

Developing the Parton Branching TMD Evolution: QED interactions

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arXiv: 2102.01494 accepted at Physics Letters B

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- 1 Recap of Parton Branching method
- 2 Determination of Photon TMD
- 3 Application of TMD photon

Recap of Parton Branching method

- Including the Δ_s in the differential form of the DGLAP eq.

$$\mu^2 \frac{\partial}{\partial \mu^2} \frac{f(x, \mu^2)}{\Delta_s(\mu^2)} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{\mathcal{P}(z)}{\Delta_s(\mu^2)} f\left(\frac{x}{z}, \mu^2\right)$$

- Integral form with a very simple physical interpretation:

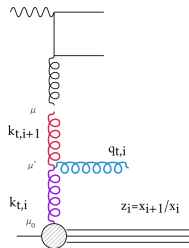
$$f(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2) + \int \frac{dz}{z} \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^R(z) f\left(\frac{x}{z}, \mu'^2\right)$$

- the solution of the integral equation has the form of a Neumann series with the following terms:

$$f_0(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2)$$

$$f_1(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2)$$

$$+ \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} \int \frac{dz}{z} P^R(z) f\left(\frac{x}{z}, \mu_0^2\right) \Delta(\mu'^2)$$



- iterating with second branching and so on to get the full solution

PDFs from PB method: QCD fit to HERA data

Convolution of kernel with starting distribution

$$\begin{aligned}xf_a(x, \mu^2) &= x \int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b(x'', \mu^2) \delta(x'x'' - x) \\&= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'} \tilde{\mathcal{A}}_a^b\left(\frac{x}{x'}, \mu^2\right)\end{aligned}$$

Fit performed using xFitter frame (with collinear coefficient functions at NLO)

[Phys. Rev. D 99 \(2019\) no. 7, 074008](#)

- Full coupled evolution with all flavors
- using full HERA+II inclusive DIS (neutral current, charged current) data
- in total 1145 data points
- $3.5 < Q^2 < 50000 \text{ GeV}^2$ & $4.10^{-5} < x < 0.65$
- $\chi^2/dof=1.21$

Can be easily extended to include any other measurement for fit.

Recap of application of PB-TMD densities

So far ...

- PB-TMDs obtained from QCD NLO fit to inclusive HERA data
- parameters of collinear initial distribution obtained
- intrinsic Gauss distribution with [Phys. Rev. D **99** \(2019\) no. 7, 074008](#)
 - $\mathcal{A}_a(x, k_t^2, \mu_0^2) = f_a(x, \mu_0^2) \cdot \exp(-|k_t^2|/\sigma^2)$
 - constrained with $\sigma^2 = q_s^2/2$ of Gauss distribution $q_s = 0.5$ GeV
- association of evolution scale with q_t
 - $\mu = q_t/(1 - z)$
- no further free parameter!

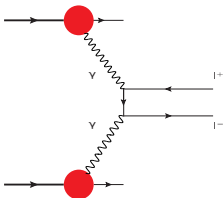
Very successful for description of inclusive processes

[Phys. Rev. D **100** \(2019\) no.7, 074027](#), [Eur. Phys. J. C **80** \(2020\) no.7, 598](#) → talk by Aleksandra Lelek

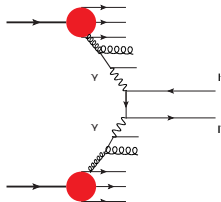
- without any adjustment of parameters : high energy/mass as well as low energy/mass DY production are reasonably well described by PB method.

