Towards DIS at N4LO

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Nikhef

Grand Goal

Calculate N4LO DIS and 4-loop splitting functions

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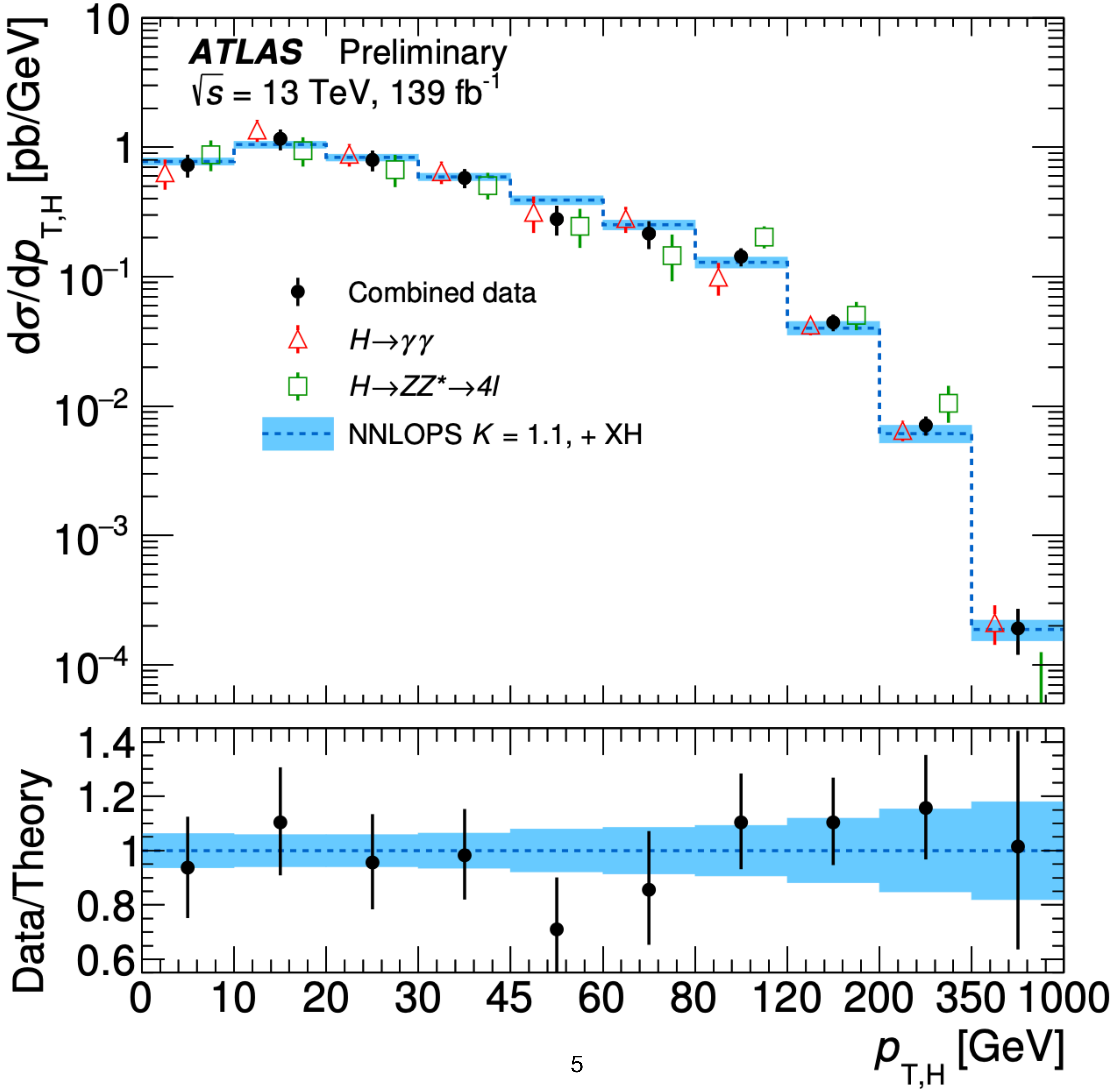
Calculate N4LO DIS and 4-loop splitting functions

