

Self-Renormalization of Quasi-Light-Front Correlators on the Lattice (Lattice Parton Collaboration (LPC))

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arXiv: [2103.02965](https://arxiv.org/abs/2103.02965)

Speaker: Yushan Su
Supervisor: Xiangdong Ji

Background

X. Ji, Phys. Rev. Lett. 110, 262002 (2013)

Large Momentum Effective Theory (LaMET) proposes a way to get parton distribution function (PDF) from lattice data:

Quasi-Light-Front correlator calculated on lattice:

$$M = \langle H | O_{\Gamma}(z) | H \rangle_{large P^z}$$

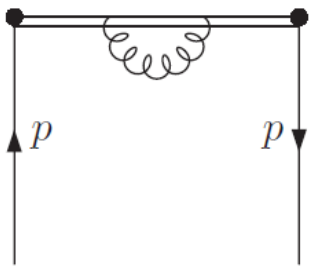
where $O_{\Gamma}(z) = \bar{\psi}(0) \Gamma U(0, z) \psi(z)$ is quasi-PDF operator (Γ is some gamma structure and $U(0, z)$ is a Wilson link), $|H\rangle_{large P^z}$ is a hadron state in a large momentum frame.



PDF

Renormalization
Fourier Transform
Matching

Self-Energy



$$\sim -\frac{\alpha_s C_F \pi}{2 a}$$

linear divergence
 a is lattice spacing



Ji, Zhang, Zhao,
Phys.Rev.Lett. 120 (2018) 11, 112001

$$M(z, a) = \text{Exp} \left[-\frac{m_{-1}}{a} z \right] f(z, a)$$

We must get rid of this linear divergence factor!

J.-W. Chen, X. Ji, and J.-H. Zhang,
Nucl. Phys. B915, 1 (2017)

- 1. Test of linear divergence
- 2. Self-renormalization procedure
- 3. Different matrix elements

RI/MOM factor $\frac{1}{4N_c} \text{Tr}[\gamma_t \langle q | O_{\gamma^t}(z) | q \rangle] \Big|_{p^2 = -\mu_R^2, p_z = p_t = 0}$

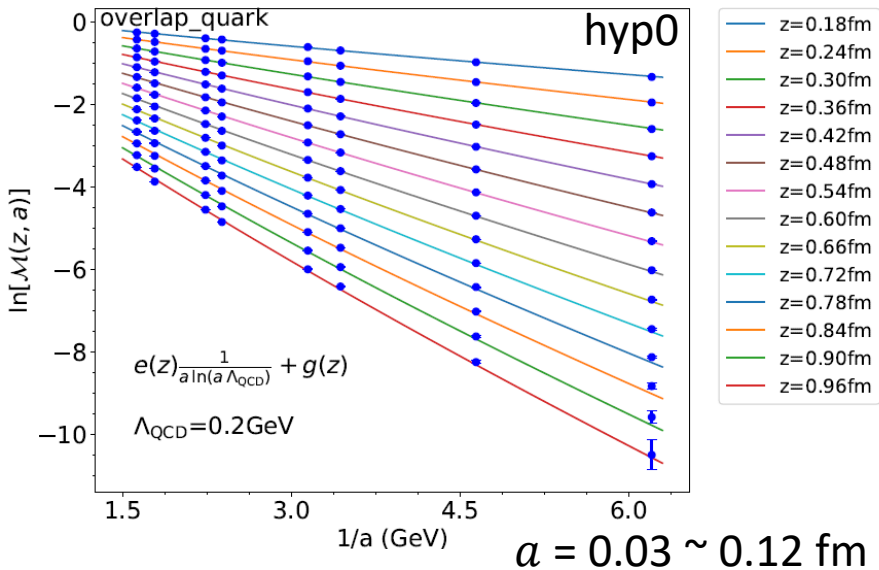
Zero-momentum hadron matrix element $\langle H | O_{\gamma^t}(z) | H \rangle_{\vec{P}=0}$

$$O_{\gamma^t}(z) = \bar{\psi}(0) \gamma^t U(0, z) \psi(z)$$

1. Test of linear divergence

e.g. for **RI/MOM factor**:

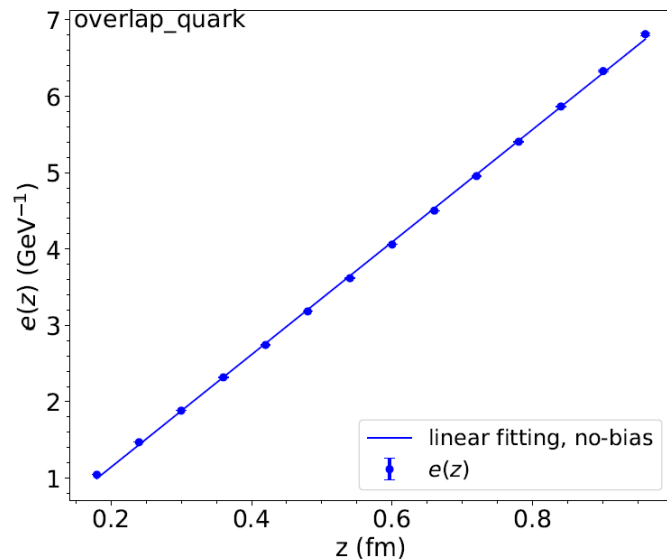
$$\mathcal{M}(z, a) = \frac{1}{4N_c} \text{Tr}[\gamma_t \langle q | O_{\gamma^t}(z) | q \rangle] \Big|_{p^2 = -\mu_R^2, p_z = p_t = 0}, \text{ where } O_{\gamma^t}(z) = \bar{\psi}(0) \gamma^t U(0, z) \psi(z)$$



Step1: for any given z , use

$$\ln[\mathcal{M}(z, a)] = \frac{e(z)}{a \ln(a \Lambda_{QCD})} + g(z)$$

to **fit dependence on a** to find $e(z)$
 ($e(z)$ and $g(z)$ are treated as fit parameters)
 (This fitting function is based on the
 perturbative renormalization theory in
 Ji, Zhang, Zhao,
Phys.Rev.Lett. 120 (2018) 11, 112001)



Step2: Plot $e(z)$ as a function of z

It shows that $e(z)$ has linear
 dependence on z

2. Self-renormalization procedure

$$\ln(\mathcal{M}) = \frac{e(z)}{a \ln(a \Lambda_{QCD})} + g(z)$$

Linear divergence, large

Residual, small



Modify

$\mathcal{M}$ is a matrix element

$$\ln(\mathcal{M}) = \frac{kz}{a \ln(a \Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b_0} \ln \left[\frac{\ln[1/(a \Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a \Lambda_{QCD})} \right]$$

Linear divergence

Residual

Discretization error

Resummation of Log divergence

Step1: For each z (z can be chosen in a large range),
extract $g(z)$ by **fitting the a dependence**:

$$\ln(M) = \frac{kz}{a \ln(a \Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a \Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a \Lambda_{QCD})} \right]$$



Step2: Fit the $m0$:

$$g(z) - \ln(Z_{\overline{MS}}) = m0 z$$

$$Z_{\overline{MS}}(z, \mu, \Lambda_{\overline{MS}}) = 1 + \frac{\alpha_s(\mu, \Lambda_{\overline{MS}}) C_F}{2\pi} \left[\frac{3}{2} \ln \frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2} \right],$$

for small z ($a < z < 0.24$ fm)