QED corrections

- LO QED splitting functions are included: P_{qq}^γ , $P_{q\gamma}$, $P_{\gamma q}$
- running LO-QED coupling with matching at quark-mass thresholds
- since $\alpha \sim \alpha_s^2$: NLO QCD splitting kernels are used
- fitting PB PDFs with QED corrections to HERA data
- photon density is generated perturbatively
 - simplicity
 - check contribution from just perturbatively generated photons in both collinear and TMD photon density



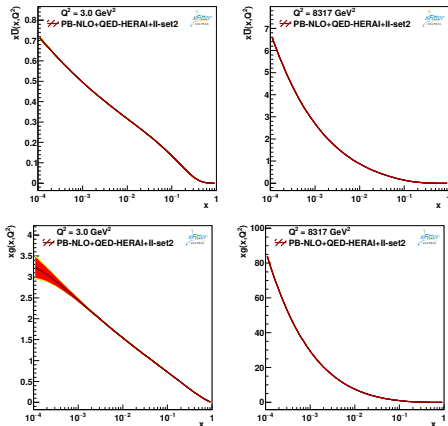
elastic photon (not considered here)



pertubatively generated photon

NLO QCD+QED fit

The first determination of the PB PDFs using LO QED + NLO QCD evolution



• $\chi^2/dof = 1.21$

$$xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g},$$

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} \left(1 + E_{u_v} x^2\right),$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}},$$

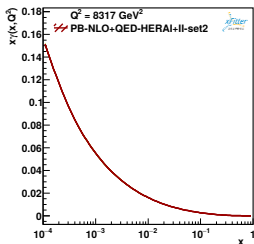
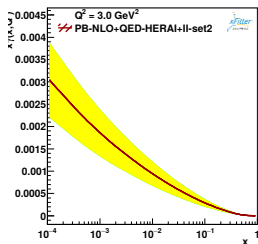
$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x),$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}.$$

- fit is as good as QCD NLO
- inclusion of a photon PDF has a negligible impact on the other PDF sets
- all PDFs were benchmarked against ones produced in 1509.00209 [S. Carrazza PhD thesis] - excellent agreement is achieved.

Collinear photon density

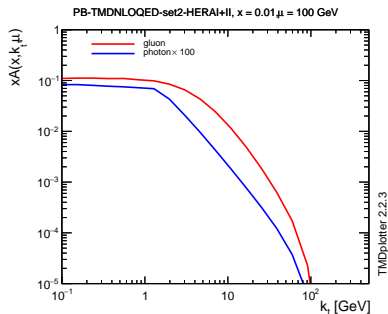
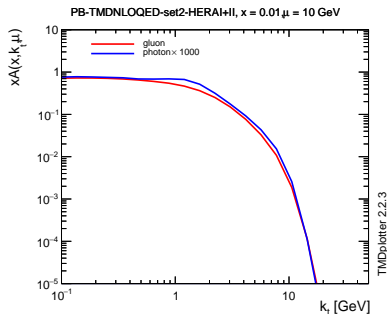
- the collinear photon PDF extracted from fit to HERA data [arXiv:2102.01494](https://arxiv.org/abs/2102.01494) [hep-ph]
- photon PDF has large experimental uncertainty coming from valence quark distribution



TMD photon and gluon

TMD parton densities can be obtained within the PB method. [arXiv:2102.01494 \[hep-ph\]](#)

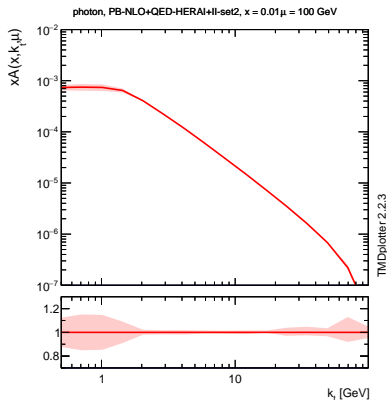
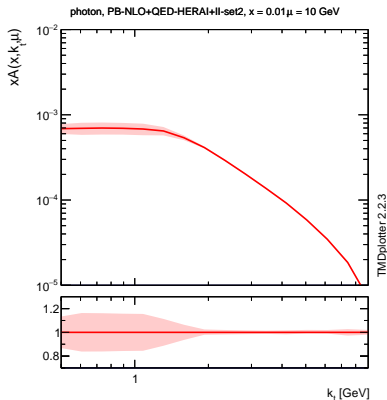
TMDs are plotted with TMDplotter [arXiv:2103.09741 \[hep-ph\]](#)



the shape of photon and gluon distributions

- at low k_t is the same.
- at large k_t is different: gluon-gluon splitting

TMD photon uncertainty



- k_t -dependent uncertainty distribution
- large uncertainty at small k_t comes from valence quark distribution at small k_t

