

# MSHT2020 PDFs

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I will discuss what we call the **MSHT2020** PDFs.

**Mass Scheme Hessian Tolerance** – now intended to be a permanent naming convention.

Includes new theoretical developments, and extended parameterisation – particularly  $\bar{d}/\bar{u}$  and strange quark – and eigenvectors sets.

More cross sections at **NNLO**.

Inclusion of new, largely **LHC** but also **HERA**, **Tevatron** data sets.

Problems with correlated uncertainties and tensions in data sets in some cases.

**NLO** clearly no longer sufficient for real precision.

## Theoretical Procedures

As before use a general mass variable flavour scheme based on the **TR** scheme, using the “optimal” choice for smoothness near threshold.  $\mathcal{O}(m_h^2/Q^2)$  corrections for  $Q^2$  slightly above  $m_h^2$  depend on the scheme choice, but only at the next order in  $\alpha_S$ .

Use deuteron and heavy nuclear corrections. Former fit using 4 parameter model, as in **MMHT14** and latter use same corrections ([arXiv:1112.6324](#)) as **MMHT14** with additional penalty-free freedom of order 1%.

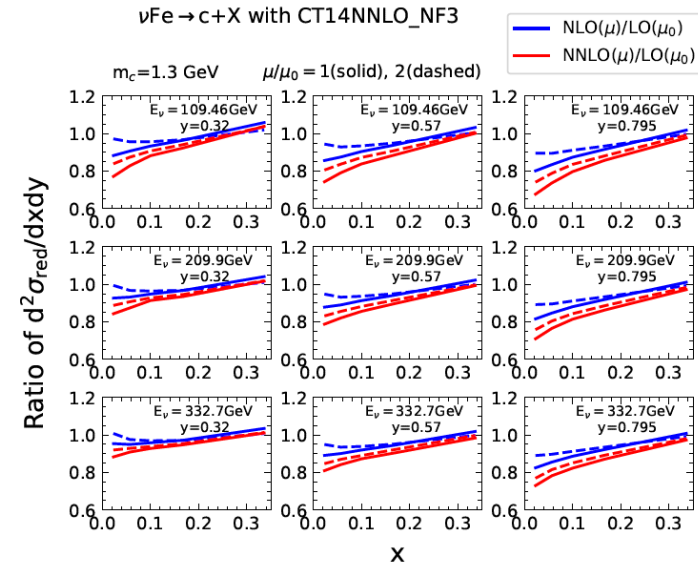
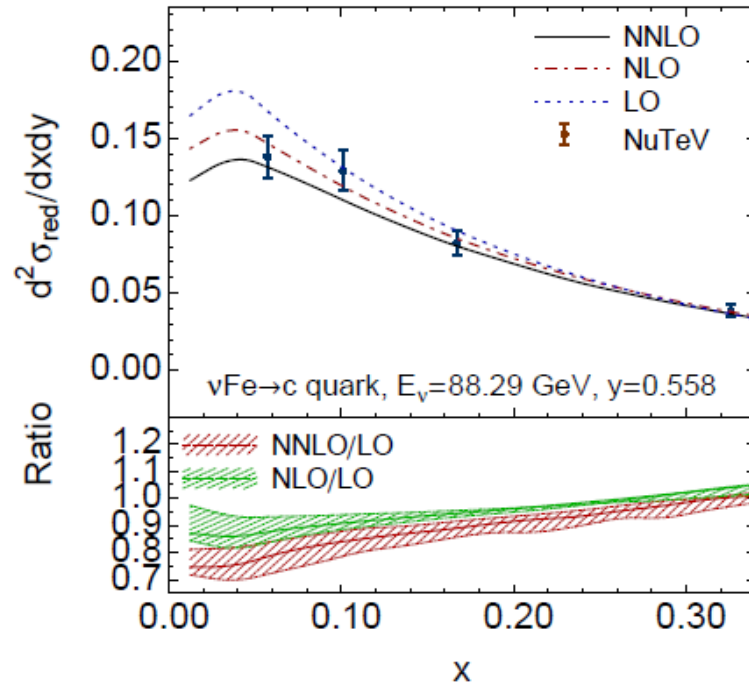
Fit data with systematics uncertainties using either nuisance parameters if possible (preferred method) or with the correlation matrix provided. Use statistical correlations whenever provided.

Some old data sets with domination of uncorrelated uncertainties and/or limited understanding of correlations have errors added in quadrature.

Fit to absolute cross sections in preference to normalized.

## Inclusion of new NNLO corrections.

NNLO corrections to dimuon production (Phys. Rev. Lett. 116 (2016), Berger *et al.*, J. Gao, arXiv:1710.04258).



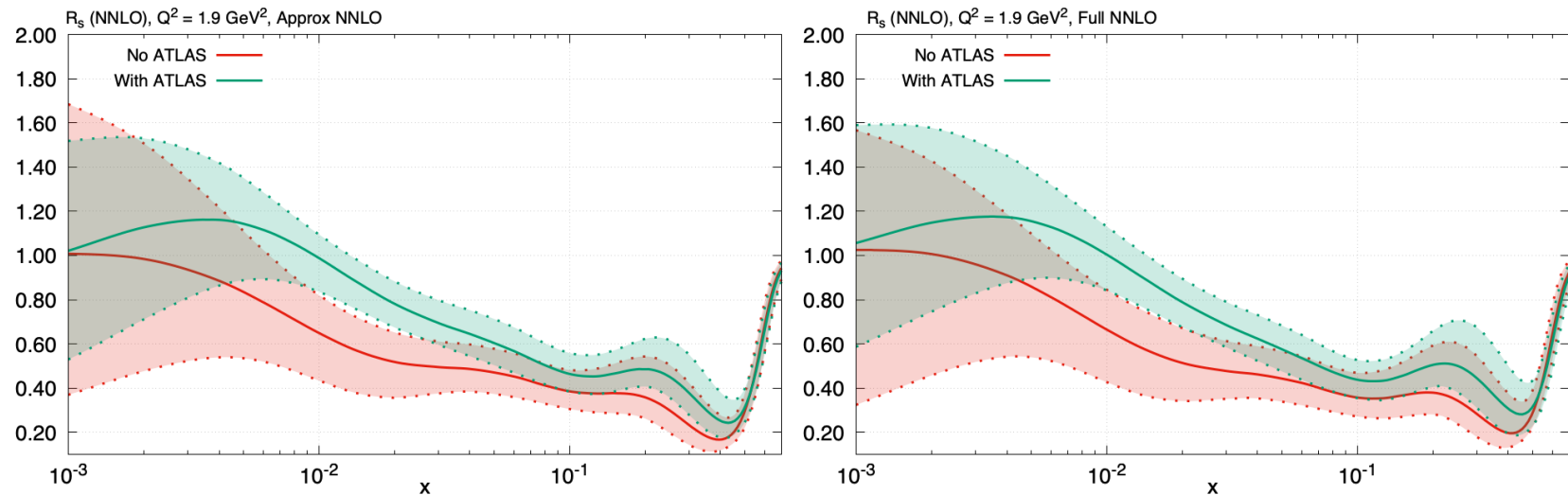
NNLO correction negative, but larger in size at lower  $x$

Impacts on tension between dimuon data and LHC  $W, Z$  data.

Now include these in fit (required some correction for charged-current VFNS scheme). Default value of  $\text{BR}(c \rightarrow \mu) = 0.092 \pm 10\%$ .

	Approx. NNLO	Full NNLO (FFNS)	Full NNLO (VFNS)
CCFR $\chi^2/N_{\text{pts}}$	0.85	0.76	0.78
NuTeV $\chi^2/N_{\text{pts}}$	0.67	0.71	0.69
CCFR + NuTeV $\chi^2/N_{\text{pts}}$	0.76	0.74	0.74
Dimuon BR ( $D \rightarrow \mu$ ).	0.079	0.088	0.089

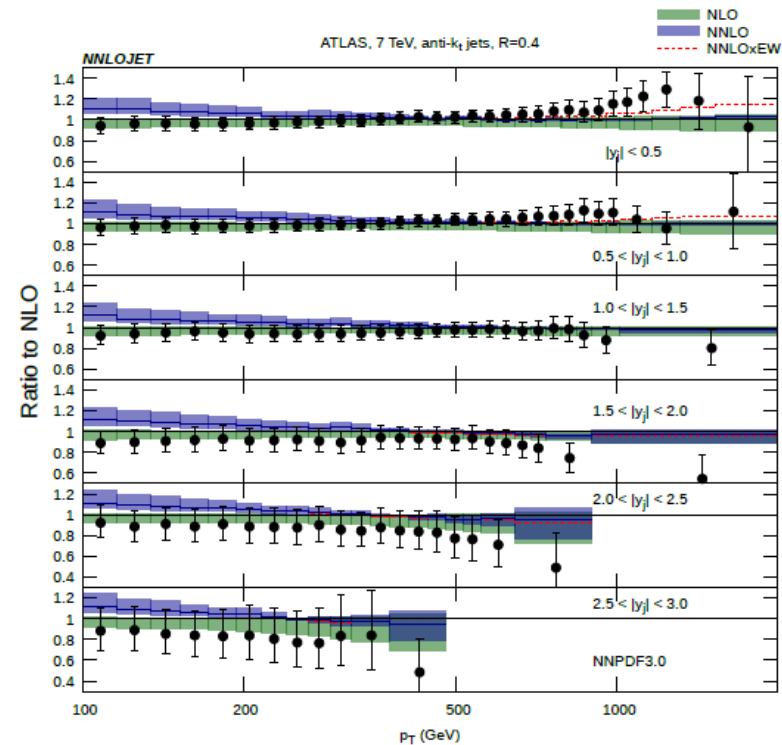
$s + \bar{s}$  illustration without full NNLO and with updated VFNS.



Nearly all other data at full  
**NNLO**.

Also now include **NNLO** cross-section calculations (Currie *et al.*, PRL 118 (2017) 072002) for all **LHC** jet data.

Use inclusive jet production at **2.76, 7 and 8 TeV**, using larger available jet radius, e.g.  $R = 0.6, 0.7$  and  $\mu_{R,F} = p_{T,jet}$ .



Older **Tevatron** data still included with threshold approx for **NNLO** - better approx. for these data, and carry little weight now.

**CMS 7 TeV  $W+c$**  data only have **NLO** theory. Correction “not expected” to be large.

Include **EW** corrections where possible and if not already subtracted from data supplied.

## Extension of parameterisation.

General parameterisation used  $A(1 - x)^\eta x^\delta (1 + \sum_{i=1}^n a_i T_i(1 - 2x^{\frac{1}{2}}))$ , where  $T_i(1 - 2x^{\frac{1}{2}})$  are Chebyshev polynomials.

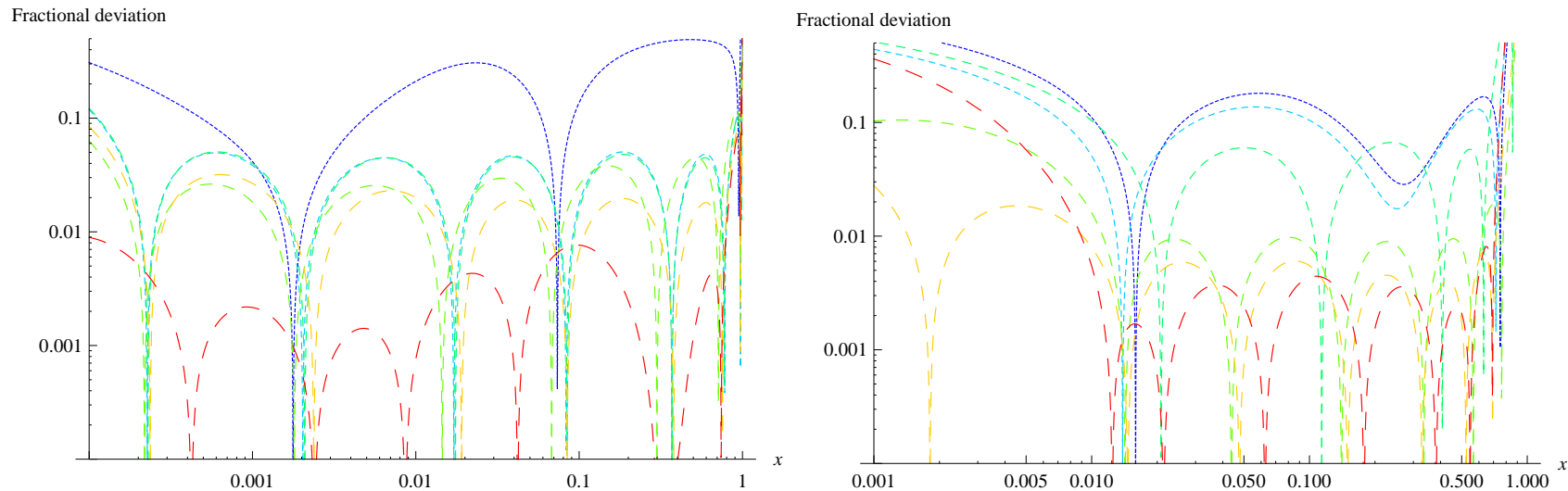


Illustration of precision possible with increasing  $n$ , sea-like (left) and valence-like (right) (where pseudo-data for  $x > 0.01$ ). Using  $n = 6$  would lead to much better than 1% precision ([arXiv:1211.1215](#)).

