Pion valence structure and form-factors from lattice QCD at physical point

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Pion valence quark PDF

Pion play a central role in the study of the strong interactions.

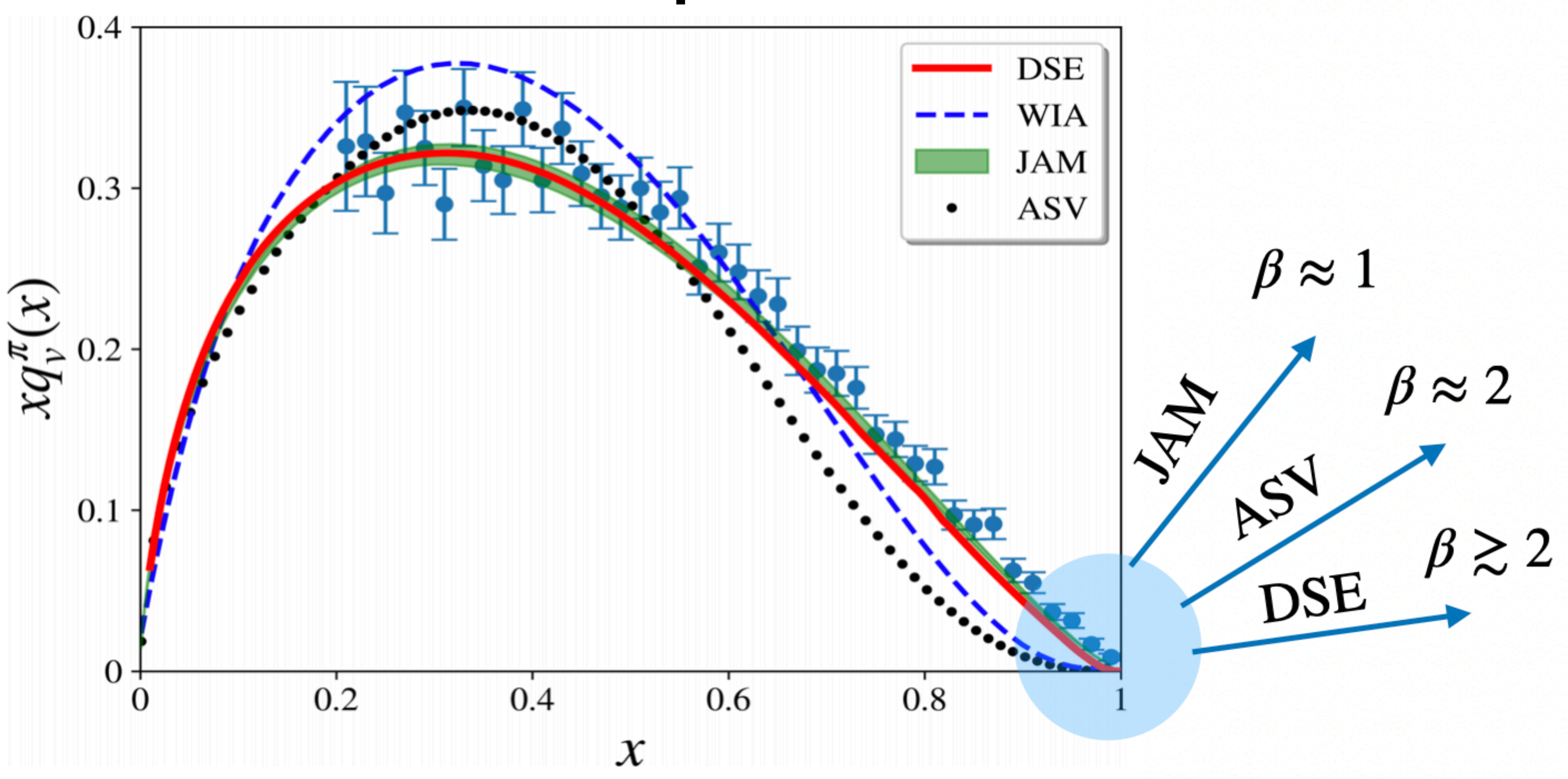
$$m_\pi \approx 140\ \text{MeV} \xrightarrow[\text{m}_\text{q}=0]{\text{chiral limit}} 0$$

- Critical ingredient for understanding the dynamical chiral symmetry breaking in QCD.
- Quarks and gluons in massless NG bosons.
- …

However, the absence of fixed pion targets has made it difficult to determine the pion's structure experimentally. One of the key physics issue is x=1 behavior:

$$\lim_{x\to 1} f_v^\pi(x) \sim (1-x)^\beta$$

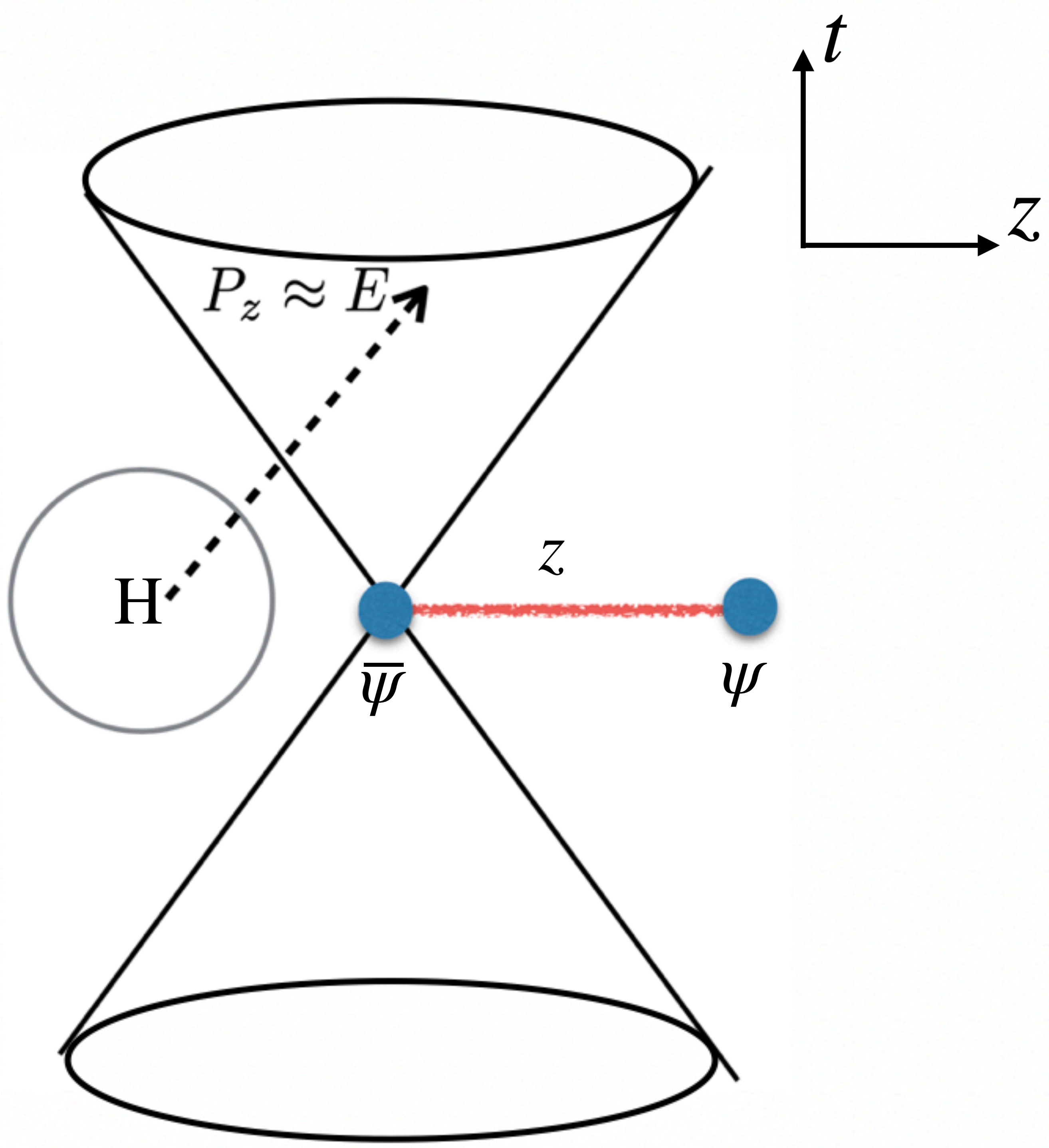
Pion valence quark PDF



I . C . Clöet et al. PRL.124.042002

Equal-time correlators and QCD factorization

$t = 0, \qquad z \neq 0$



Short distance Factorization in coordinate space:

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin, PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

$$\langle P | \tilde{O}_\Gamma(z,\mu) | P \rangle = \sum_n C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \int_{-1}^{1} dy\, y^n q(y,\mu) + \mathcal{O}(z^2 m_h^2, z^2 \Lambda_{QCD}^2)$$

Moments of PDF

Wilson coefficients

Small z and large P_z is essential

Ratio scheme renormalization:

$$\mathcal{M}(z, P_z, P_z^0) = \frac{\langle P_z | \tilde{O}_\Gamma(z,a) | P_z \rangle}{\langle P_z^0 | \tilde{O}_\Gamma(z,a) | P_z^0 \rangle}$$

$$\tilde{O}_\Gamma(z,\mu) = Z_{\psi,z} e^{\delta m |z|} \tilde{O}_\Gamma(z,\epsilon)$$

- **Hadron state independent**

Quasi PDF:

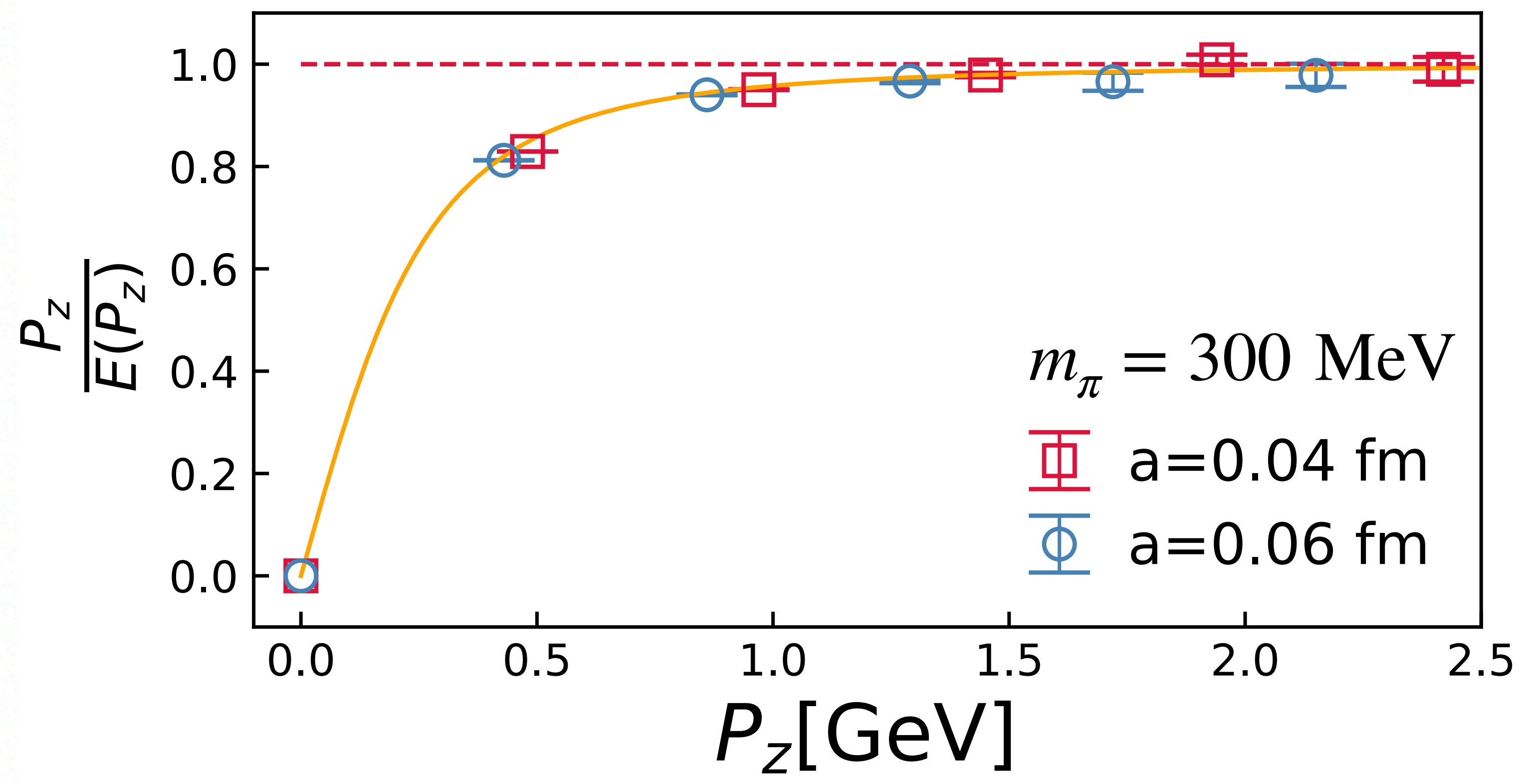
- X. Ji, PRL 110 (2013); SCPMA57 (2014);

$$\tilde{q}(x) \equiv \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P | \tilde{O}_\Gamma(z,\epsilon) | P \rangle,$$

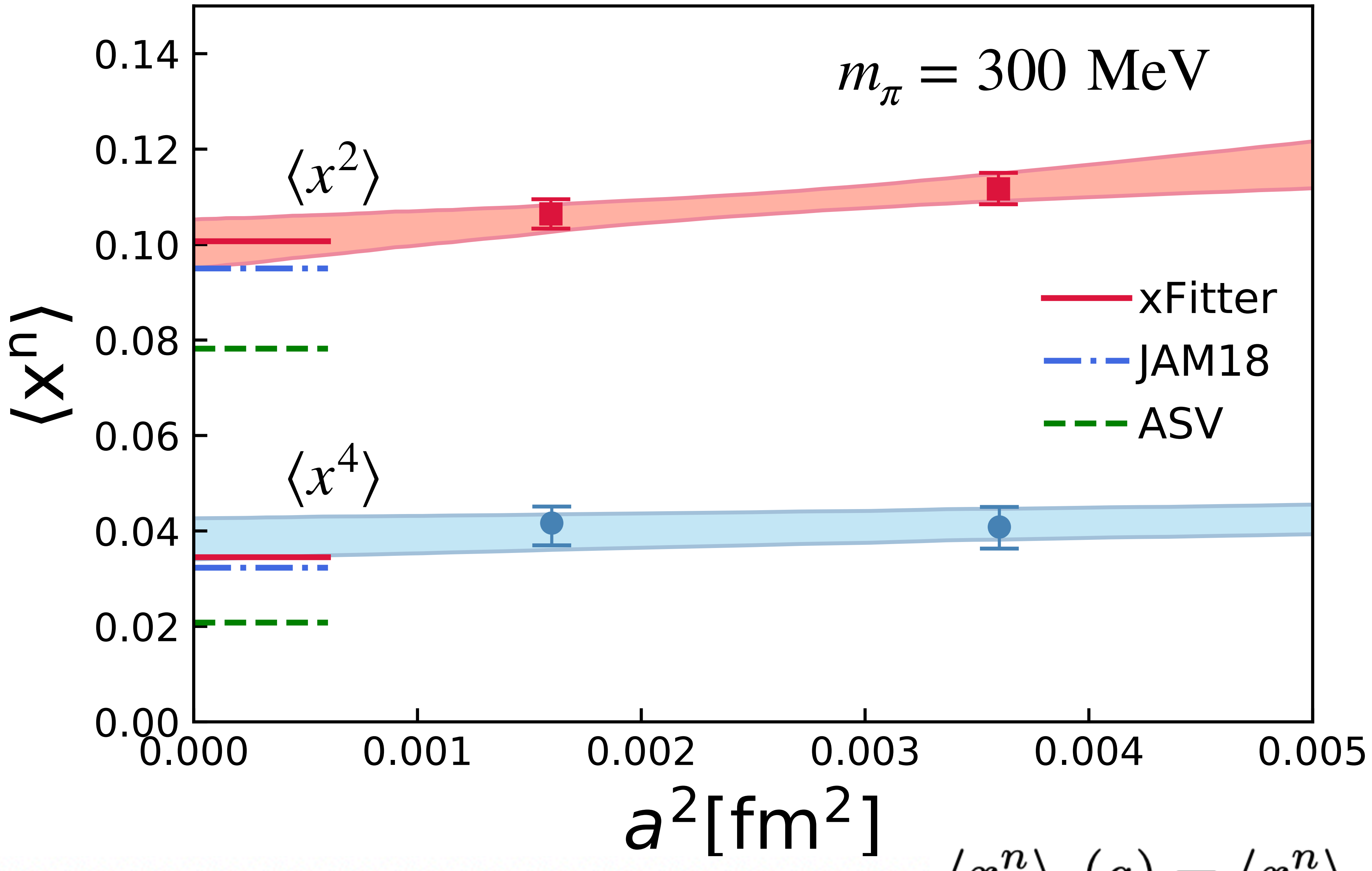
$$\tilde{O}_\Gamma(z,\epsilon) = \overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)$$