TMD photon application: high mass DY spectrum

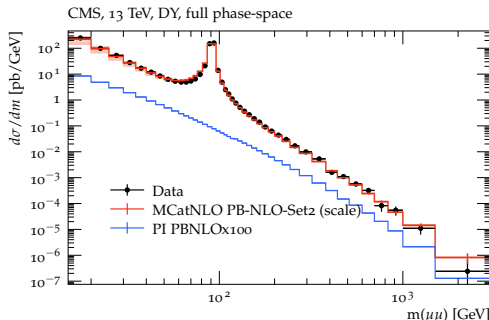
Measurement of the differential Drell-Yan cross section in proton-proton collisions at 13 TeV (CMS-2018-I1711625) JHEP 12 (2019), 059

Matrix Elements: MC@NLO

● Standard Drell-Yan : $pp \rightarrow l^+ l^-$

● PI process : $\gamma\gamma \rightarrow l^+ l^-$

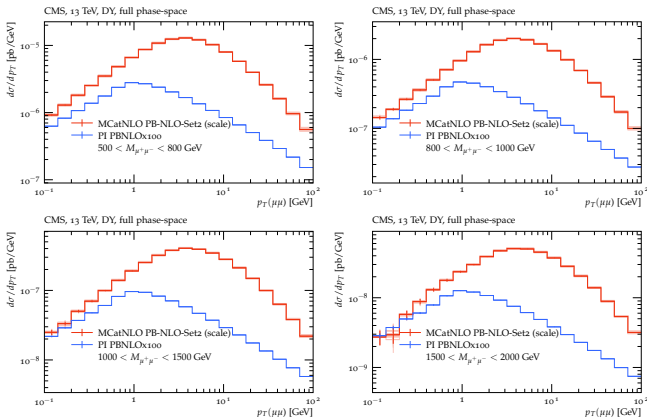
PDFs



- whole mass region is well described by collinear PB-QED (SET2)
- the fraction of PI process is generally less than 1%
- our PI prediction is 1% of the other approaches with intrinsic photon distribution - For the integrated photon density the perturbative piece is 10%
- special feature of PB method: the transverse momentum spectrum of DY and PI process can be predicted for high mass DY pairs

TMD photon application: p_t spectrum at large DY mass

- p_t -distribution of high mass DY pair at different energies [CASCADE3 \[arXiv:2101.10221 \[hep-ph\]\]](#)



- huge difference in the scale from high mass DY pair down to few GeV is predicted by PB formalism with no tuning
- the transverse momentum contribution of PI processes predicted with a TMD photon density
- non-zero p_t of lepton pair can come only from perturbatively generated photon
- the p_t -spectrum of PI process is different from standard DY

Conclusion

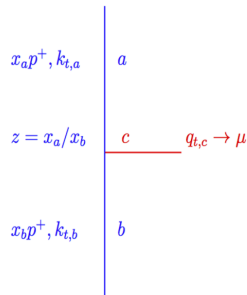
- PB method to solve DGLAP equation at LO, NLO, NNLO.
 - advantages of PB method (angular ordering)
 - method directly applicable to determine k_t -distribution
 - **collinear and TMD photon** density recently determined with the PB method
- TMD densities are used to predict the mass and **transverse momentum** spectra of very high mass lepton pairs from both **Drell-Yan production and Photon-Initiated lepton processes** at the LHC
 - dilepton mass measurements well described in the range 15 to 3000 GeV at $\sqrt{s} = 13$ TeV - there is no significant contribution from perturbatively generated photon
 - a measurement can be made to constrain the photon TMD distribution
- outlook:
 - PDFs for heavy gauge bosons