Step3: $m0 z$ subtraction (The result works in a large range of z):

$$\text{Exp}[g(z) - m0 z]$$

Fixed parameters:

$k = 7.4 \text{ GeV}^{-1} \text{ fm}^{-1}$ (for hyp0),

$\Lambda_{\overline{MS}} = 0.3 \text{ GeV}$, $\mu = 2 \text{ GeV}$

Fitted parameters: k (for hyp1),

Λ_{QCD} , $g(z)$, $f1(z)$, $f2(z)$, $m0$;

Fine-tuning parameter: d

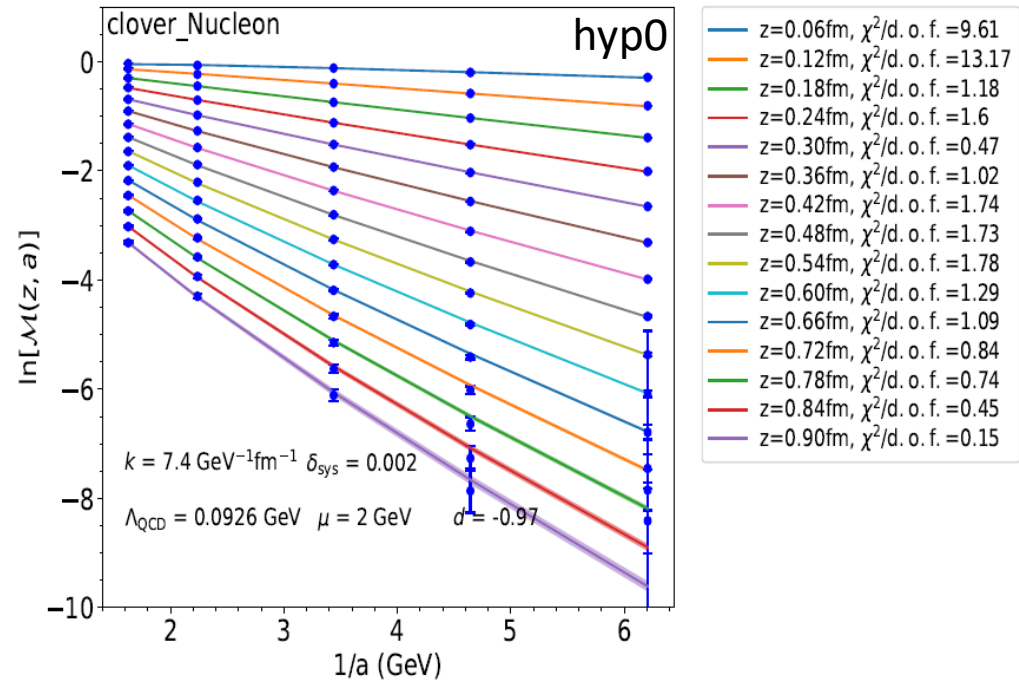
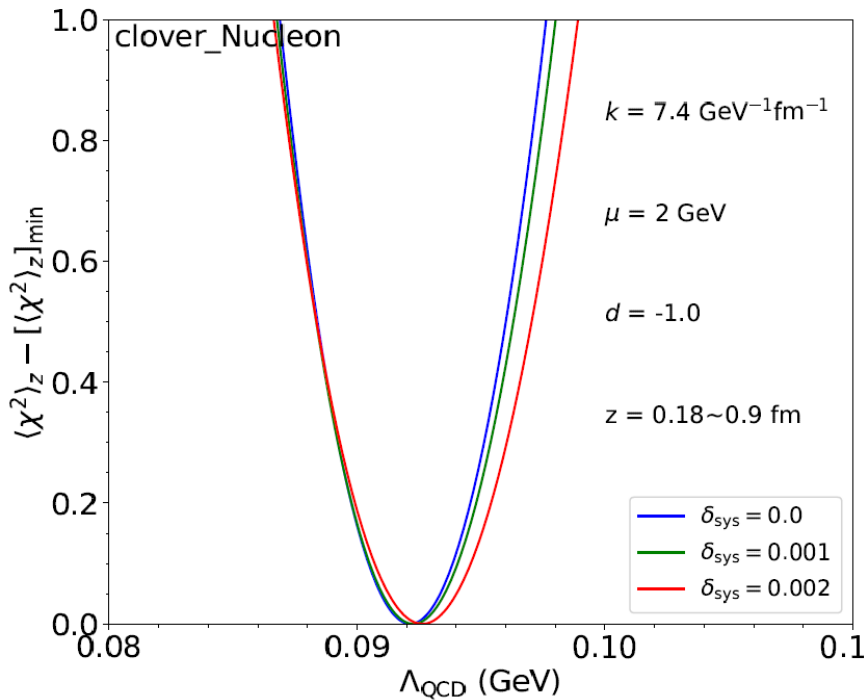
Residual
intrinsic physics



e.g. To self-renormalize a zero-momentum nucleon state
quasi-light-front correlator: $M(z, a) = \langle N | O_{\gamma_t}(z) | N \rangle_{\vec{P}=0}$

Step1: For each z (z can be chosen in a large range),
extract $g(z)$ by **fitting the a dependence**:

$$\ln(M) = \frac{kz}{a \ln(a \Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a \Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a \Lambda_{QCD})} \right]$$

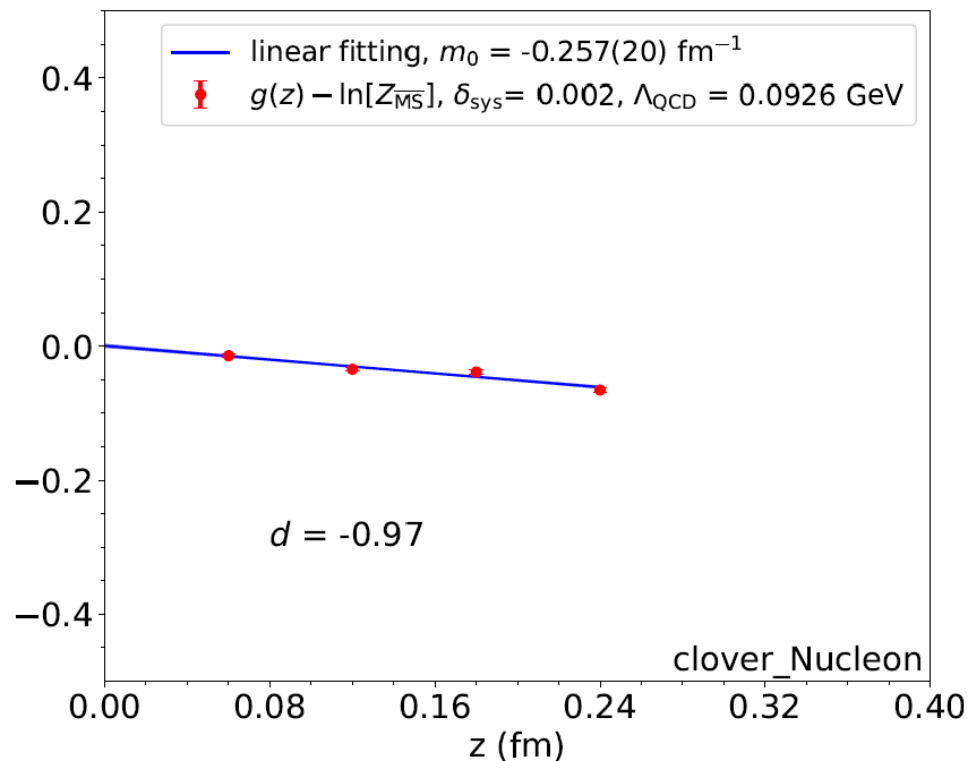


$$\sigma_{\ln M_{\text{new}}} = \sqrt{(\sigma_{\ln M_{\text{old}}})^2 + (\delta_{\text{sys}} a \mu)^2}$$

