Nucleon valence quark distribution functions from Lattice QCD

An exploratory study of Distillation in the pseudo-PDF framework

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HadStruct Collaboration

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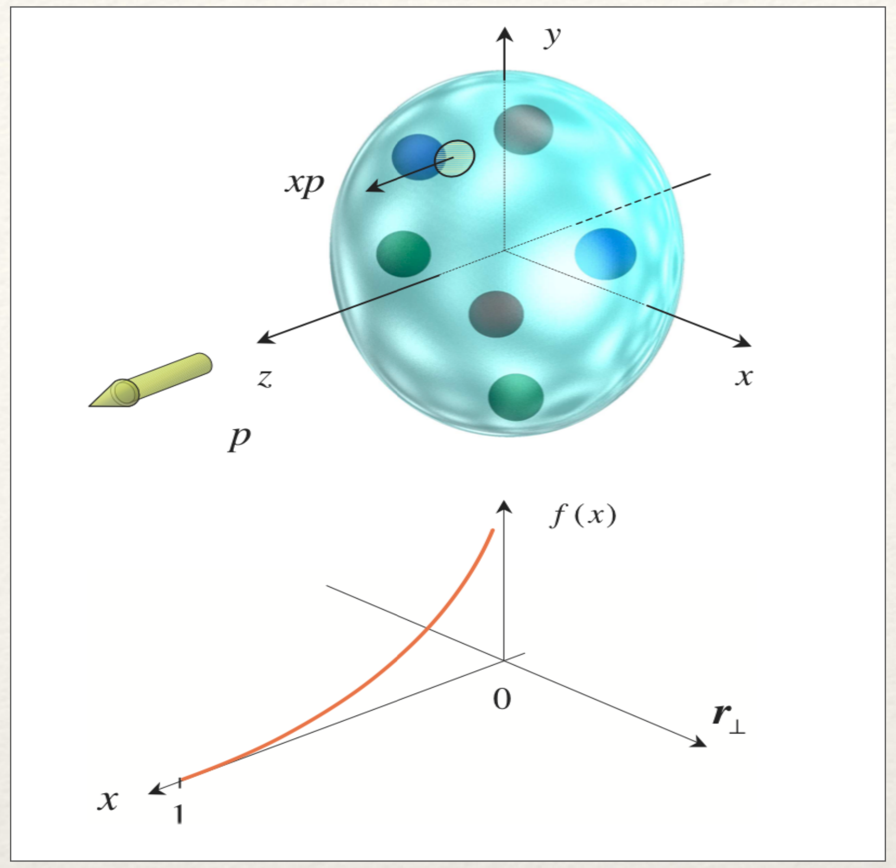
Roadmap

- intro & motivation
- pseudo-PDF: matrix elements
- lattice evaluation
- full picture: PDF x-dependence
- summary & outlook

Introduction | Why the PDFs? Why on the Lattice?

Parton Distribution Functions:

- $1d$-object: number density of "partons" with longitudinal momentum fraction x of nucleon's momentum
- accessible in deep inelastic and semi-inclusive deep inelastic scattering → intricate procedure
  - Still, good precision exists for unpolarized PDFs



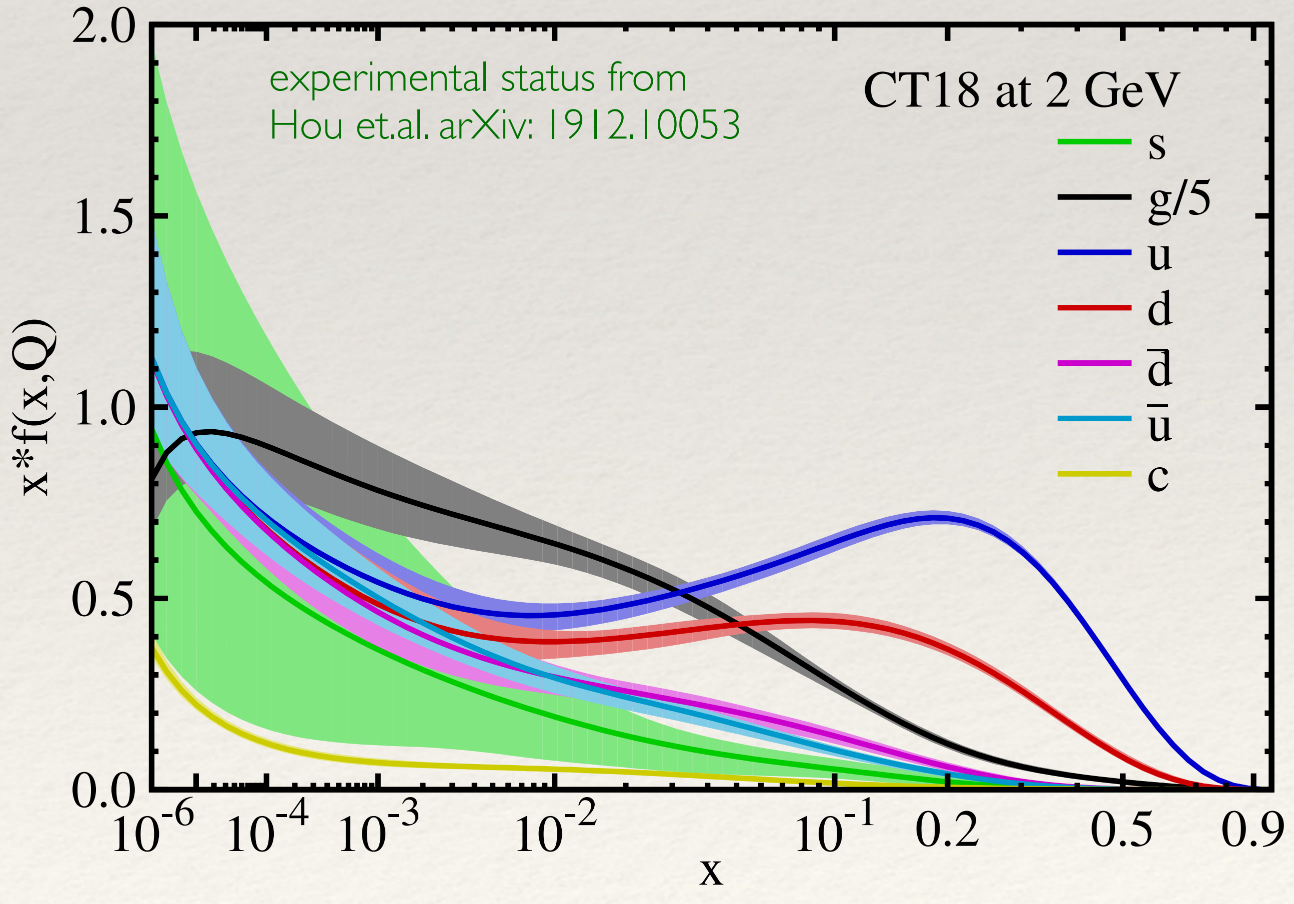
Important LHC applications

Gao et.al., arXiv: 1709.04922

- W-boson mass
- Strong coupling constant
- Higgs boson couplings and cross-sections (BSM physics)

Lattice Evaluation

- PDFs: "hot" topic
- first principles calculation
- unpolarized PDFs → benchmark quantity
  - transversity PDFs: not well constrained
  - GPDs and TMDs: more difficult to access



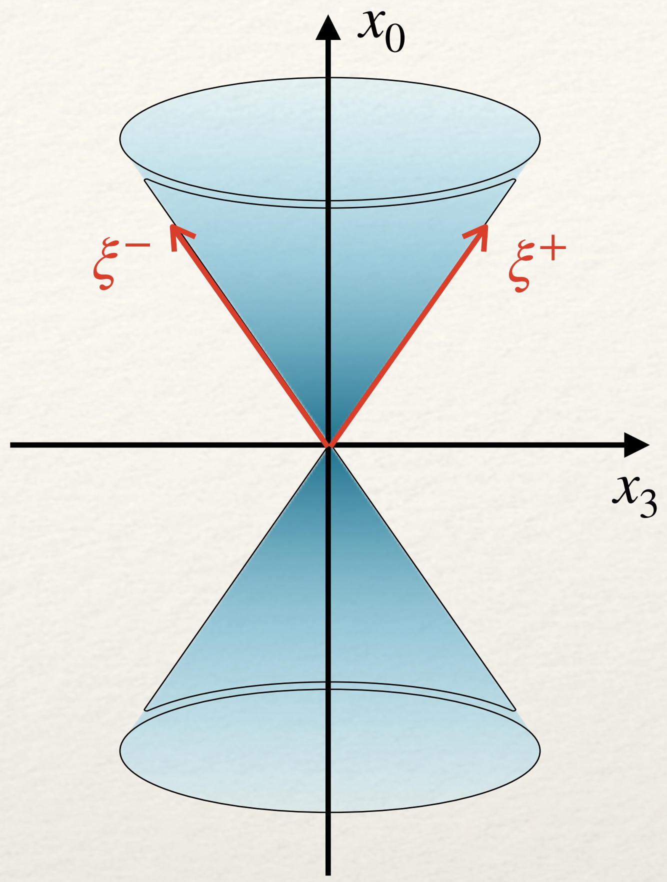
pseudo-PDFs | Matrix elements

Unpolarized PDF: non-local matrix element on the light-cone in Minkowski space

$$q(x) = \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle N(P)|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|N(P)\rangle \qquad v^{\pm} = \frac{v^0 \pm v^3}{\sqrt{2}}$$

ξ^+, ξ^-: light-cone coordinates

light-cone distances shrink to origin in Euclidean space-time



Inaccessible on the lattice!