VERY HARD

Current/short term Goal

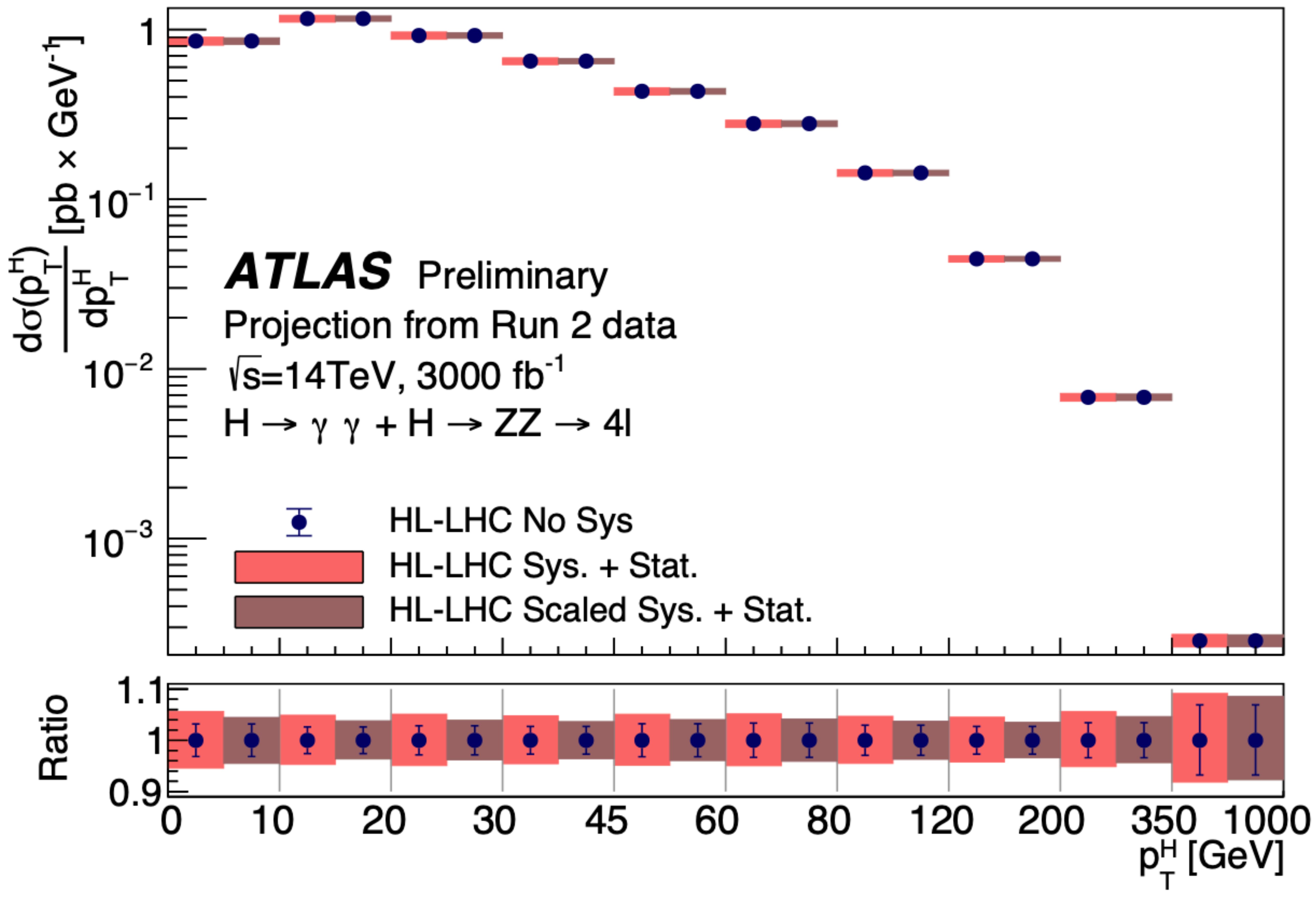
- Validate algorithm
- Check 3-loop results:
  - DIS coefficient functions
  - *Polarised splitting functions*

Motivation



ATLAS Preliminary
√s = 13 TeV, 139 fb-1
Combined data
H→γγ
H→ZZ*→4l
NNLOPS K = 1.1, + XH
dσ/dp_T,H [pb/GeV]
Data/Theory
p_T,H [GeV]
10
1
10−1
10−2
10−3
10−4
1.4
1.2
1
0.8
0.6
0
10
20
30
45
60
80
120
200
350
1000

Motivation



ATLAS Preliminary
Projection from Run 2 data
$\sqrt{s}$=14TeV, 3000 fb^{-1}
H → γ γ + H → ZZ → 4l
HL-LHC No Sys
HL-LHC Sys. + Stat.
HL-LHC Scaled Sys. + Stat.
$\frac{d\sigma(p_T^H)}{dp_T^H}$ [pb × GeV^{-1}]
1
10^{-1}
10^{-2}
10^{-3}
Ratio
1.1
1
0.9
0
10
20
30
45
60
80
120
200
350
1000
p_T^H [GeV]

Motivation

- Decreasing experimental uncertainty → require better predictions

  $\Delta\sigma_H \sim$ 8% (**run 2**) VS expected 3% (**high luminosity**) [arxiv:2003.01700]

  N3LO predictions → **4-loop splitting functions**

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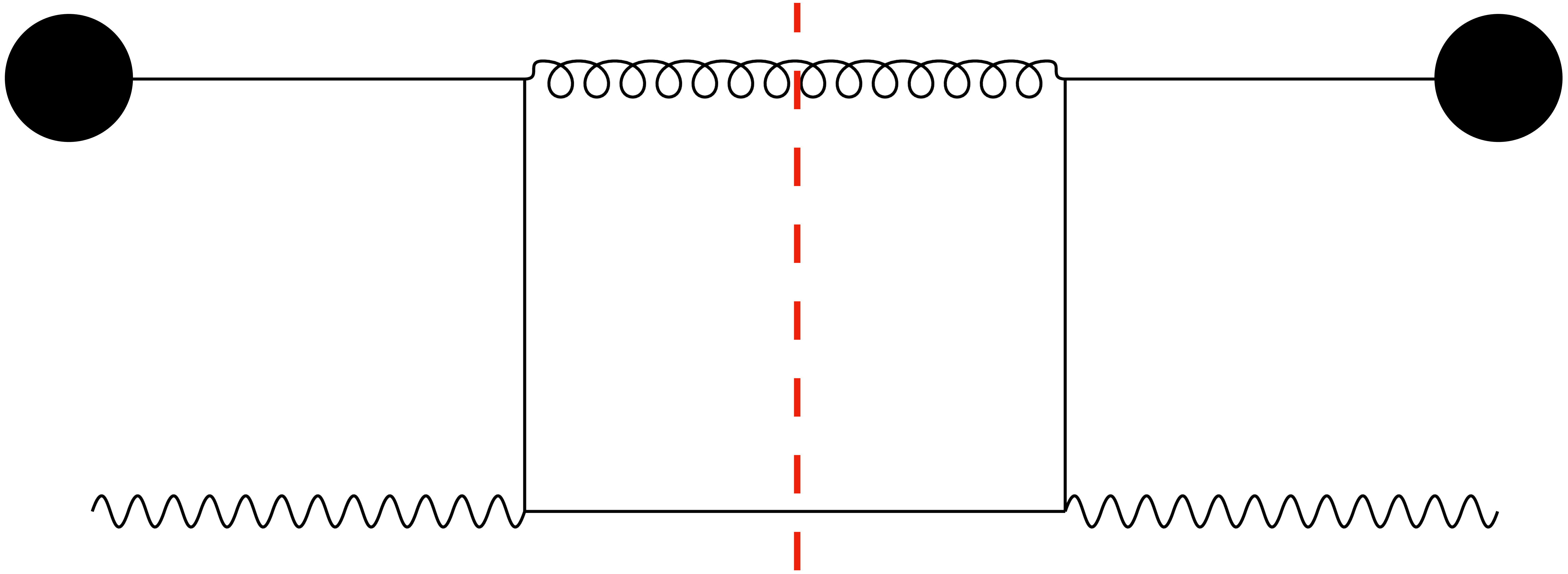
  N3LO predictions → **4-loop splitting functions**