Step2: Fit the m_0 :

$$g(z) - \ln(Z_{\overline{MS}}) = m_0 z$$

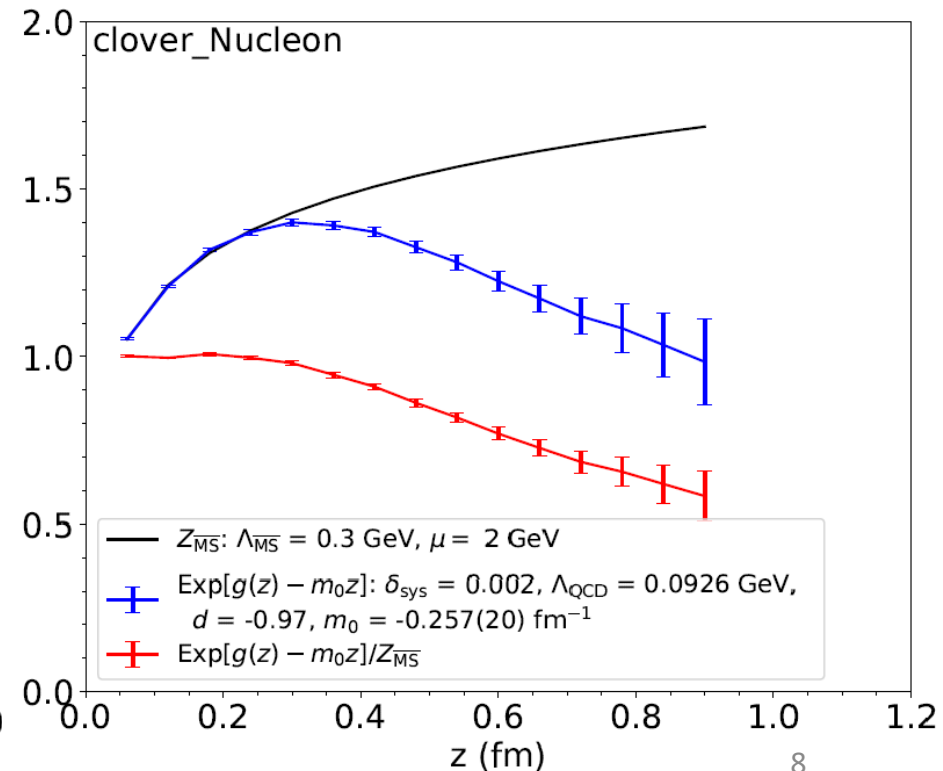
$$Z_{\overline{MS}}(z, \mu, \Lambda_{\overline{MS}}) = 1 + \frac{\alpha_s(\mu, \Lambda_{\overline{MS}}) C_F}{2\pi} \left[\frac{3}{2} \ln \frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2} \right], \text{ for small } z \text{ (} 0.06 \text{ fm} < z < 0.24 \text{ fm)}$$



Step3: m_0 z subtraction (The result works in a large range of z):

$$\text{Exp}[g(z) - m_0 z]$$

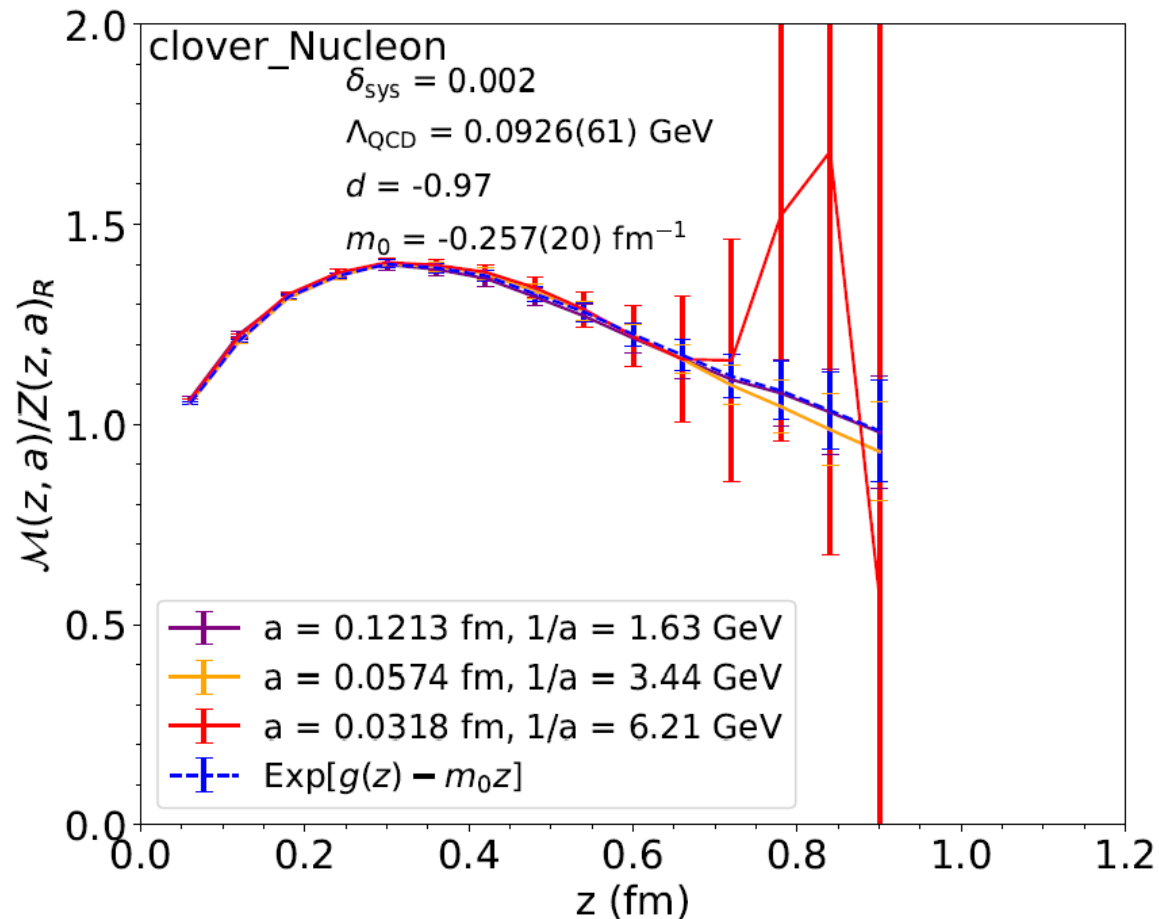
Residual intrinsic physics



Test of Self-renormalization method

Renormalization factor:

$$Z(z, a)_R = \text{Exp} \left\{ \frac{kz}{a \ln(a \Lambda_{QCD})} + m_0 z + \left\{ \begin{array}{l} f1(z)a, \text{MILC} \\ f2(z)a, \text{RBC} \end{array} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a \Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a \Lambda_{QCD})} \right] \right\}$$