Reviews: K. Cichy, M. Constantinou arXiv: 1811.07248, M.Constantinou arXiv: 2010.02445

Methods for accessing x-dependence on the lattice:

- hadronic tensor K.F. Liu et.al. arXiv: hep-ph/9306299
- auxiliary quarks U. Aglietti et.al. arXiv: hep-ph/9806277, W. Detmold et al. arXiv: hep-lat/0507007, V. Braun et. al. arXiv: 0709.1348
- quasi-distributions X. Ji arXiv: 1305.1539
- "good lattice cross sections" Y-Q. Ma and J. Qiu arXiv: 1404.6860, 1412.2688, 1709.03018
- "OPE without OPE" A.J. Chambers et.al. arXiv: 1703.01153
- **pseudo-distributions**

  A. Radyushkin arXiv: 1705.01488, 1612.05170, 1710.08813, 1711.06031, 1801.02427, 1912.04244

Steps towards x-dependence within pseudo-PDF:

- compute appropriate matrix elements
- resolve divergences, renormalization
- perturbative evolution → light-cone distribution
- Fourier Transform: reconstruct x-dependence

pseudo-PDFs | Matrix elements

Euclidean space

Wilson line

boosted nucleon states

$$M^\alpha(P,z) \equiv \langle N(P)|\bar\psi(z)\gamma^\alpha W(z,0)\psi(0)|N(P)\rangle$$

vector current: unpolarized PDFs

Lorentz decomposition

$$M^\alpha(P,z) = 2P^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

leading twist
+ $\mathcal{O}(z^2\Lambda^2_{\rm QCD})$

higher twist

kinematics

$P = (E,0,0,P_z)$
$z = (0,0,0,z_3)$
$\alpha = 0$

Ioffe-time distributions

$$\mathcal{M}(\nu, z_3^2) \sim \langle N(P_z)|\bar\psi(z_3)\gamma^0 W(z_3,0)\psi(0)|N(P_z)\rangle$$

Lorentz invariant!
ν, z_3 : Lorentz scalars

$\nu \equiv P_z \cdot z_3$: "Ioffe time"

B. Ioffe, Phys.Lett. 30B, 123 (1969)

F.T.: Along constant z_3 line

$$\mathcal{M}(\nu, z_3^2) = \int_{-1}^{1} dx e^{i\nu x}\mathcal{P}(x, z_3^2) \Rightarrow \mathcal{P}(x, z_3^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, z_3^2)$$

pseudo-PDF

$$q(x) = \lim_{z_3\to 0}\mathcal{P}(x, z_3^2)$$ (ignoring divergences)

PDF

$\mathcal{M}(\nu, z_3^2)$ features:

- standard log. divergence
- power divergence (Wilson line)
- mult. renormalizable to all orders in PT

T. Ishikawa et.al. arXiv: 1609.02018
J.W. Chen et. al. arXiv: 1707.03107

Lattice evaluation | Correlation functions: “Distillation” method

Smearing: Increase overlap of hadron interpolating fields with ground state

$$\mathcal{X}_B(\vec{x},t) = \epsilon^{abc}\mathcal{S}_{\sigma_1\sigma_2\sigma_3}(\mathcal{D}_1 q_1)^a_{\sigma_1}(\vec{x},t)(\mathcal{D}_2 q_2)^b_{\sigma_2}(\vec{x},t)(\mathcal{D}_3 q_3)^c_{\sigma_3}(\vec{x},t) \qquad q_{\rm sm}(\vec{x},t) = \mathcal{F}[U(\vec{x},t);\vec{y}]q(\vec{y},t)$$

Standard techniques: Gauge-covariant, based on 3D lattice Laplacian

$$-\nabla^2_{\vec{x},\vec{y}}(t) = 6\delta_{\vec{x}\vec{y}} - \sum_{j=1}^{3}\left[U_j(\vec{x},t)\delta_{\vec{x}+\hat{j},\vec{y}} + U_j^\dagger(\vec{x}-\hat{j},t)\delta_{\vec{x}-\hat{j},\vec{y}}\right] \qquad \mathcal{F}_{\alpha_s,N_s} = \left(1+\frac{\alpha_s\nabla^2}{N_s}\right)^{N_s}$$

M. Peardon et. al. arXiv: 0905.2160

Distillation: Smearing operator → low-rank eigenvector representation

$$\Box_{\vec{x},\vec{y}}(t) \equiv \sum_{k=1}^{N} v_k(\vec{x},t)v_k^\dagger(\vec{y},t) = V(t)V^\dagger(t)\ ,\ -\nabla^2 v_k = \lambda_k v_k \ \longrightarrow\ q_{\rm sm}(\vec{x},t) = \Box_{\vec{x},\vec{y}}(t)q(\vec{y},t)$$

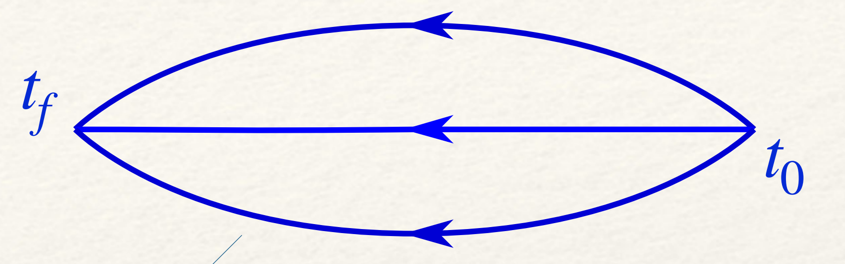
perambulators: $\tau(t_f,t_0) = V^\dagger(t_f)M^{-1}(t_f,t_0)V(t_0)$

elementals: $\Phi^{i,j,k}_{\sigma_1\sigma_2\sigma_3}(t) = \epsilon^{abc}\mathcal{S}_{\sigma_1\sigma_2\sigma_3}(\mathcal{D}_1 v_i(t))^a(\mathcal{D}_2 v_j(t))^b(\mathcal{D}_3 v_k(t))^c$

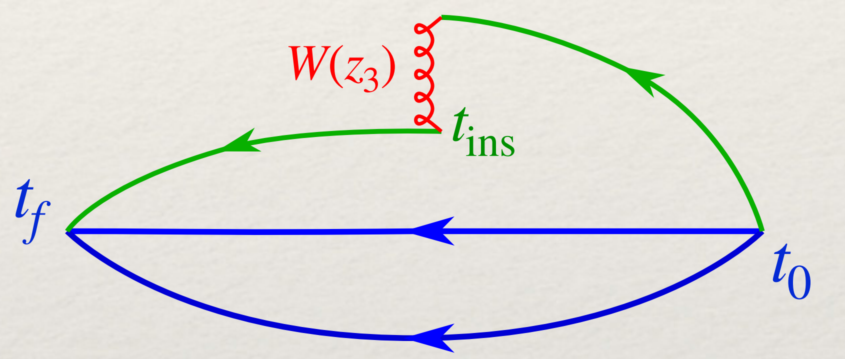
gen. perambulators: $\tilde{\tau}(t_f,t_{\rm ins},t_0) = V^\dagger(t_f)M^{-1}(t_f,t_{\rm ins})\Gamma(t_{\rm ins})M^{-1}(t_{\rm ins},t_0)V(t_0)$

inversions: $S^{(i)}_\alpha(y;t_0) = M^{-1}(y;\vec{z},t)_{\alpha\beta}v_i(\vec{z})\delta_{\beta\beta_0}\delta_{tt_0} = M^{-1}(y;\vec{z},t_0)_{\alpha\beta_0}v_i(\vec{z})$

$$C_{\rm 2pt}(t_f,t_0) = \sum_{\vec{x},\vec{y}}\langle\mathcal{X}(\vec{x},t_f)\bar{\mathcal{X}}(\vec{y},t_0)\rangle$$



$$C_{\rm 3pt}(t_f,t_{\rm ins},t_0;\Gamma) = \sum_{\vec{x},\vec{z},\vec{y}}\langle\mathcal{X}(\vec{x},t_f)\ [\bar{q}\gamma^0 W q](\vec{z},t)\ \bar{\mathcal{X}}(\vec{y},t_0)\rangle$$



Pros:

- factorization of correlation functions
- various spin structures *without* additional inversions
- mom. projection at sink *and* source
- reusability of building blocks

Cons:

- extra step: compute eigenvectors of ∇^2
- big files/storage

Lattice evaluation | Simulation details

Lattice action / configurations: W&M / JLab

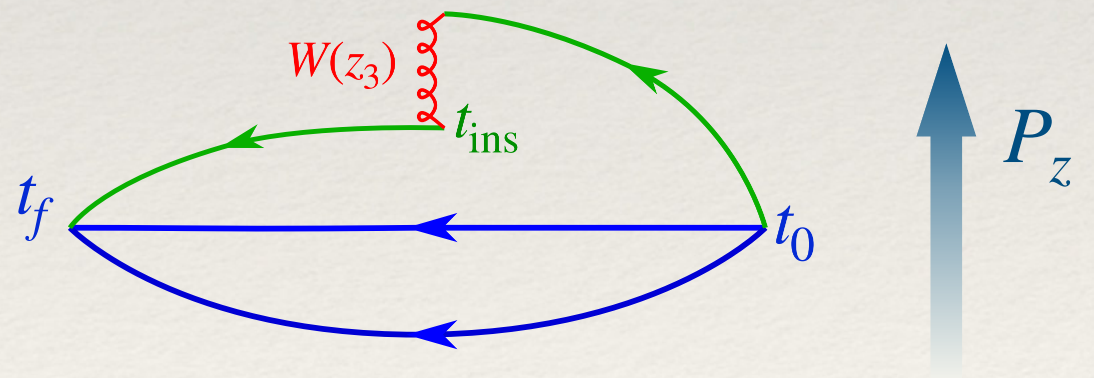
- Wilson-Clover fermion action, $N_f = 2 + 1$
- lattice volume: $32^3 \times 64$
- $\beta = 6.3\,,\, a = 0.091\,\mathrm{fm}\,(a^{-1} = 2.16\,\mathrm{GeV})$
- $m_\pi = 0.350\,\mathrm{GeV}, m_N = 1.160\,\mathrm{GeV}$
- $L = 2.9\,\mathrm{fm}, m_\pi L = 5.15$

Runs performed at JLab computer clusters and at Frontera (TACC) using the Chroma software suite and in-house analysis packages

Parameters and statistics:

- 350 gauge configurations
- 4 time sources $t_0/a = [t_0^i, +T/4, +T/2, +3T/4]$
- boost momentum $P_z = 2\pi/L \times [0, \pm 1, \ldots, \pm 6] \rightarrow 0\ldots 2.55\,\mathrm{GeV}$
- displacements $z_3 = 0, \pm 1a, \ldots, \pm 12a \rightarrow 0\ldots 1.1\,\mathrm{fm}$
- **5600 statistics**
- distillation vectors: $N_{ev} = 64$
- 6 $t_{sep} \equiv (t_f - t_0) = [4a, 6a, \ldots, 14a] \rightarrow 0.4\ldots 1.3\,\mathrm{fm}$