- Better **PDFs** → require better **extraction DIS data**

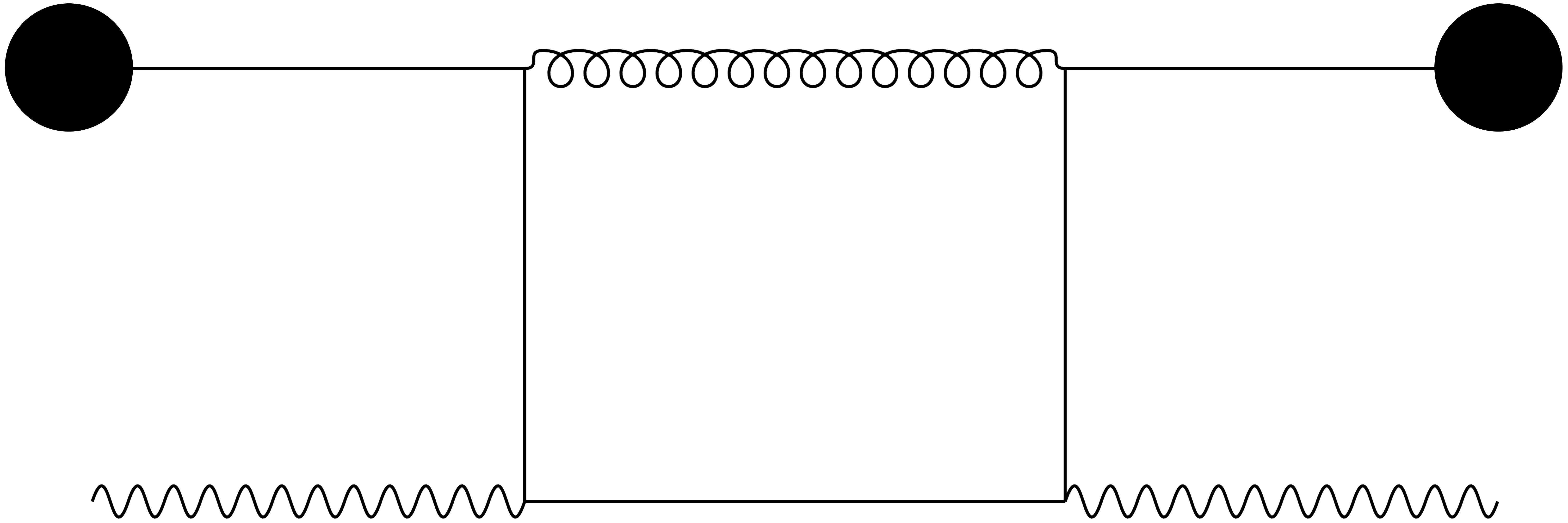
Setup

- Hadronic tensor $W_{\mu\nu}$



Setup

- Hadronic tensor $W_{\mu\nu}$
- Optical theorem $W_{\mu\nu}(p, q) = 2 \text{ Im } T_{\mu\nu}(p, q)$



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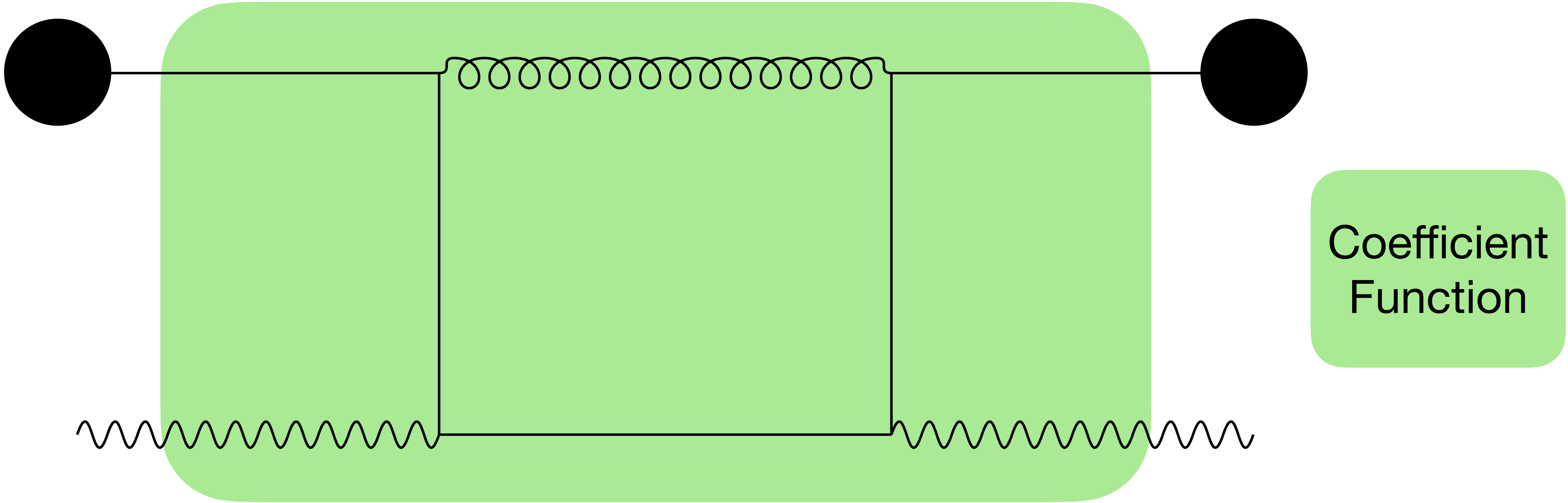