3. Different matrix elements

Linear divergence parameters k and Λ_{QCD}

$$M = \langle state | O_{\gamma_t}(z) | state \rangle_{p^z=0}$$

$$\ln M = \frac{kz}{a \ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \dots$$

hyp0 (Unsmearing)

Cases	k	$\Lambda_{QCD}(\text{GeV})$	d	$m_0(\text{GeV})$
overlap quark	1.46	0.109(02)	-1.29	0.0462(16)
clover quark	1.46	0.135(02)	-1.35	0.1513(16)
overlap pion	1.46	0.093(10)	-1.17	-0.0357(46)
clover pion	1.46	0.086(14)	-0.92	-0.0715(50)
clover nucleon	1.46	0.093(06)	-0.97	-0.0508(40)

hyp1 (one-step of HYP smearing)

Cases	k	$\Lambda_{QCD}(\text{GeV})$
overlap quark	0.5521(07)	0.39
clover quark	0.6328(05)	0.39
overlap pion	0.5191(14)	0.39
clover pion	0.5178(18)	0.39
clover nucleon	0.5139(54)	0.39

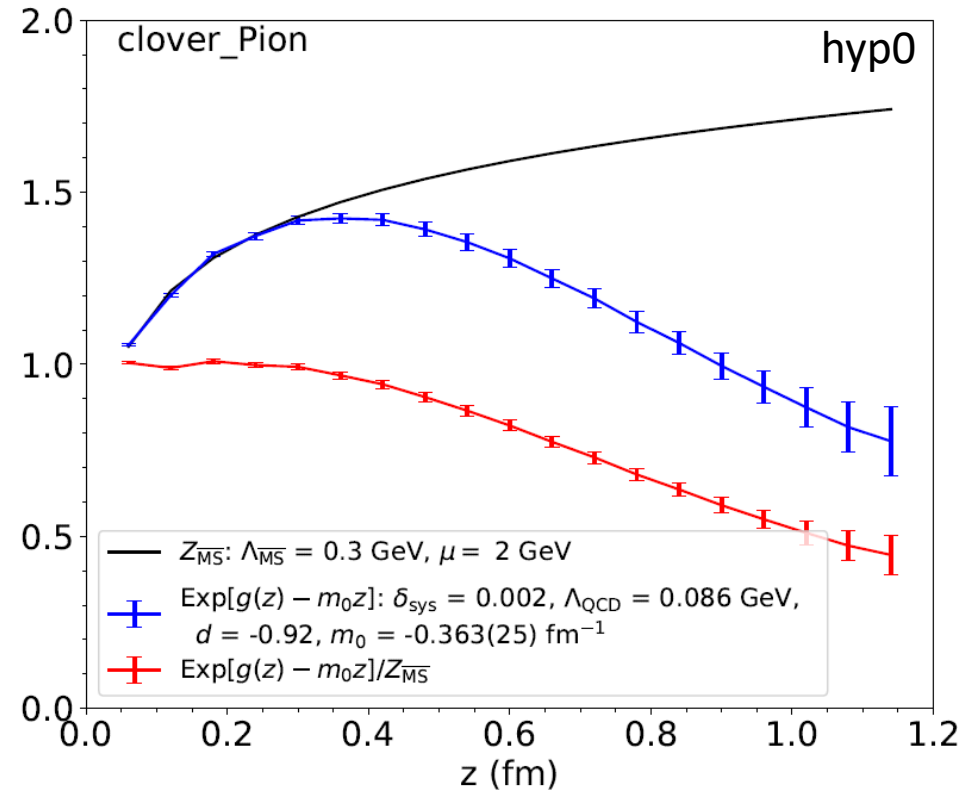
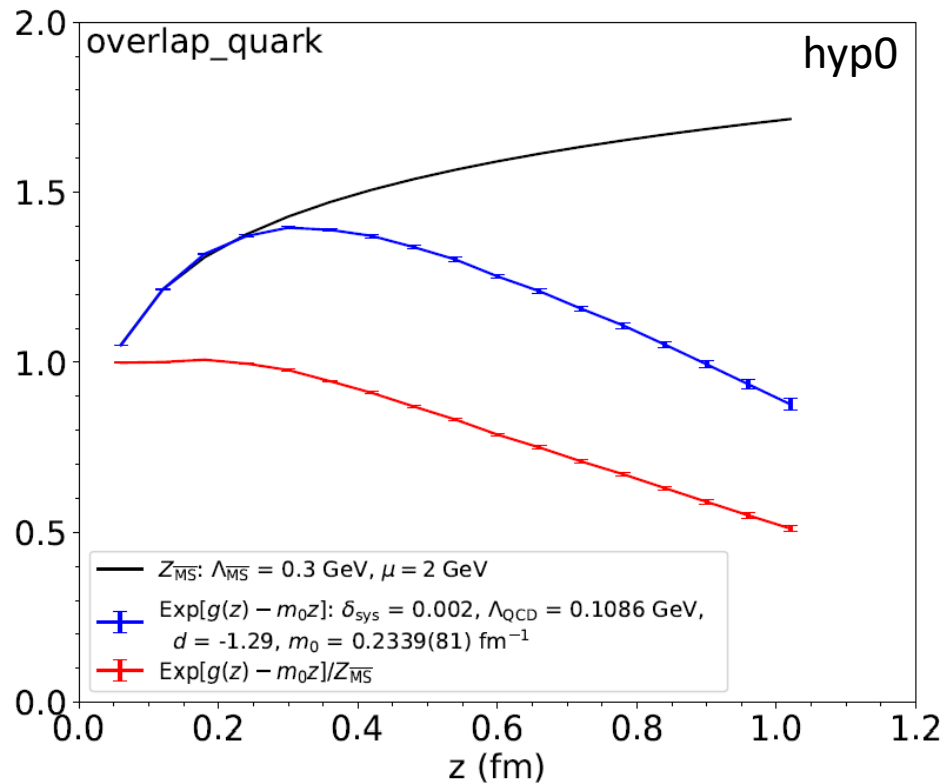
Valence
fermion
action

External state
(quark means
RI/MOM factor)

Linear divergences for hadron matrix elements are similar to each other.

Linear divergences for RI/MOM factors are different from those of hadron matrix elements.