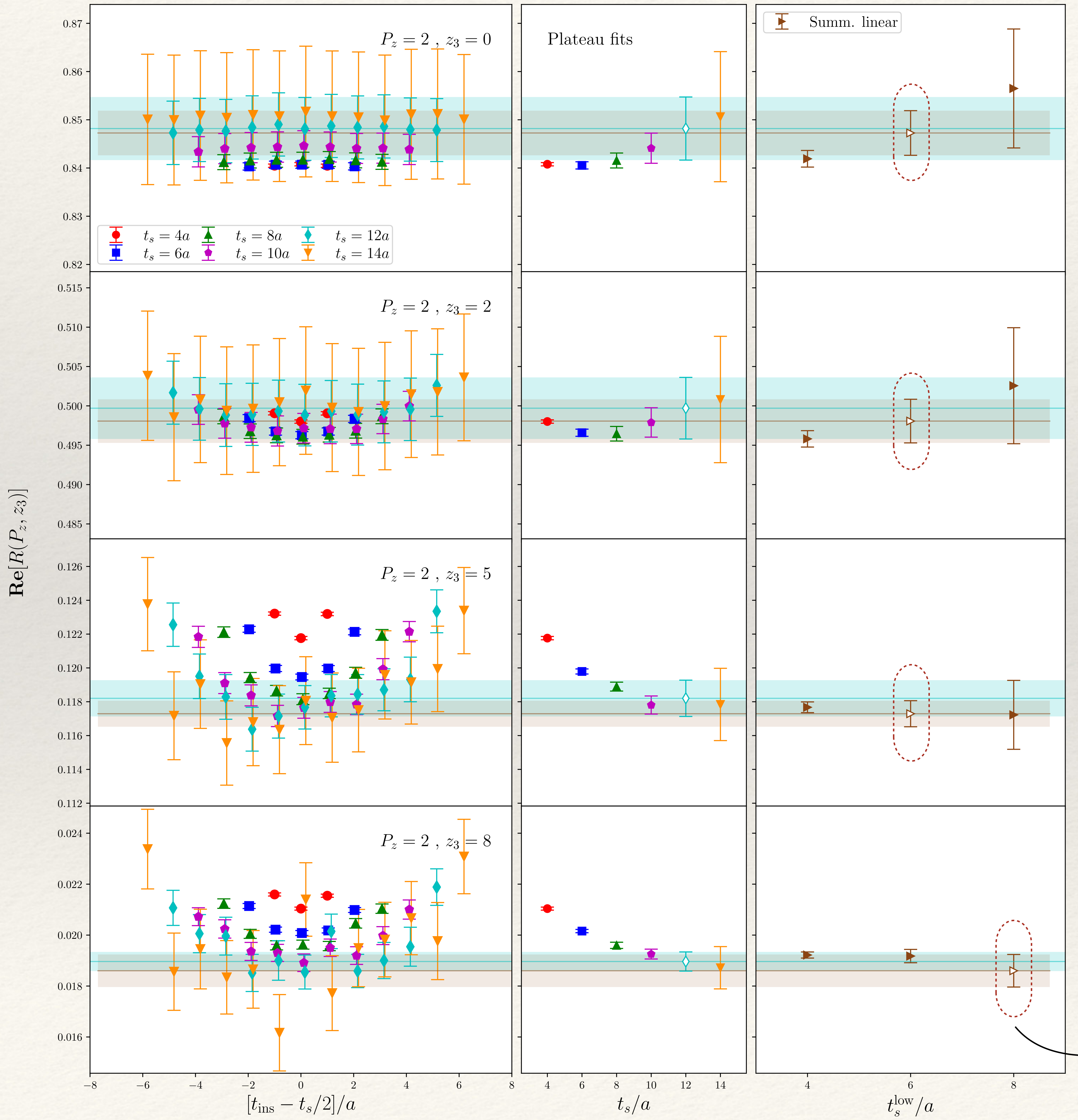
$$C_{3\mathrm{pt}}(t_f, t_{\mathrm{ins}}, t_0; \Gamma) = \sum_{\vec{x},\vec{z},\vec{y}} \langle \mathcal{X}(\vec{x}, t_f)\, [\bar{q}\gamma^0 W q](\vec{z}, t)\, \bar{\mathcal{X}}(\vec{y}, t_0)\rangle$$



Lattice evaluation | Matrix elements: reaching the ground state

Construct ratio: $R(P_z, z_3; t_{sep}, t_{ins}) = \dfrac{C_{3\text{pt}}(P_z, z_3; t_{sep}, t_{ins})}{C_{2\text{pt}}(P_z, t_{sep})} \xrightarrow[t_{ins}-t_0 \gg a]{t_{sep}-t_{ins} \gg a} \mathcal{M}(\nu, z_3^2)$

Valence quark distribution → Real part



Two methods for ensuring ground state dominance

i) Plateau method
ii) Summation method

Perform fits from which the matrix element is extracted (ask for more!)

Choose summation method results as final

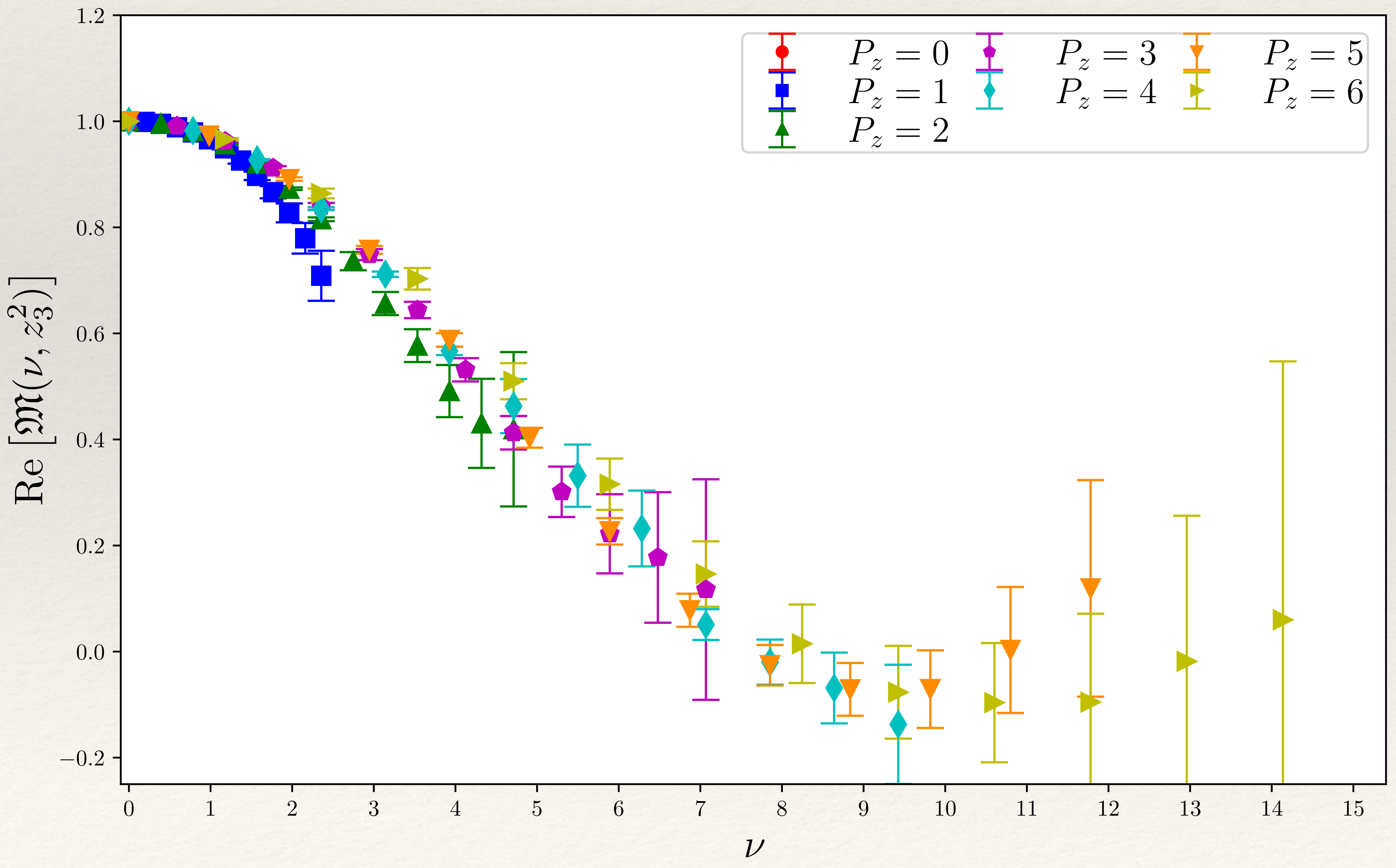
final value for $\mathcal{M}(P_z z_3, z_3^2)$

Lattice evaluation | Ioffe-time Distributions

Cancel divergences - Renormalization

Divide $\mathcal{M}(P_z z_3, z_3^2)$ with appropriate factor $D(z_3)$:

- $D(0) = 1$
- similar z_3-dependence
- no ν-dependence ($P_z = 0$)
- good choice: $\mathcal{M}(0, z_3^2)$



K. Orginos et.al. arXiv: 1706.05373
J. Karpie et. al. arXiv: 1710.08288
J. Karpie et. al. arXiv: 1807.10933

$$\mathfrak{M}(\nu, z_3^2) = \frac{\mathcal{M}(P_z z_3, z_3^2)}{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0}}$$