Method

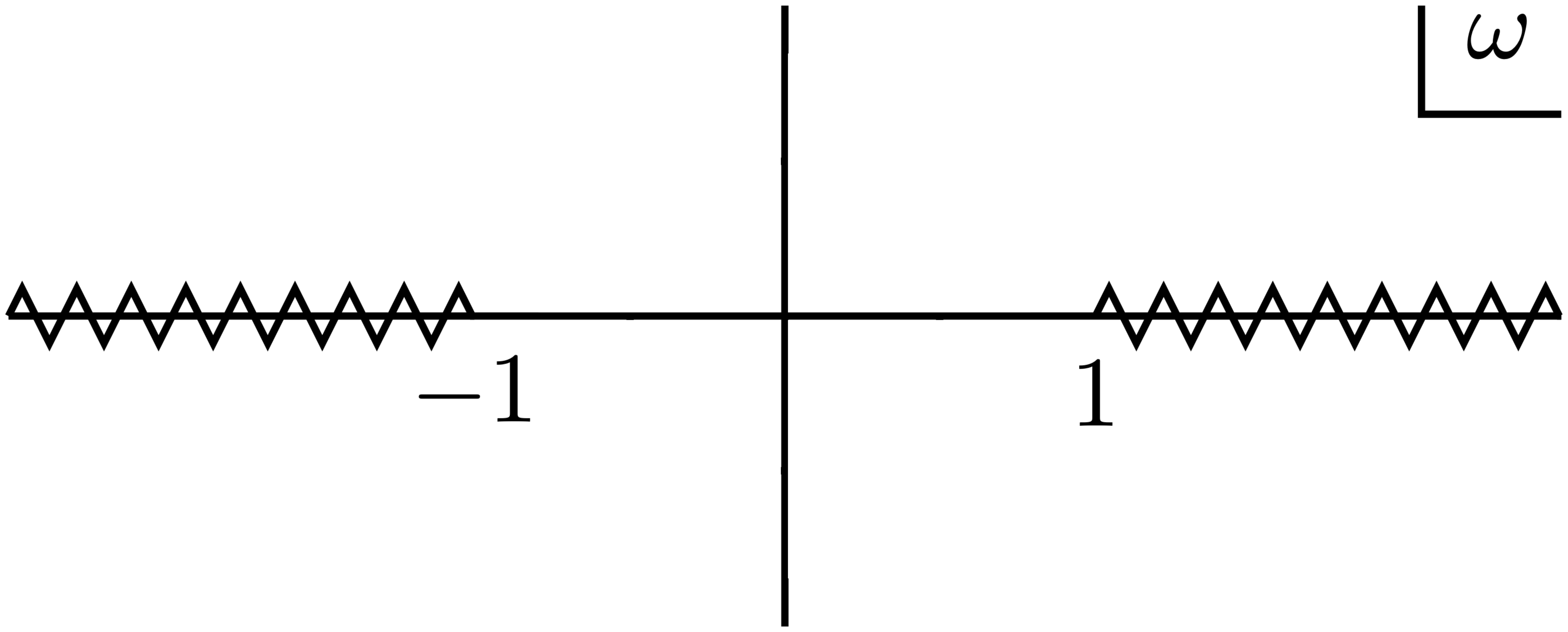
Method

- Reconstruct coefficient function from finite #Mellin moments
- Obtain Mellin moments recursively
  - Reduction to masters integrals
  - Exploiting differential equations and analytic structure

