Systematic Extraction of Pion Parton Distributions Using Threshold Resummation and Applications to Pion TMDs

Patrick Barry, Nobuo Sato, Wally Melnitchouk, Chueng-Ryong Ji, Leonard Gamberg, and Astrid Hiller Blin

DIS 2021: Structure function and parton densities 4/15/2021

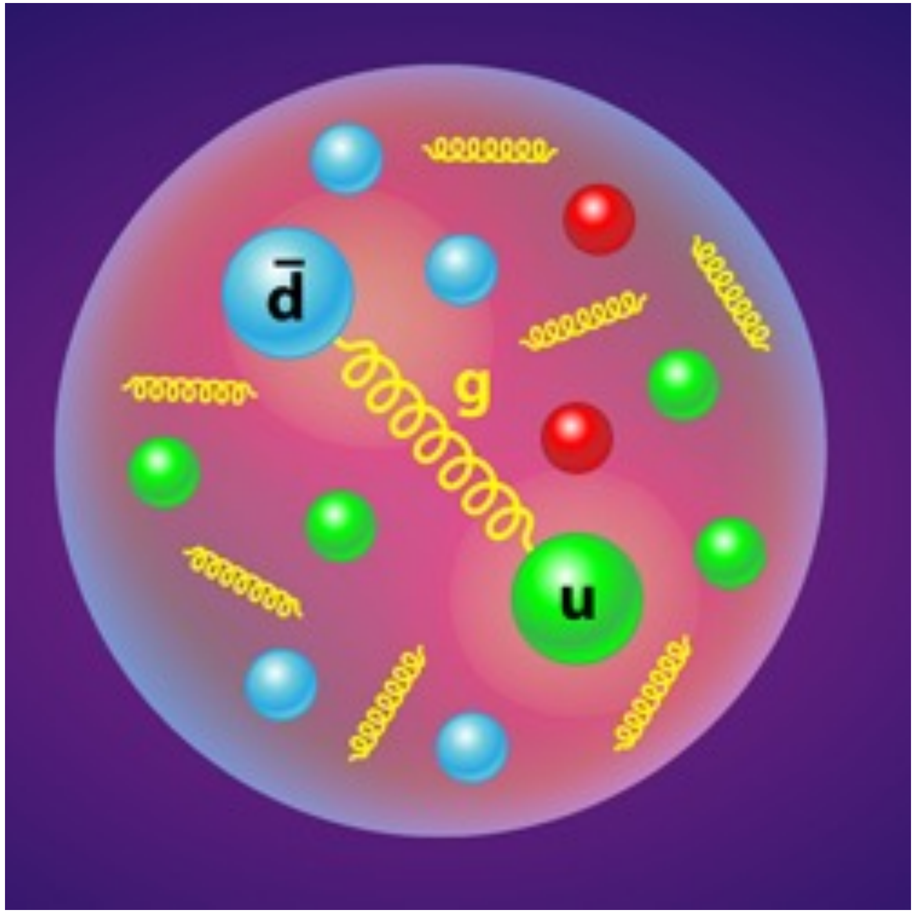
Motivation

What to do:

- Define a structure of hadrons in terms of quantum field theories
- Identify theoretical observables that factorize into non-perturbative objects and perturbatively calculable physics
- Perform global QCD analysis as structures are universal and are the same in all subprocesses

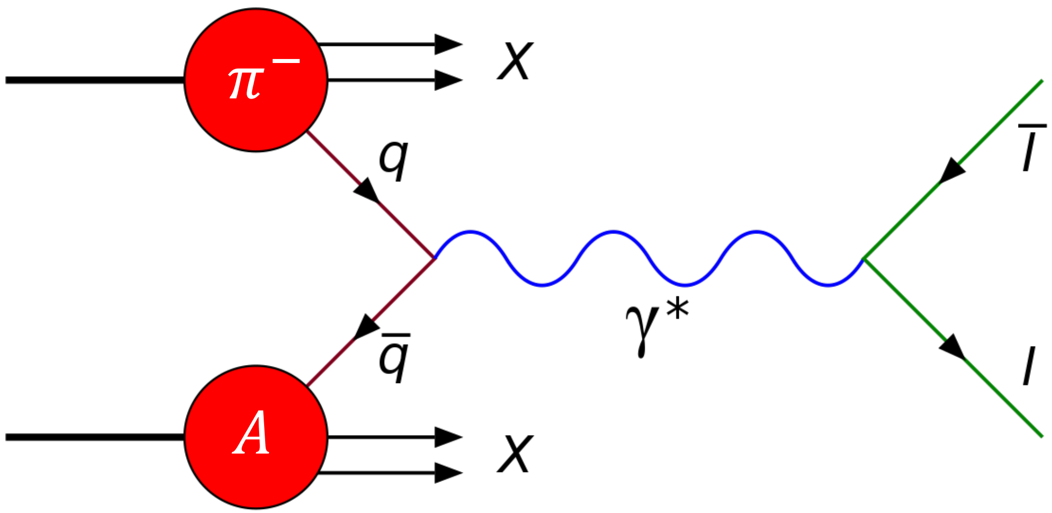
Pions

- Pion is the Goldstone boson associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry
- Lightest hadron as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies
- Simultaneously a pseudoscalar meson made up of q and $\bar{q}$ constituents



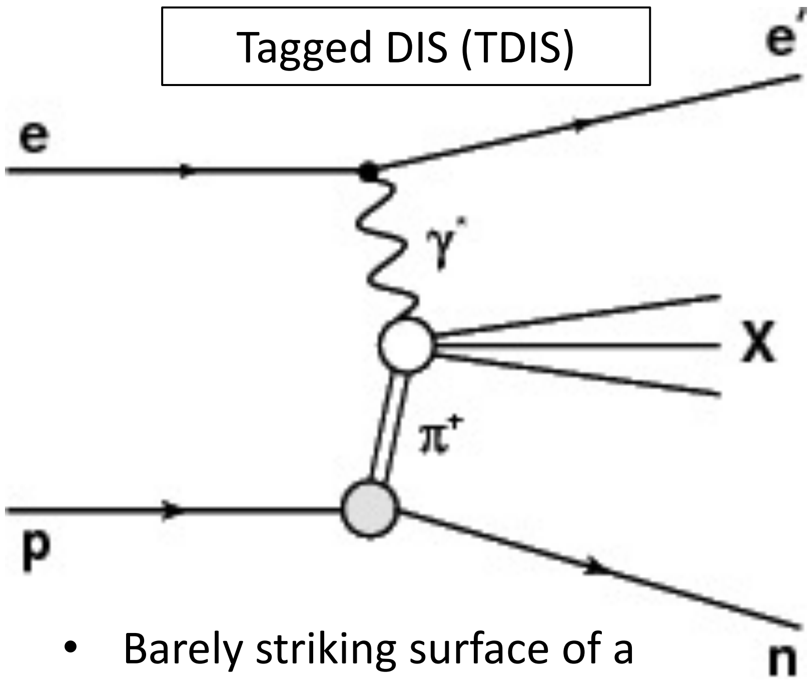
Experiments to Probe Pion Structure

- Drell-Yan (DY)



- Accelerating pion allows for time dilation and longer lifetime

- Leading Neutron (LN)



- Barely striking surface of a target proton knocks out an almost on-shell pion to probe

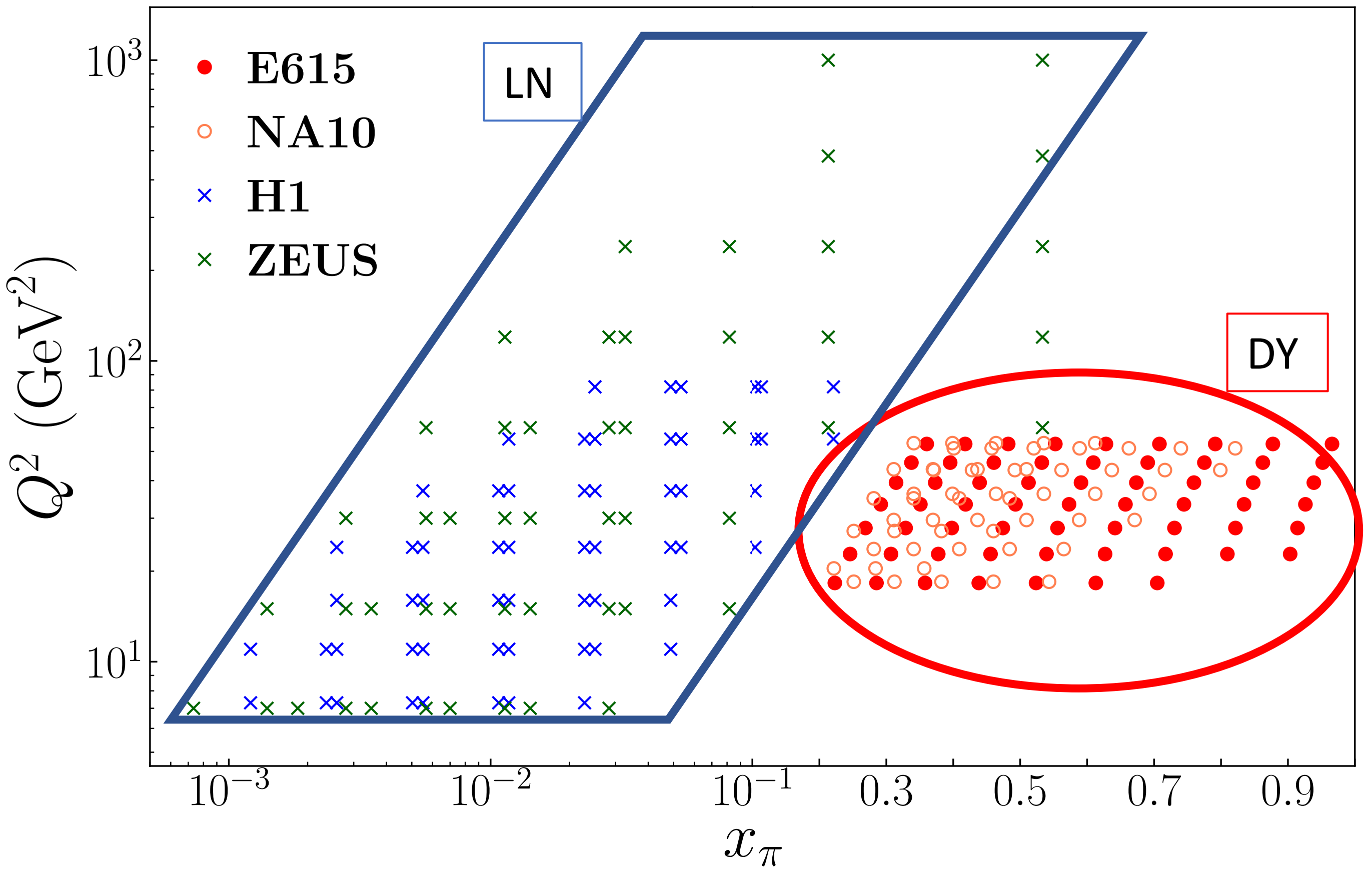
Datasets -- Kinematics

- Large x_π -- Drell-Yan (DY)
- Small x_π -- Leading Neutron (LN)
- Not much data overlap
- In DY:

$$x_\pi = \frac{1}{2}\left(x_F + \sqrt{x_F^2 + 4\tau}\right)$$

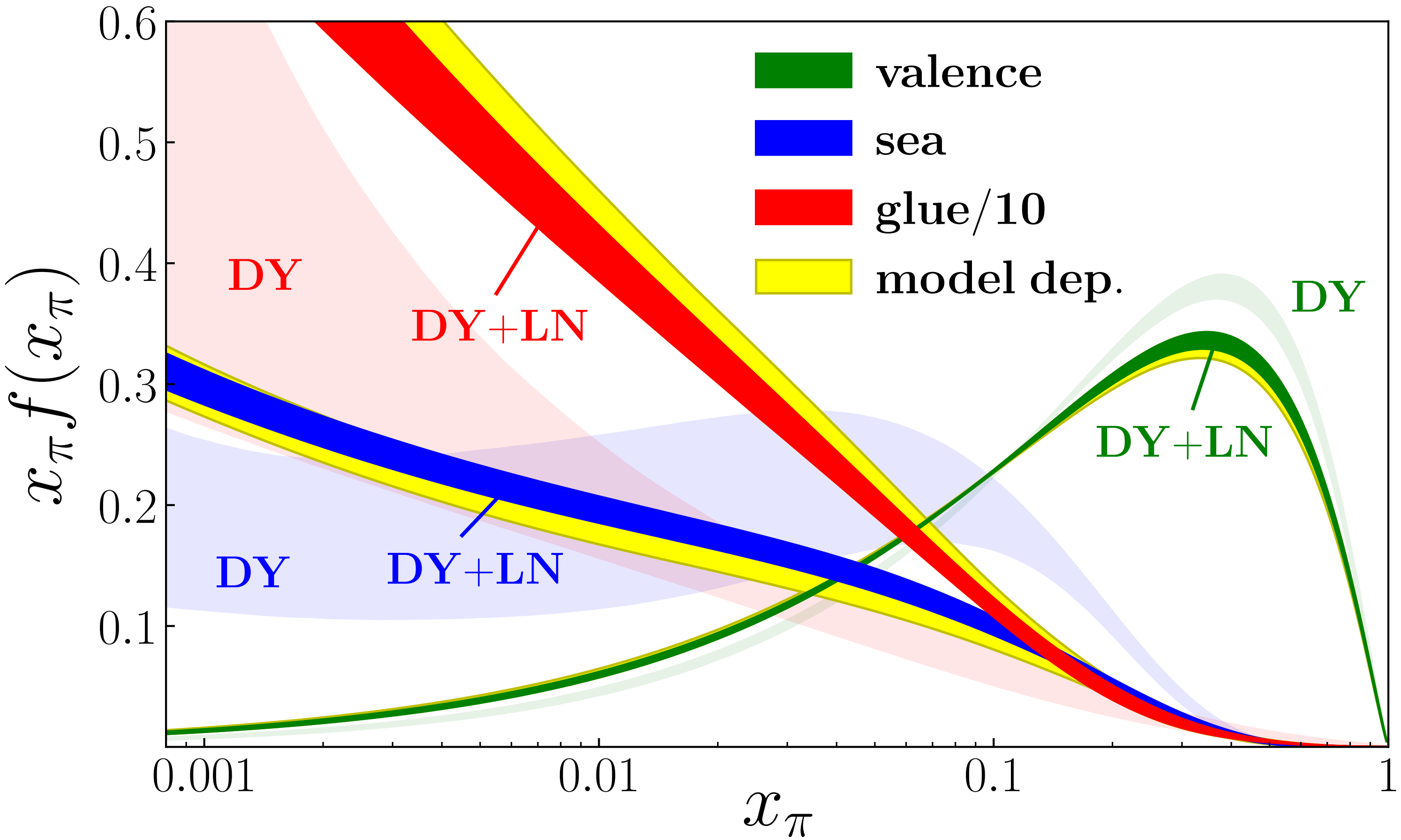
- In LN:

$$x_\pi = x_B/\bar{x}_L$$



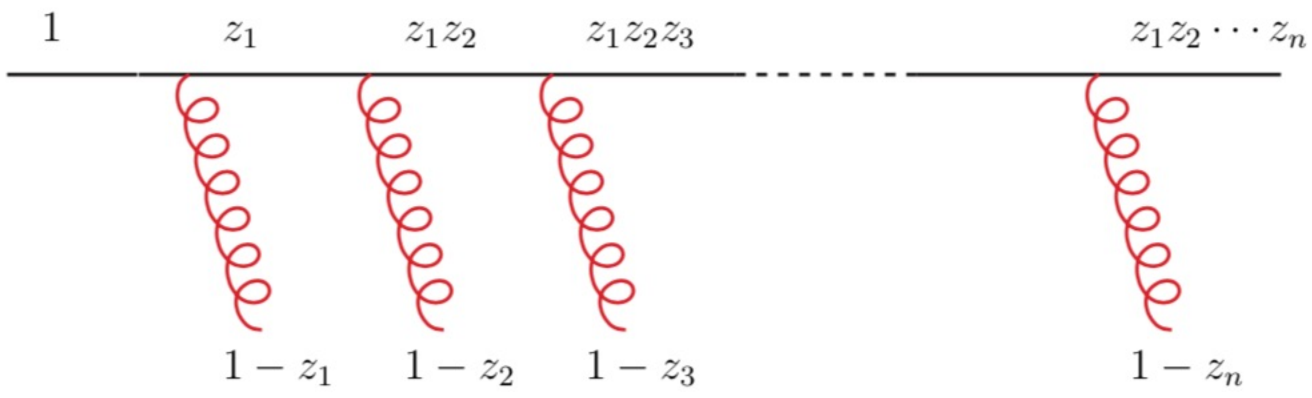
JAM18 Pion PDFs

- Lightly shaded bands – only Drell-Yan data
- Darkly shaded bands – fit to both Drell-Yan and LN data



PCB, N. Sato, W. Melnitchouk and Chueng-Ryong Ji, Phys. Rev. Lett. **121**, 152001 (2018).

Soft Gluon Resummation



- Fixed-target Drell-Yan notoriously has large-x_F contamination of higher orders
- Large logarithms may spoil perturbation
- Focus on corrections to the most important $q\bar{q}$ channel
- Resum contributions to all orders of α_s

Issues with Perturbative Calculations

$$\hat{\sigma} \sim \delta(1-z) + \alpha_S(\log(1-z))_+ \longrightarrow \hat{\sigma} \sim \delta(1-z)[1 + \alpha_S \log(1-\tau)]$$

- If τ is large, can potentially spoil the perturbative calculation
- Improvements can be made by resumming $\log(1-z)_+$ terms

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

An NLO calculation gathers the $\mathcal{O}(\alpha_S)$ terms

	LL	NLL	…	N^{p}LL
LO	1	--	…	--
NLO	$\alpha_S \log(N)^2$	$\alpha_S \log(N)$	…	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2(\log(N)^2, \log(N)^3)$	…	--
…	…	…	…	…
N^{k}LO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k\left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	…	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Add the columns to the rows

	LL	NLL	…	N^pLL
LO	1	--	…	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	…	--
NNLO	$\alpha_s^2 \log(N)^4$	$\alpha_s^2 (\log(N)^2 , \log(N)^3)$	…	--
…	…	…	…	…
N^kLO	$\alpha_s^k \log(N)^{2k}$	$\alpha_s^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	…	$\alpha_s^k \log(N)^{2k-2p} + \cdots$

barryp@jlab.org

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Make sure only counted once!
- Subtract the matching

	LL	NLL	...	N^p^LL
LO	1	--	...	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	...	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2(\log(N)^2, \log(N)^3)$	...	--
...	...	...	...	...
N^k^LO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k\left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	...	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Origin of Landau Pole

$$\alpha_S C^{(1)}_{\text{soft}}(N) = 2\frac{C_F}{\pi}\int_0^1 dz \frac{z^{N-1}-1}{1-z}\int_{Q^2}^{(1-z)^2Q^2}\frac{dk_\perp^2}{k_\perp^2}\alpha_S(k_\perp^2)$$