V^0 renormalization

$$Z_V(P_z) = \frac{1}{\mathcal{M}(P_z z_3, z_3^2)|_{z_3=0}}$$

$$\mathfrak{M}(\nu, z_3^2) = \left(\frac{\mathcal{M}(P_z z_3, z_3^2)}{\mathcal{M}(P_z z_3, z_3^2)|_{z_3=0}}\right) \times \left(\frac{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0, z_3=0}}{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0}}\right)$$

reduced Ioffe-time Distribution

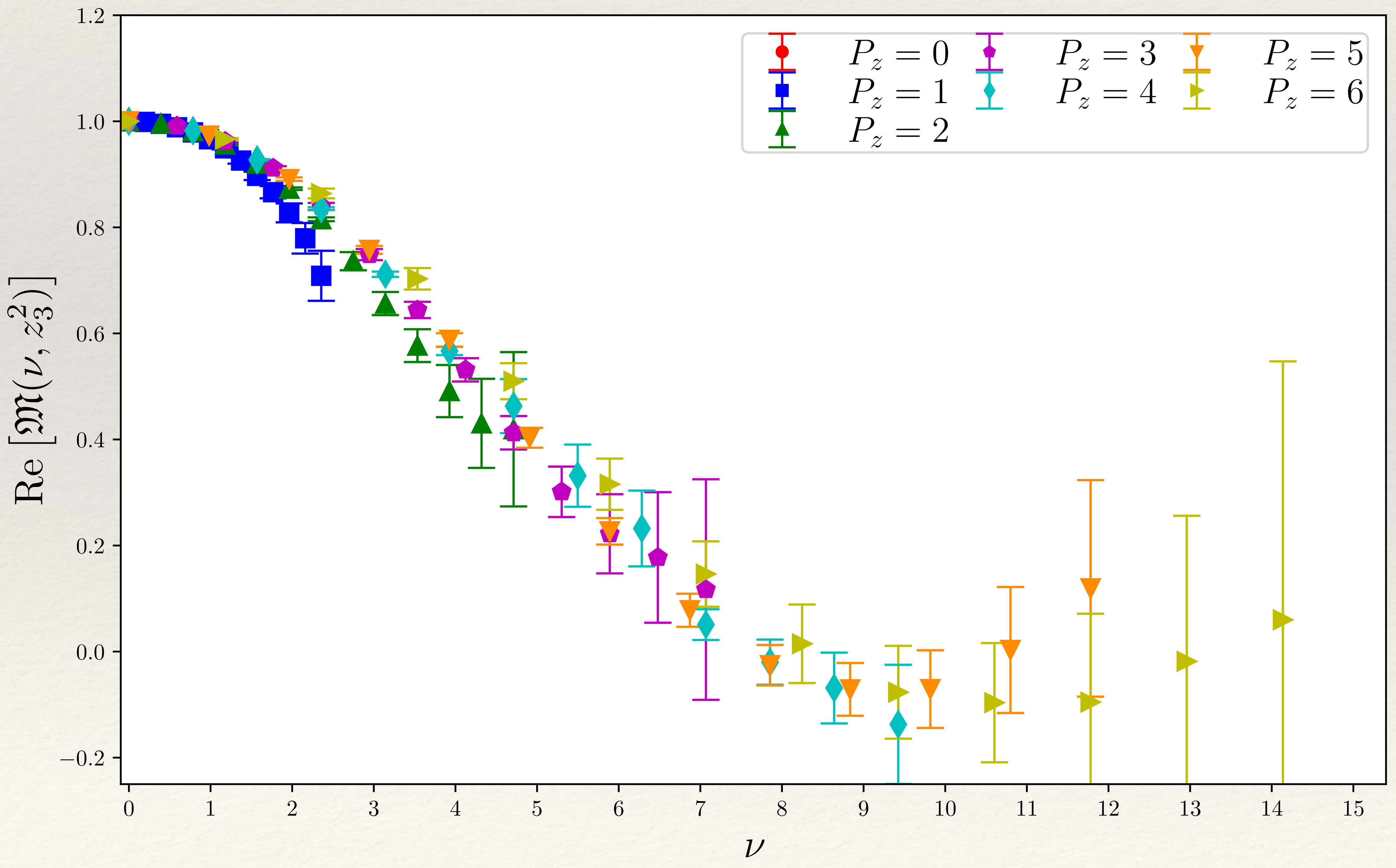
- cancel mult. factor from gauge link renormalization
- higher-twist contaminations $\mathcal{O}(z_3^2 \Lambda_{QCD}^2)$ partially suppressed

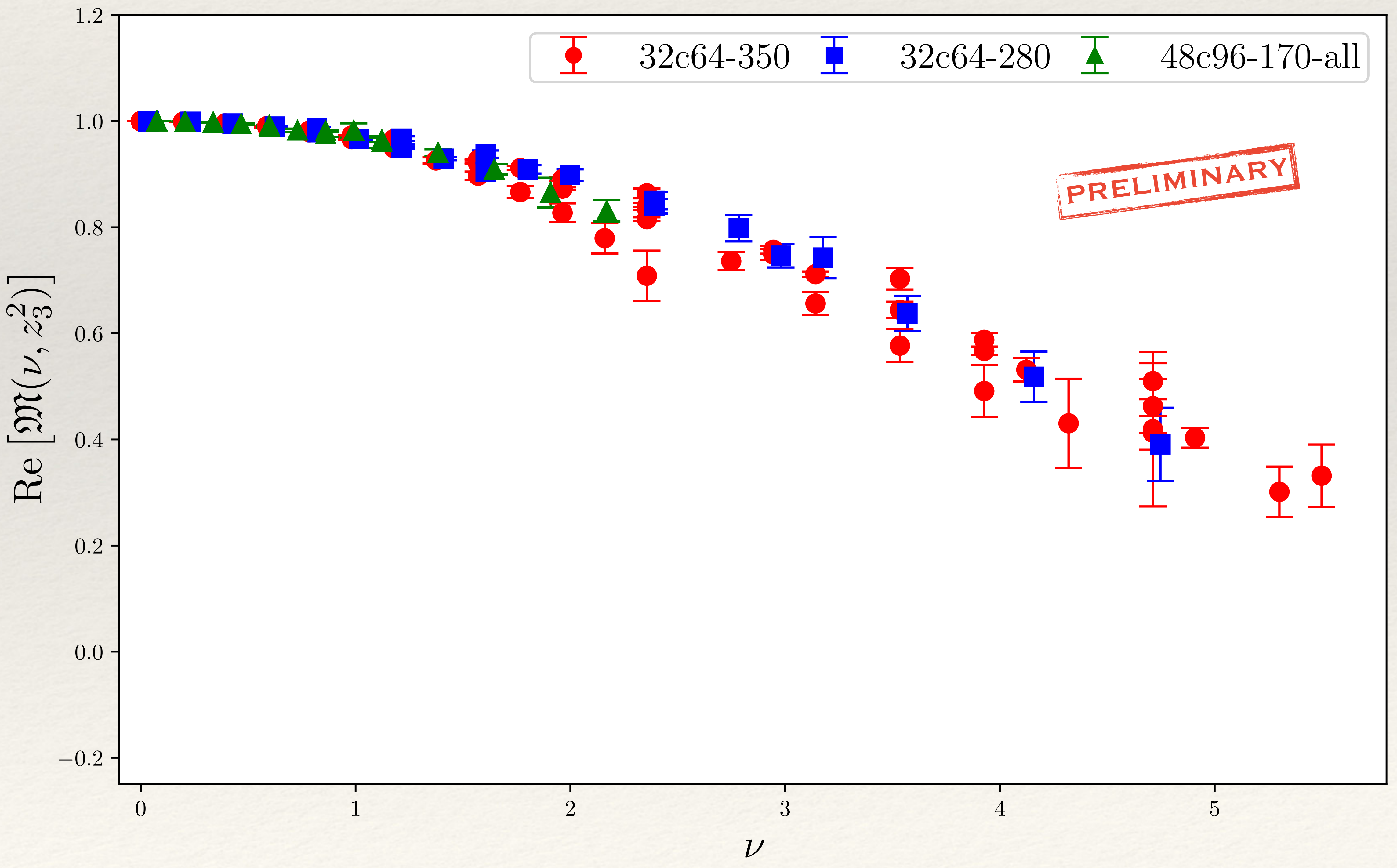
Lattice evaluation | Ioffe-time Distributions

Two more ensembles on the way

$$\mathfrak{M}(\nu, z_3^2) = \left(\frac{\mathcal{M}(P_z z_3, z_3^2)}{\mathcal{M}(P_z z_3, z_3^2)|_{z_3=0}} \right) \times \left(\frac{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0, z_3=0}}{\mathcal{M}(P_z z_3, z_3^2)|_{P_z=0}} \right)$$

	32c64-280	48c96-170
Lattice volume	$32^3 \times 64$	$48^3 \times 96$
Pion mass	$m_\pi = 280$ MeV	$m_\pi = 170$ MeV
Physical volume	$L = 2.9$ fm	$L = 4.4$ fm
max P_z	$2\pi/L \times 3$	$2\pi/L \times 3$





Lattice evaluation | Perturbative evolution: Light-cone ITDs

$$\mathcal{Q}(\nu,\mu^2) = \mathfrak{M}(\nu,z^2) - \frac{\alpha_s C_F}{2\pi}\int_0^1 du\,\mathfrak{M}(u\nu,z^2)\left[\ln\left(z^2\mu^2\frac{e^{2\gamma_E+1}}{4}\right)\mathbf{B}[u] + \mathbf{L}[u]\right]$$