For most PDFs  $n = 4$  is default for **MMHT2014** -  $g(x, Q_0^2)$  has a negative term,  $s^+(x, Q_0^2)$  has two parameters tied to the sea and  $(\bar{d} - \bar{u})(x, Q_0^2)$  and  $s^-(x, Q_0^2)$  have fewer parameters. 36 parameters in total.

Try extending parameters of other PDFs sequentially using  $n = 6$ :

Also, now parameterize  $(\bar{d}/\bar{u})$  instead of  $(\bar{d} - \bar{u})$ .  $(\bar{d}/\bar{u}) \rightarrow \text{constant}$  as  $x \rightarrow 0$ .

$$u_v(x, Q_0^2) = A_u(1-x)^{\eta_u} x^{\delta_u} (1 + \sum_{i=1}^6 a_{i,u} T_i(1-2x^{\frac{1}{2}})); A_u \text{ fixed by } \int_0^1 u_v dx = 2$$

$$d_v(x, Q_0^2) = A_d(1-x)^{\eta_d} x^{\delta_d} (1 + \sum_{i=1}^6 a_{i,d} T_i(1-2x^{\frac{1}{2}})); A_d \text{ fixed by } \int_0^1 d_v dx = 1$$

$$sea(x, Q_0^2) = A_S(1-x)^{\eta_S} x^{\delta_S} (1 + \sum_{i=1}^6 a_{i,S} T_i(1-2x^{\frac{1}{2}}));$$

$$s^+(x, Q_0^2) = A_s(1-x)^{\eta_s} x^{\delta_s} (1 + \sum_{i=1}^6 a_{i,s} T_i(1-2x^{\frac{1}{2}})); (a_{i,s} \neq a_{i,S}, i = 5, 6)$$

$$(\bar{d}/\bar{u})(x, Q_0^2) = A_{\text{rat}}(1-x)^{\eta_{\text{rat}}} (1 + \sum_{i=1}^6 a_{i,\text{rat}} T_i(1-2x^{\frac{1}{2}}));$$

$$g(x, Q_0^2) = A_g(1-x)^{\eta_g} x^{\delta_g} (1 + \sum_{i=1}^4 a_{i,g} T_i(1-2x^{\frac{1}{2}})) - A_{g-}(1-x)^{\eta_{g-}} x^{\delta_{g-}};$$

$$s^-(x, Q_0^2) = A_{s-}(1-x)^{\eta_{s-}} (1-x_o/x) x^{\delta_{s-}}. x_o \text{ fixed by } \int_0^1 s^- dx = 0, \delta_{s-} \text{ fixed.}$$

Change of to a maximum of 51 parton parameters.



## Improvements in Global Fit.

Main improvements after:

- change to  $(\bar{d}/\bar{u})(x, Q_0^2) - -\Delta\chi_{tot}^2 = 18$
- additionally introducing  $d_V(x, Q_0^2) - -\Delta\chi_{tot}^2 \sim 32$
- $u_V(x, Q_0^2)$  not significant
- $g(x, Q_0^2) - -\Delta\chi_{tot}^2 \sim 50$
- $sea(x, Q_0^2)$  and  $s^+(x, Q_0^2) - -\Delta\chi_{tot}^2 = 73$ .

Improves fit to high- $x$  fixed-target data.

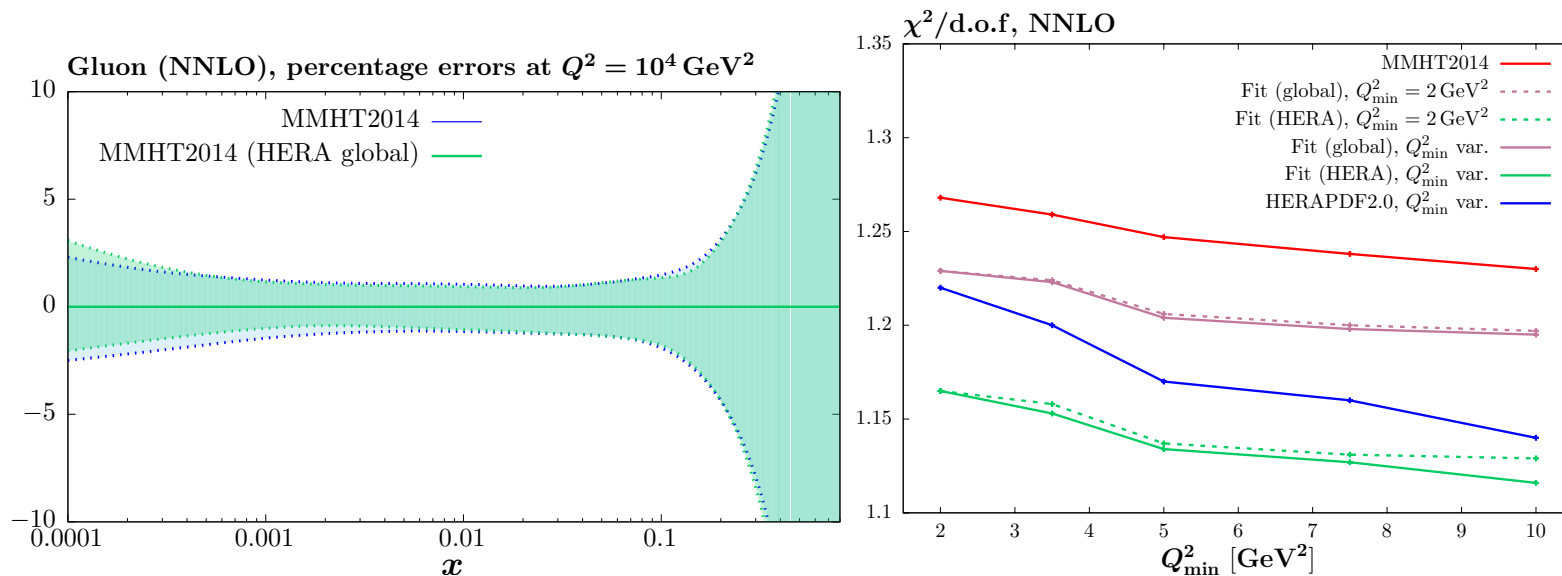
Reduces tension between E866 DY ratio data and LHC data.

LHC lepton asymmetry improved.

Gluon-induced improvement only partially from HERA data.

## Inclusion of final HERA data

Final total cross sections [Eur.Phys.J.C75 \(2015\), 580](#) already studied in [Eur.Phys.J.C76 \(2016\), 186](#). Limited effect on PDFs, but some trouble fitting lower  $Q^2, x$  data.



Now also  $\tilde{\sigma}^{c\bar{c}}$  and  $\tilde{\sigma}^{b\bar{b}}$  data [Eur.Phys.J.C78 \(2018\), 473](#).

Best fit  $\chi^2 = 132/79$ .

No tension with other data within global fit. Inclusive HERA data carries enormously more weight on relevant PDFs.

Fit at low  $Q^2$  is not optimal, but similar results seen in other PDF studies.

## Additional new data sets - **D0** electron/ $W$ asymmetry

Previously fit **D0**  $e$  asymmetry [Phys. Rev. D 91, 032007 \(2015\)](#). Good agreement with **MMHT**.

Alternatively use  $W$ -asymmetry [Phys. Rev. Lett. 112, 151803 \(2014\)](#).

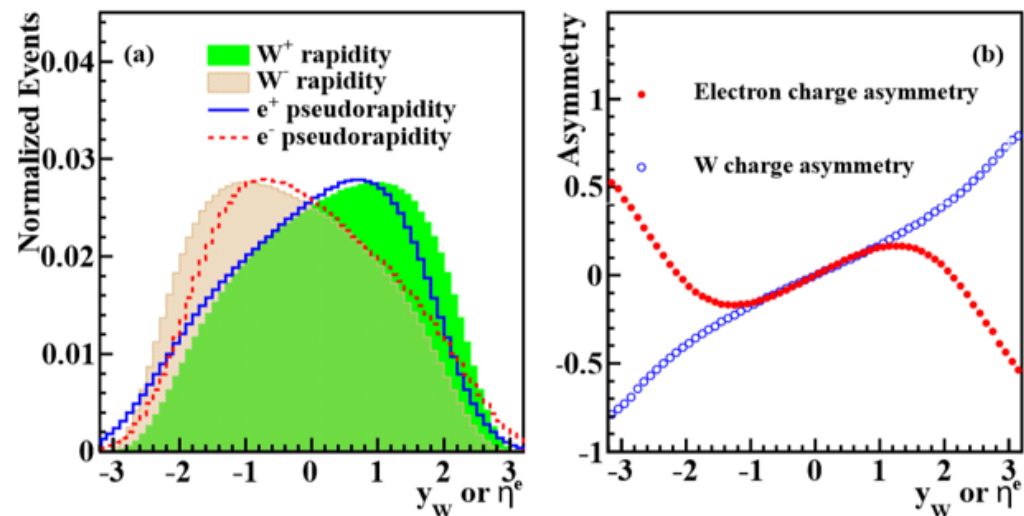
$W^{+/-}$  produced preferentially in proton/antiproton direction.

$V - A$  structure of lepton decay means  $e^{+/-}$  emitted preferentially opposite to  $W^{+/-}$ .

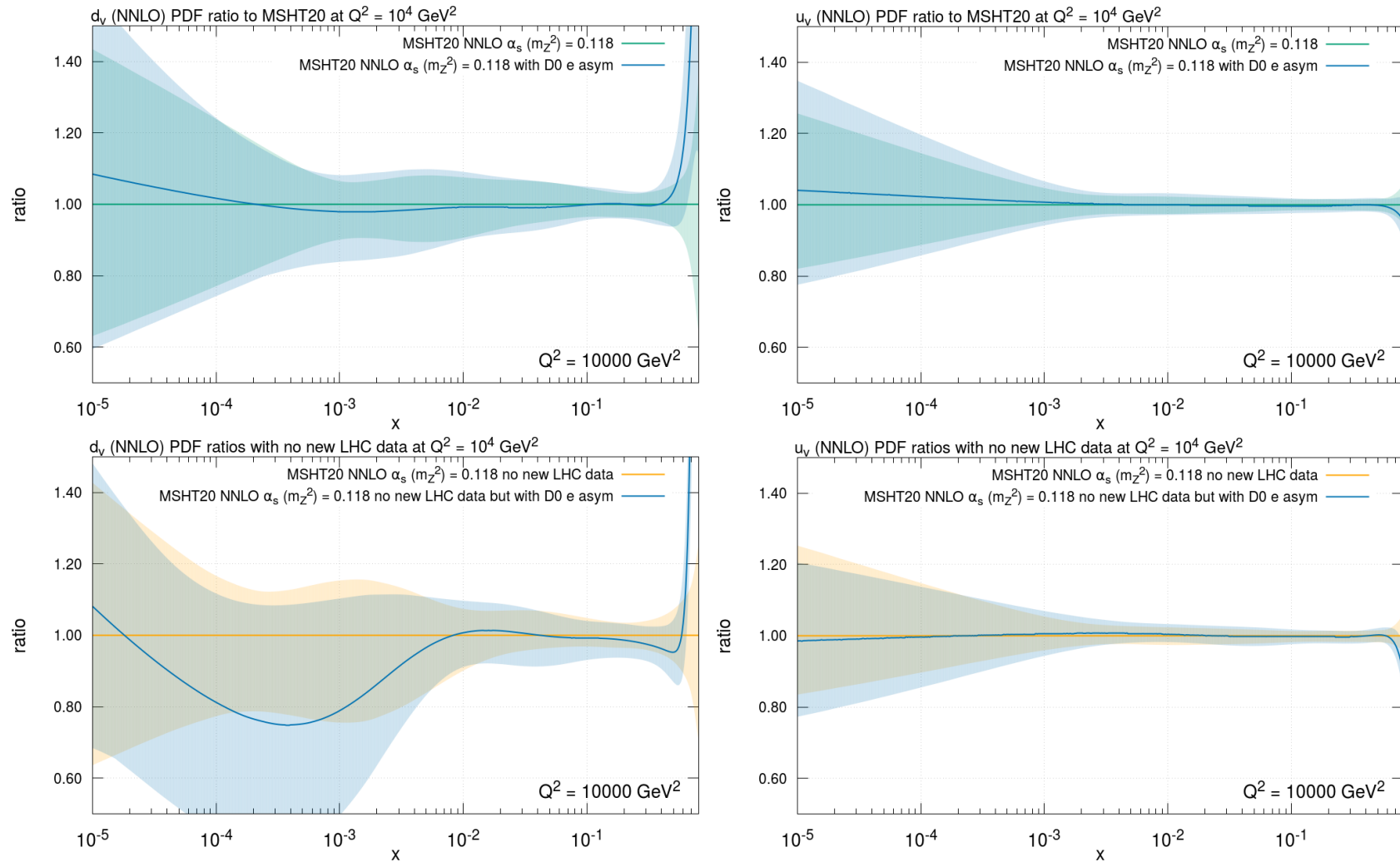
Leptons at particular  $\eta^e$  come from range of  $\eta^W$  values and dilute direct constraint on PDFs at given  $x$ .