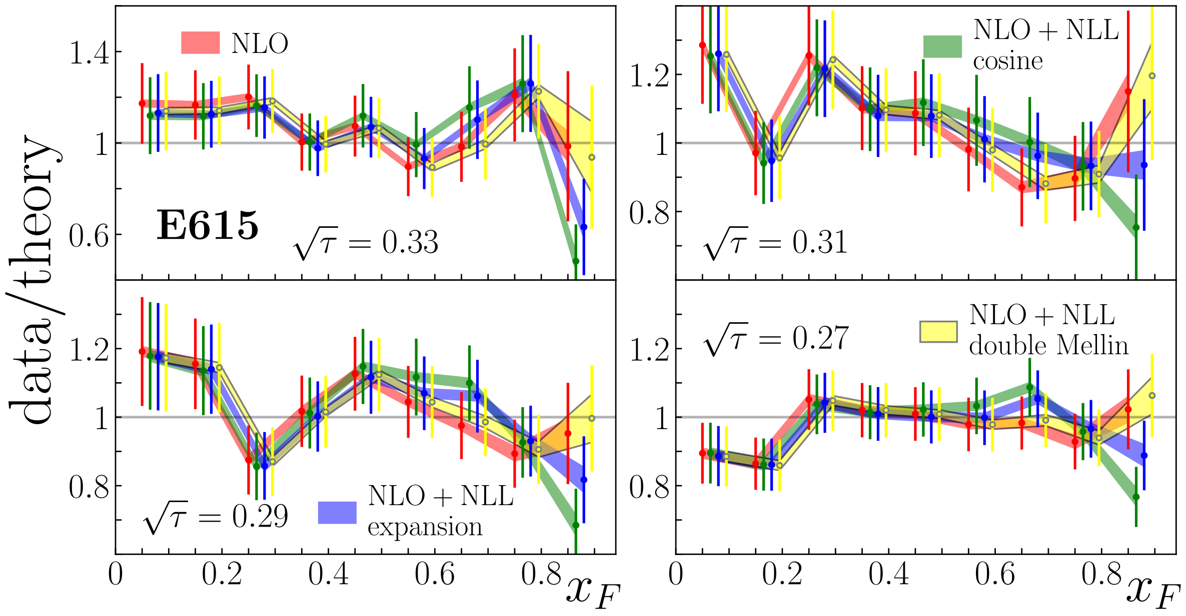
- $\boxed{\text{Upper Limits}}$ imply that $k_\perp^2$ will go to 0
- $\alpha_S(\mu^2 = 0)$ is NOT well-defined
- Ambiguities on how to deal with this provide needs for prescriptions

Methods of Resummation

- Make use of the Minimal Prescription to avoid Landau Pole
- Rapidity distribution $\frac{d\sigma}{dQ^2 dY}$ adds more complications
- We can perform a Mellin-Fourier transform to account for the rapidity
  - A cosine appears while doing Fourier transform; options:
    1) Take first order expansion, cosine ≈ 1
    2) Keep cosine intact
- Can additionally perform a Double Mellin transform
- Explore the different methods and analyze effects

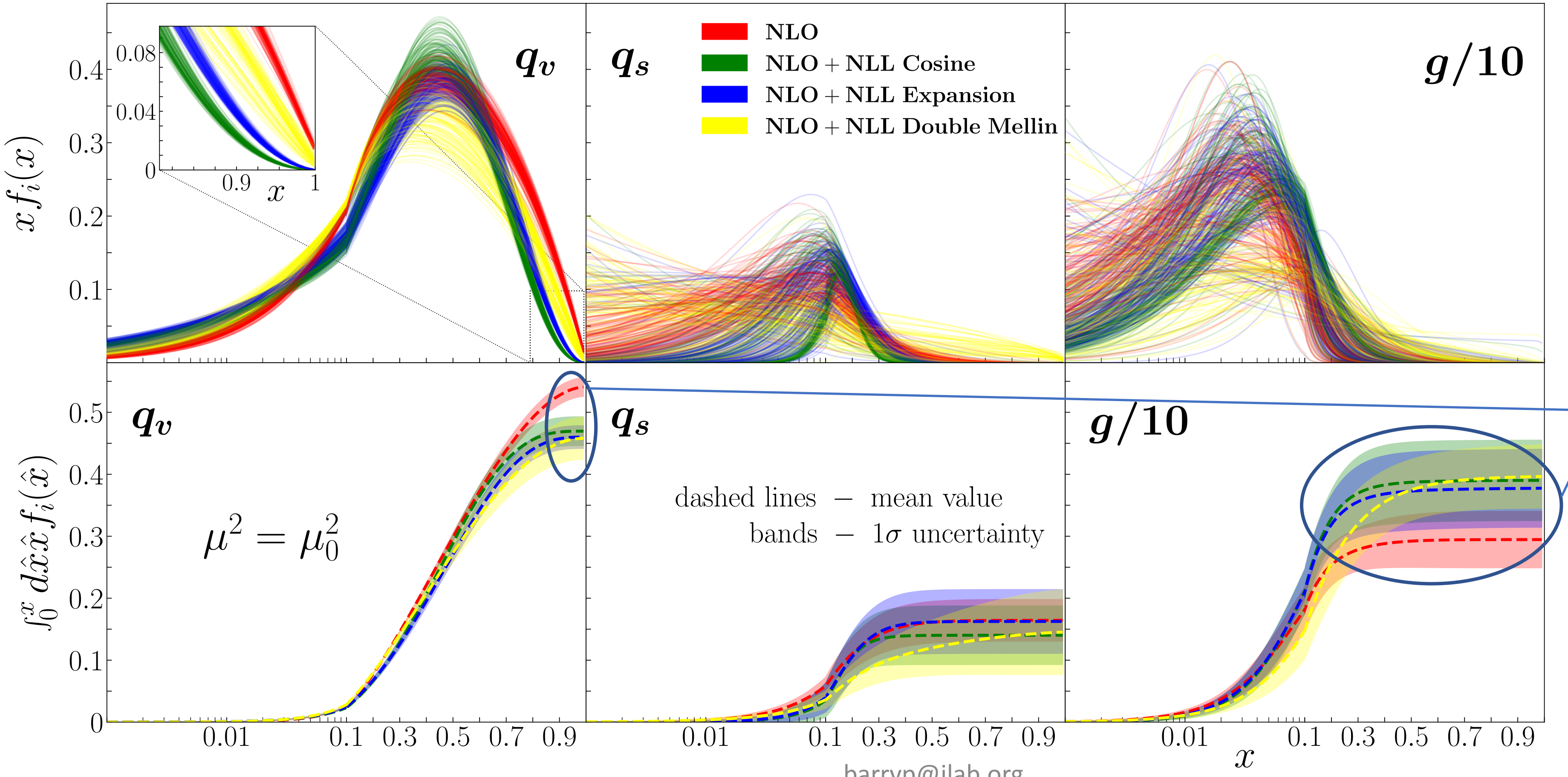
Data and Theory Comparison – Drell-Yan

- Cosine method tends to overpredict the data at very large x_F
- Double Mellin method is qualitatively very similar to NLO
- Resummation is largely a high-x_F effect



PDF Results

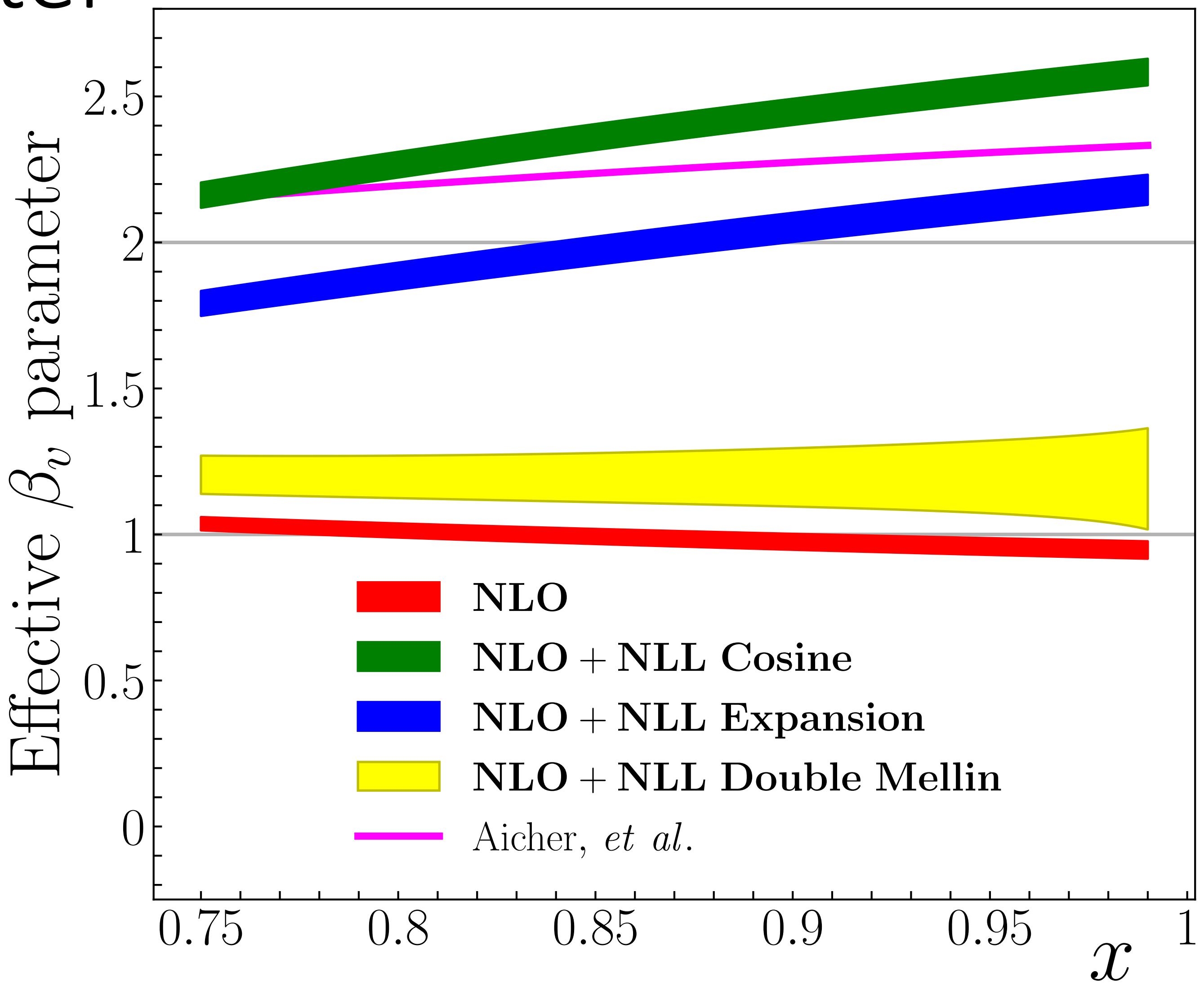
- Large x behavior in valence depends on prescription