light-cone ITD

evolve to common scale $1/z_0$

convert to $\overline{\mathrm{MS}}$ scheme

$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

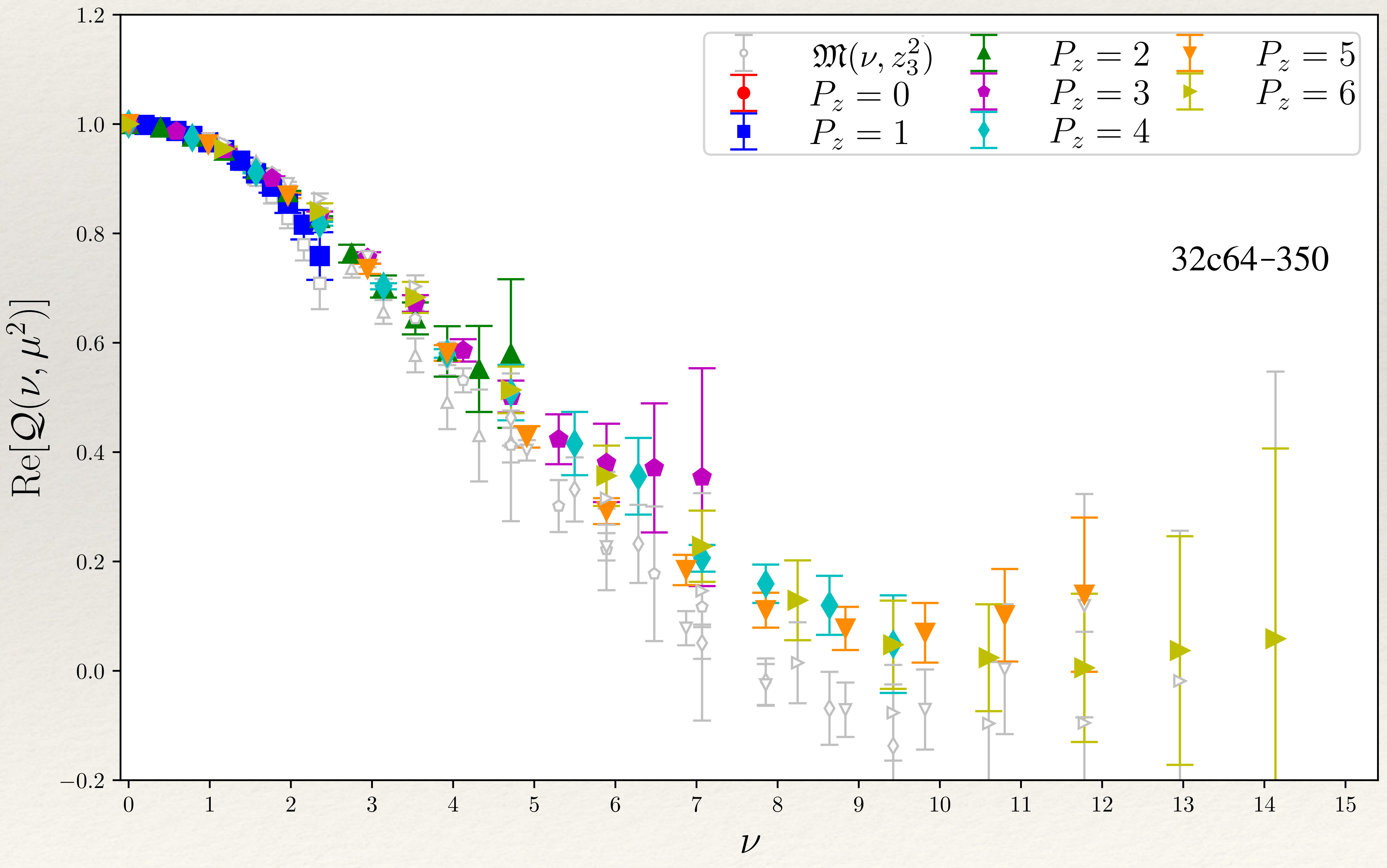
$$L(u) = \left[4\frac{\ln(1-u)}{1-u} - 2(1-u)\right]_+$$

$$\int_0^1 \mathfrak{M}(u\nu)[f(u)]_+ = \int_0^1 (\mathfrak{M}(u\nu) - \mathfrak{M}(\nu))\,f(u)$$

A. Radyushkin arXiv: 1801.02427
J.-H.Zhang et. al. arXiv: 1801.03023
T. Izubuchi et. al. arXiv: 1801.3917

Common scale: $z_0(\mu = 2\mathrm{GeV}) = 0.067\,\mathrm{fm}\,,\ \frac{1}{z_0} = 2.94\mathrm{GeV}$

- treat each z_3 independently
- need $\mathfrak{M}(u\nu,\mu^2) \to$ polynomial parametrization w.r.t. ν
- evolution/matching processes separately or combined can give rise to different systematics
- residual discrepancies: evolution fails? higher twist effects?



Full picture | PDF x-dependence

discrete data in ν-space

A. Radyushkin arXiv: 1705.01488

$$\mathcal{Q}(\nu,\mu^2) = \int_{-1}^{1} dx e^{i\nu x} q(x,\mu^2)$$

continuous function in x-space

Inverse problem! Way around:

- choose a physically meaningful functional form for $q(x,\mu^2)$
- introduce model bias in determination
- general form:

$$q_v(x) = \frac{x^\alpha(1-x)^\beta(1+\delta x)}{B(\alpha+1,\beta+1)+\delta B(\alpha+2,\beta+1)}$$

i) fit for $z_3 \le 8$

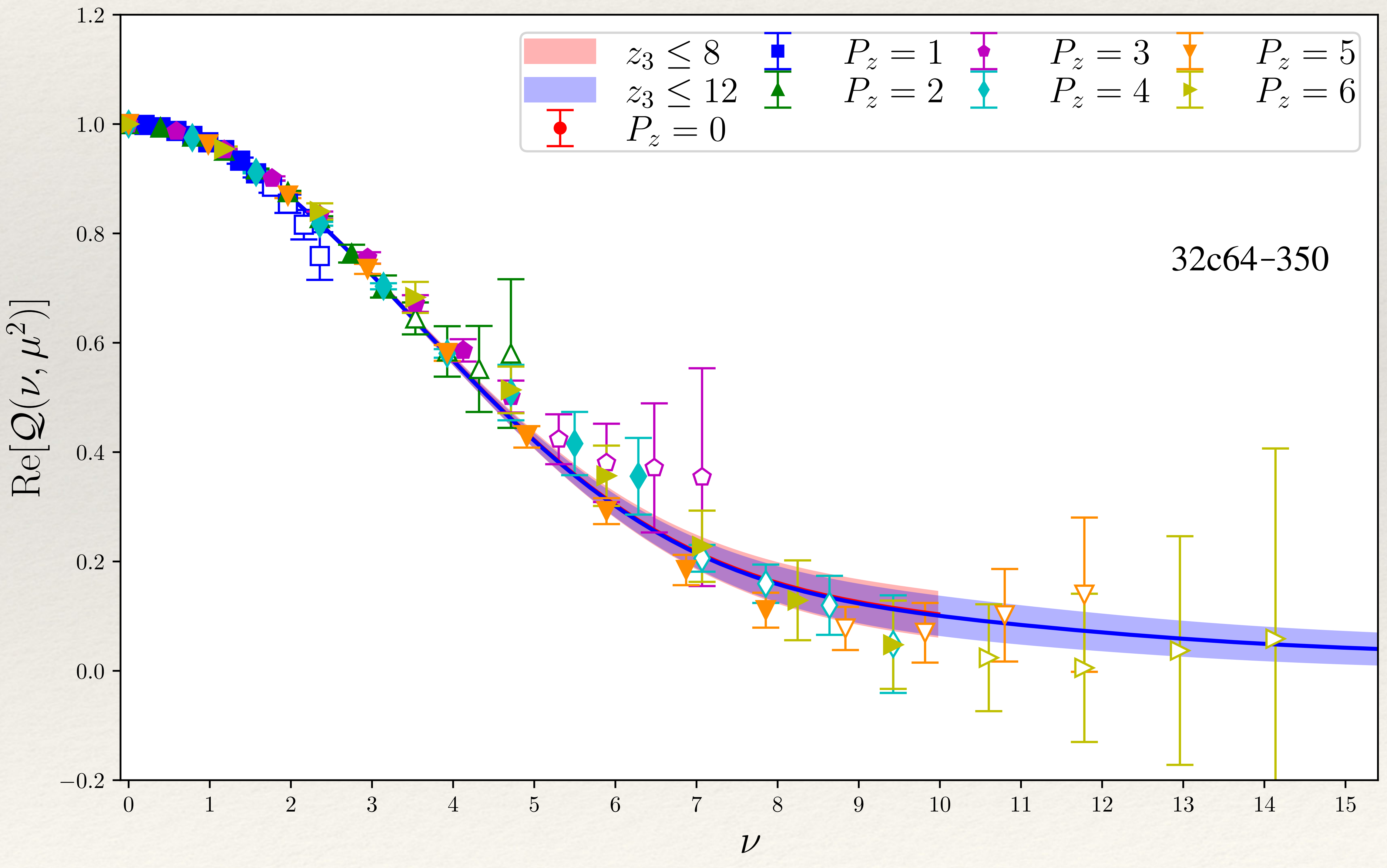
ii) fit for $z_3 \le 12$

$B(i,j)$: beta-function

$$\chi^2 = \sum_{\nu=0}^{\nu_{\max}} \frac{Q(\nu,\mu^2) - Q_{\rm FT}(\nu,\mu^2)}{\sigma_Q^2}\ , \ Q_{\rm FT}(\nu,\mu^2) = \int_0^1 dx \cos(\nu x) q_v(x)$$

valence quark distribution:

$$\mathrm{Re}\,\mathcal{Q}(\nu,\mu^2) = \int_0^1 dx \cos(\nu x) q_v(x,\mu^2)$$



Full picture | PDF x-dependence

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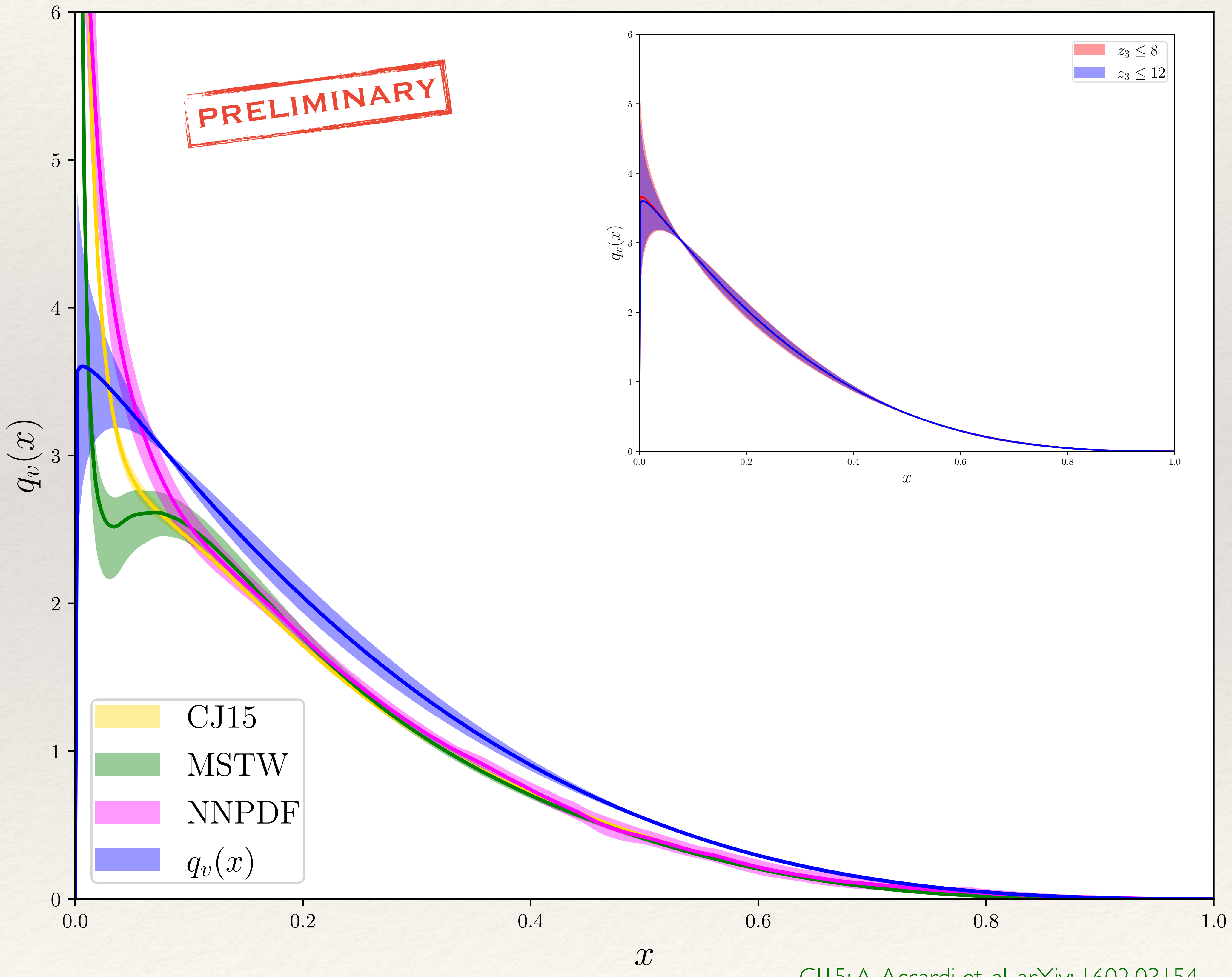