Thank you

Backup

Transverse Momentum Dependence

- Parton Branching evolution generates every single branching:
 - kinematics can be calculated at every step
- give physics interpretation of evolution scale:
 - in high energy limit: p_T -ordering:
 $\mu = q_T$
 - angular ordering:
 $\mu = q_T/(1 - z)$

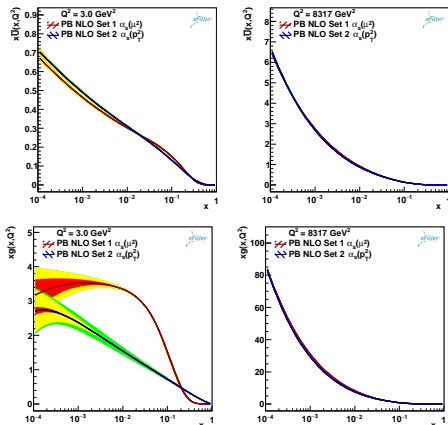


PDFs from PB method: fit to HERA data

- two angular ordered sets with different argument in α_s (either μ or q_t)
- q_{cut} in, $\alpha_s(\max(q_{cut}^2, |q_{t,i}^2|))$, to avoid the non-perturbative region, $|q_{t,i}^2| = (1 - z_i)^2 \mu_i^2$
- for both LO & NLO:
 - $\mu_0^2 = 1.9 \text{ GeV}^2$ for set1 (as in HERAPDF)
 - $\mu_0^2 = 1.4 \text{ GeV}^2$ for set2 (the best χ^2/dof)
- fits to HERA measurements performed using χ^2/dof minimization
- the experimental uncertainties defined with the Hessian method with $\Delta\chi^2 = 1$.
- the model dependence obtained by varying charm and bottom masses and μ_0^2 .
- the uncertainty coming from the q_{cut} in set2

	Central value	Lower value	Upper value
PB Set1 μ_0^2 (GeV ²)	1.9	1.6	2.2
PB Set 2 μ_0^2 (GeV ²)	1.4	1.1	1.7
PB Set 2 q_{cut} (GeV)	1.0	0.9	1.1
m_c (GeV)	1.47	1.41	1.53
m_b (GeV)	4.5	4.25	4.75

Standard 5FL-NLO full fit



- Set1- $\alpha_s(\mu^2) \rightarrow \chi^2/dof = 1.21$
- Set2- $\alpha_s(p_T^2) \rightarrow \chi^2/dof = 1.21$

$$xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g},$$

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2),$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}},$$

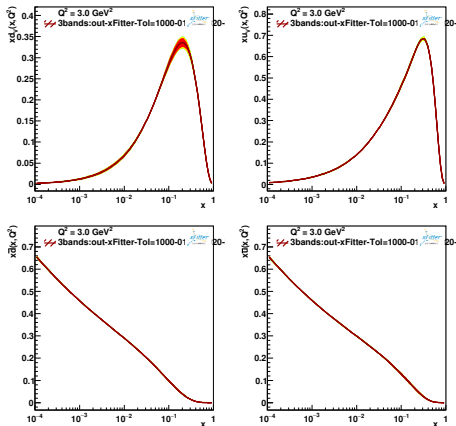
$$x\bar{u}(x) = A_{\bar{u}} x^{B_{\bar{u}}} (1-x)^{C_{\bar{u}}} (1 + D_{\bar{u}} x),$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}.$$

- fits are as good as HERAPDF2.0.
- very different gluon distribution obtained at small Q^2
- the differences are washed out at higher Q^2

Phys. Rev. D **99**, no. 7, 074008 (2019).