Large x momentum fraction in the valence shifts to low x gluon momentum fraction

Effective β_v parameter

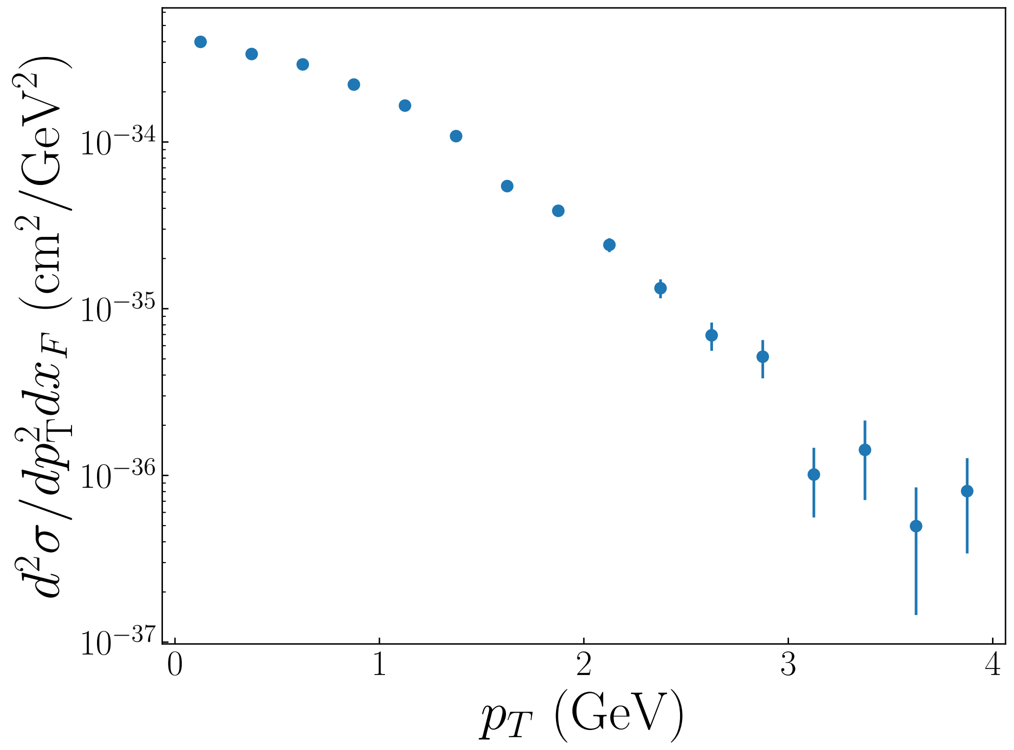
- $q_v(x) \sim (1-x)^{\beta_v}$ as $x \to 1$
- Threshold resummation does not give universal behavior of β_v
- NLO and double Mellin give $\beta_v \approx 1$
- Cosine and Expansion give $\beta_v > 2$



Transverse Momentum-dependent Drell-Yan

p_T-dependent spectrum for pion data

- Small-p_T data – TMD factorization – partonic transverse momentum
- Large-p_T data – collinear factorization – recoil transverse momentum

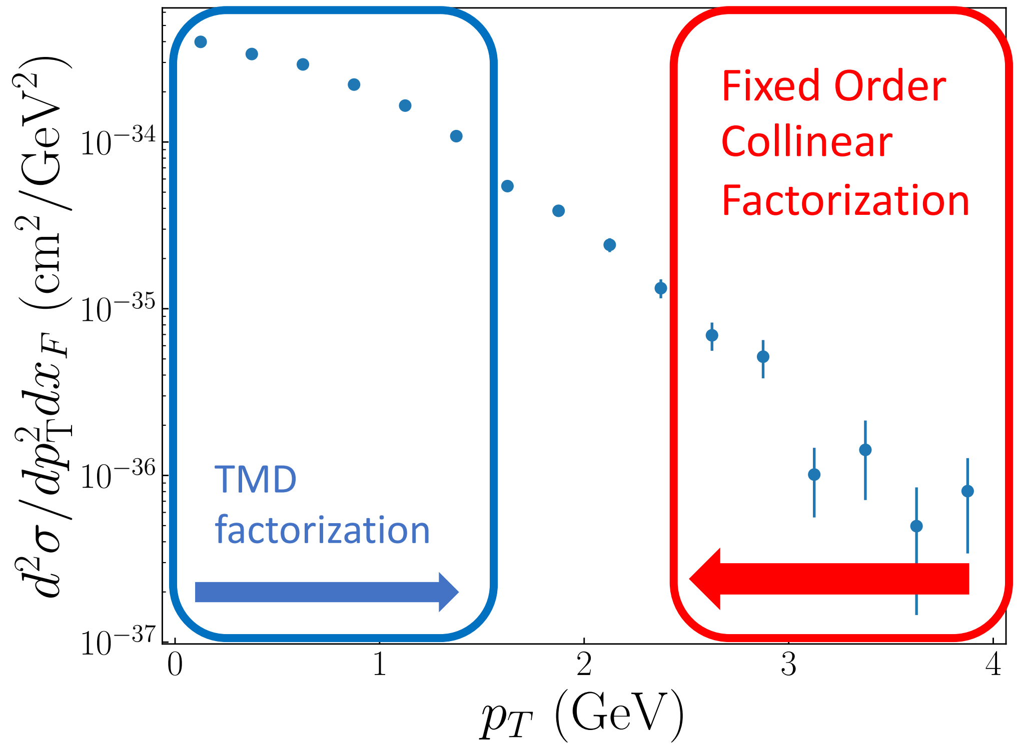


E615 πW Drell-Yan

Phys. Rev. D **39**, 92 (1989).

p_{T}-dependent spectrum for pion data

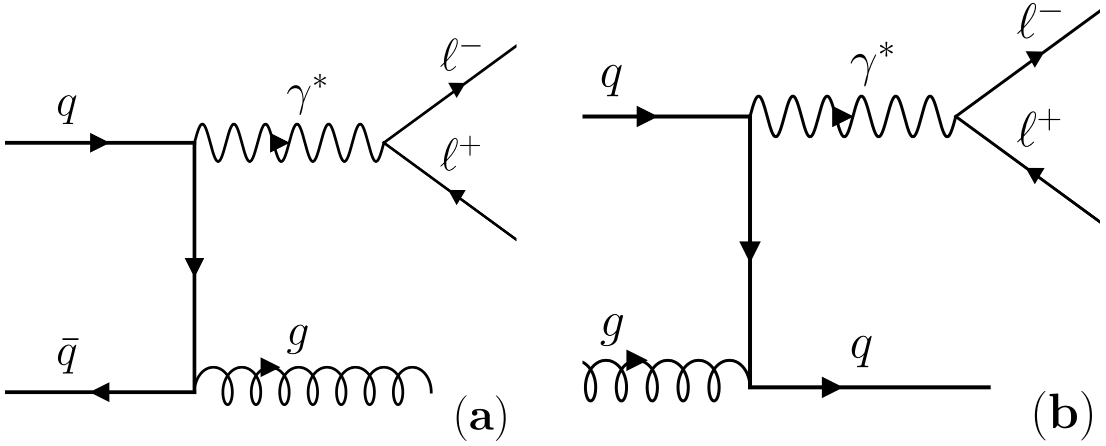
- Various factorization theorems break down in certain regions of p_{T}
- Errors are related with $\mathcal{O}(p_{\mathrm{T}}/Q)$ (low-p_{T}) or $\mathcal{O}(m/p_{\mathrm{T}})$ (large-p_{T})

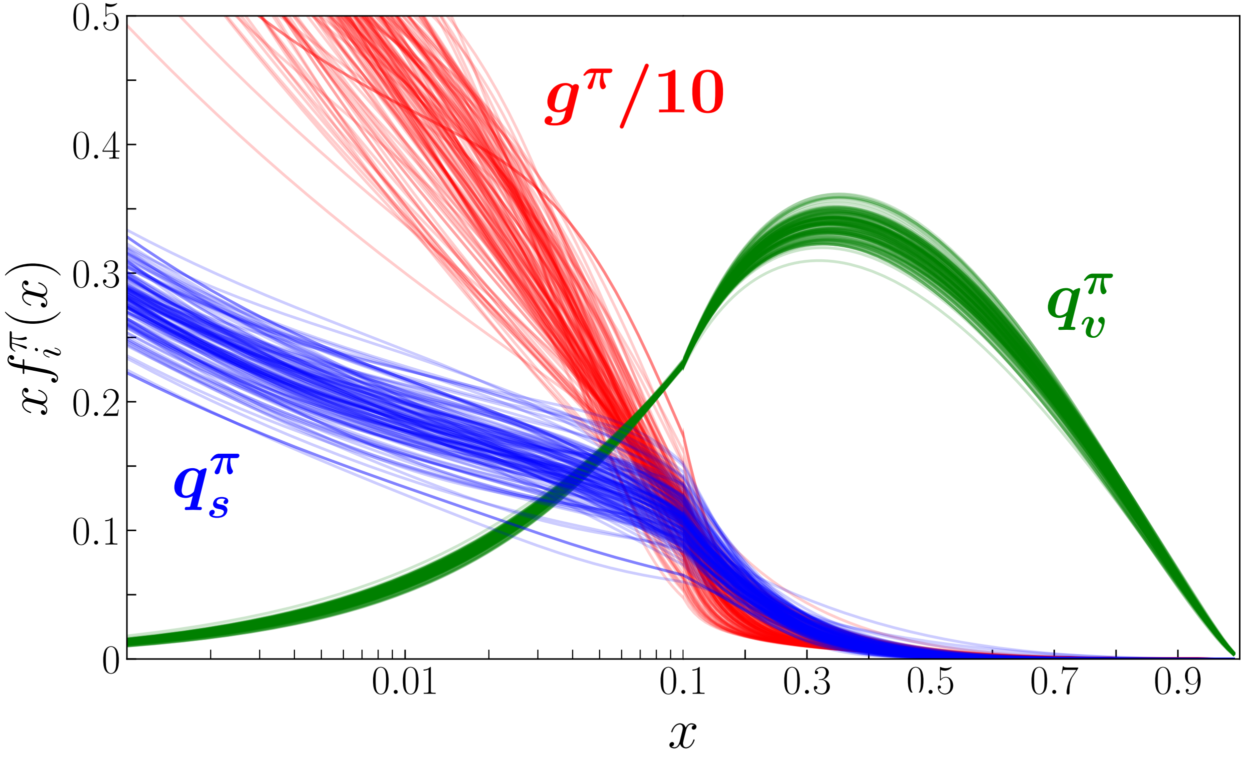


E615 πW Drell-Yan

Phys. Rev. D **39**, 92 (1989).