Forward scattering amplitude

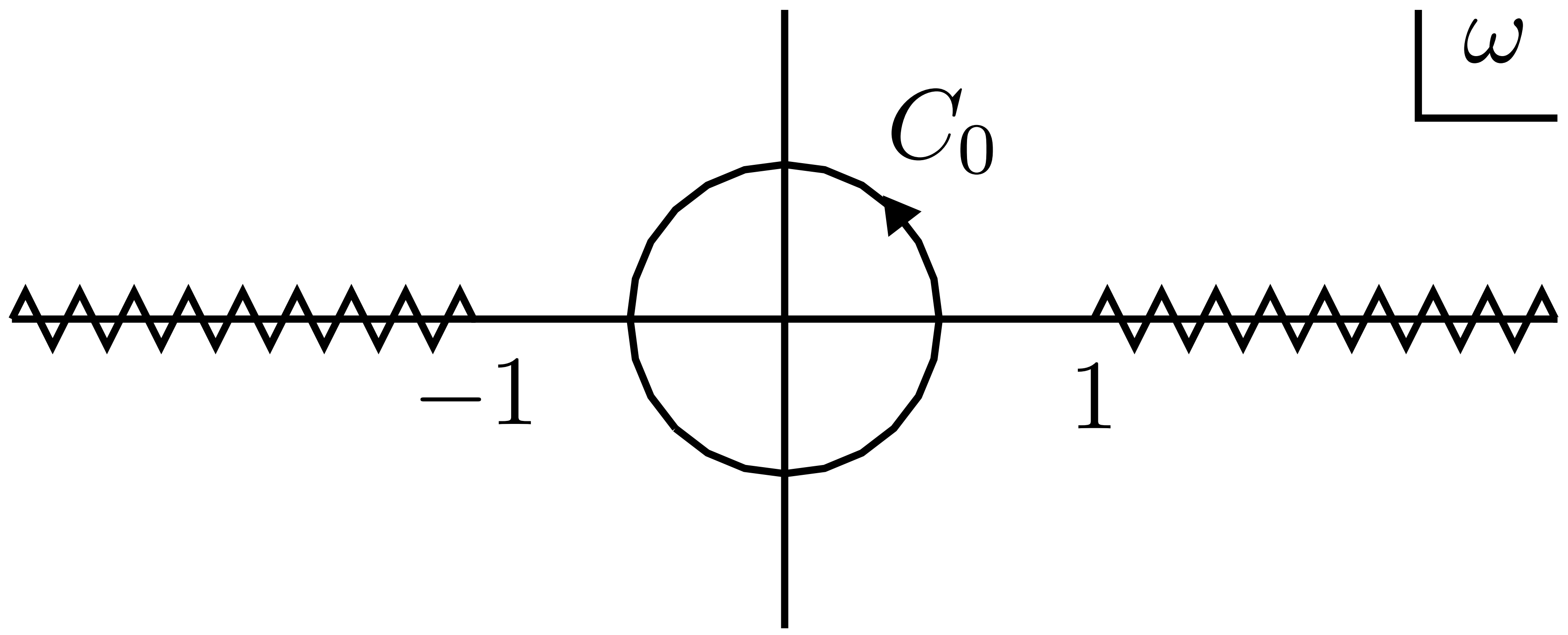
$$T_{\mu\nu}(p,q) = -\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) T_1(\omega, Q^2) + \left(p_\mu - \frac{q_\mu p \cdot q}{q^2}\right)\left(p_\nu - \frac{q_\nu p \cdot q}{q^2}\right) T_2(\omega, Q^2)$$