Pion valence quark PDF: NLO results

Boosted pion state on the lattice

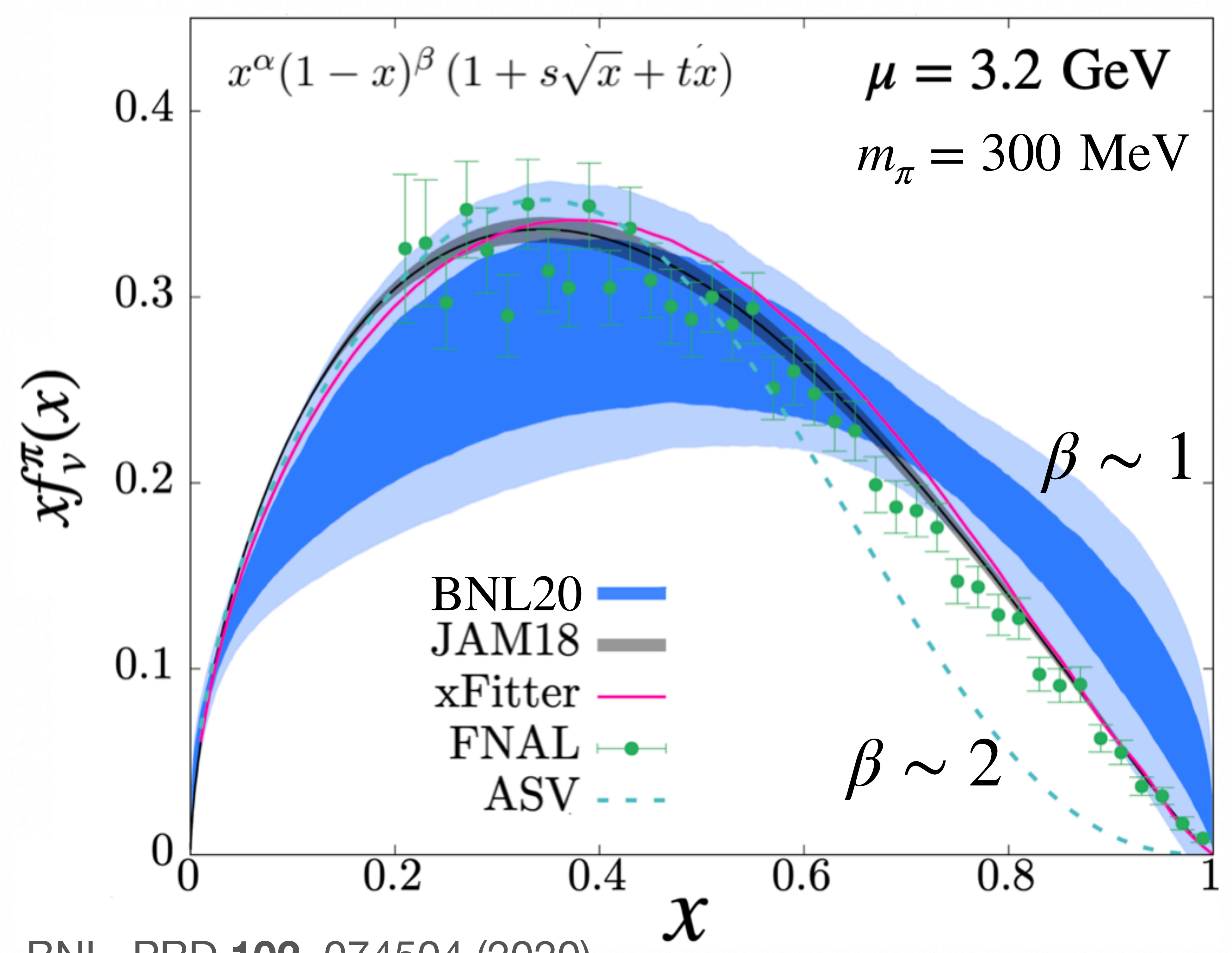


Moments of pion valence quark PDF



$$\langle x^n \rangle_v(a) = \langle x^n \rangle_v + d_n a^2$$

Pion valence quark PDF



BNL, PRD **102**, 074504 (2020)

Improvement:

- Matching formula beyond one-loop.
- Computation with physical pion mass.
- Extract PDFs information from chiral fermions.

Pion valence quark PDF: NNLO

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- Matching formula beyond one-loop.
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- **NNLO matching**

  - Li, Ma and Qiu, PRL 126 (2021)

$$C_n(z^2\mu^2) = 1 + \alpha_s(\mu)C_n^{(1)}(z^2\mu^2) + \alpha_s^2(\mu)C_n^{(2)}(z^2\mu^2) + \mathcal{O}(\alpha_s^3)$$

$$= 1 + \frac{\alpha_s(\mu)C_F}{2\pi}[(\frac{3+2n}{2+3n+n^2} + 2H_n)\ln(z_0^2\mu^2) + \ldots] + \ldots$$