JAM20 Pion PDFs





N.Cao, PCB, N. Sato, and W. Melnitchouk
arXiv:2103.02159 [hep-ph]

- For the first time, we included large p_{T}-dependent Drell-Yan data, which follows collinear factorization
- Large p_{T} does not dramatically affect the PDF
- Successfully describe data with a scale $\mu = p_{\mathrm{T}}/2$

TMD factorization

- In small-p_{T} region, Use the CSS formalism for TMD evolution

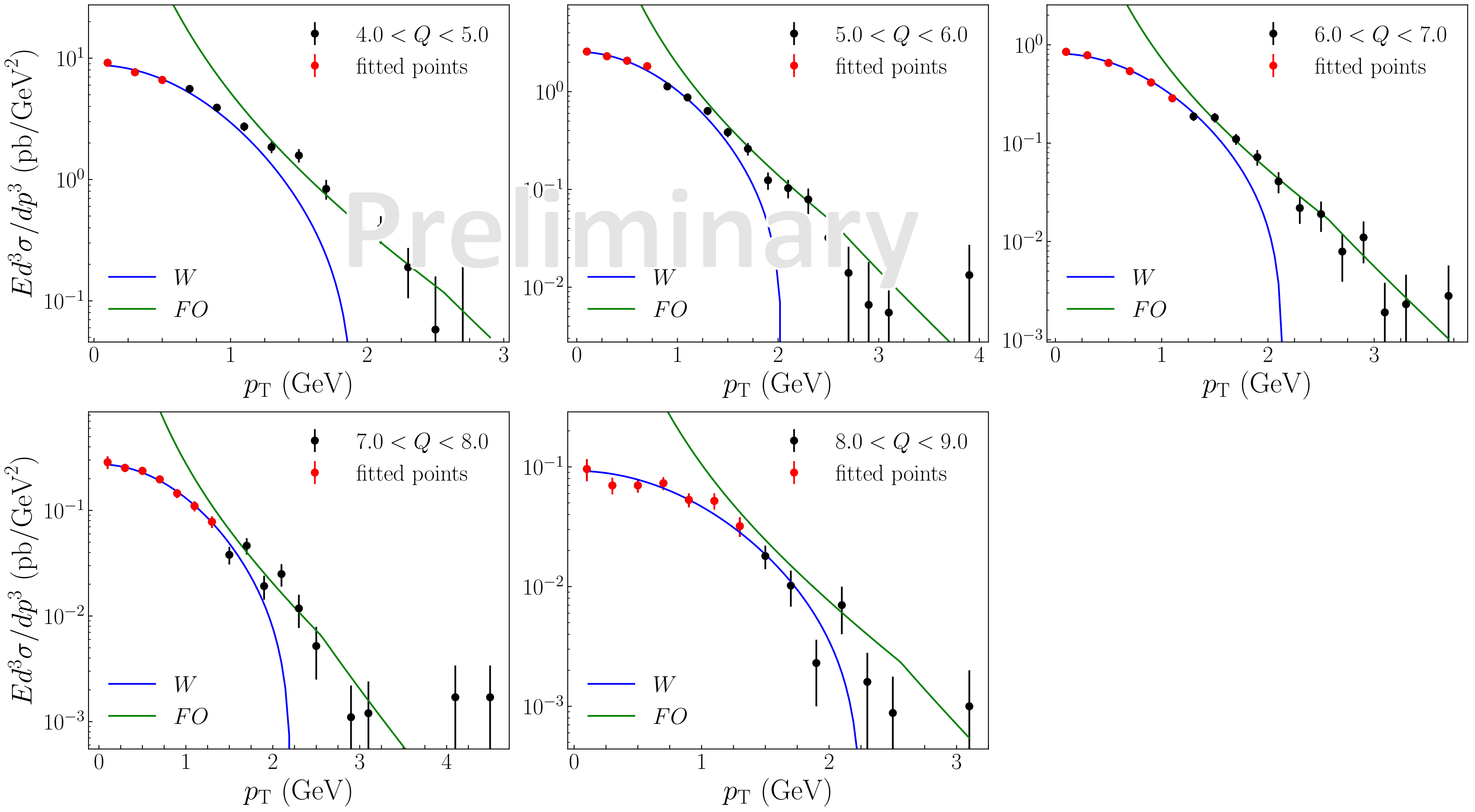
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H_{j\bar{j}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}$$
$$\times\, e^{-\boxed{g_{j/A}(x_A,b_{\mathrm{T}};b_{\max})}} \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}_{j/j_A}^{\mathrm{PDF}}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\, e^{-\boxed{g_{\bar{j}/B}(x_B,b_{\mathrm{T}};b_{\max})}} \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}_{\bar{j}/j_B}^{\mathrm{PDF}}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$
$$\times\, \exp\left\{-\boxed{g_K(b_{\mathrm{T}};b_{\max})}\ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Non-perturbative TMDs to extract

- Approach with leading order to diagnose current situation

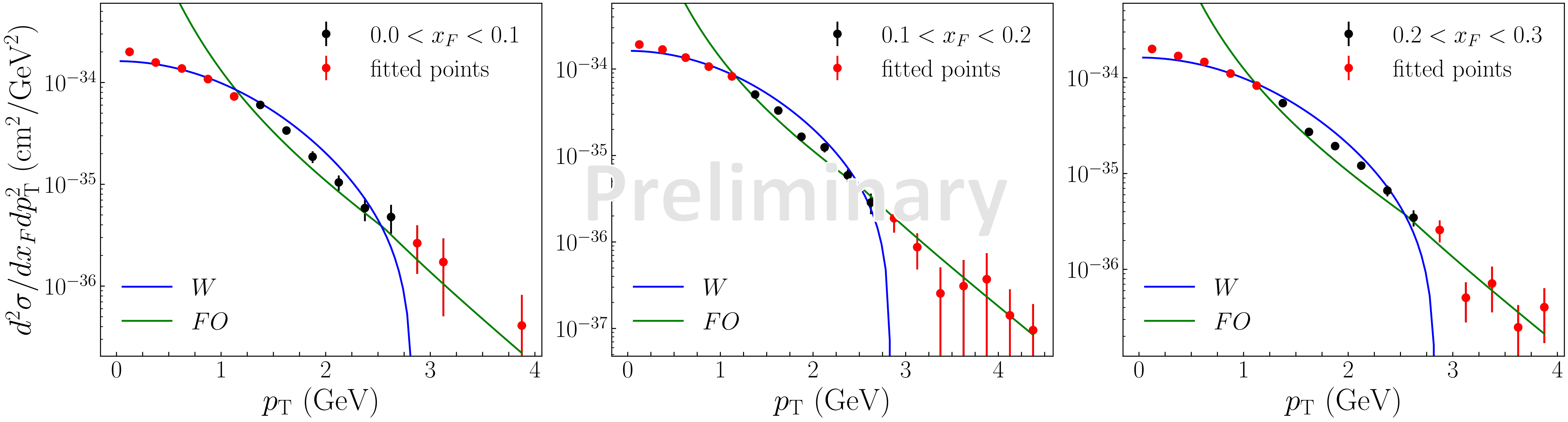
Results – E288 pp data

- Perform simultaneous fit of small-p_{T} TMDs to E288 (pp) and E615 (πA) data
- FO is prediction using collinear PDFs and scale $\mu = p_{\mathrm{T}}/2$



Results – E615 πW data

- Proton and pion TMDs are different, but g_K are same
- Perform simultaneous fit of small-p_{T} TMDs to E288 (pp) and E615 (πA) data



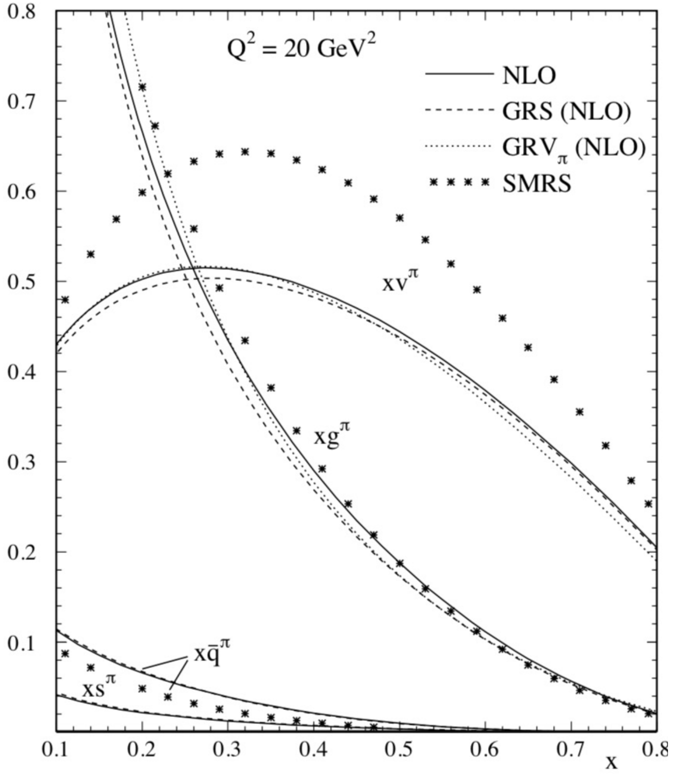
Future Work

- Investigate high-x behavior of valence PDF through constraints from the lattice data
- Perform a simultaneous extraction of pion PDFs and TMDs using available low- and high-p_T data
- Explore matching procedure between W and FO terms in p_T spectrum