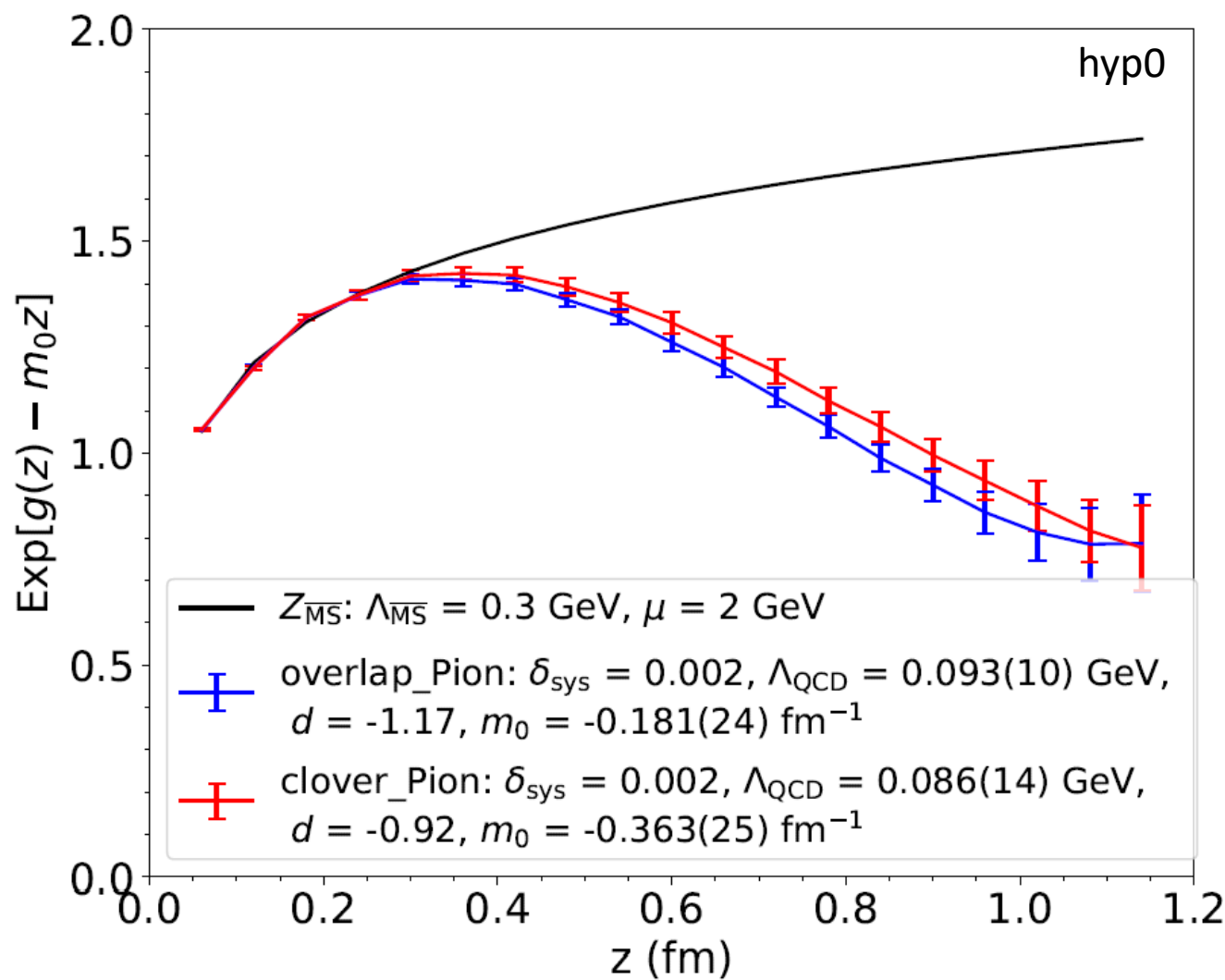
Residual intrinsic physics



Large extra non-perturbative effect at large distance in RI/MOM and Ratio scheme, which supports Hybrid renormalization method

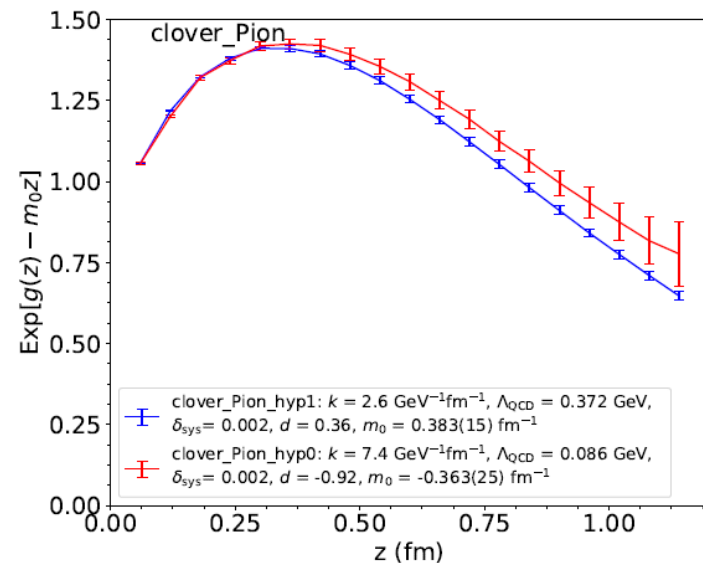
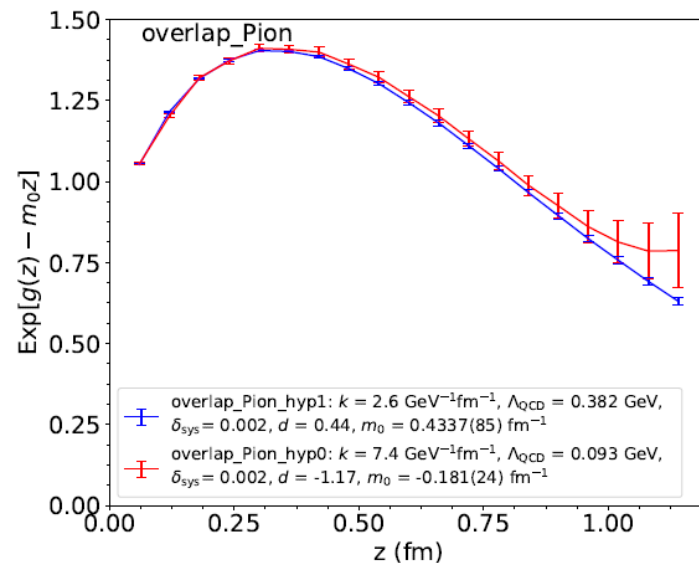
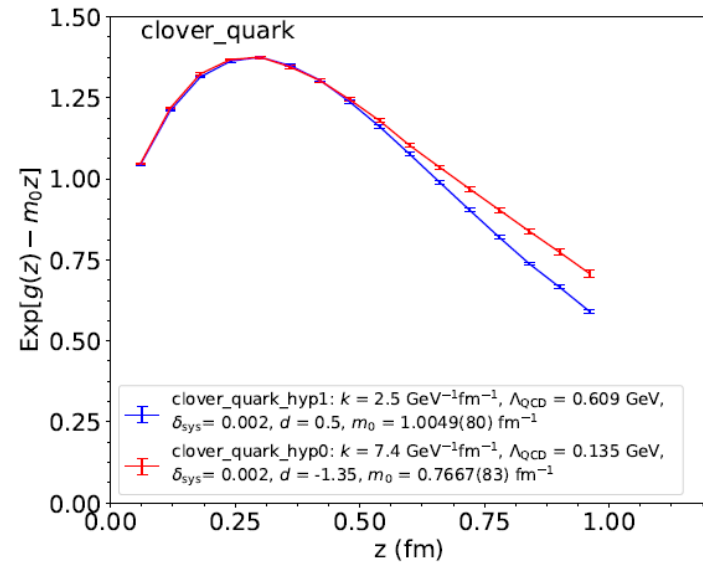
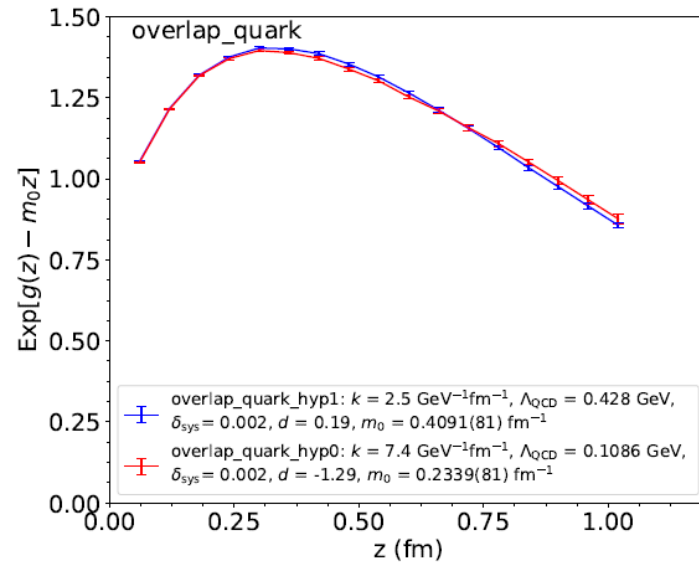
X. Ji, Y. Liu, A. Schafer, W. Wang, Y.-B. Yang, J.-H. Zhang, and Y. Zhao, (2020), arXiv:2008.03886