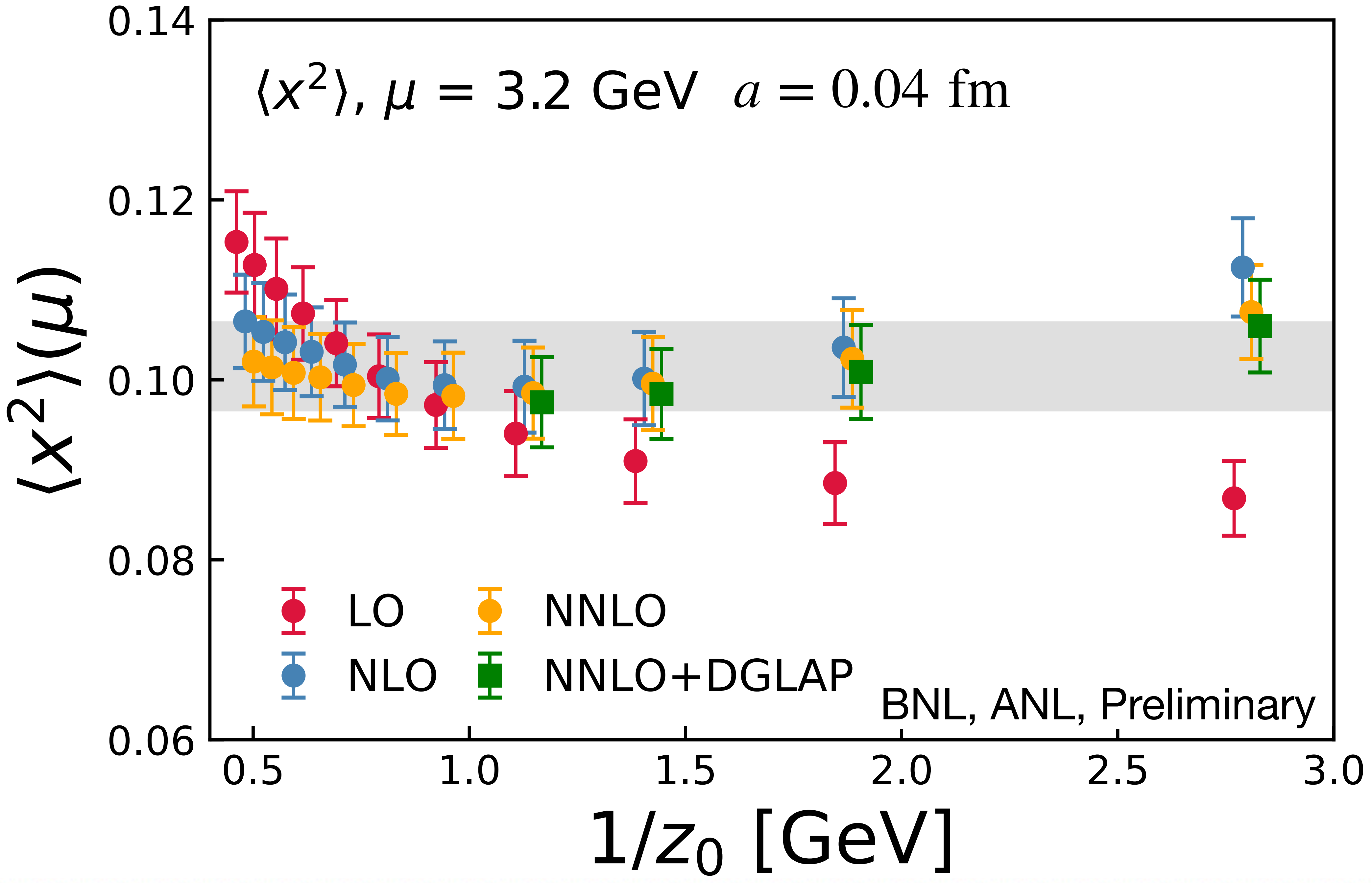
$$z_0^2 = z^2 e^{2\gamma_E}/4$$

When $\ln(z_0^2\mu^2)$ become large, one may need to include the DGLAP evolution:

$$\left[\frac{\partial}{\partial \ln \mu^2} + \beta(\alpha_s(\mu))\frac{\partial}{\partial \alpha_s} - \gamma_n\right] C_n^{evo} = 0$$

- A. V. Radyushkin, PLB 781 (2018)
- BNL, ANL, arXiv: 2102.01101

Moments extracted at fixed z by varying P_z



- Clear z_0 dependence can be observed at **LO**.
- Moments evolved from $1/z_0$ to μ from **NNLO** are consistent with **NLO** with current statistics but more flat, and agree with the **DGLAP** improved case.

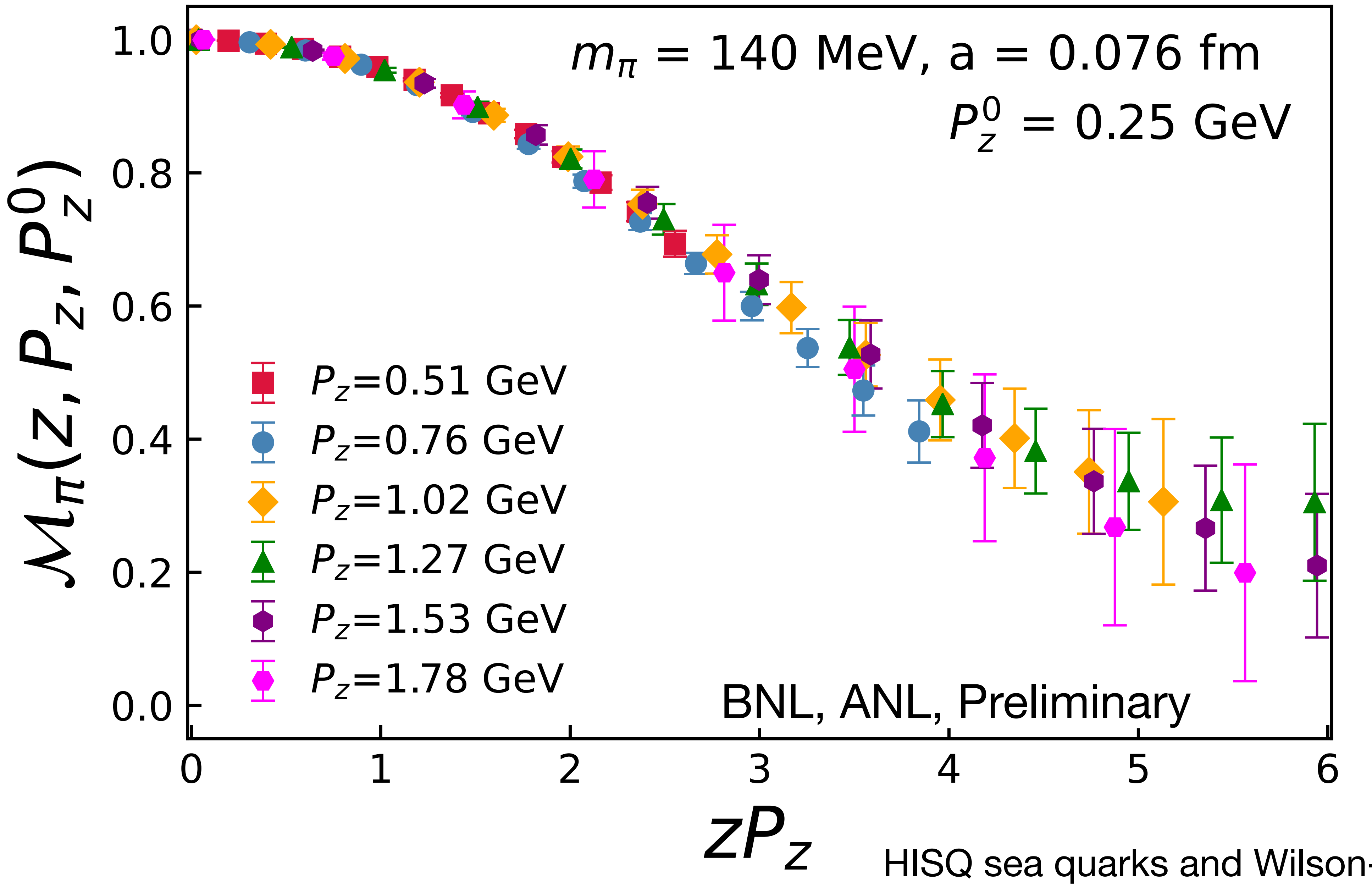
Pion valence quark PDF: Improvement

Improvement:

- Matching formula beyond one-loop.
- Computation with physical pion mass.
- Extract PDFs information from chiral fermions.