with $\omega \equiv x^{-1}$,
$Q^2 = -q^2$



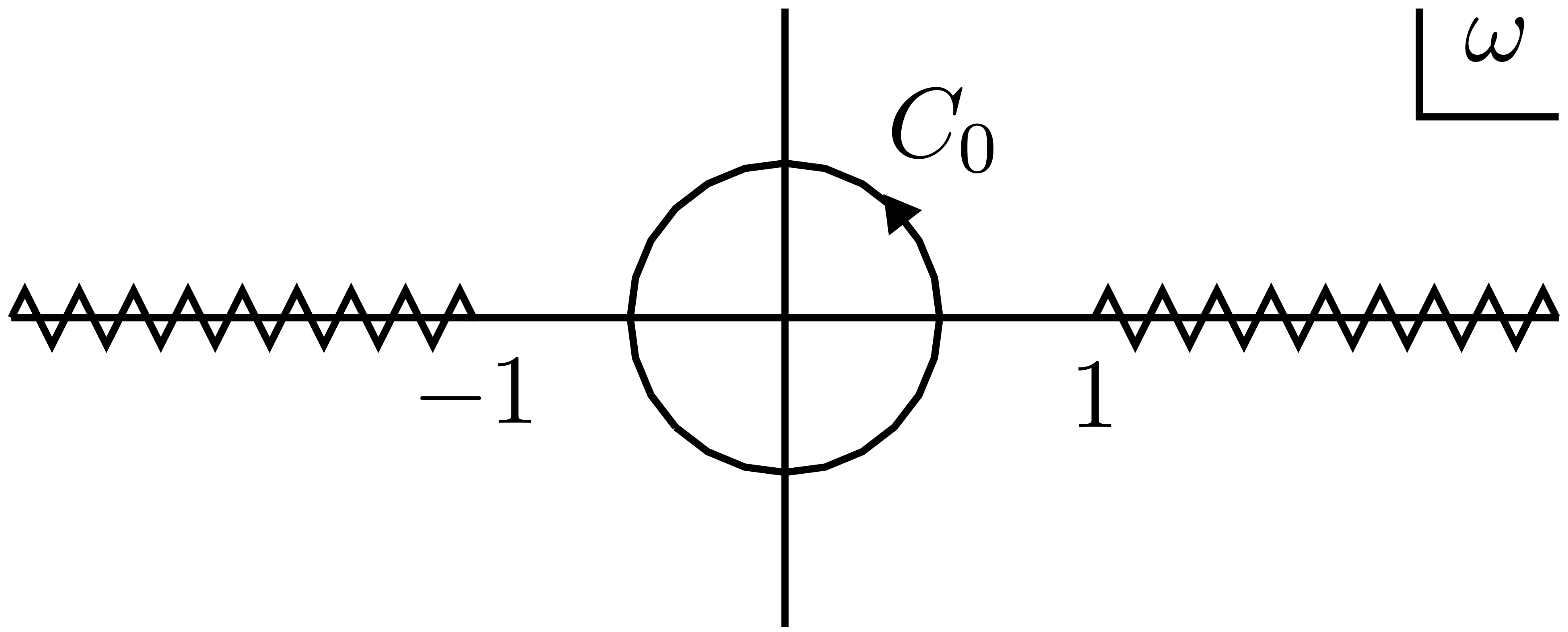
Cauchy's theorem

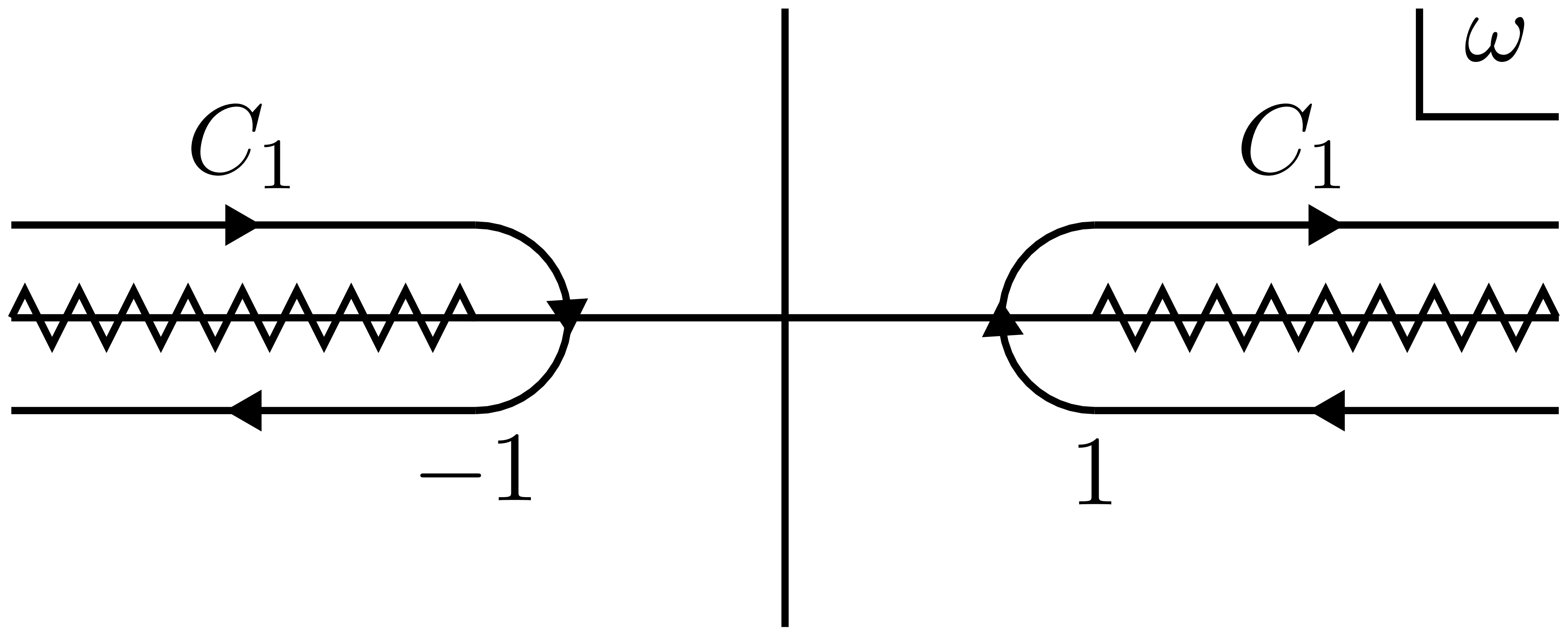
Relate derivatives at $\omega = 0$ to contour integrals



$$T_i^{(n)}(Q^2) = \frac{1}{n!}\frac{d^n T_i(\omega, Q^2)}{d\omega^n}\bigg|_{w=0} = \oint_{C_0} \frac{d\omega}{2\pi i}\frac{T_i(\omega, Q^2)}{\omega^{n+1}}$$

Deforming contour





Analytic structure

- Such that $T_i^{(n)}(Q^2) = \frac{(1+(-1)^n)}{2\pi i}\int_1^\infty d\omega \frac{Disc_\omega T_i(\omega, Q^2)}{\omega^{n+1}}$, $Disc_\omega f(\omega) = \lim_{\epsilon \to 0}[f(\omega + i\epsilon) - f(\omega - i\epsilon)]$
- The discontinuity across the branch cuts is the imaginary part of $T_{\mu\nu}$
- Again using the optical theorem for even n, going back to x

Mellin Moments

$$T_i^{(n)}(Q^2) = \frac{1}{\pi}\int_0^1 dx\ x^{n-1} W_i(x, Q^2) = \frac{1}{\pi}\mathscr{M}_n[W_i(Q^2)]$$

- **Finite** #Mellin moments ⇒ **Full** ω **dependance** coefficient functions
  - Moch, Vermaseren **2-loop** [hep-ph/9912355]
  - Moch, Vermaseren, Vogt **3-loop** [hep-ph/0403192] (non-singlet)
  - Moch, Vermaseren, Vogt **3-loop** [hep-ph/0404111] (singlet)
  - Moch et al, **4-loop** [1707.08315] (non-singlet in planar limit, using OPE)
  - Vogt et al , **4-loop** [1808.08981] (numerical approximations singlet, using moments with <N=10)
  - Herzog et at, **5-loop** [1812.11818] (N=2,3 moments of non-singlet at 5 loops with R*)
- We: **Exhaust** the analytic behaviour at $\omega \to 0$