Mapping lepton to  $W$  asymmetry requires PDF-dependent modelling with **small** uncertainty.

Better constraint from  $W$  asymmetry.



See reduced uncertainty on  $d/u$  compared to  $e$  asymmetry, but less relative effect on top of LHC data.



Marked effect at very high  $x$ , where  $d_v$  reduced.

## New LHC data fit.

Extremely high precision data on  $W, Z$  at 7 TeV from ATLAS, and high precision  $W^{+/-}$  data and double differential  $Z$  data at 8 TeV.

CMS 8 TeV precise data on the  $W^{+,-}$  rapidity distribution.

LHCb data at 7 and 8 TeV on  $W, Z$  rapidity distributions at higher rapidity.

$W + c$  jets data at 7 TeV from CMS.

ATLAS high mass Drell Yan data at 8 TeV.

ATLAS data on  $W^{+/-} + \text{jets}$  at 8 TeV.

$Z$   $p_T$  distributions at 8 TeV.

New data on  $\sigma_{t\bar{t}}$  at 8 TeV plus ATLAS single differential distributions in  $p_{T,t}, M_{t\bar{t}}, y_t, y_{t\bar{t}}$  and CMS double differential distributions in  $p_{T,t}, y_t$  both at 8 TeV.

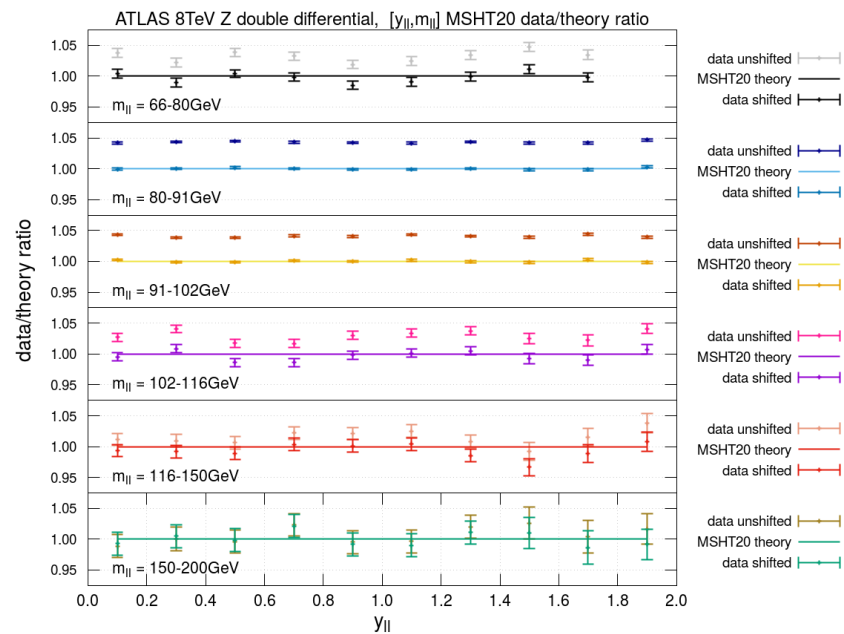
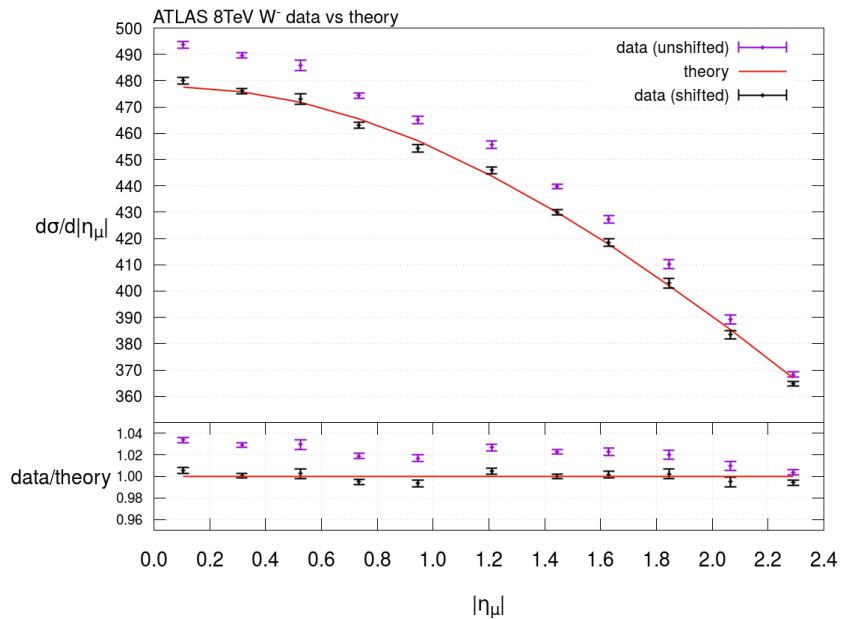
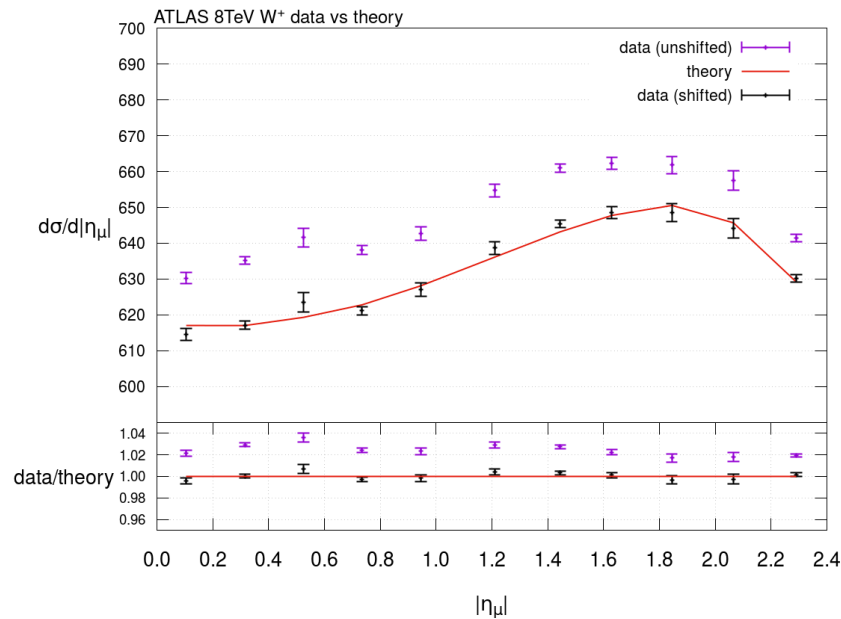
Inclusive jet data from ATLAS at 7 TeV and CMS at 2.76, 7 and 8 TeV.

Include all our recent **LHC** data updates in the fit at **NNLO** (for default  $\alpha_S(M_Z^2) = 0.118$ ).

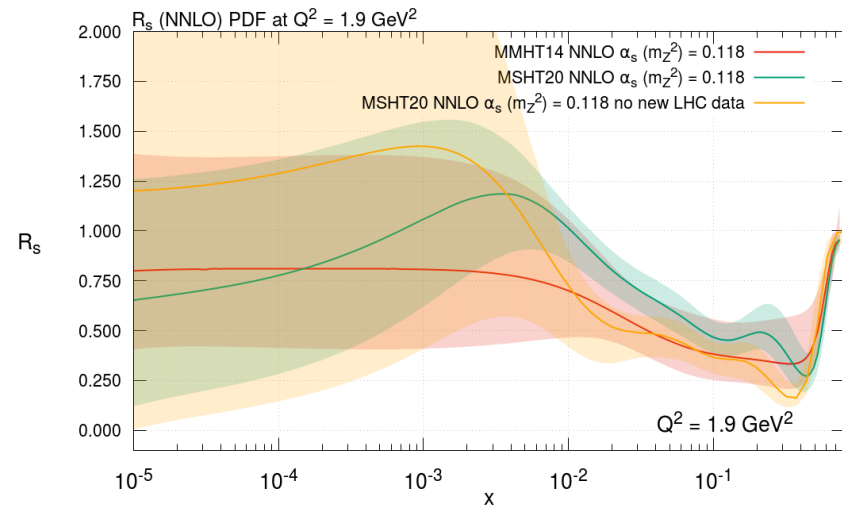
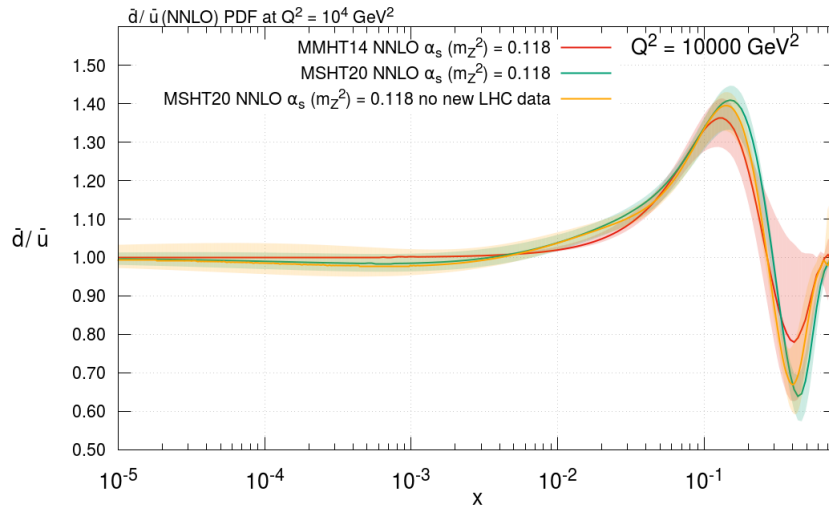
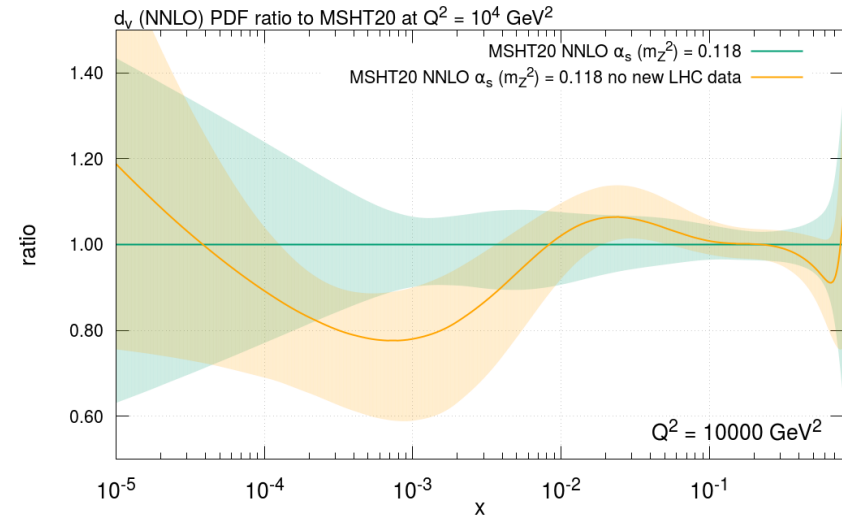
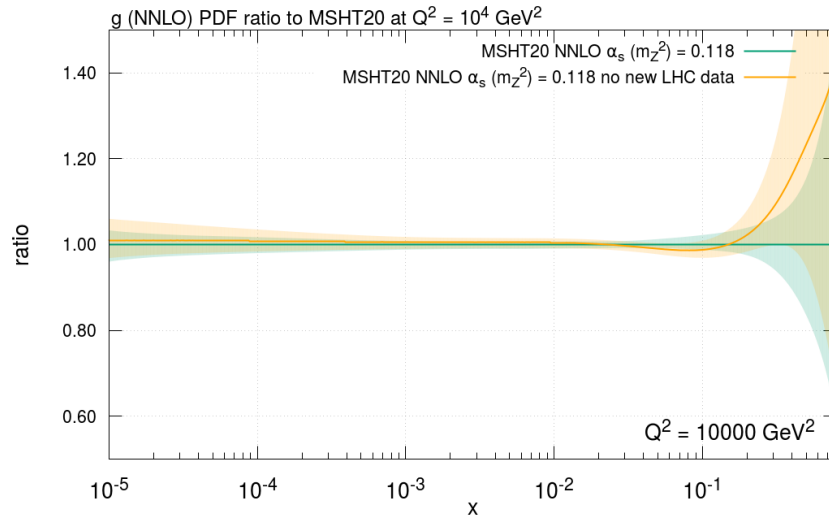
	no. points	NNLO $\chi^2/N_{\text{pts}}$
D0 $W$ asymmetry	14	0.86
$\sigma_{t\bar{t}}$ Tevatron +CMS+ATLAS 7, 8TeV	17	0.85
LHCb 7+8 TeV $W + Z$	67	1.48
LHCb 8 TeV $e$	17	1.54
CMS 8 TeV $W$	22	0.58
ATLAS 7 TeV jets $R = 0.6$	140	1.59
CMS 7 TeV $W + c$	10	0.86
ATLAS 7 TeV $W, Z$	61	1.91
CMS 7 TeV jets $R = 0.7$	158	1.11
ATLAS 8 TeV $Z p_T$	104	1.81
CMS 8 TeV jets	174	1.50
ATLAS 8 TeV $t\bar{t} \rightarrow l + j$ single-diff	25	1.02
ATLAS 8 TeV $t\bar{t} \rightarrow l^+ l^-$ single-diff	5	0.68
ATLAS 8 TeV high-mass Drell-Yan	48	1.18
ATLAS 8 TeV $W^{+,-} + \text{jet}$	32	0.60
CMS 8 TeV $(d\sigma_{t\bar{t}}/dp_{T,t} dy_t)/\sigma_{t\bar{t}}$	15	1.50
ATLAS 8 TeV $W^+, W^-$	22	2.61
CMS 2.76 TeV jets	81	1.27
CMS 8 TeV $t\bar{t} y_t$ distribution	9	1.47
ATLAS 8 TeV double differential $Z$	59	1.45
Total, <b>LHC</b> data	1328	1.33
Total, all data	4363	1.17