Residual intrinsic physics



Clover and overlap valence fermion actions
lead to similar residual intrinsic physics

Residual intrinsic physics



HYP smearing does not change
residual intrinsic physics

Conclusion

1. We propose the self-renormalization method. It works well in disentangling the renormalization factor (containing linear divergence, log divergence, discretization effect, renormalon effect) from residual intrinsic physics;
2. Linear divergences for RI/MOM factors are different from those of hadron matrix elements;
3. Large non-perturbative effect at large distance in RI/MOM and Ratio scheme;
4. Clover and overlap, hyp0 and hyp1 lead to similar residual intrinsic physics.

Appendix

A. Explanation of each term in the fitting function

For each z (z can be chosen in a large range),
extract $g(z)$ by fitting the a dependence:

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda_{QCD})} + g(z) + \left\{ \begin{array}{l} f1(z)a, MILC \\ f2(z)a, RBC \end{array} \right. + \frac{3C_F}{b_0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a\Lambda_{QCD})} \right]$$



$g(z)$ is a fitted parameter for each z

Fit the m_0 :

$$g(z) - \ln(Z_{\overline{MS}}) = m_0 z$$

$$Z_{\overline{MS}}(z, \mu, \Lambda_{\overline{MS}}) = 1 + \frac{\alpha_s(\mu, \Lambda_{\overline{MS}}) C_F}{2\pi} \left[\frac{3}{2} \ln \frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2} \right],$$

for small z ($a < z < 0.2$ fm)



$m_0 z$ subtraction:

$$g(z) - m_0 z$$

Our final result,
the physical result

Λ_{QCD} as a fitted parameter

We only consider leading order term, but there should be higher order terms, and the higher order terms may be different for different actions and ensembles.

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda_{QCD})} + g(z) + \left\{ \begin{array}{l} f1(z)a, MILC \\ f2(z)a, RBC \end{array} \right. + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a\Lambda_{QCD})} \right]$$

Choose a proper Λ_{QCD} , we can use the leading order term to represent the results up to higher order corrections:

$$\alpha_s(Q, \Lambda_{QCD}) \sim \alpha_s(Q, \Lambda) + c1 * \alpha_s(Q, \Lambda)^2 + c2 * \alpha_s(Q, \Lambda)^3 \dots$$

[G.Peter Lepage \(Cornell U., LNS\), Paul B. Mackenzie \(Fermilab\), On the viability of lattice perturbation theory, DOI: \[10.1103/PhysRevD.48.2250\]\(#\)](#)

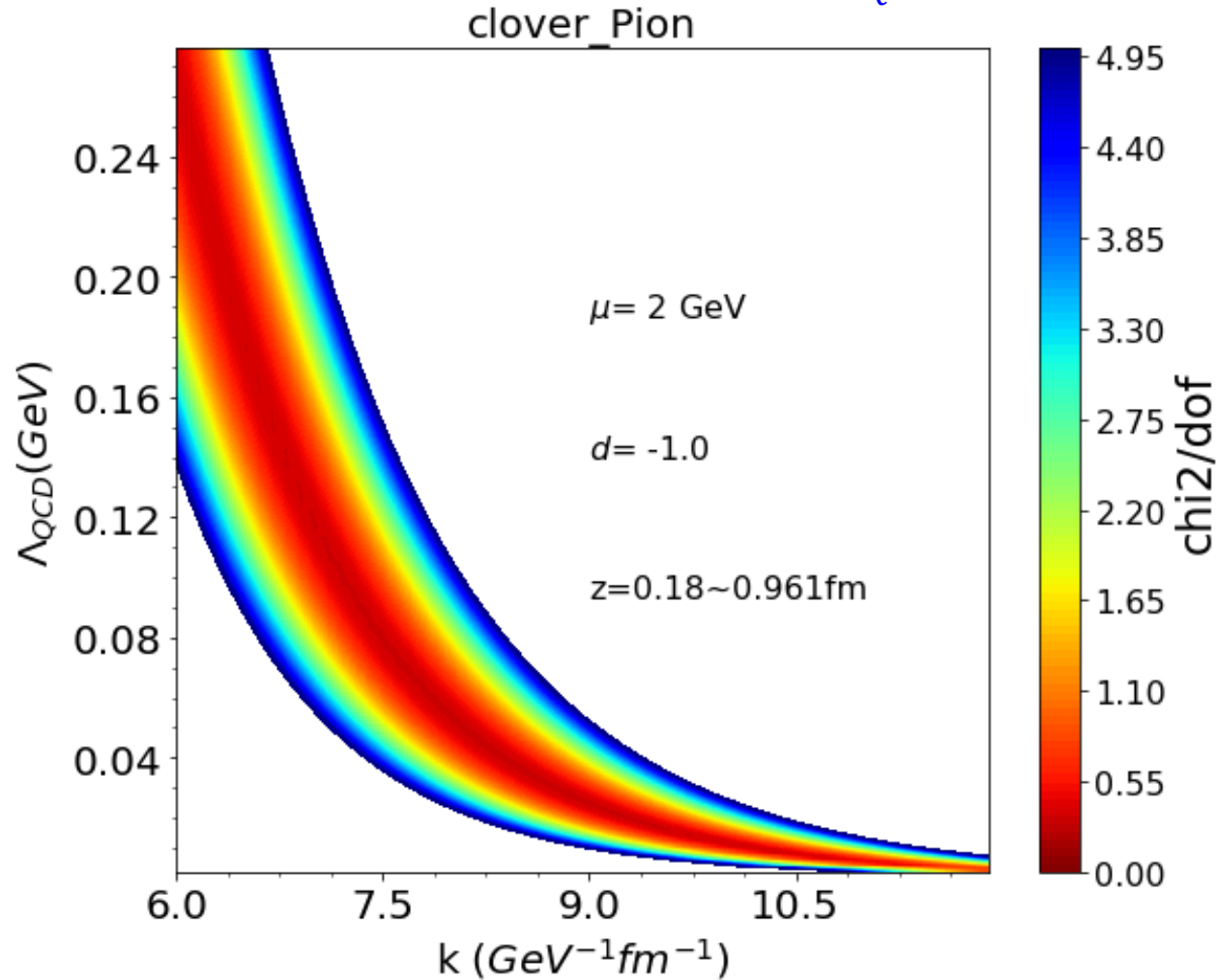
Let the data help us choose the Λ_{QCD} !