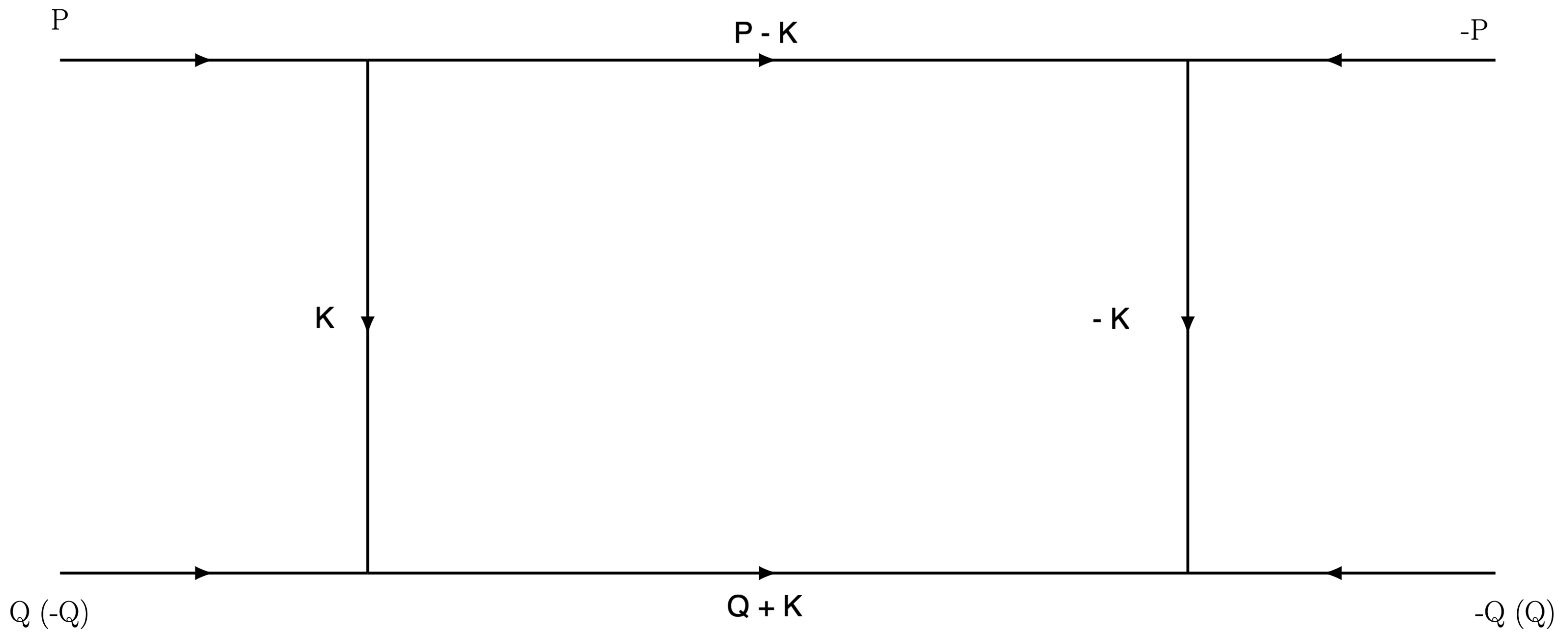
Recursive Algorithm

Topology

1-loop

$$\text{Topo}(A1,A2,A3) = \int d^D K \left(\frac{1}{(K^2)^{A1}}\right)\left(\frac{1}{((P-K)^2)^{A2}}\right)\left(\frac{1}{((Q+K)^2)^{A3}}\right)$$

- **IBP** → Reduce to set of masters M, **Kira** [Maierhöfer, Usovitsch, Uwer (2018)]
- $M_1 = \text{Topo}(1,0,1)$, $M_2 = \text{Topo}(0,1,1)$
- **DE** $\dfrac{\partial M_2}{\partial \omega} = -\dfrac{\epsilon}{1+\omega} M_2$, used reduction tables
- **BC** $M_2(\omega \to 0) = M_1$



$$M_2(\omega, \epsilon) = \sum_{n=0}^{\infty} \omega^n a_n(\epsilon) \Rightarrow a_n = -\frac{(\epsilon + n - 1)\, a_{n-1}}{n}$$

So let's tackle 4 loops?

Multiple loops

- 3 problems
  1. Boundaries
  2. Poles in A
  3. Reductions

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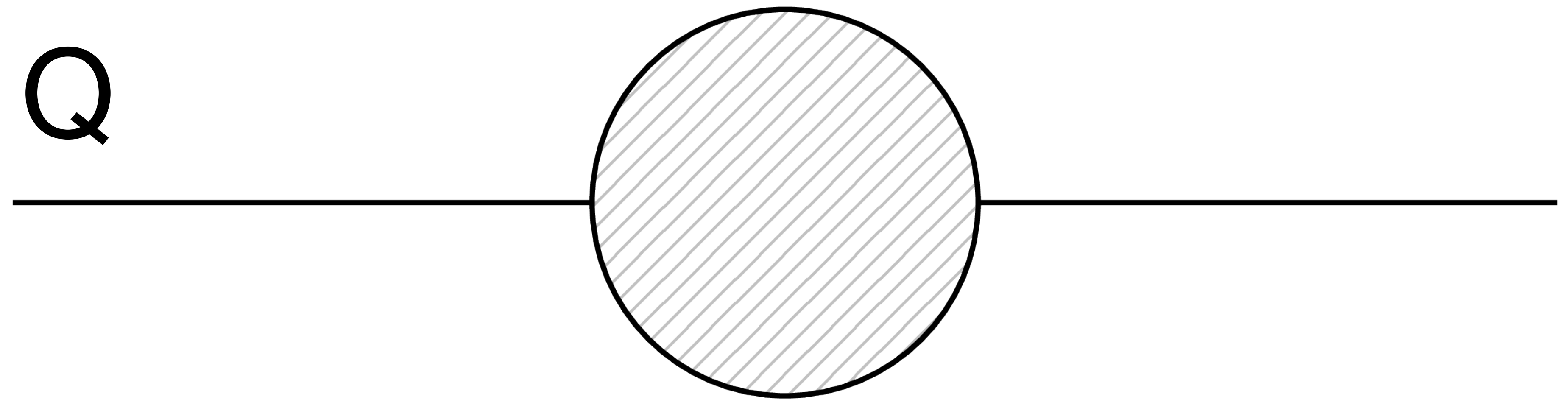
Massless self energies