- **Physical pion mass**

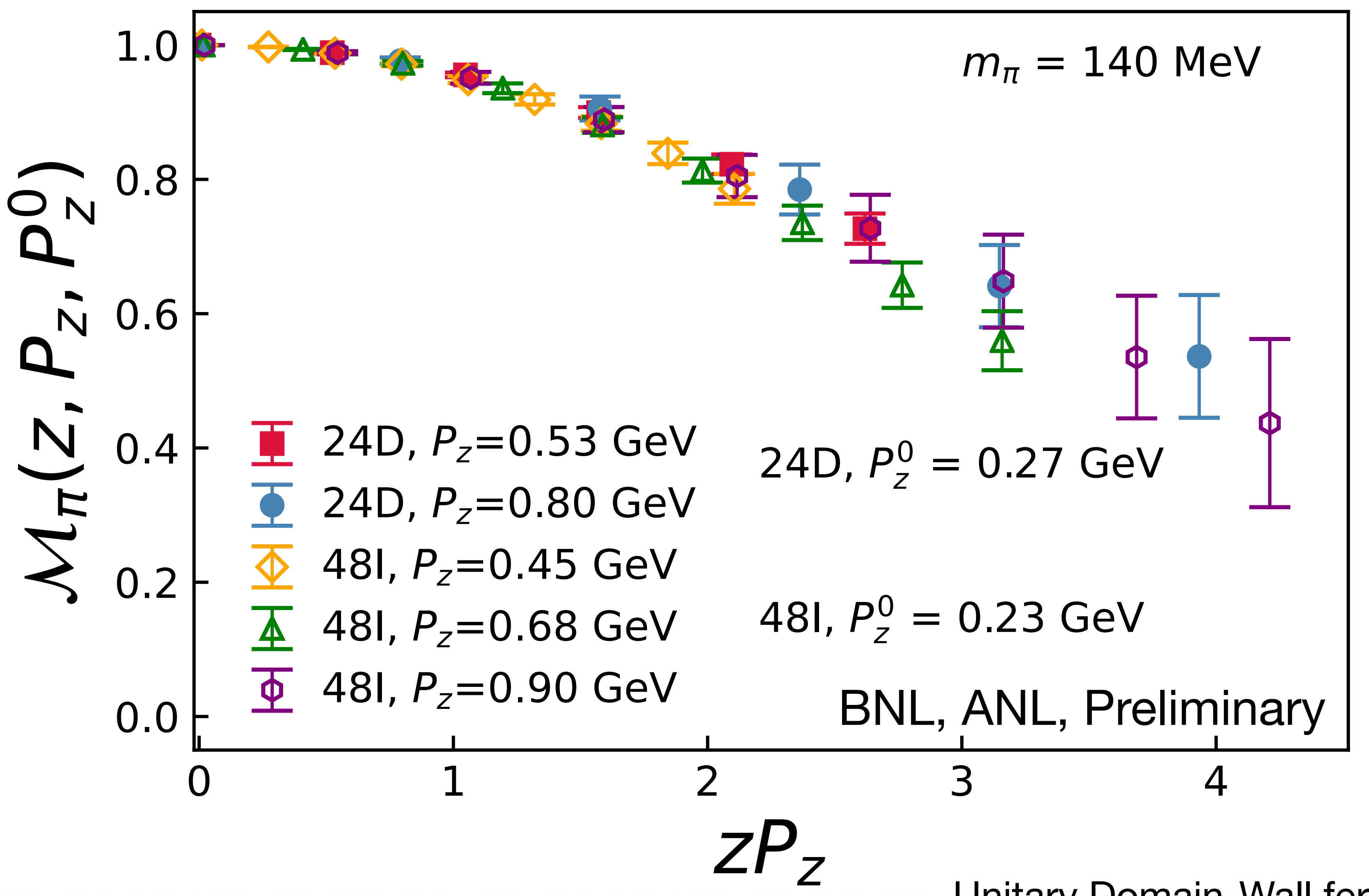
Ratio scheme renormalized matrix elements



HISQ sea quarks and Wilson-Clover valence quarks.