Collinear PDF at $Q^2 = 3 \text{ GeV}^2$



$$P_{qq} = e_q^2 \frac{1+z^2}{[1-z]_+} + \frac{3}{2} e_q^2 \delta(1-z)$$

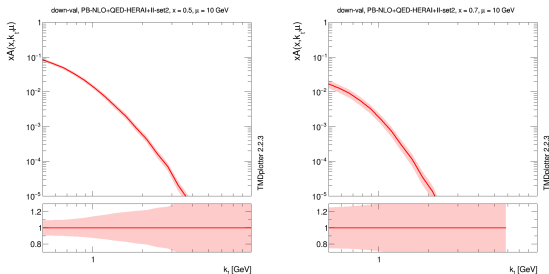
$$P_{q\gamma} = N e_q^2 (z^2 + (1-z)^2)$$

$$P_{\gamma q} = e_q^2 \frac{1+(1-z)^2}{z}$$

$$P_{\gamma\gamma} = -\frac{N}{3} \sum_q e_q^2 \delta(1-z)$$

- the probability to radiate a photon off quark is large at small z
- small- x photon is radiated from large- x values of quarks, where the probability of valence quarks is large
- ⇒ the valence quark uncertainty is reflected to photon uncertainty

Valence TMD



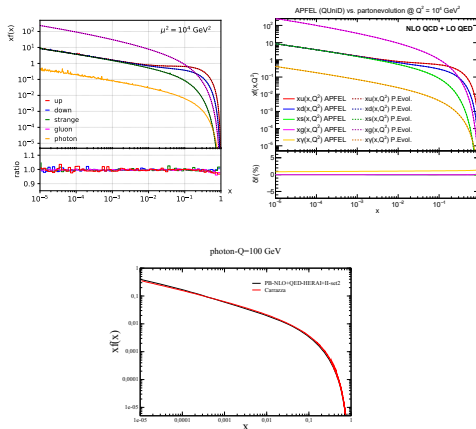
- the uncertainty of valence quark at small- k_t is large

$$\begin{aligned} \mathcal{A}_a(x, \mathbf{k}, \mu^2) &= \Delta_a(\mu^2) \mathcal{A}_a(x, \mathbf{k}, \mu_0^2) + \sum_b \int \frac{d^2 \mathbf{q}'}{\pi \mathbf{q}'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mathbf{q}'^2)} \Theta(\mu^2 - \mathbf{q}'^2) \Theta(\mathbf{q}'^2 - \mu_0^2) \\ &\times \int_x^{z_M} dz z P_{ab}^{(R)}(\alpha_s, z) \mathcal{A}_b\left(\frac{x}{z}, \mathbf{k} + (1-z)\mathbf{q}', \mathbf{q}'^2\right) \end{aligned}$$

- at small $k_t \rightarrow$ the uncertainty comes from the starting distribution at x
- at large $k_t \rightarrow$ the uncertainty comes from the starting distribution at $x/z \gg x$

a benchmark test

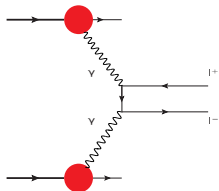
- a benchmark test by taking the same parametrization and initial scale in the FFN scheme with only four active quarks is performed.
- with the same assumption of $\gamma(x, Q^2) = 0$, an excellent agreement is achieved for all flavors, photon and gluon PDFs.



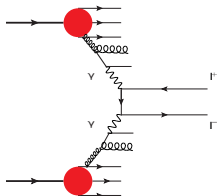
Photon TMD application: DY production

lepton pair p_t can come from two PI processes:

- elastic \rightarrow rapidity gap + lepton pair (can be distinguished experimentally)
- inelastic \rightarrow hadronic activity + lepton pair (More similar to normal DY production)



a) elastic photon (not considered here)



b) perturbatively generated photon