CJ15: A. Accardi et. al. arXiv: 1602.03154
MSTW: A.D.Martin etl al. arXiv: 0901.0002
NNPDF: R.D.Ball et. al. arXiv: 1706.00428

Summary & Outlook

- Lattice QCD via pseudo-PDF framework: in good position to give reliable insight on nucleon PDFs
  - Distillation: valuable alternative to standard smearing techniques
  - controllable statistical uncertainties, excited state effects
- Some stretch between lattice and phenomenological x-dependence exists:
  - lattice artifacts → finite volume and lattice spacing, non-physical simulations
  - parametrization of PDFs → be systematic: try model-independent approaches, compare with other lattice evaluations

On the horizon:

- finalize results from other ensembles → study volume, pion mass effects
- analyze ensembles with smaller lattice spacing → continuum extrapolation
- helicity, transversity distributions & GPDs on the way!
- coming soon: disconnected diagrams

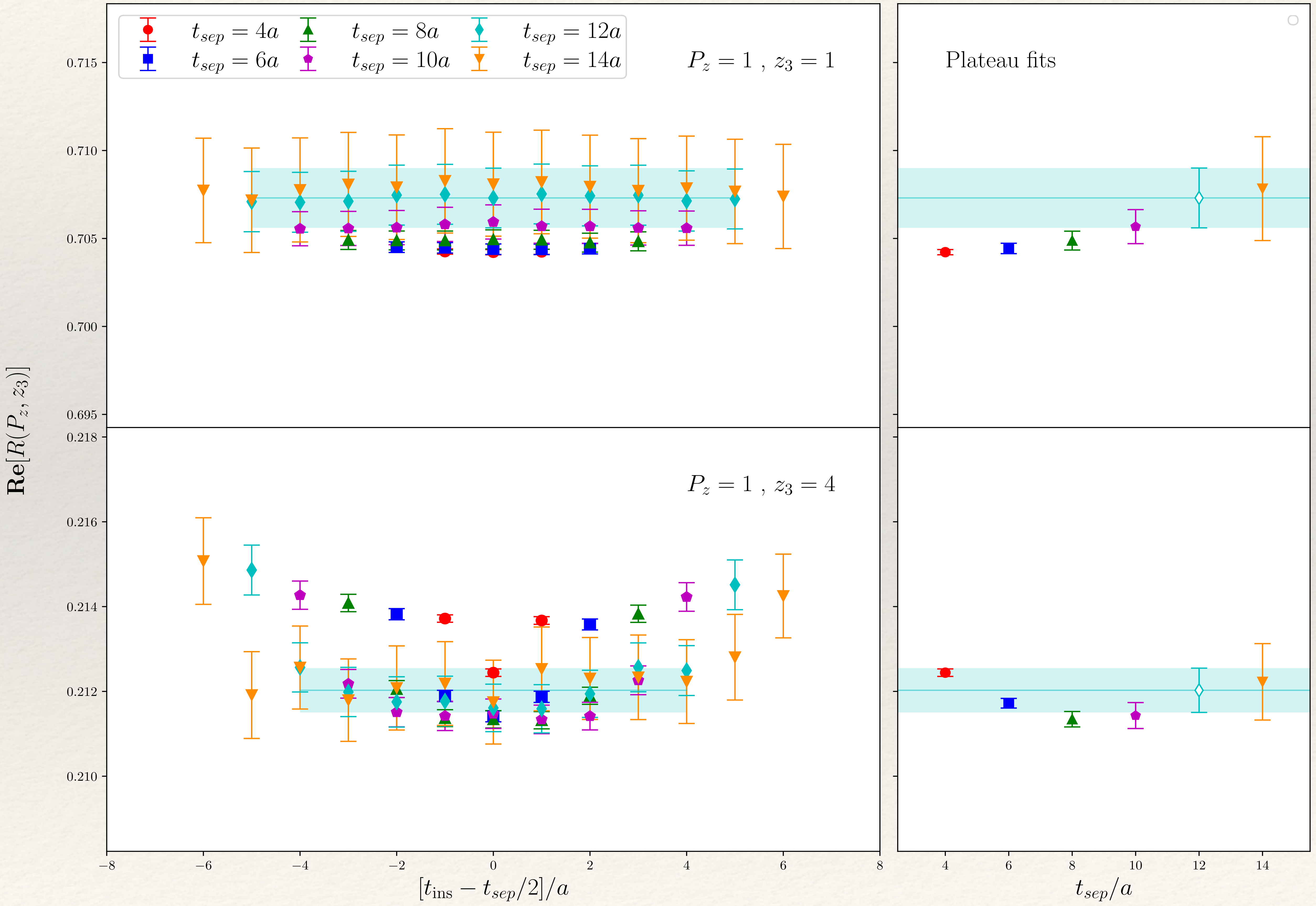
Bonus!

(Bonus!) Lattice evaluation | Matrix elements: reaching the ground state

Construct ratio: $R(P_z, z_3; t_{sep}, t_{ins}) = \dfrac{C_{3\text{pt}}(P_z, z_3; t_{sep}, t_{ins})}{C_{2\text{pt}}(P_z, t_{sep})} \xrightarrow[t_{ins}-t_0 \gg a]{t_{sep}-t_{ins} \gg a} \mathcal{M}(\nu, z_3^2)$

i) Plateau method

- for each t_{sep} , perform constant fits for multiple ranges
- decrease each fit range by one point on each side
- best fit: longest for which $\chi^2 \leq 0.9$