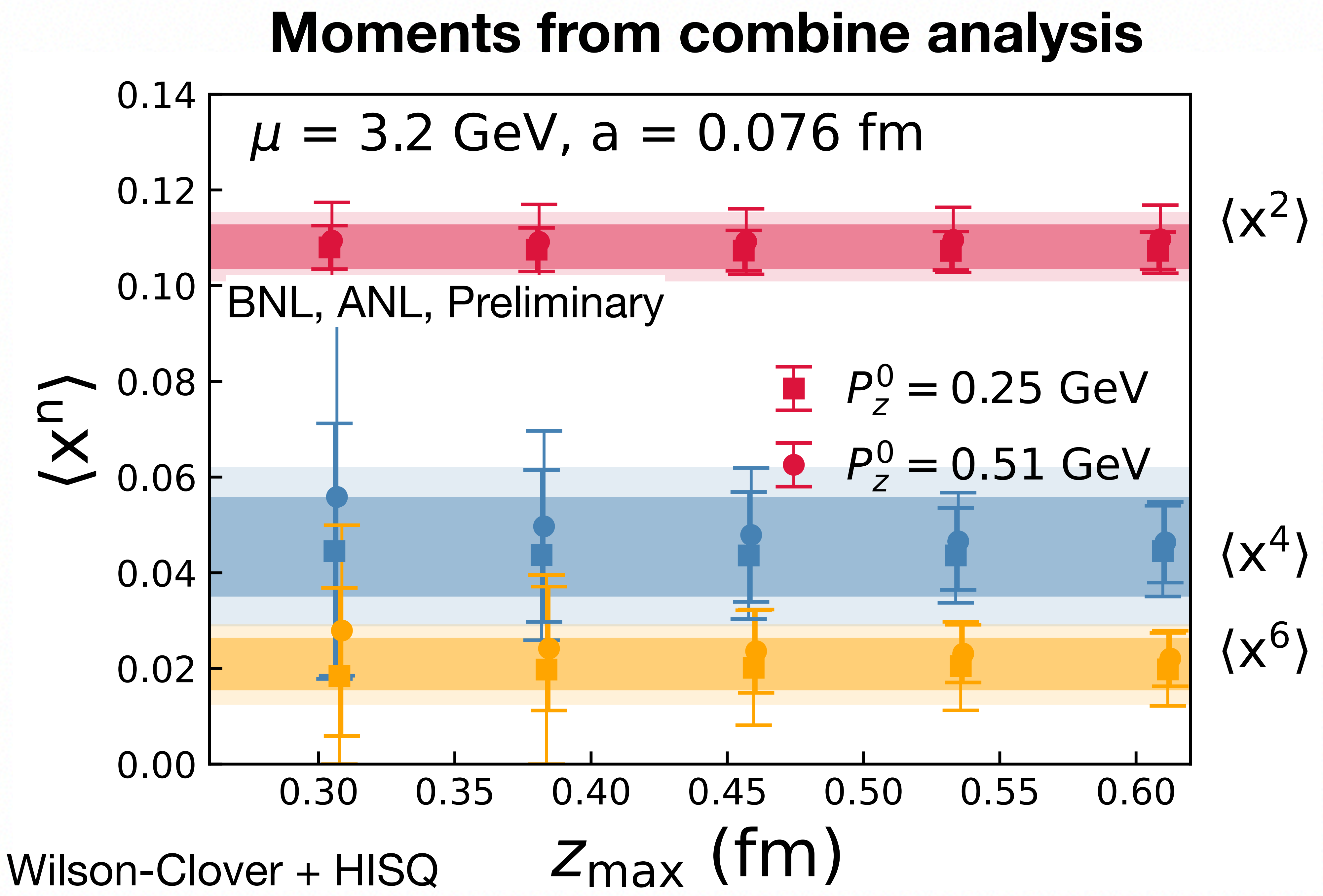
- **Chiral fermion**

Ratio scheme renormalized matrix elements



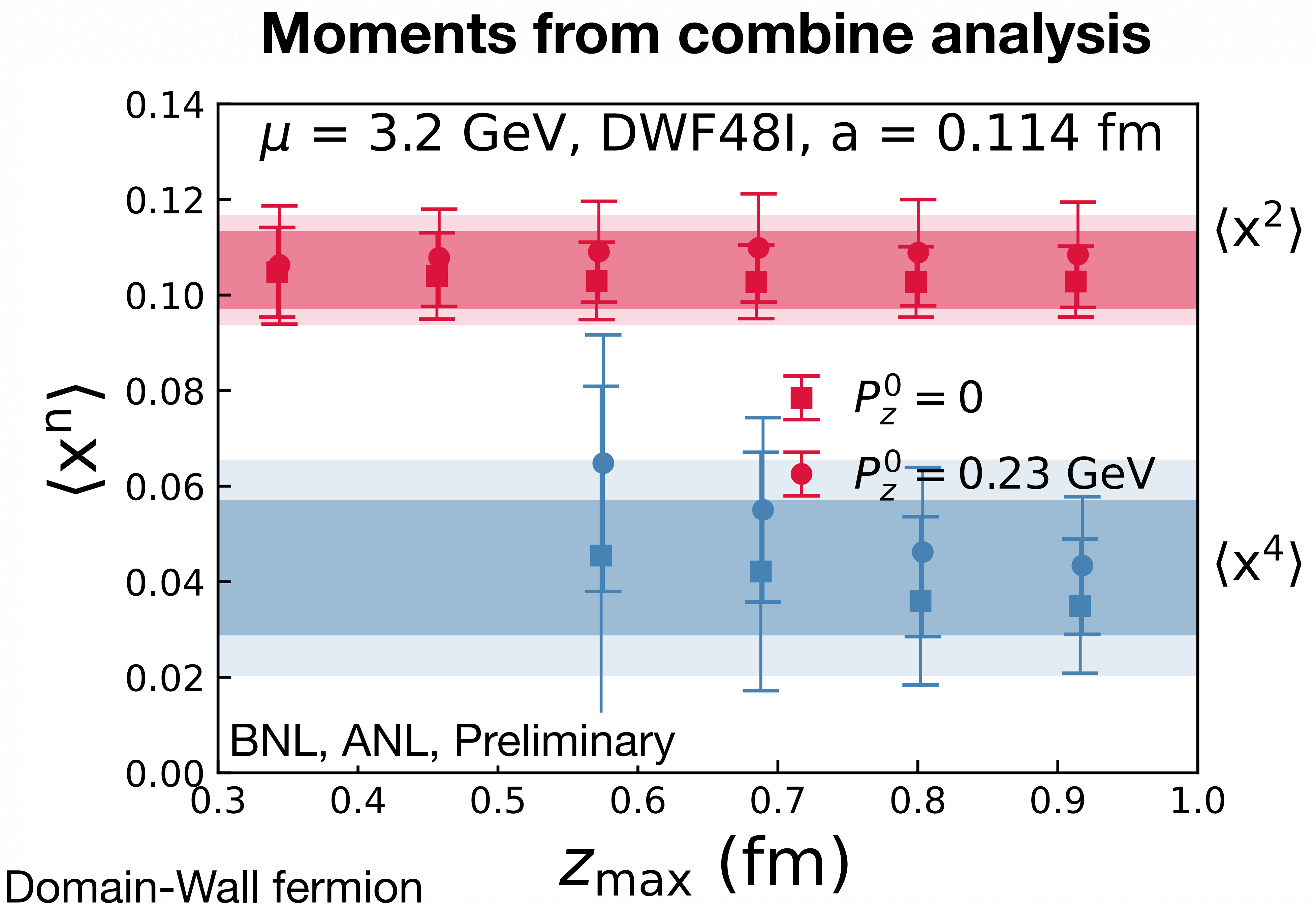
Unitary Domain-Wall fermion calculation.

Pion valence quark PDF: Moments



Wilson-Clover + HISQ

$m_\pi = 140$ MeV



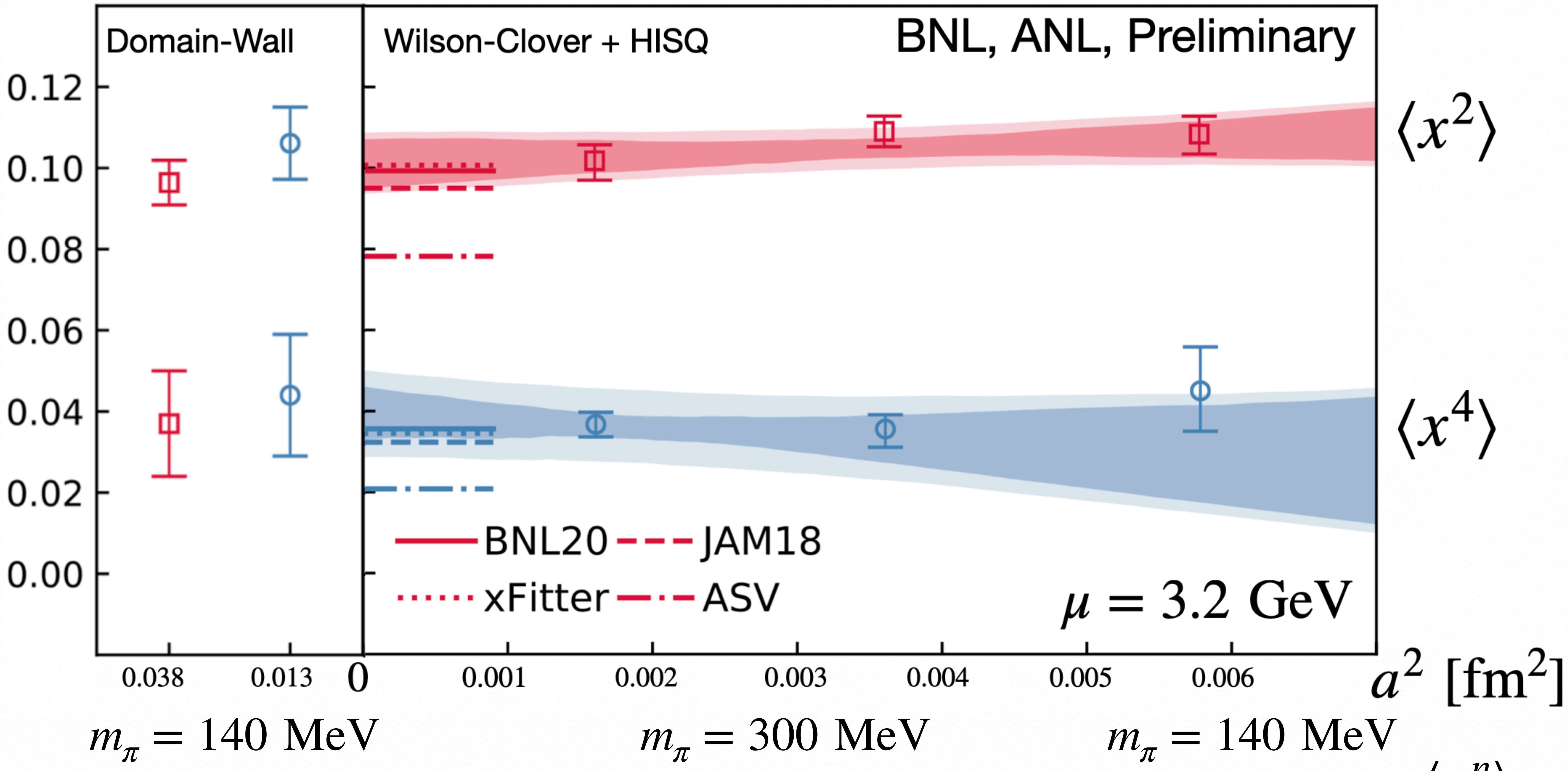
Domain-Wall fermion

$m_\pi = 140$ MeV

- To stabilize the fit and extract higher moments, we perform **combine analysis** with data in range $[3a, z_{\rm max}]$ using **NNLO** matching coefficients.