$$\omega \to 0 \Rightarrow P \to 0$$



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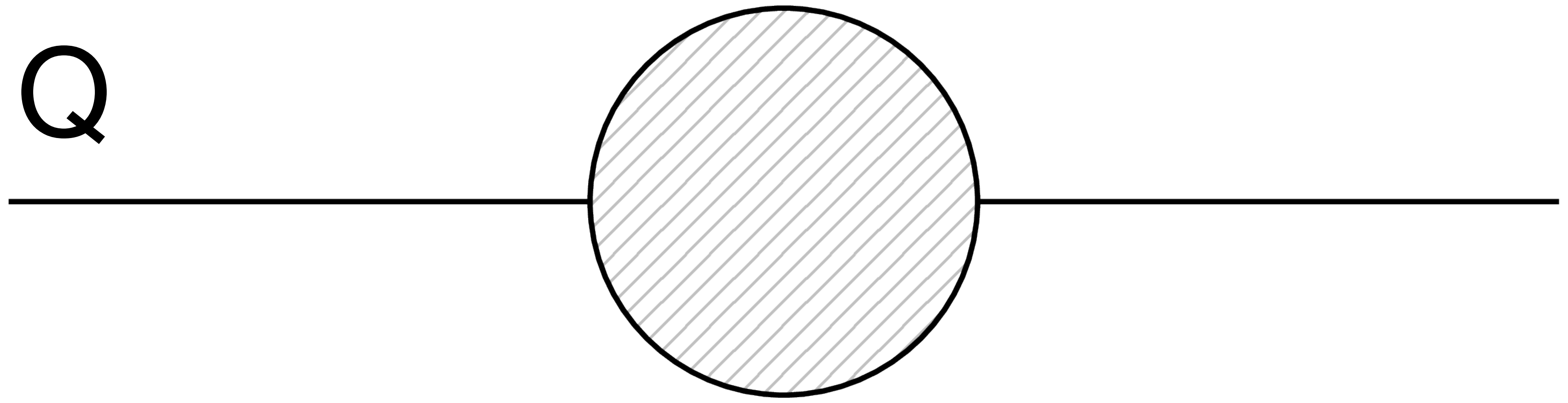

FORCER

[1607.07318]

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Transforming the A matrix

- Toy topology with 3 masters

$$\frac{d}{d\omega} M_i = A_{ij}\, M_j$$

- Scale away poles up till $O(\omega^{-1})$

- Transform order by order

$$\vec{M}' = T\vec{M}$$

$$A' = TAT^{-1} - T\partial_\omega T^{-1}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ \frac{a}{\omega^4} & 1 & 0 \\ \frac{b}{\omega^3} & 0 & \frac{c}{\omega} \end{pmatrix}$$

Transforming the A matrix

$O(\omega^{-4})$

- Remove highest order pole
- T_{ij} diagonal matrix in which the i^{th} entry equals (negative) exponent in ω of the highest order pole in the i^{th} row in A_{ij}

$$T_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \quad A' = \begin{pmatrix} 1 & 0 & 0 \\ a & \frac{\omega+4}{\omega} & 0 \\ \frac{b}{\omega^3} & 0 & \frac{c}{\omega} \end{pmatrix}$$

Transforming the A matrix

$O(\omega^{-3})$

- Subsequently scale away $O(\omega^{-3})$ term with new T matrix

$$T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega^3 \end{pmatrix} \Rightarrow A'' = \begin{pmatrix} 1 & 0 & 0 \\ a & \frac{\omega+4}{\omega} & 0 \\ b & 0 & \frac{c+3}{\omega} \end{pmatrix}$$

Transforming the A matrix

$O(\omega^{-1})$

- Left with $O(\omega^{-1})$ poles, impossible to be scaled way
- Final transformation matrix

  $T = T_3 \cdot T_4$

- Consequently M_2'' starts at $O(\omega^4)$

  and M_3'' at $O(\omega^3)$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^4 & 0 \\ 0 & 0 & \omega^3 \end{pmatrix}$$

Conjecture:

Diagonal T sufficient to map away deeper poles from A

Modified recurrence relation

General form

- Convenient to pull out $1/\omega$ from A, such that

$$\frac{d}{d\omega}M_i = \frac{A_{ij}}{\omega}M_j$$

- $$c_l^{(k)} = \left(B_{li}^{(k)}\right)^{-1} \sum_{0\leq k'\leq(k-1)} A_{ij}^{(k-k')}c_j^{(k')} \qquad B_{ij}^{(k)} \equiv \left(k\ \delta_{ij} - A_{ij}^{(0)}\right)$$

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RECURRENCE

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RECURRENCE **FORM** [math-ph/0010025]

Caveat for B…

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Number of topos

Non-singlet

#loops	#topos
1	1
2	3
3	32
4	364

Reduction time

#loops	time
1	3.7s (8 cores)
2	90s (24 cores)
3	~ week (64 cores)
4	?

Number of masters

#loops	#masters
1	2
2	31
3	2898
4	?

Outlook

- Toughest calculation so far:
  topo 31 at 3-loop (156 masters), 10 hours to calculate 100 coefficients (58 cores)
- 4-loop reduction unsure on Torre (64 cores, 1TB RAM memory)
- NF^2
- Collaboration w/ Johann Usovitsch (Author Kira), new methods

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Conclusion

- Background DIS
  - Mellin moments $\leftrightarrow$ forwards scattering
- Recursive method
  - IBP, DE's series solution
  - Transform system, modify recurrence relation
- Finished calculation masters up till 3-loop, need to put everything together and check against previous results
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Thank you!