Renormalon Uncertainty

Fit the matrix element with the formula:

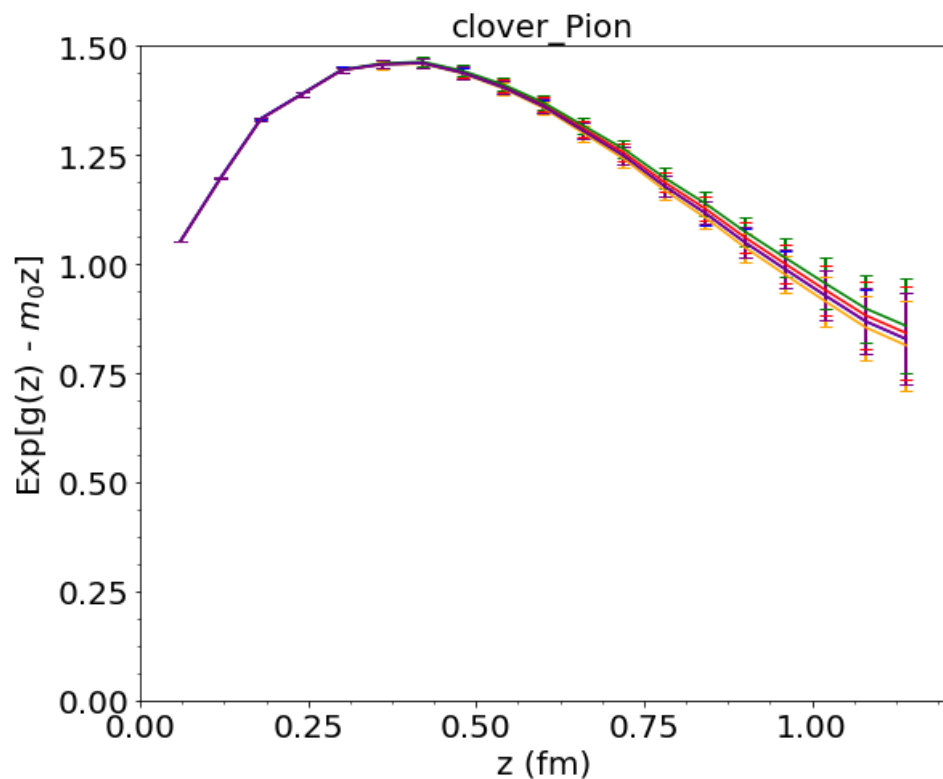
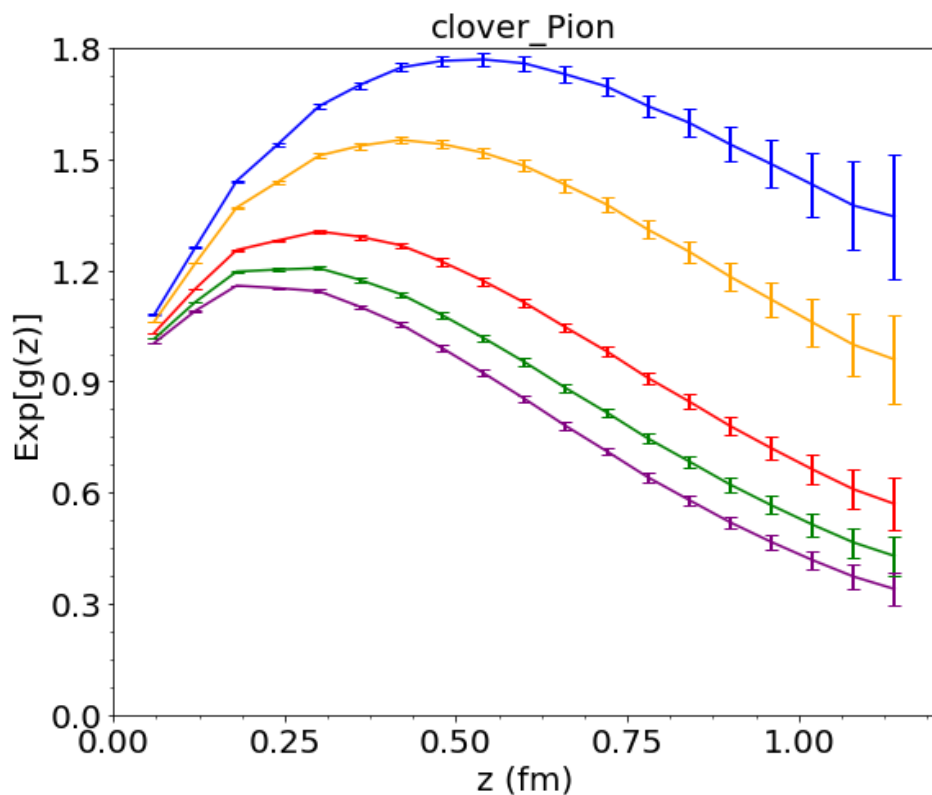
$$\ln(Z) = \frac{kz}{a \ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a\Lambda_{QCD})} \right]$$

Chisqmap with respect to k and Λ_{QCD} :