Pion valence quark PDF: Moments

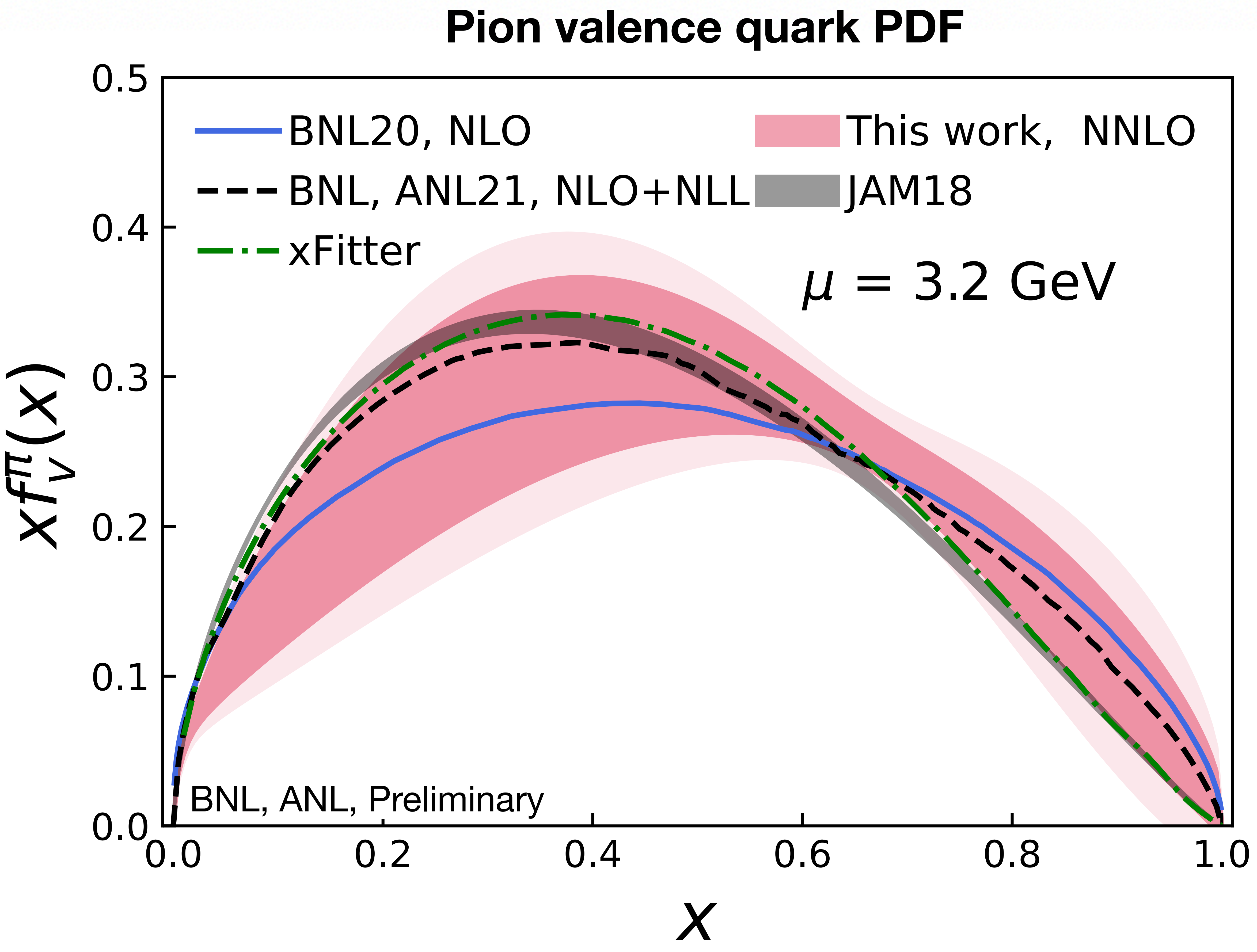
Moments: NNLO matching, physical point, chiral fermion



$$\langle x^n \rangle = \langle x^n \rangle_{\text{Ext}} + d_n a^2$$

- The mass dependence is mild for pion valence PDF.
- Chiral fermion shows good agreement with Wilson-Clover + HISQ fermion with fine lattice spacings.

Pion valence quark PDF



Preliminary results of the large-x behavior from model $x^{\alpha}(1-x)^{\beta}(1+t\sqrt{x}+sx)$:

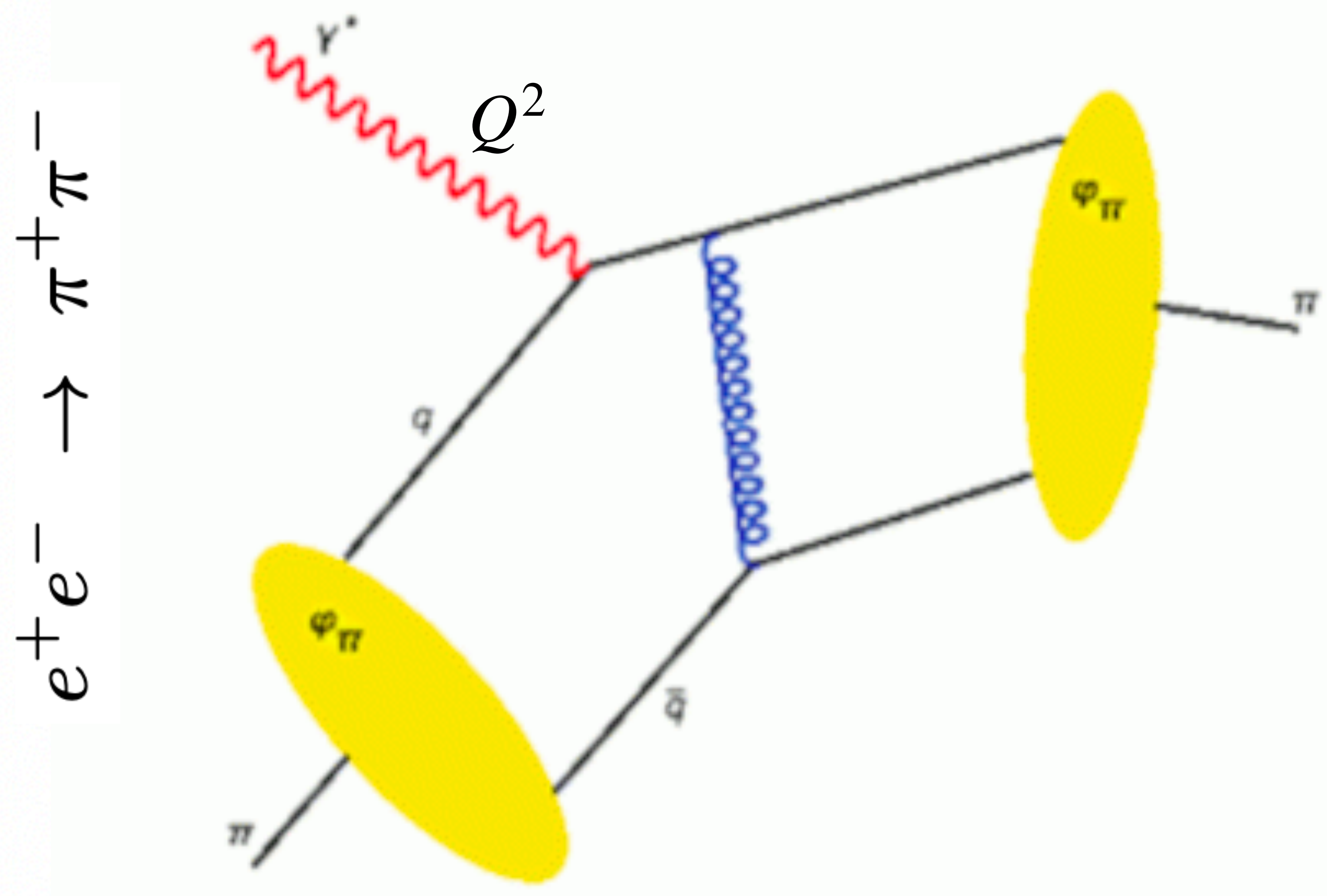
$$\beta = 1.07(37)(29),$$

which shows good agreement with JAM18, xFitter.

More improvement:

- Resummation in perturbative matching. For example, NLO+NLL **threshold resummation** (BNL, ANL 21 arXiv: 2102.01101).
- More statistics and large momentum to extract **higher moments**.