Fit quality generally good. Relatively poor  $\chi^2$  values for some sets seemingly observed by other groups.

# Precision 8 TeV ATLAS $W, Z$ data – 1711.03296, 171005167.



# Effect of new LHC data



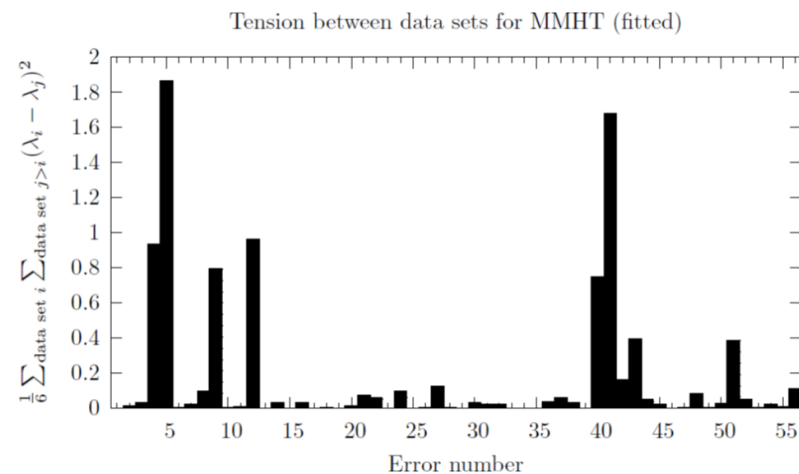
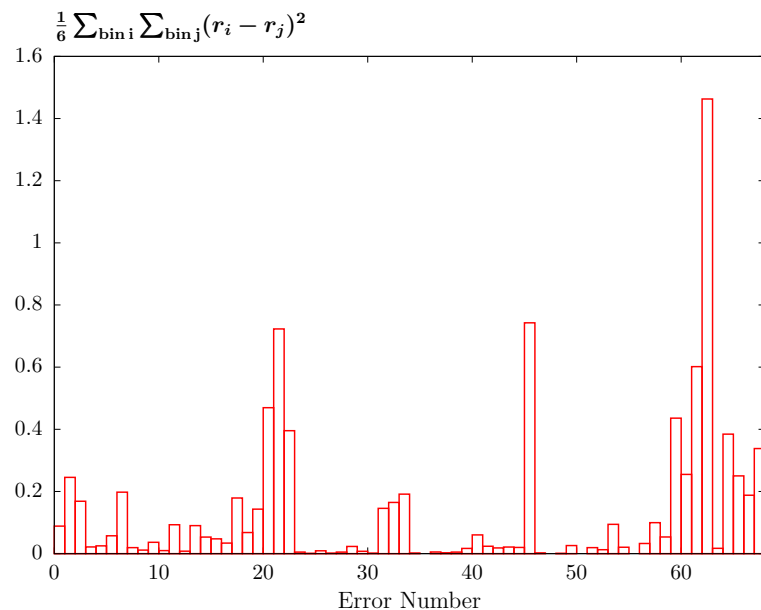
Main effect on details of flavour, i.e.  $d_v$  shape, increase in strange quark for  $0.001 < x < 0.3$  and  $\bar{d}, \bar{u}$  details, though also partially from parameterisation change. Decrease in high- $x$  gluon.



## Inclusion of **LHC** data - systematic uncertainties

Generally the fit is good. However, most straightforward approach gives distinctly poor fit quality to some data sets.

Tensions between different kinematic regions (e.g. rapidity bins) or differential distributions. Sometimes clearly related to modelling-type systematics uncertainties. (Particularly jet and  $t\bar{t}$  data.)



Discussed in detail in [Eur.Phys.J.C 78 \(2018\) 248](#), [80 \(2020\) 60](#).

For some sets use the sort of smooth decorrelation advocated in [ATLAS – JHEP 09 020 \(2017\)](#).

# Eigenvectors

When determining uncertainties go from 25 eigenvector pairs to 32 - one extra parameter for each PDF and two for  $s + \bar{s}$ .

Mean tolerance  $T \sim 3 - 4$

About half constraints primarily by precision electroweak collider data, largely D0  $W$  asymmetry, 7 TeV and 8 TeV ATLAS  $W, Z$  and CMS  $W$ .

$\sim 8-10$  eigenvectors – E866 Drell-Yan ratio vital for  $\bar{d}/\bar{u}$  constraint.

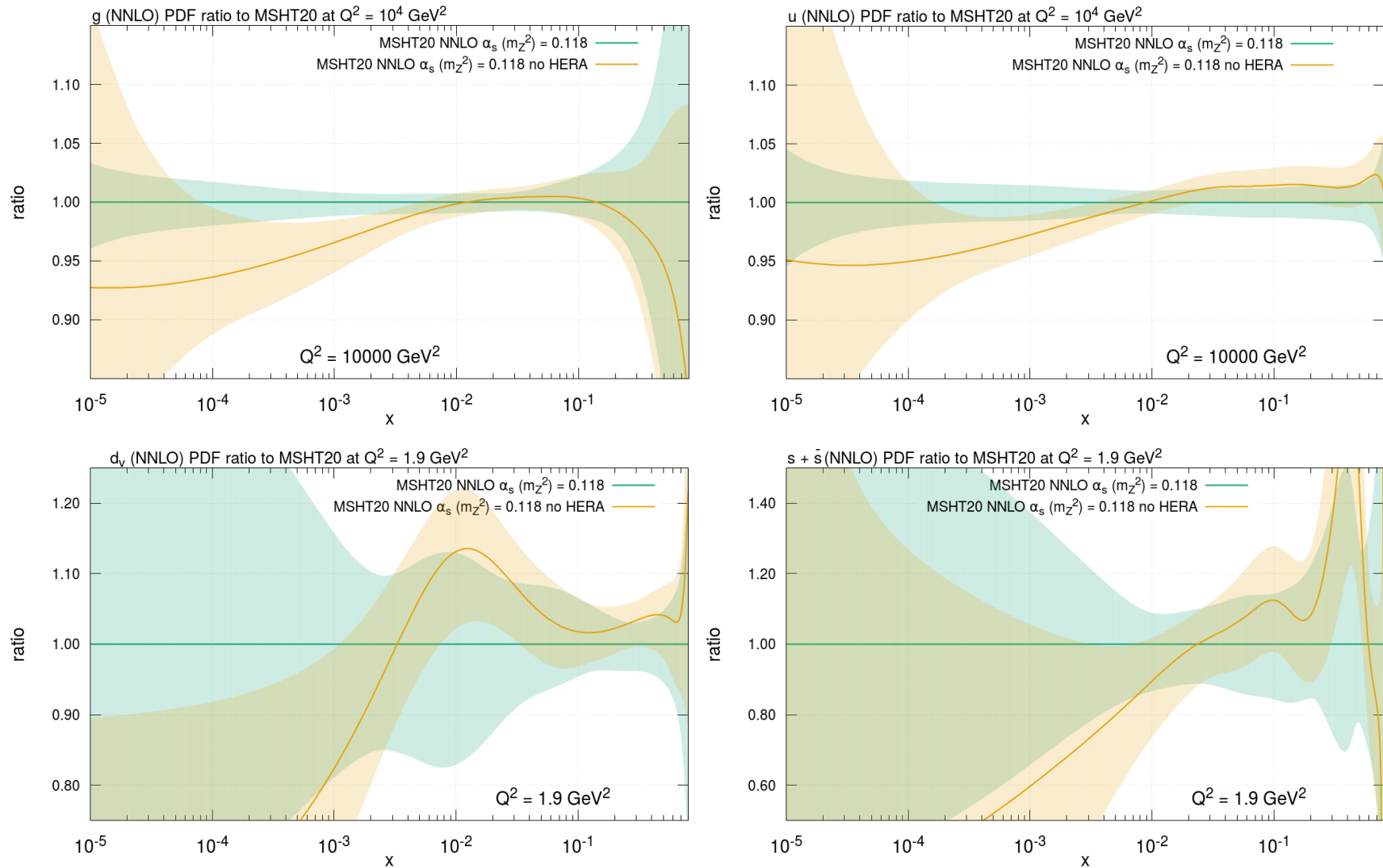
$\sim 10$  eigenvectors – fixed target DIS data (BCDMS, NMC, NuTeV, CCFR) still constrains mainly high- $x$ .

$\sim 10$  eigenvectors – CCFR, NuTeV dimuon data - main constraint still on strange quark and asymmetry.

Fully global fit necessary for full constraint with (almost) no assumptions/models.