Backup

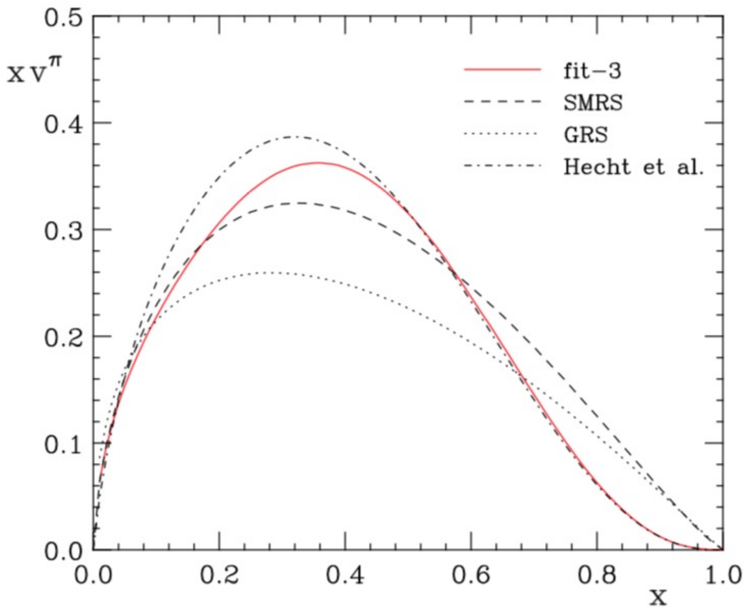
Previous Pion PDFs

- Fits to Drell-Yan, prompt photon, or both



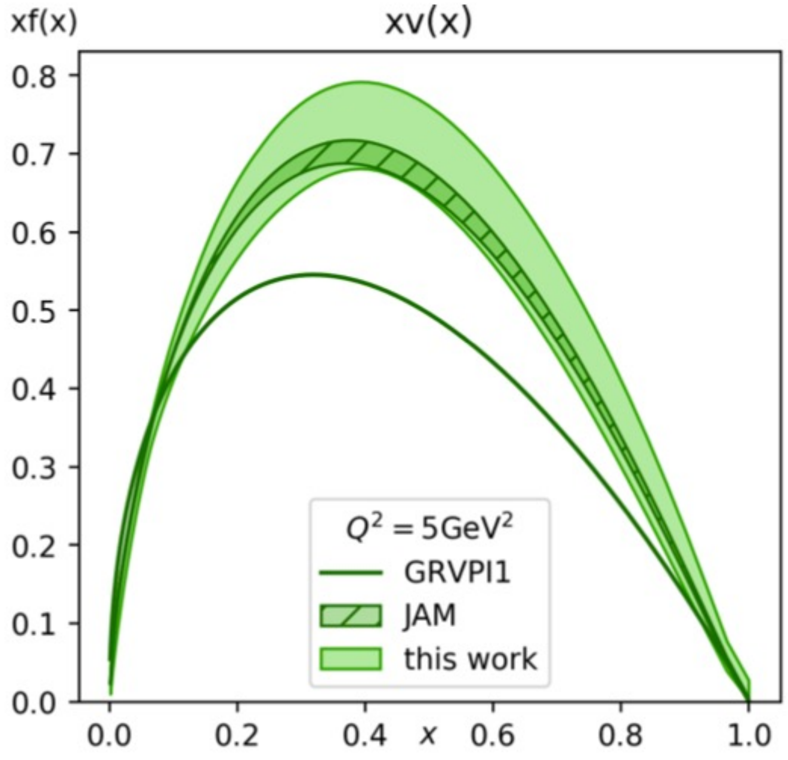
GRS, GRV, and SMRS

Z. Phys. C **67**, 433 (1995).
Eur. Phys. J. C **10** 313 (1997).
Phys. Rev. D **45** 2349 (1992).



Aicher's valence PDF

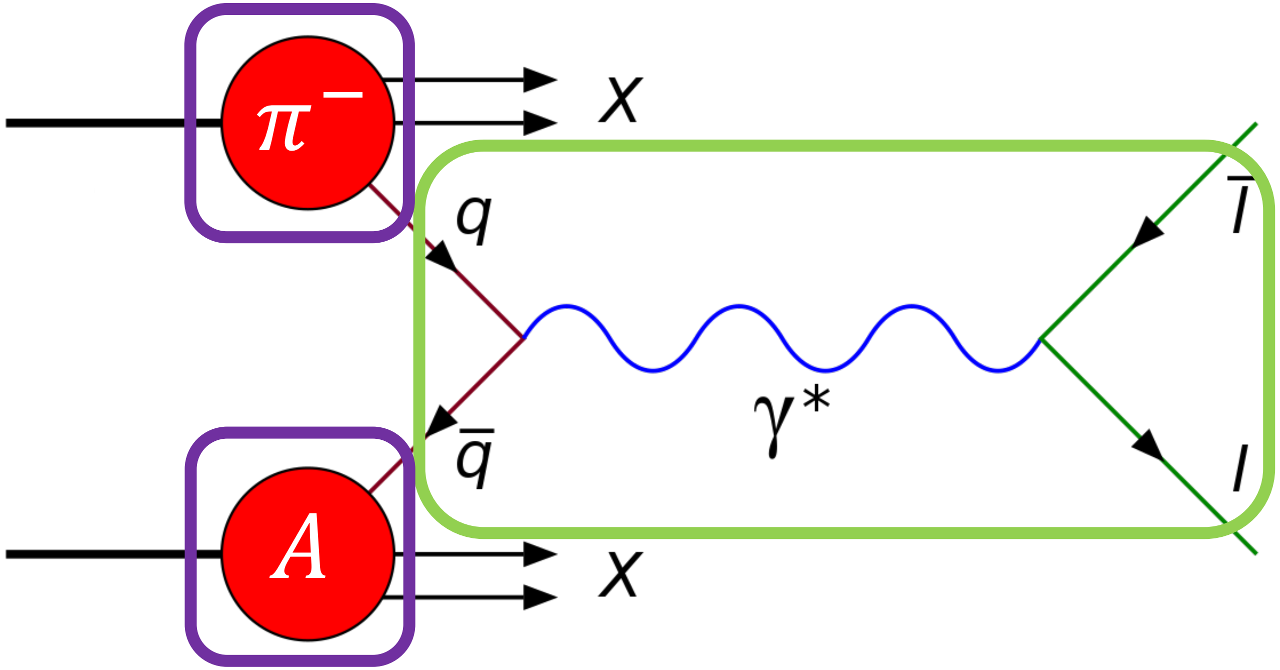
Phys. Rev. Lett. **105**, 114023 (2011).



xFitter

Phys. Rev. D **102**, 014040 (2020).

Drell-Yan (DY)



$$\sigma \propto \sum_{i,j} f_i^{\pi}(x_\pi, \mu) \otimes f_j^{A}(x_A, \mu) \otimes \hat{\sigma}_{i,j}(x_\pi, x_A, Q/\mu)$$

Drell-Yan (DY) Definitions

Hadronic variable

$$\tau = \frac{Q^2}{S}$$

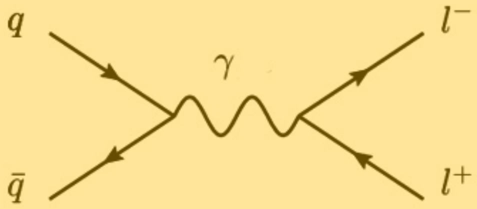
Partonic variable

$\hat{S}$ is the center of mass momentum squared of incoming partons

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{x_1 x_2}$$

LO

LO: $\mathcal{O}(1)$

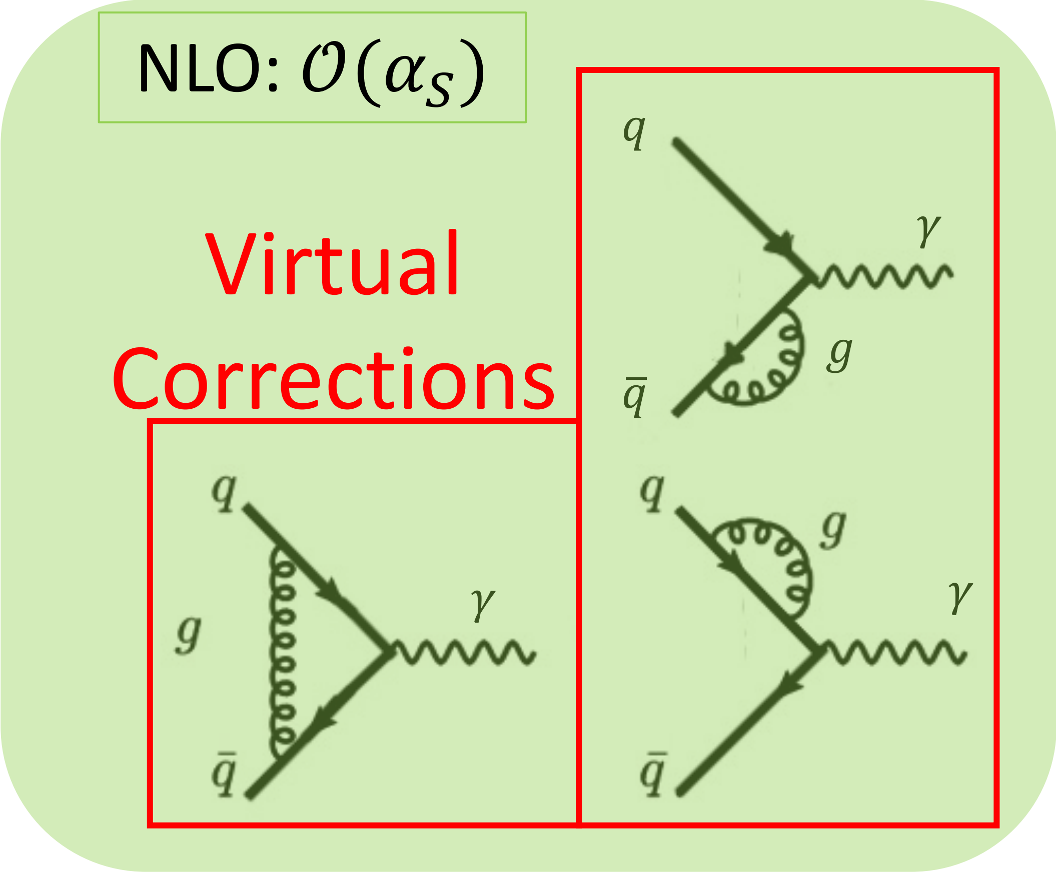


$$C_{q\bar{q}} = \delta(1-z)\frac{\delta(y)+\delta(1-y)}{2}$$

- $z = 1$ corresponds to partonic threshold
- All $\hat{S}$ is equal to Q^2
- All energy of hard partons turns into virtuality of photon