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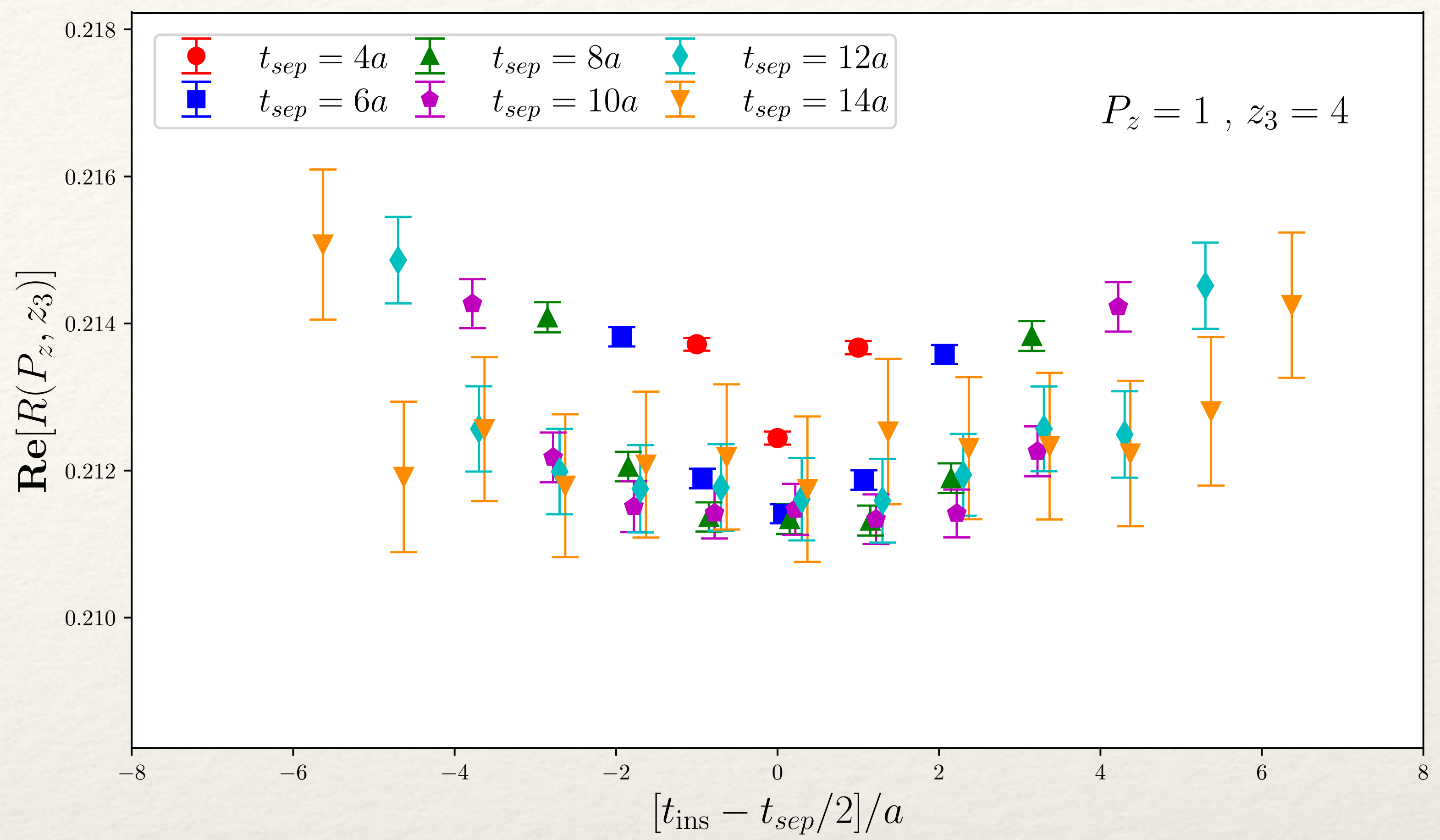
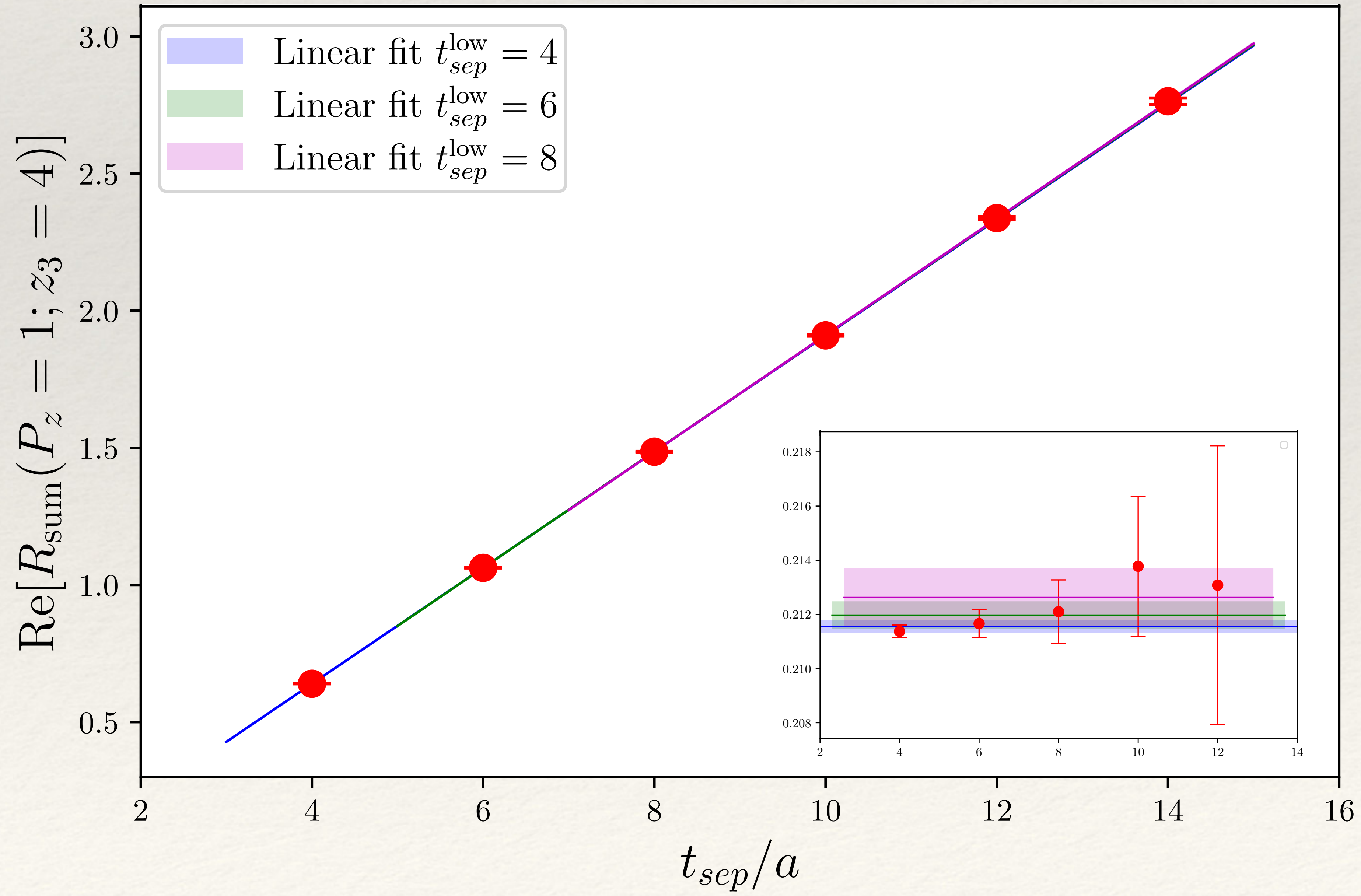


ii) Summation method

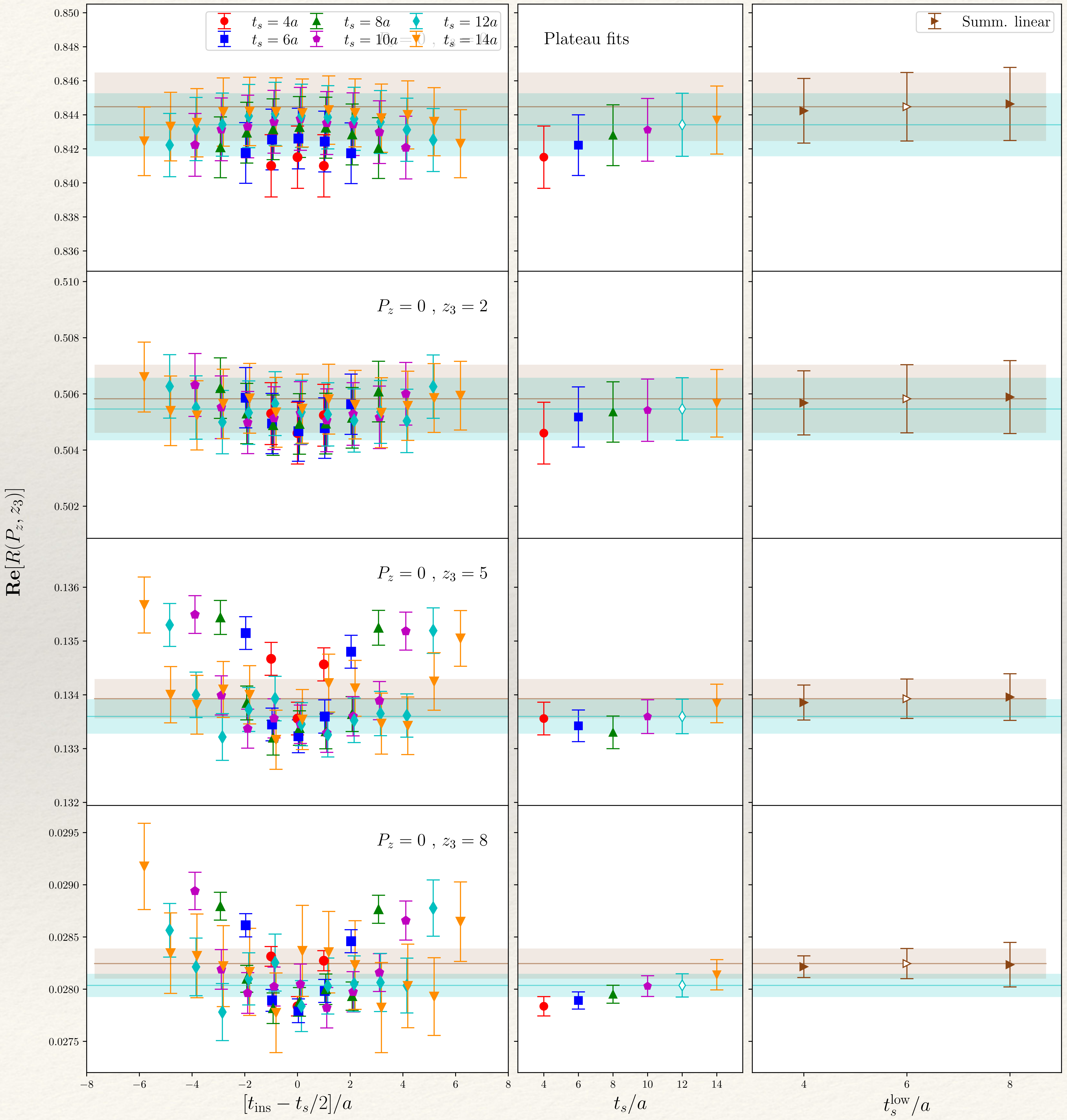
$$R_{\mathrm{sum}}(P_z, z_3; t_{sep}) = \sum_{t_{ins}=a}^{t_{sep}-a} R(P_z, z_3; t_{sep}, t_{ins})$$

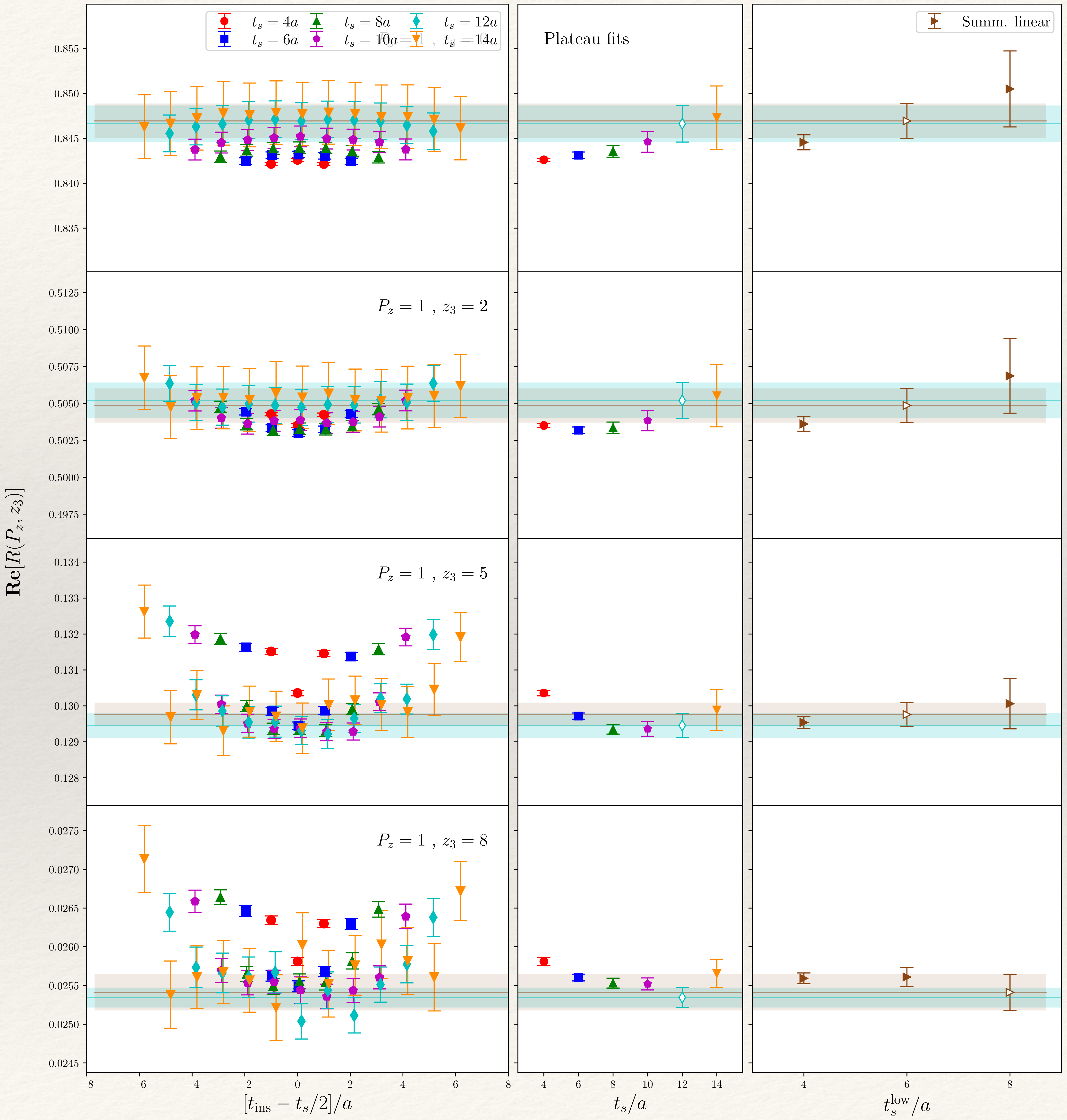
$$= c_1 + \mathcal{M}(P_z, z_3) \times t_{sep} + \mathcal{O}\left(e^{-\Delta E t_{sep}}\right)$$

