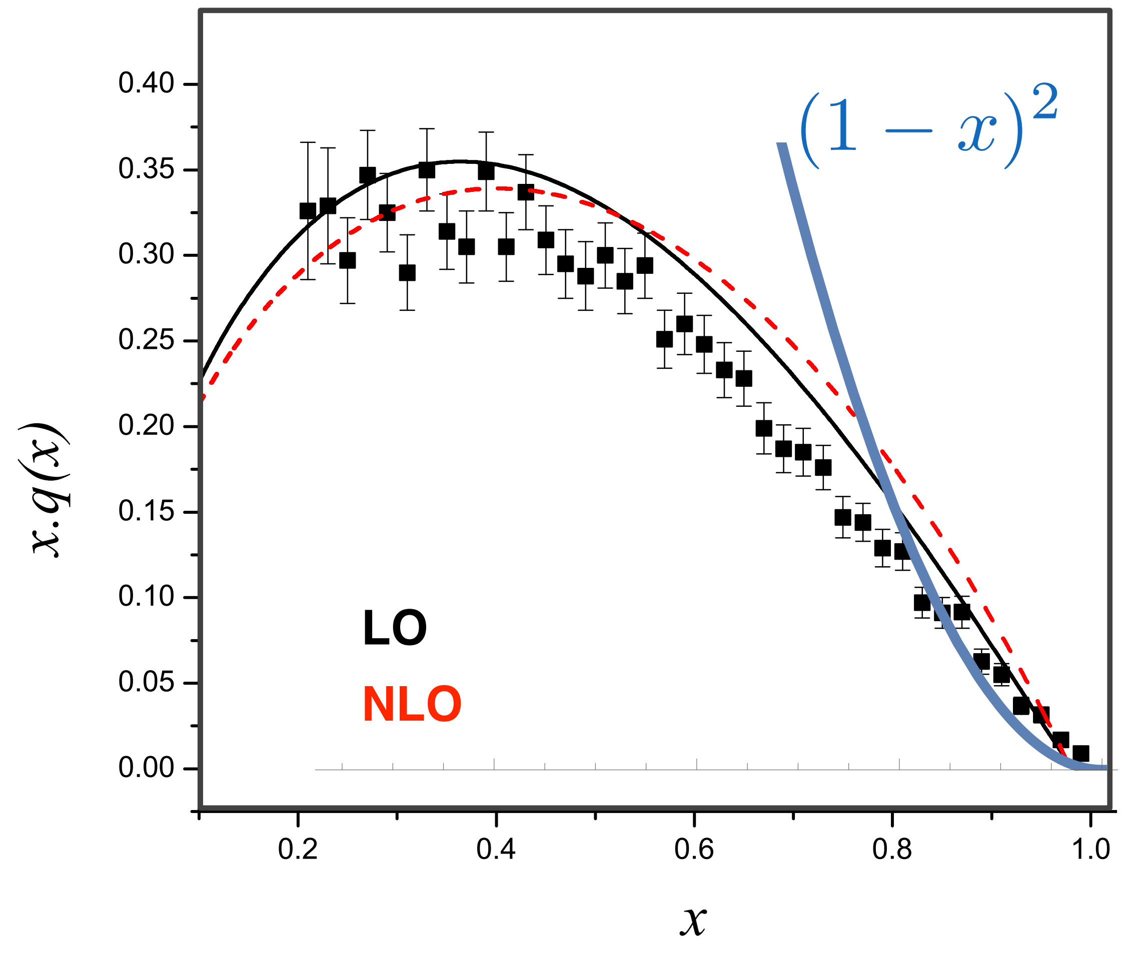

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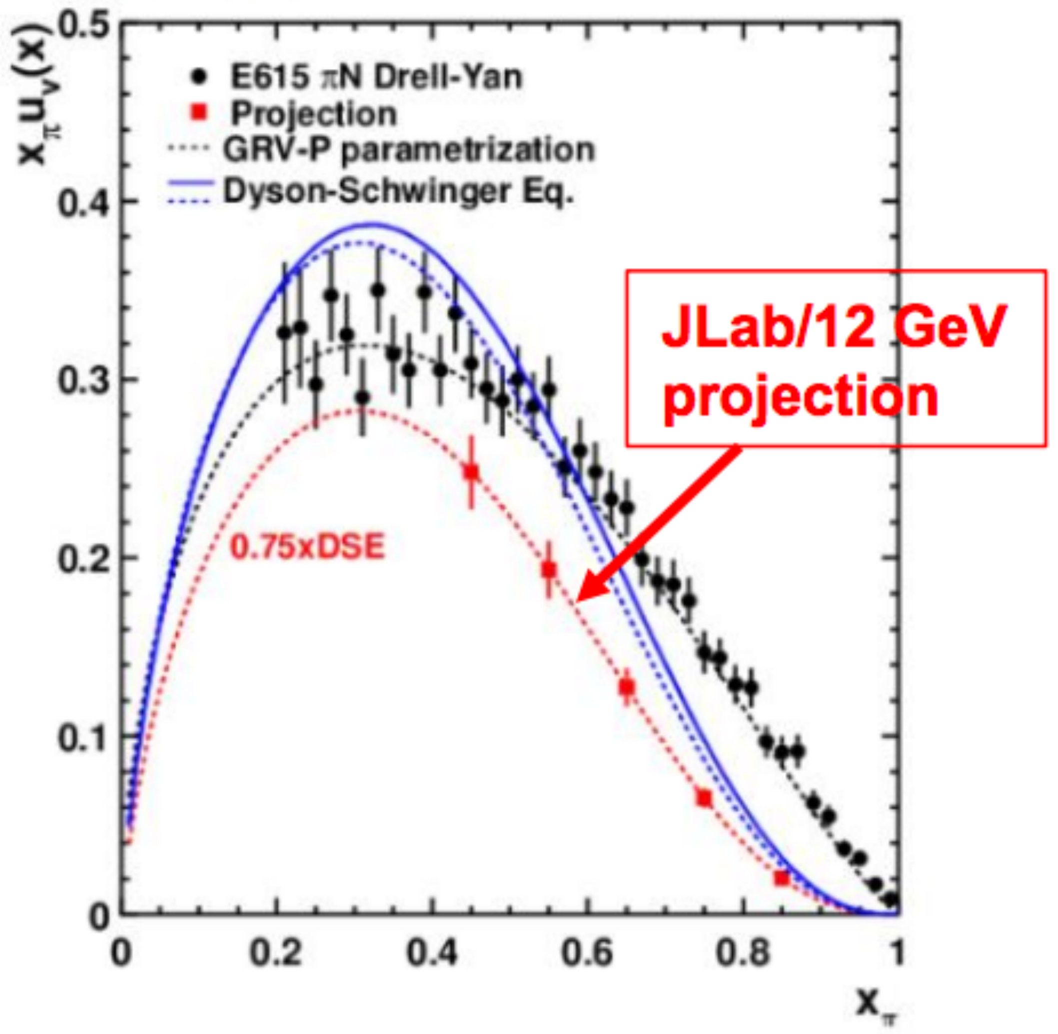

Plot by R. Ent

AMBER meeting 12/20

(1-x)-tail often appears in global analyses

- explained by mimicry, process dependence, QCD corrections,…
- nonperturbative approaches can also be affected by mimicry.

Can large-x PDFs be used to validate a manifestation of QCD dynamics?

there exists a necessary —agreement with data—but no sufficient condition — due to mimicry.

Pion PDF analysis
in talk by P. Barry

Conclusions

We have analyzed the quark counting rules for the CT18NNLO global fit for the proton PDFs.

We have addressed the question of their universality for processes, flavors as well as Structure Function vs. PDFs.

- ⇨ The Q^2 dependence of the (1-x)-power is not negligible — supported by other global fits and by pQCD.
- ⇨ Global analyses rely on complex processes: underlying hadronic activity.
- ⇨ The universality of Quark Counting Rules for PDFs depends on the validity of factorization —O(M/Q) terms.
- ⇨ **Mimicry** reconciles many parametrizations of PDFs with measurements.
- ⇨ The **uncertainties** must be estimated from both the nonperturbative and the pheno side
    - ⇨ highlight on the parametrization dependence!

The interpretative **effective** (1-x)-exponent at $x \to 1$ contains information from nonperturbative manifestations to high energy observables.

HADRON 2021

19TH INTERNATIONAL CONFERENCE ON HADRON SPECTROSCOPY AND STRUCTURE

Hosted by UNAM (Mexico City) from 26th of July to 1st of August 2021

May 1, 2021: Registration and abstract submission opening

www.nucleares.unam.mx/hadron2021

The main topics of this conference include:

- Meson spectroscopy
- Baryon spectroscopy
- Exotic hadrons and candidates
- Hadron decays, production and interactions
- Analysis tools
- QCD and hadron structure
- Hadrons in hot and nuclear environment including hypernuclei

Conveners:

- Carlota Andrés
- Charlotte van Hulse
- Cristina Aguilar
- Martin Hentschinski