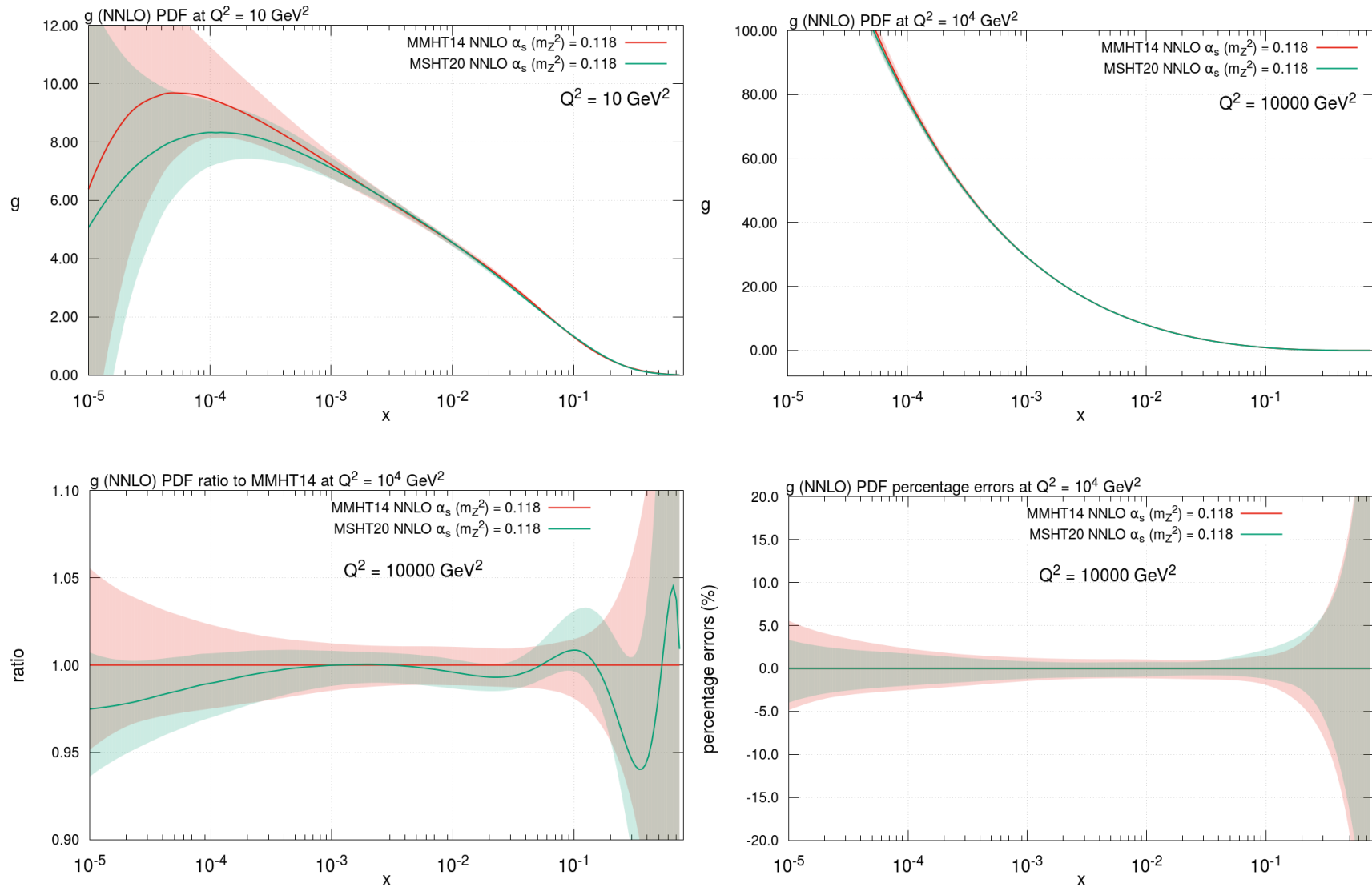
HERA data provides good constraints on widest variety of PDF parameters, mainly gluon and light sea (almost none on a few) – rarely the best. Strong constraint on best fit PDFs.

# Impact of HERA data.



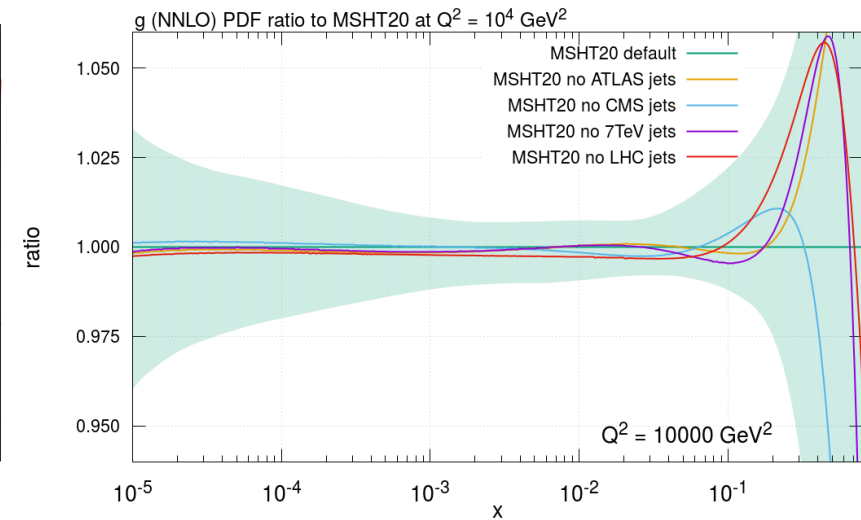
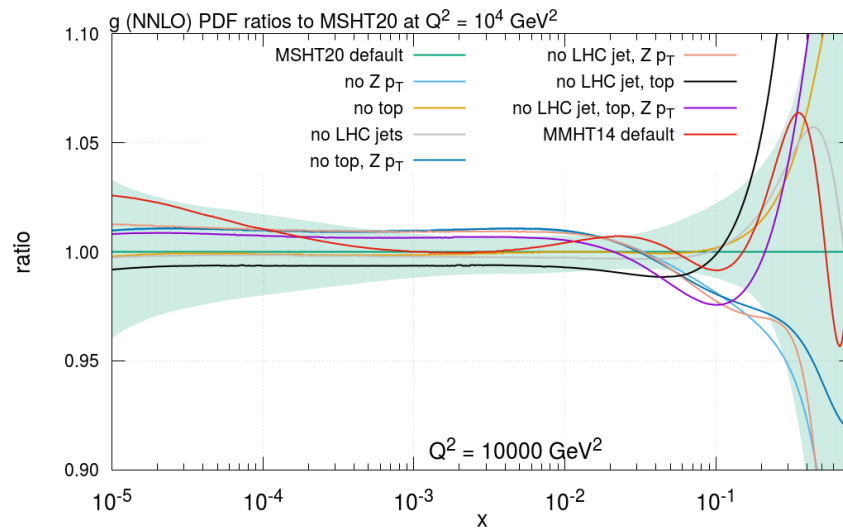
Central values and uncertainties at small  $x$  strongly constrained by HERA data. However, quark normalization at high- $x$  also affected - related to sum rules.

# New PDFs compared to MMHT2014 at NNLO.



No significant change in gluon, though uncertainty reduced.

# Gluon tensions at high $x$ .

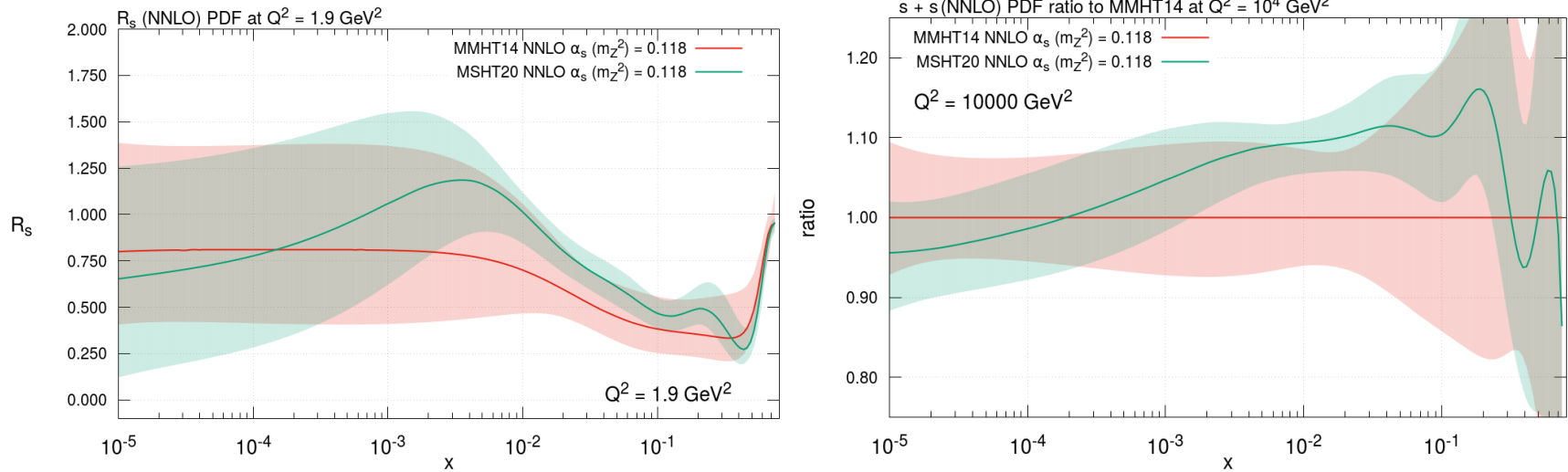


Details in shape near and above  $x = 0.1$  due to LHC jet,  $Z p_T$  and differential  $t\bar{t}$  data.

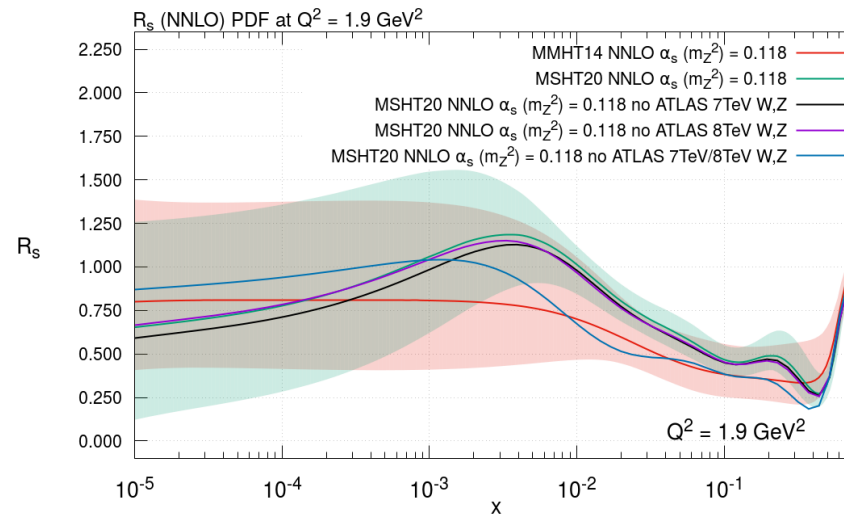
$Z p_T$  pulls gluon up, differential  $t\bar{t}$  data pulls the gluon down.

Affects lower  $x$  normalization via momentum sum rule.

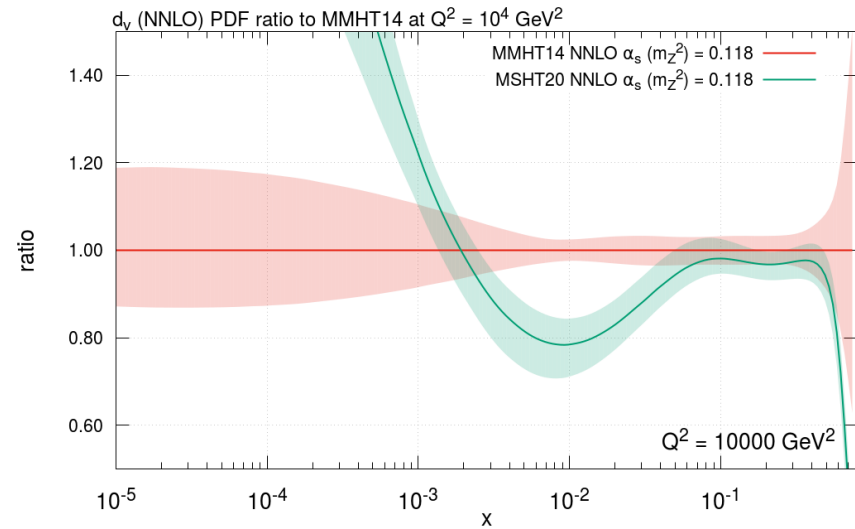
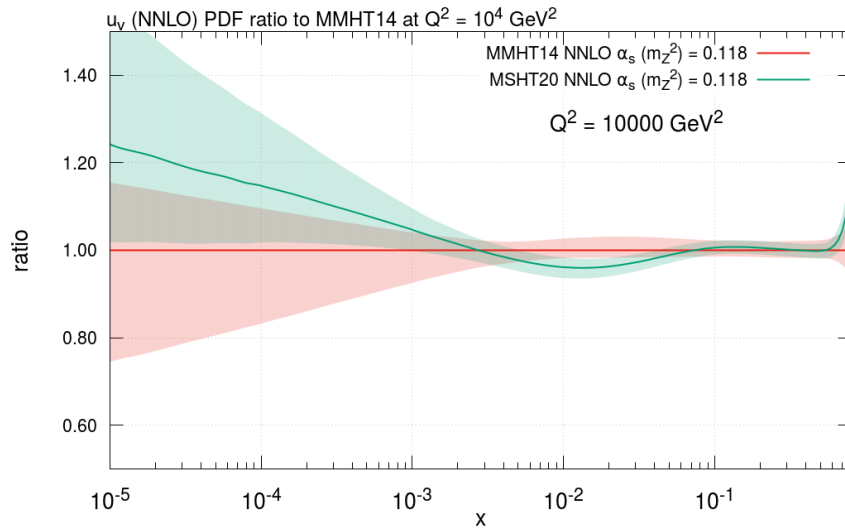
Not all jet data pulls in the same direction.