NLO Virtual

- Virtual corrections at NLO are proportional to $\delta(1-z)$
  - Exhibit Born kinematics



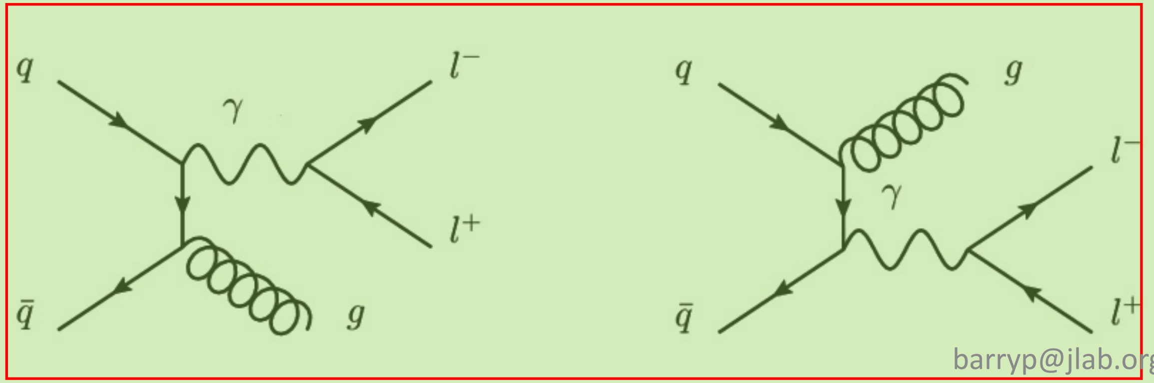
$$C_{q\bar{q}}^{\text{virtual}} = \delta(1-z)\frac{\delta(y)+\delta(1-y)}{2}\left[\frac{C_F\alpha_S}{\pi}\left(\frac{3}{2}\ln\frac{Q^2}{\mu^2}+\frac{2\pi^2}{3}-6\right)\right]$$

NLO Real Emission

- Next to leading order, real gluon emissions

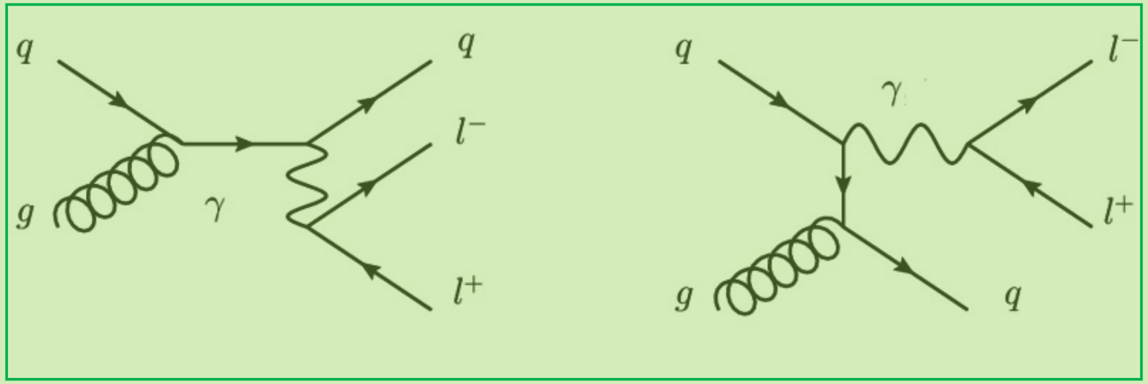
$$
C^{\text{real}}_{q\bar{q}} = \frac{C_F \alpha_S}{\pi}\left[\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left(\frac{1}{1-z}\ln\frac{Q^2(1-z)^2}{\mu^2 z}\right)_+ + 1 - z\right] + \frac{1}{2}\left[\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right) - 2(1-z)\right]\right]
$$

Real emissions



NLO Real Emission

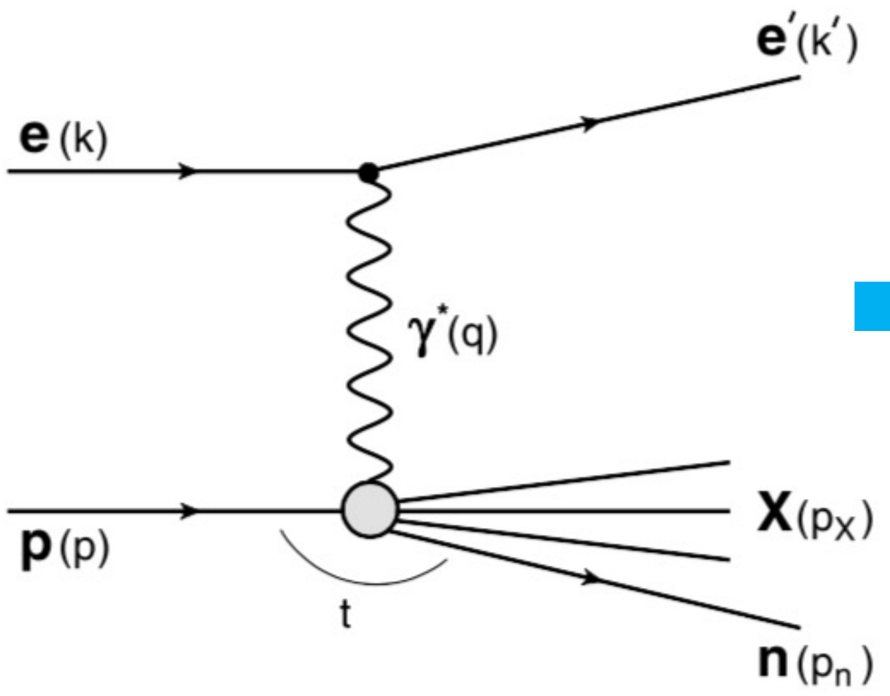
Real emissions

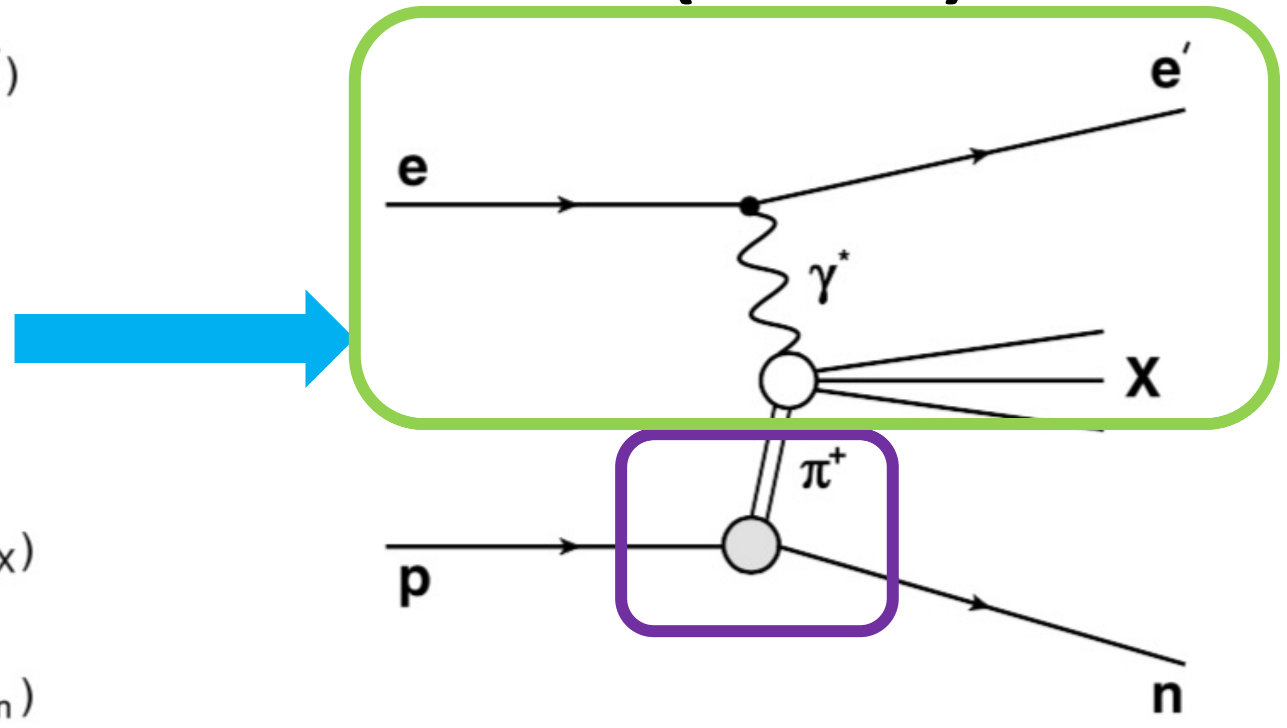


$$C_{qg}^{\text{real}} = \frac{T_F \alpha_S}{2\pi} \left[\delta(y) \left[(z^2 + (1-z)^2) \ln \frac{Q^2 (1-z)^2}{\mu^2 z} + 2z(1-z) \right] + \left[1 + \frac{(1-z)^2}{z} y(1-y) \right] \left[(z^2 + (1-z)^2) \left(\frac{1}{y} \right)_+ + 2z(1-z) + (1-z)^2 y \right] \right]$$

- Real quark emissions
- $C_{qg} = C_{gq}|_{y \to 1-y}$

Leading Neutron (LN)