Perform linear fits, increase lower value of t_{sep} until convergence



(Bonus!) Lattice evaluation | Matrix elements: select data





(Bonus!) Lattice evaluation | Ioffe-time Distributions (Imag)

Two more ensembles on the way

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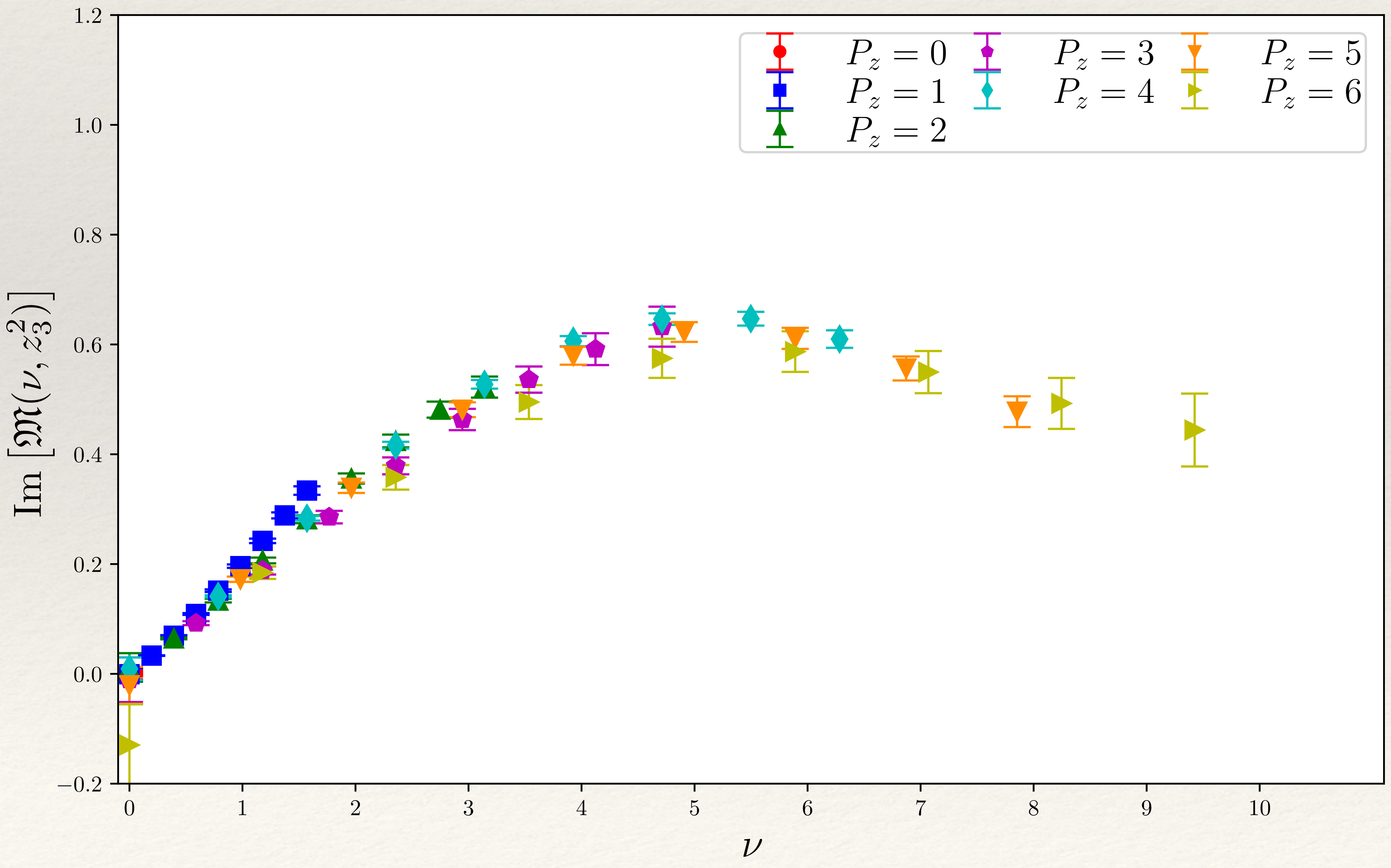

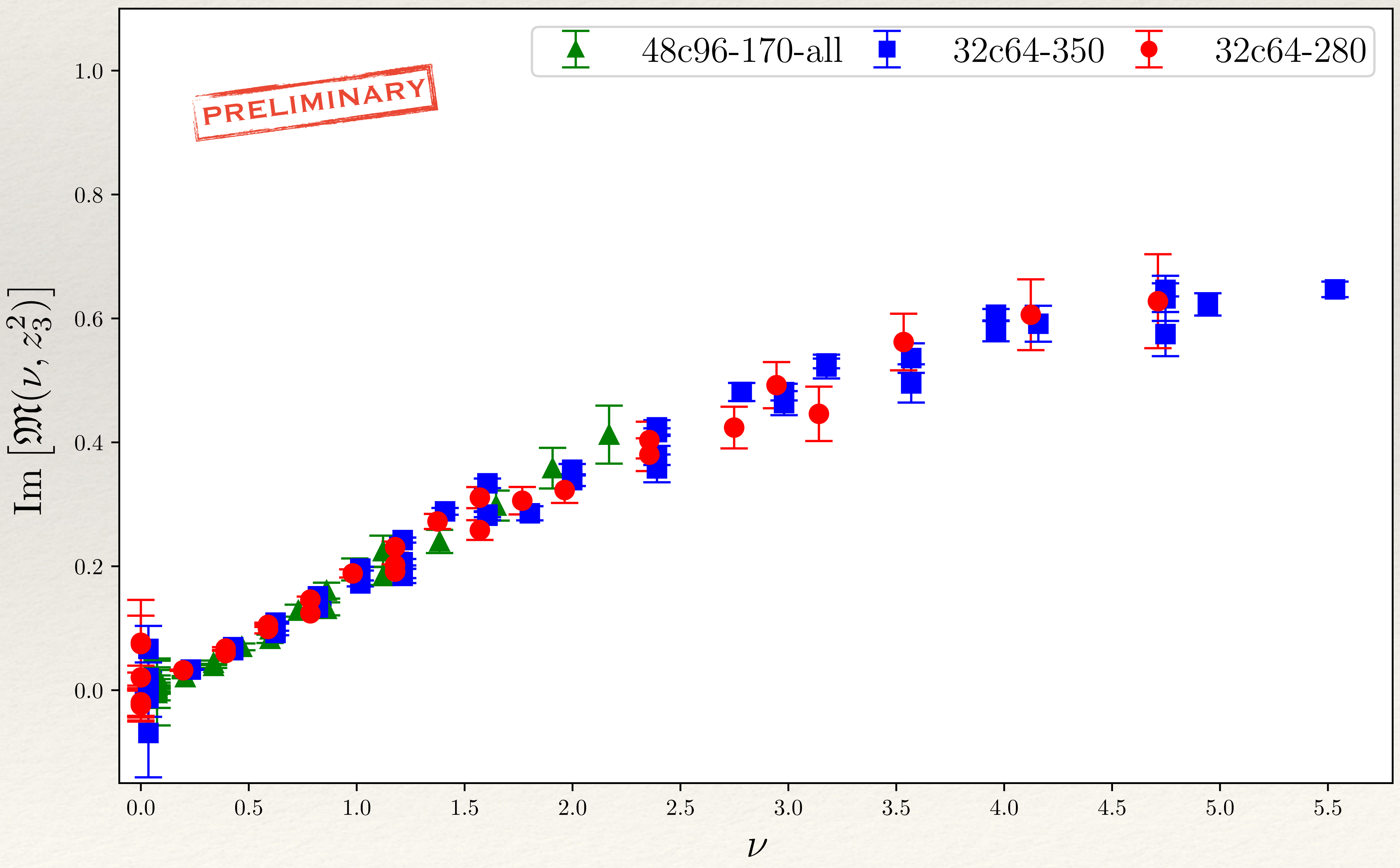

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light-cone ITD

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$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

$$L(u) = \left[4\frac{\ln(1-u)}{1-u} - 2(1-u)\right]_+$$

$$\int_0^1 \mathfrak{M}(u\nu)[f(u)]_+ = \int_0^1 (\mathfrak{M}(u\nu) - \mathfrak{M}(\nu))\, f(u)$$

A. Radyushkin arXiv: 1801.02427
J.-H.Zhang et. al. arXiv: 1801.03023
T. Izubuchi et. al. arXiv: 1801.3917

Common scale: $z_0(\mu = 2\mathrm{GeV}) = 0.067\,\mathrm{fm}\,,\quad \frac{1}{z_0} = 2.94\mathrm{GeV}$

- treat each z_3 independently
- need $\mathfrak{M}(u\nu,\mu^2) \rightarrow$ polynomial parametrization w.r.t. ν
- evolution/matching processes separately or combined can give rise to different systematics
- residual discrepancies: evolution fails? higher twist effects?