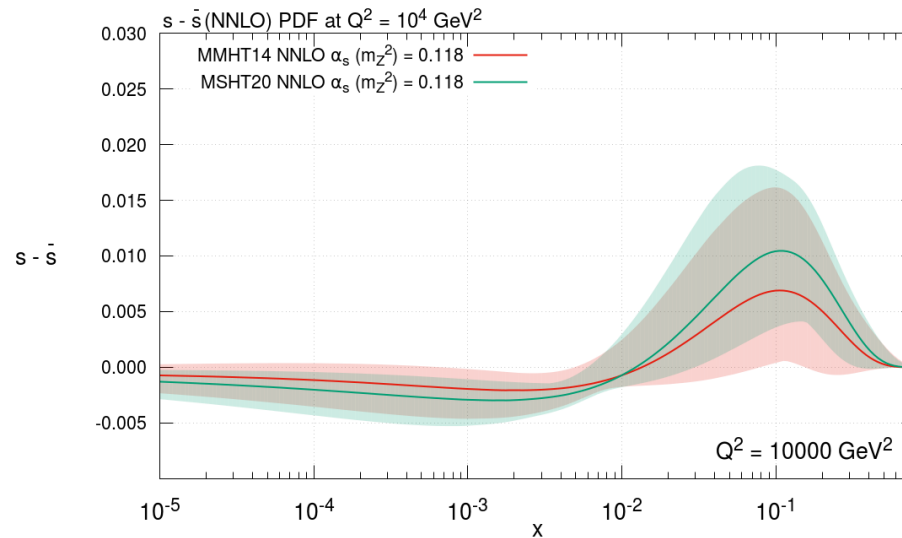
Increase in strange quark below  $x = 0.1$  due to  $W, Z$  data. Mainly ATLAS 7, 8 TeV) – influence PDFs similarly.



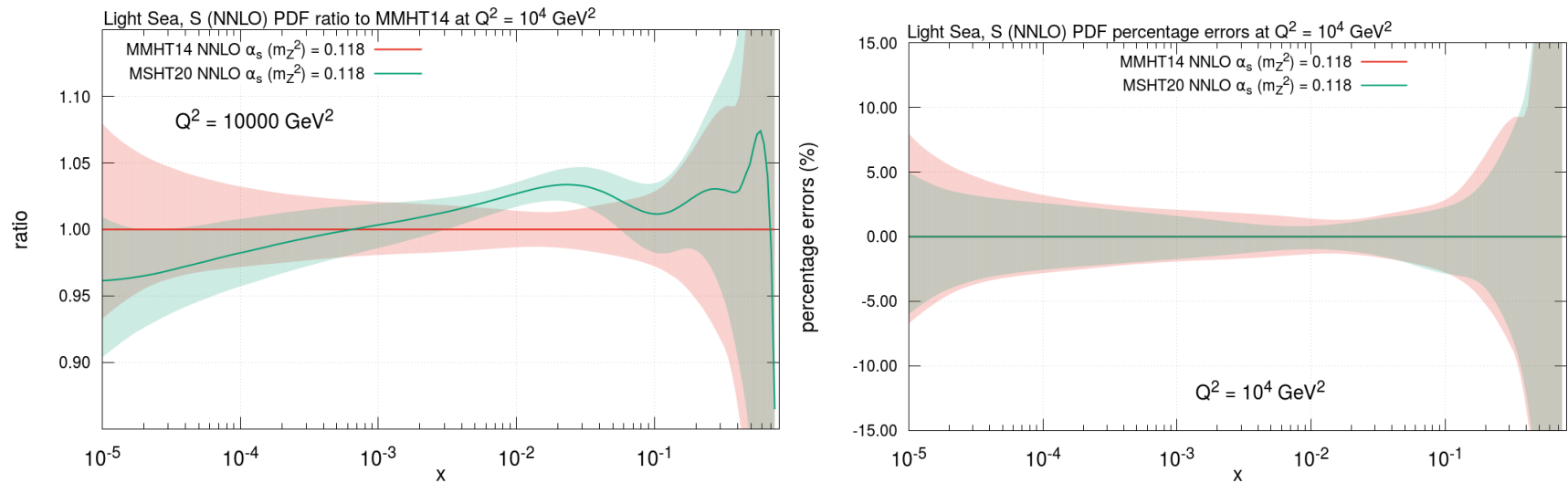
However,  $R_s$  consistent with 1, or lower, at very small  $x$ .



Significant change in shape in valence quarks, most notably  $d_v$ . Due to LHC data on  $W, Z$  and improved parameterisation flexibility.



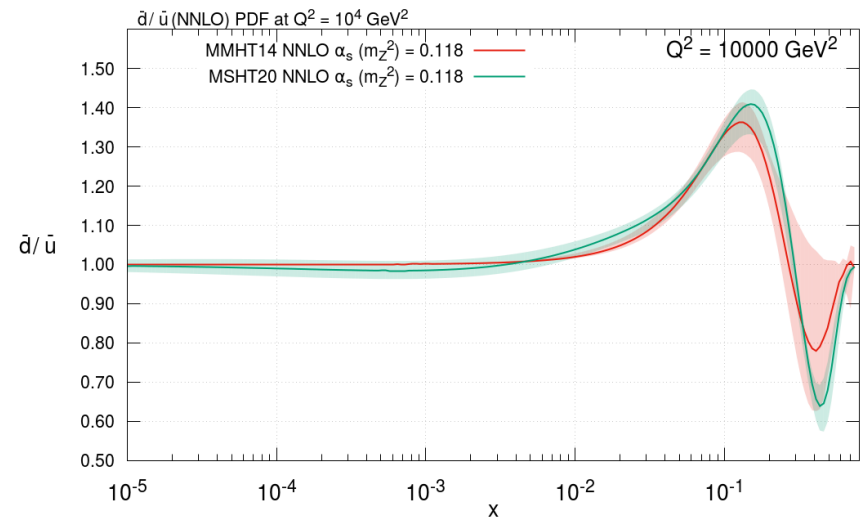
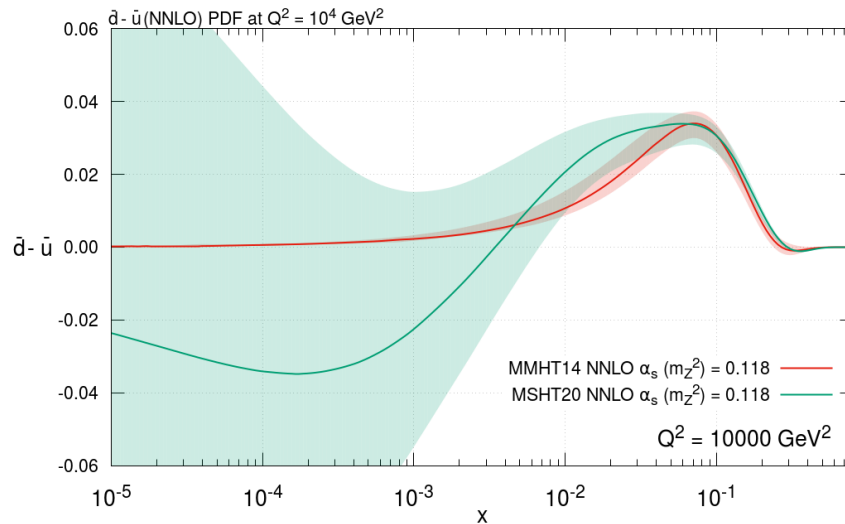
Strange asymmetry similar, but now non-zero outside uncertainties.



Change in details of light antiquarks at high- $x$  where constraints weak.

Slight decrease at low  $x$  due to compensation for increase in strange quark.





Details of  $\bar{u}, \bar{d}$  difference completely changed due to new type of parameterisation.

Huge increase in uncertainty at small  $x$ , and slight tendency for negative  $\bar{d} - \bar{u}$ .

However, different impression looking at  $\bar{d}/\bar{u}$ . Ratio  $\rightarrow 1$  as  $x \rightarrow 0$  without constraint.

## Fits at NLO

We also produce PDFs at NLO (and also still at LO - fit very poor).

Start to notice significant deterioration in fit quality for some of the precision LHC data. NNLO now much preferred.

Data set	Points	NLO $\chi^2/N_{pts}$	NNLO $\chi^2/N_{pts}$
DØ $W$ asymmetry	14	0.94	0.86
$\sigma_{t\bar{t}}$	17	1.34	0.85
LHCb 7+8 TeV $W + Z$	67	1.71	1.48
LHCb 8 TeV $Z \rightarrow ee$	17	2.29	1.54
CMS 8 TeV $W$	22	1.05	0.58
CMS 7 TeV $W + c$	10	0.82	0.86
ATLAS 7 TeV jets $R = 0.6$	140	1.62	1.59
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ATLAS 8 TeV high-mass DY	48	1.79	1.18
ATLAS 8 TeV $W^+ W^- +$ jets	30	1.13	0.60
CMS 8 TeV $(d\sigma_{t\bar{t}}/dp_{T,t} dy_t)/\sigma_{t\bar{t}}$	15	2.19	1.50
ATLAS 8 TeV $W^+ W^-$	22	3.85	2.61
CMS 2.76 TeV jets	81	1.53	1.27
CMS 8 TeV $\sigma_{t\bar{t}}/dy_t$	9	1.43	1.47
ATLAS 8 TeV double differential $Z$	59	2.67	1.45
Total, LHC data in MSHT20	1328	1.79	1.33
Total, non-LHC data in MSHT20	3035	1.13	1.10

## Constraints on $\alpha_S(M_Z^2)$ and quark masses.

For **MMHT2014**  $\alpha_S(M_Z^2) = 0.1172 \pm 0.0013$ .

Final current value from global fit near to  $\alpha_S(M_Z^2) = 0.1174$ .

Constraints from a variety of new **LHC** data, but in different directions – in general jet data slightly lower,  $W, Z$  data slightly higher.

No single new set constrains more strongly than a number of older sets.

At **NLO** for **MMHT2014**  $\alpha_S(M_Z^2) = 0.1201 \pm 0.0015$ . Now find  $\alpha_S(M_Z^2) = 0.1204$ , so very similar picture to **NNLO**.