Pion form factor and charge radius



$e^+\pi^+ \to e^+\pi^+$

$$\mathcal{M}_{\text{scatt.}} = \frac{1}{q^2}\, e\,\overline{u}(k_2)\gamma_\mu u(k_1)\,\langle \pi^+(p_2)|J^\mu_\pi(0)|\pi^+(p_1)\rangle$$

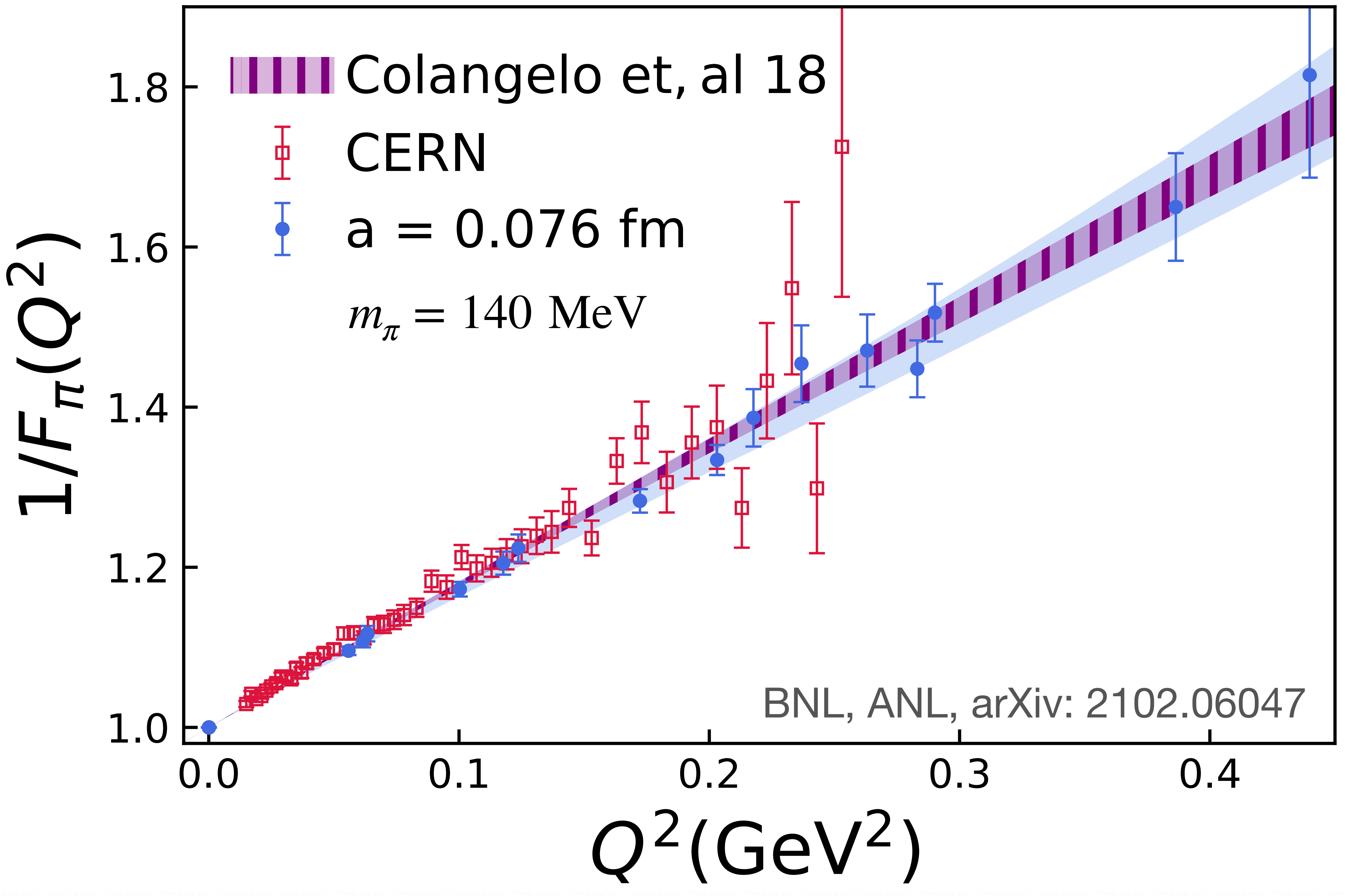
$$(p_1 + p_2)^\mu F_\pi(Q^2)$$

Pion elastic electromagnetic form factor $F_\pi(Q^2)$

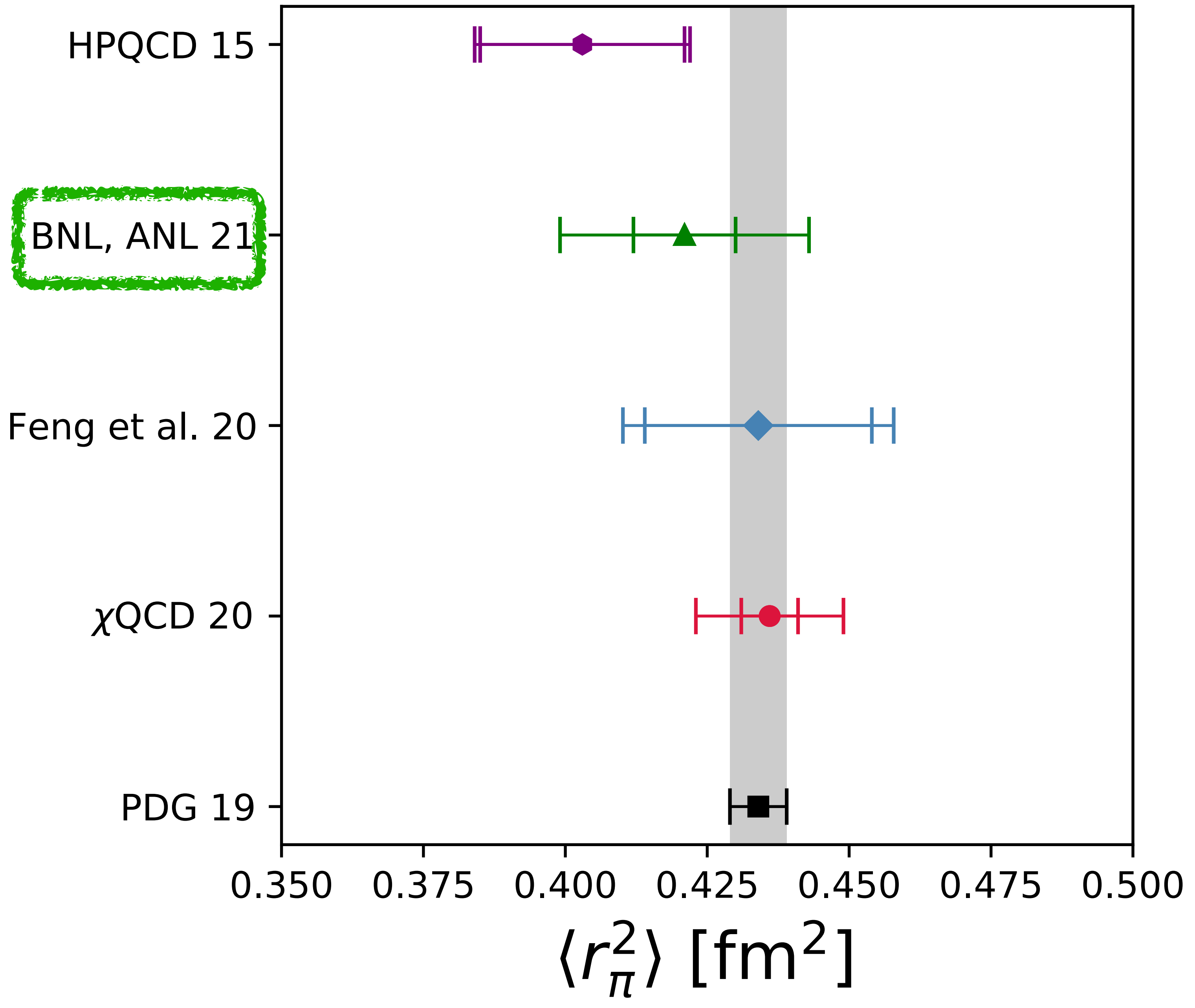
- The 3-D generalized PDFs (**GPDs**) combine the information contained in **PDFs** and **form factors**.
- The mean charge radius is related to form factors at low Q^2 .
- EIC facility will allow higher Q^2 up to 30 GeV^2, make contact with pQCD.

Pion form factor and charge radius

Pion form factors at physical point



Pion charge radius



z-expansion fit:

$$F_\pi(Q^2) = \sum_{k=0}^{k_{max}} a_k z^k$$

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}, \; t = -Q^2$$

$$r_\pi^2 = -6 \frac{dF_\pi(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

Summary

- We studied pion valence quark PDF with multiple lattices and pion mass.
- The mass dependence of pion valence PDF is mild.
- The usage of Wilson-Clover fermion didn’t bring trouble to the determination of pion valence structure.
- We calculate the pion electromagnetic form factors which show good agreement with experimental data.