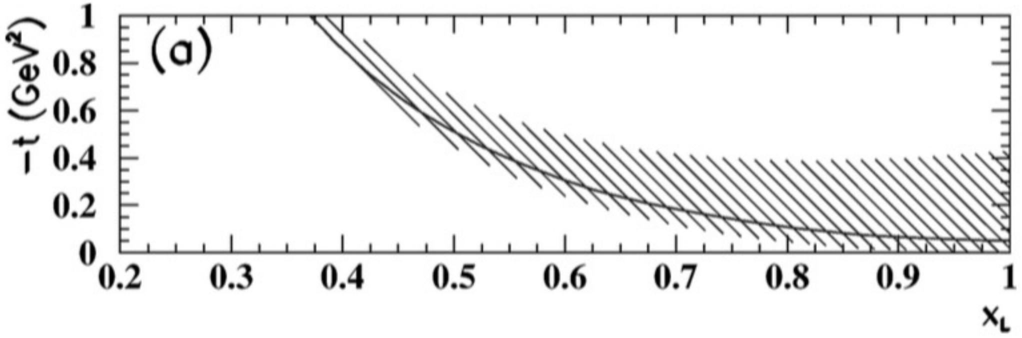


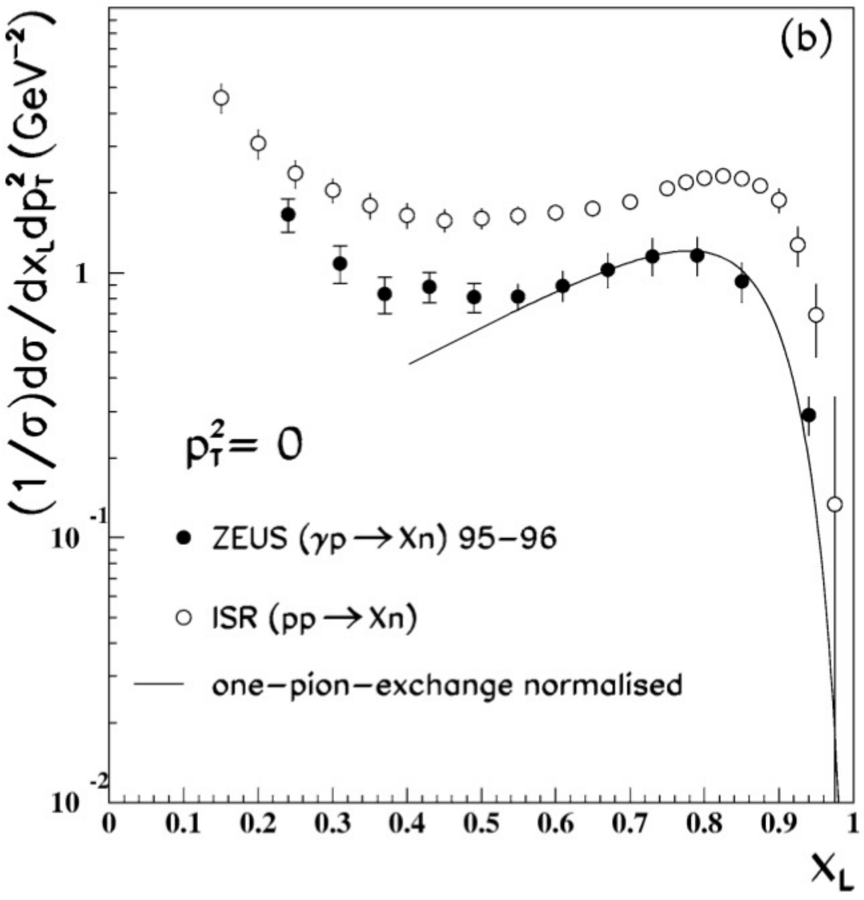


$$\frac{d\sigma}{dx\,dQ^2\,dy} \sim f_{p\to\pi^+ n}(y) \times \sum_q \int_{x/y}^{1} \frac{d\xi}{\xi} C(\xi)\, q\left(\frac{x/y}{\xi}, \mu^2\right)$$

Large x_L

- x_L is fraction of longitudinal momentum carried by neutron relative to initial proton
- For t to be close to pion pole, has to go near 0 – happens at large x_L
- In this region, one pion exchange dominates





Splitting Function and Regulators

Amplitude for proton to dissociate into a π^+ and neutron:

$$f_{\pi N}(\bar{x}_L) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{\bar{x}_L \left[k_\perp^2 + \bar{x}_L^2 M^2\right]}{x_L^2 D_{\pi N}^2} |\mathcal{F}|^2,$$

$$D_{\pi N} \equiv t - m_\pi^2 = -\frac{1}{1-y}[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]$$

Best fit

$$\mathcal{F} = \begin{cases} \text{(i)} & \exp\left((M^2 - s)/\Lambda^2\right) & s\text{-dep. exponential} \\ \text{(ii)} & \exp\left(D_{\pi N}/\Lambda^2\right) & t\text{-dep. exponential} \\ \text{(iii)} & (\Lambda^2 - m_\pi^2)/(\Lambda^2 - t) & t\text{-dep. monopole} \\ \text{(iv)} & \bar{x}_L^{-\alpha_\pi(t)} \exp\left(D_{\pi N}/\Lambda^2\right) & \text{Regge} \\ \text{(v)} & \left[1 - D_{\pi N}^2/(\Lambda^2 - t)^2\right]^{1/2} & \text{Pauli-Villars} \end{cases}$$

- We examine five regulators, and we fit Λ
- $\mathcal{F}$ is a UV regulator, which the data chooses

Bayesian Inference

- Minimize the χ^2 for each replica

Normalization parameter

$$\chi^2(\boldsymbol{a}, \text{data}) = \sum_e \left(\sum_i \left[\frac{d_i^e - \sum_k r_k^e \beta_{k,i}^e - t_i^e(\boldsymbol{a})/n_e}{\alpha_i^e} \right]^2 + \left(\frac{1-n_e}{\delta n_e} \right)^2 + \sum_k \left(r_k^e \right)^2 \right)$$

- Perform N total χ^2 minimizations and compute statistical quantities

Expectation value $$\mathrm{E}[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(\boldsymbol{a}_k),$$

Variance $$\mathrm{V}[\mathcal{O}] = \frac{1}{N} \sum_k \left[\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}] \right]^2,$$

TMD Non-Perturbative Parametrization

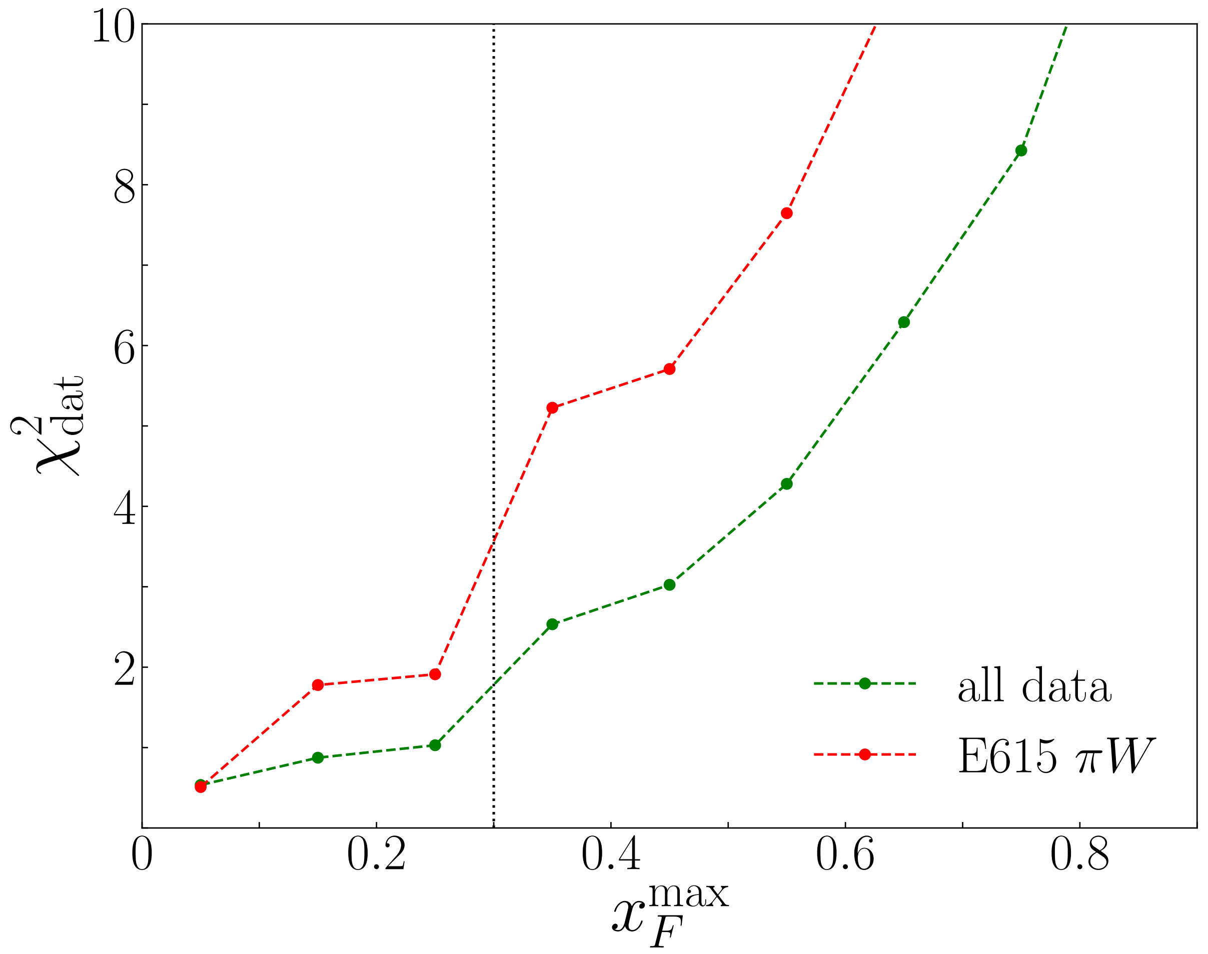
- Parametrize g_j for each hadron and g_K as in Slide 21

$$g_{j/h}(x, b_T^2) = -b_T^2 g_1 \left(\frac{1}{2} + g_3 \log(10x)\right)$$

$$g_K(b_T) = -b_T^2 \frac{g_2}{2}$$

χ^2 as a function of $x_F^{\mathbf{max}}$

- Phenomenologically determine where TMD factorization breaks down in large x_F data



E288 pp

- Calculate large-p_T FO term with $\mu = Q$
- Not a good description