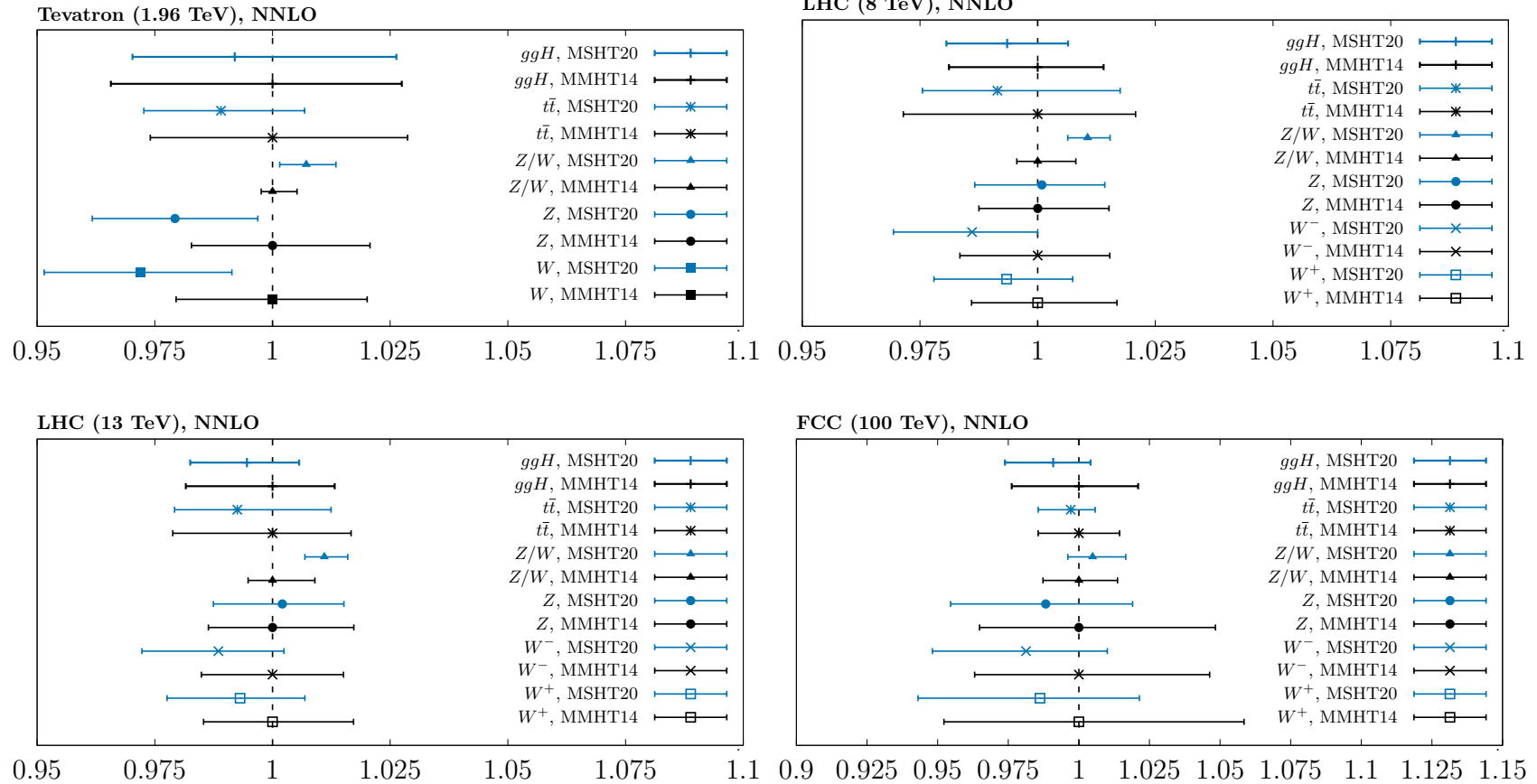
For quark masses, unlike previous results (**Eur.Phys.J. C76 (2016)**) which preferred lower values ( $m_c^{\text{pole}} \sim 1.25\text{GeV}$ ), default choice of  $m_c^{\text{pole}} = 1.4\text{GeV}$  close to optimal.

Global fit min near  $m_c^{\text{pole}} \approx 1.35\text{GeV}$ .  $\tilde{\sigma}^{c\bar{c}}$  best very near  $m_c^{\text{pole}} = 1.4\text{GeV}$

No strong pull from default  $m_b^{\text{pole}} = 4.75\text{GeV}$ , though slightly lower values weakly preferred.

# Predictions for Benchmark Processes.

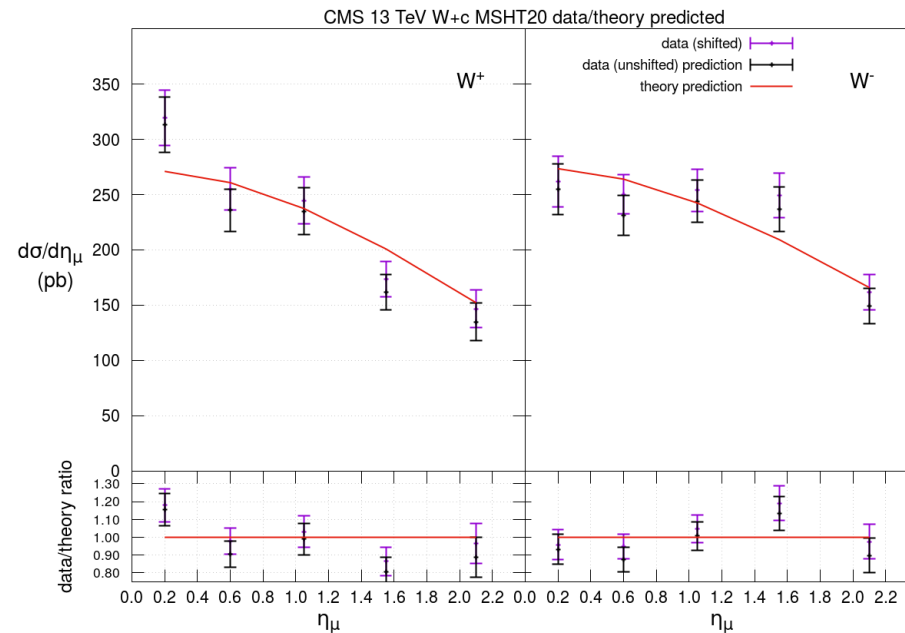
Some changes in  $\sigma_W, \sigma_Z$  and particularly their ratio largely due to changes in strange quarks.



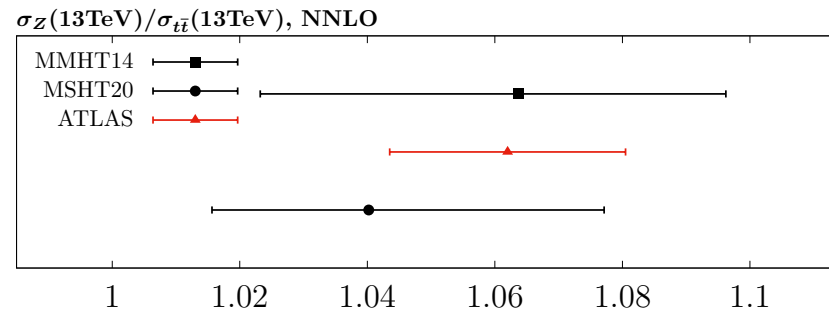
For gluon initiated top and Higgs cross sections an improvement in uncertainties but central values stable.

## Predictions for other data

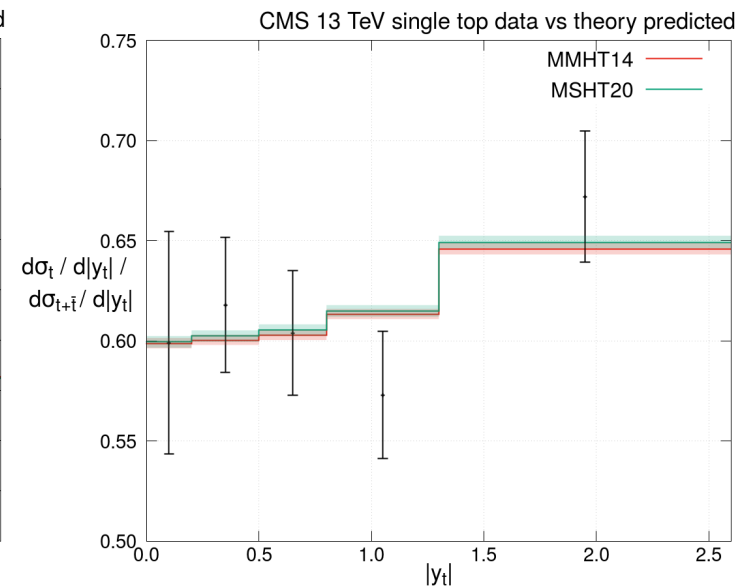
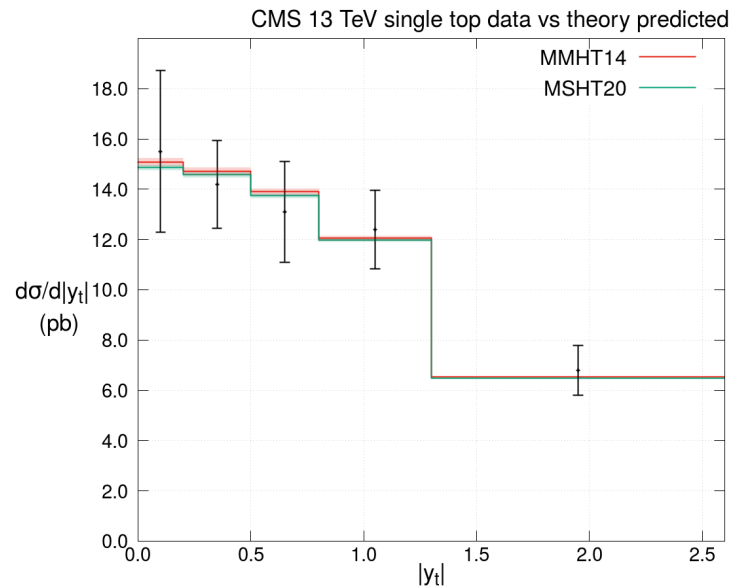
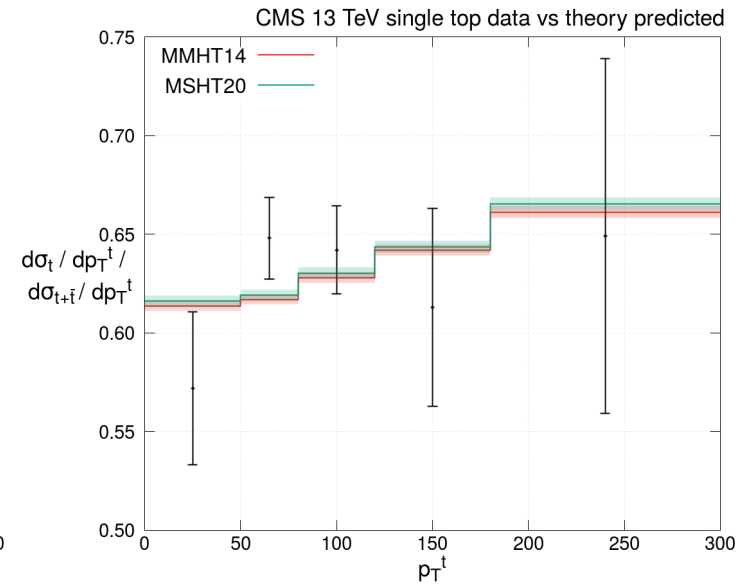
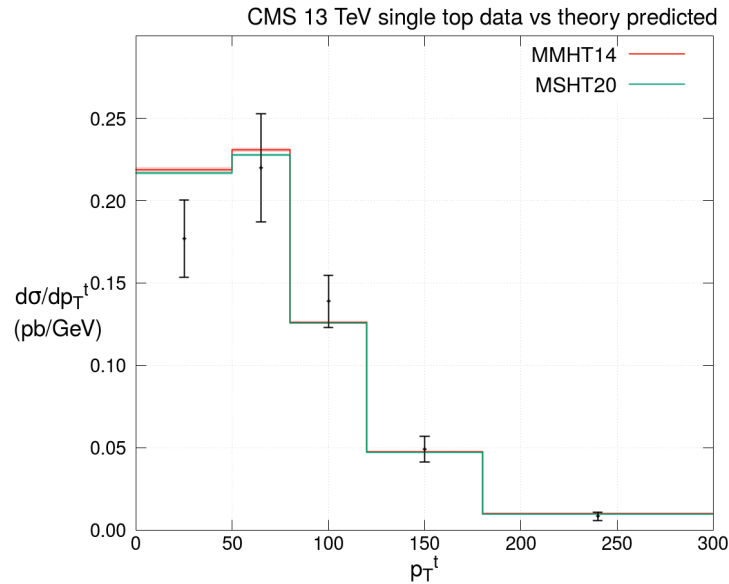
Good prediction for **CMS 13 TeV  $W + c$**  data (**Eur.Phys.J.C 79 (2019), 269**) – mainly dependent on strange quarks (prediction at **NLO**).



Also good agreement with **13 TeV** cross section ratios, e.g.  $\sigma_Z/\sigma_{t\bar{t}}$  shown below.



Single top data not fit (uncertainties much larger than PDF uncertainties), but good predictions (e.g. [Eur.Phys.J.C 80 \(2020\), 370](#)).



## Conclusions

LHC data starting to have a very significant impact on PDF extractions.

Theory catching up for precision data, e.g. NNLO jets, differential top,  $Z, W p_T$  ...

Improvements in parameterisation. Better fit to data - improves some (not all) data tensions and increases some uncertainties in extreme kinematic regions.

Significant changes in  $\bar{d}, \bar{u}$  difference, in  $s + \bar{s}$  distribution and small- $x$   $d_v(x)$  - uncertainties and central values. Largely stability for other PDFs, e.g. gluon, but uncertainty reduction in PDFs/benchmark processes.

Precision data and theory causing problems in cases where correlated systematics (which increasingly dominate) are important. Improved interplay between theory/experiment on these seems a priority.

Additional PDFs with varying  $\alpha_S(M_Z^2)$ , quark masses, photon distribution, and associated studies, to follow soon.

Theory uncertainties a bit longer.

# Back-up



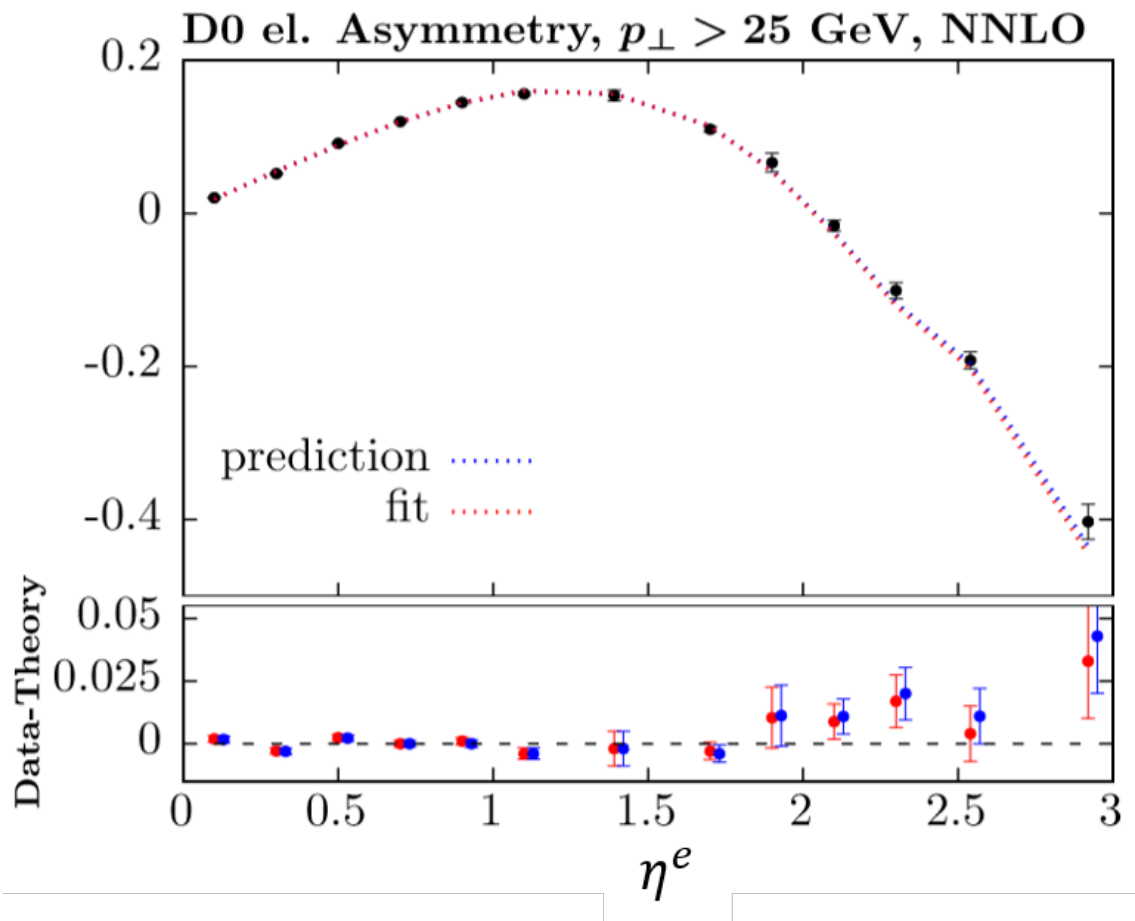
## Additional new data sets - D0 electron/ $W$ asymmetry

Already mentioned D0  $e$  asymmetry [Phys. Rev. D 91, 032007 \(2015\)](#).

Good agreement with [MMHT](#).

Slight undershoot at high  $\eta^e$ , implying smaller  $d_v(x)$  not favoured by other data.

Can alternatively use  $W$ -asymmetry for same data [Phys. Rev. Lett. 112, 151803 \(2014\)](#).



## Inclusion of LHC data – tensions.

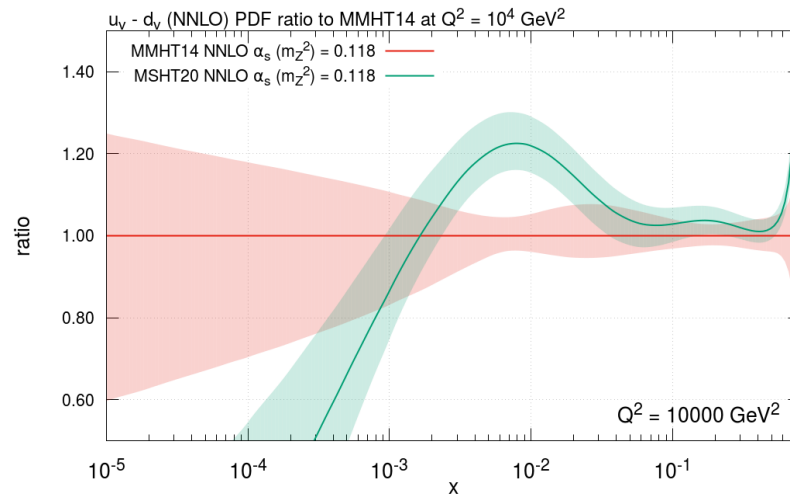
See some distinct tension between new LHC data and older data sets in the global fit, e.g. ATLAS 7 TeV  $W, Z$  and CCFR/NuTeV dimuon data - strange quark. (Better parameterisation removes tension with E866 Drell-Yan ratio data.)

Also between some different new LHC data sets, e.g. ATLAS 7 TeV  $W, Z$  and ATLAS  $Zp_T$  data (Eur.Phys.J. C76 (2016) 291) - gluon.

In general new data sets quite compatible (within context of global fit) with all other data

Some data not included due to inability to get reasonable fit, i.e. at worst 2-3 per point, in some regions, e.g. 8 TeV CMS data on  $d\sigma/dp_T^{ll}$  in rapidity bins for invariant mass near  $Z$  peak (Phys.Lett. B749 (2015) 187), and 8 TeV CMS double differential Drell-Yan data.

Made more difficult by use of covariance matrix only rather than nuisance parameters.



Significant change in small- $x$   $u_V - d_V$ .

Allowed from increased flexibility in parameterisation, and driven mainly by  $W^+, W^-$  asymmetry data at low rapidity.

## Studied by **NNPDF** - smaller strange enhancement.

PDF set	$R_s(x = 0.023, Q = 1.65 \text{ GeV})$	$R_s(x = 0.013, Q = M_Z)$
NNPDF3.0	$0.47 \pm 0.09$	$0.79 \pm 0.04$
NNPDF3.1	$0.62 \pm 0.12$	$0.83 \pm 0.05$
NNPDF3.1 collider-only	$0.86 \pm 0.17$	$0.94 \pm 0.07$
NNPDF3.1 HERA + ATLAS W, Z	$0.96 \pm 0.20$	$0.98 \pm 0.09$
ATLAS W, Z 2011 xFitter (Ref. [93])	$1.13^{+0.11}_{-0.11}$	-
ATLAS W, Z 2010 HERAFitter (Ref. [120])	$1.00^{+0.25}_{-0.28} (*)$	$1.00^{+0.09}_{-0.10} (*)$

👤 **Confirmed the strange symmetric fit** preferred by the ATLAS W,Z 2011 measurements, though we find PDF uncertainties larger by a factor 2

👤 The **global fit** accommodates both the neutrino data and the ATLAS W,Z 2011 (  $\chi^2_{\text{nutev}}=1.1$ ,  $\chi^2_{\text{AWZ11}}=2.1$  ) finding a compromise value for  $R_s=0.62 \pm 0.12$

👤 **Mild tension** in the global fit (1.5-sigma level at most) when simultaneously included neutrino data, CMS W+charm and ATLAS W,Z 2010+2011

$$\sigma_W \propto c\bar{s}, \quad \sigma_Z \propto g_s * s\bar{s} + g_c * c\bar{c}, \quad \text{where } g_s > g_c.$$

Smaller strange correlated with smaller charm, i.e.  $\sigma_Z/\sigma_W$  rises with smaller charm.

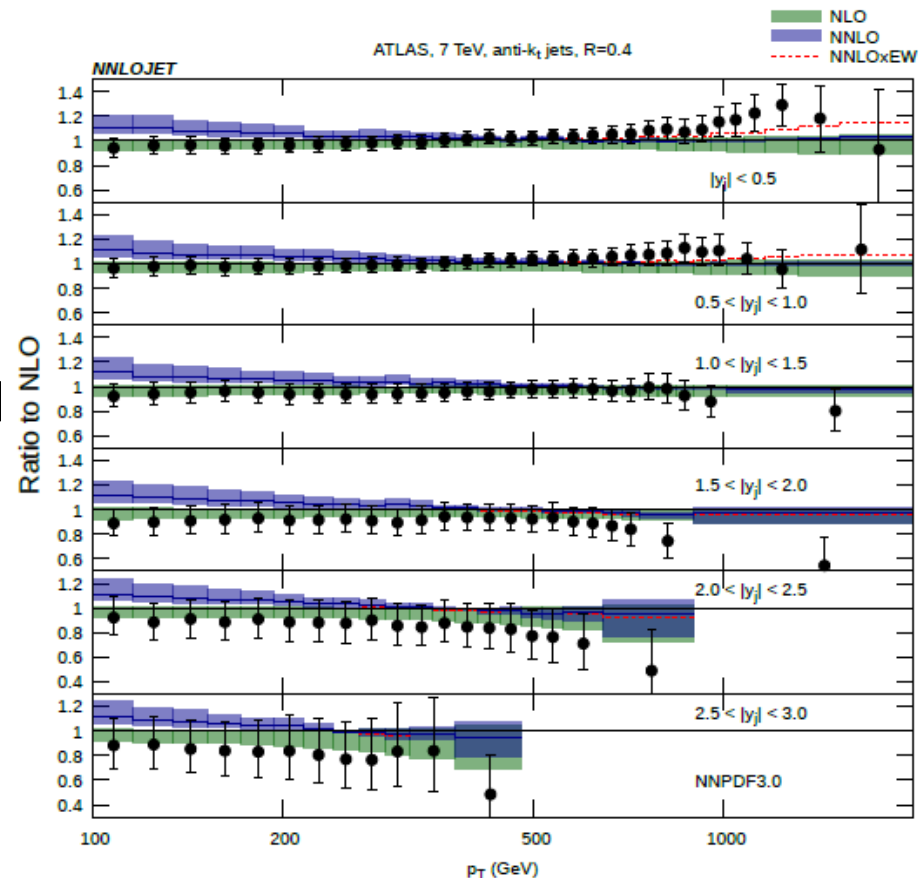
Improved fit to older **ATLAS W, Z** data with larger  $m_c$  evident in **MMHT2014**. Usually interplay with fitting **HERA** data.

## NNLO corrections

Now calculated Currie *et al* Phys.Rev.Lett. 118 (2017) 072002.

Fit quality can slightly improve or decrease compared to NLO depending on choices.

Electroweak corrections to jets different in different bins, but much smaller than systematic effect.



Exact form dependent on  $R$  and on scale choice, e.g.  $\mu = p_{T,1}$  or  $p_T$ . Up to 20% at low  $p_T$ .

Authors now recommend using more physical scale,  $\hat{p}_T$  – sum of parton  $p_T$  (arXiv:1807.03692), improved convergence criteria properties.