$g(z), f1(z), f2(z)$ fitted for each set of $\{k, \Lambda_{QCD}\}$

Choose some sets of parameters on the small chisq band

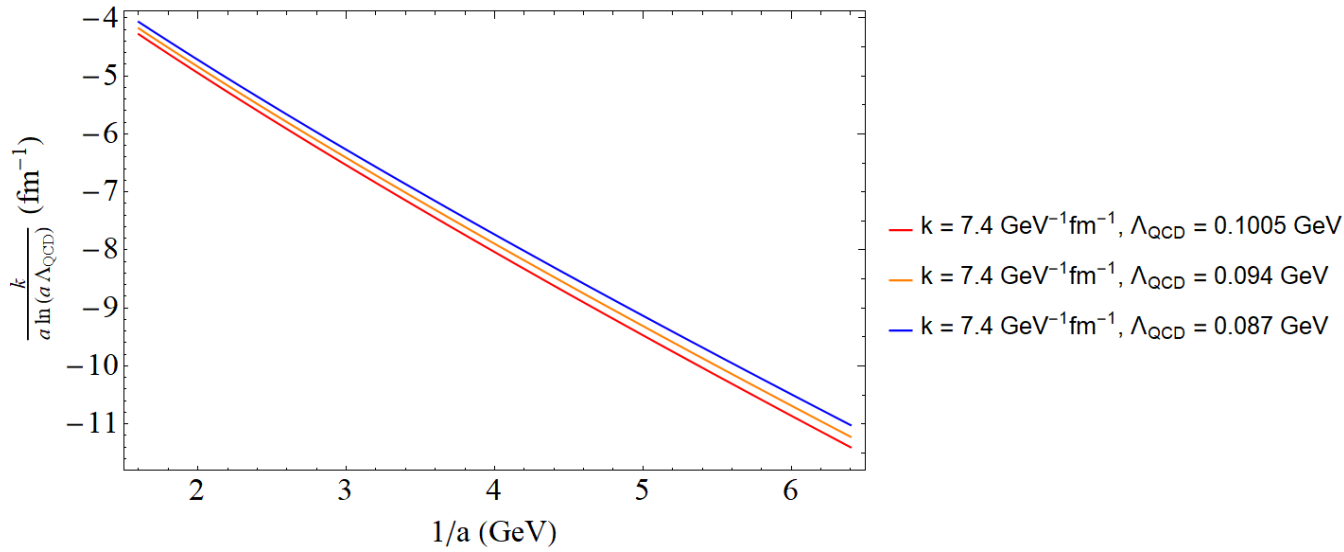
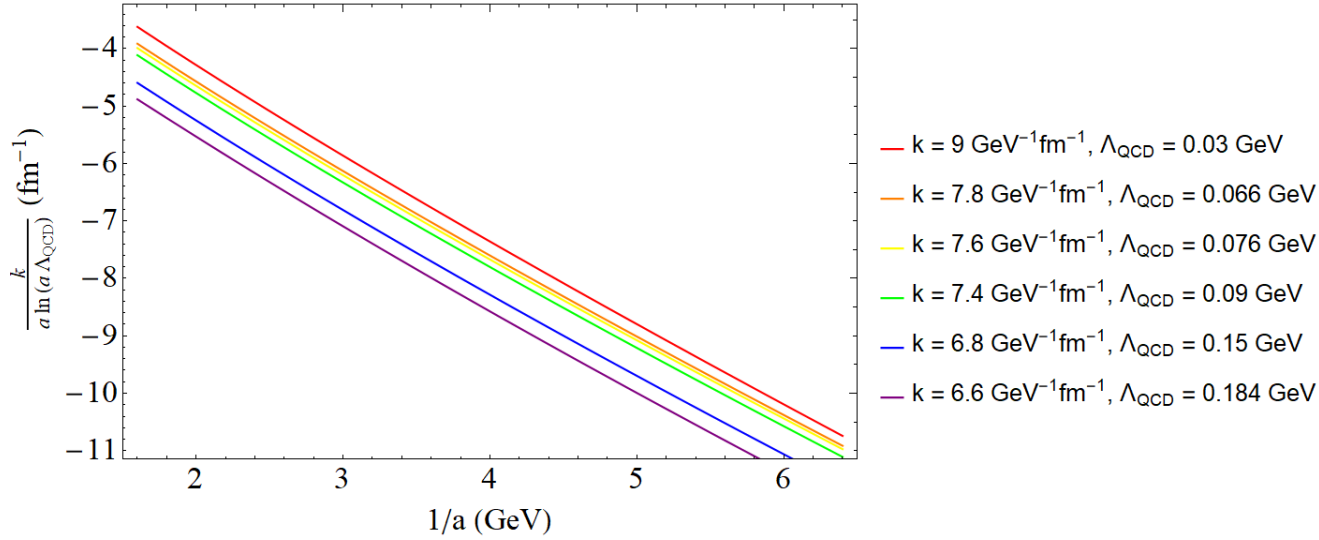


- +— $k = 7.4 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.09 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $d = -0.931$
- +— $k = 7.4 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.11 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $d = -0.845$
- +— $k = 7.4 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.08 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $d = -0.98$
- +— $k = 6.6 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.184 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $d = -0.6$
- +— $k = 9 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.03 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $d = -1.42$

- +— $k = 7.4 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.09 \text{ GeV}$, $d = -0.931$, $\Lambda_{MS} = 0.3 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $m_0 = -0.3423(95) \text{ fm}^{-1}$
- +— $k = 7.4 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.11 \text{ GeV}$, $d = -0.845$, $\Lambda_{MS} = 0.3 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $m_0 = 0.1438(95) \text{ fm}^{-1}$
- +— $k = 7.4 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.08 \text{ GeV}$, $d = -0.98$, $\Lambda_{MS} = 0.3 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $m_0 = -0.6067(95) \text{ fm}^{-1}$
- +— $k = 6.6 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.184 \text{ GeV}$, $d = -0.6$, $\Lambda_{MS} = 0.3 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $m_0 = 0.4243(95) \text{ fm}^{-1}$
- +— $k = 9 \text{ GeV}^{-1} \text{ fm}^{-1}$, $\Lambda_{QCD} = 0.03 \text{ GeV}$, $d = -1.42$, $\Lambda_{MS} = 0.3 \text{ GeV}$, $\mu = 2 \text{ GeV}$, $m_0 = -0.7795(95) \text{ fm}^{-1}$

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a\Lambda_{QCD})} \right]$$

In our data range (lattice spacing range), varying k and Λ_{QCD} along the small chisq band just translates $\frac{k}{a \ln(a\Lambda_{QCD})}$ by a constant:



$$\ln(Z) = \frac{kz}{a \ln(a \Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a \Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a \Lambda_{QCD})} \right]$$

varying k and Λ_{QCD} along
the small chisq band



In our data range

$$\frac{kz}{a \ln(a \Lambda_{QCD})} \rightarrow \frac{kz}{a \ln(a \Lambda_{QCD})} + C^*z$$



$$g(z) \rightarrow g(z) - C^*z$$



The physical result
 $\text{Exp}(g(z) - m_0 z)$
will not change

We cannot determine k
and Λ_{QCD} from our data.
But the uncertainty of k
and Λ_{QCD} doesn't
influence the physical
result.

We can fix k at the one-loop
perturbative value with $n_f=3$:

$$k = \frac{(2\pi i)^2}{3 \left(11 - \frac{2}{3} * n_f \right)} \frac{1}{0.1973269631 \text{ GeV fm}} \\ = 7.4 \text{ GeV}^{-1} \text{ fm}^{-1}$$

Ji, Zhang, Zhao,
Phys.Rev.Lett. 120 (2018) 11, 112001

Then choose Λ_{QCD} as the
best fitted value

Resummation of the logarithmic divergence term

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a\Lambda_{QCD})} \right]$$

logarithmic divergence term from wavefunction and vertex renormalizations at one-loop order:

$$Z_{\psi,z} = 1 + \frac{3 C_F \alpha_s}{2\pi} \frac{1}{4-d}$$

Ji, X., Liu, Y.-S., Liu, Y., Zhang, J.-H., and Zhao, Y., "Large-Momentum Effective Theory", [arXiv:2004.03543v1](https://arxiv.org/abs/2004.03543v1), 2020.

Solve the renormalization group equation to get the QCD rescaling factor:

$$\frac{dZ_Q}{d\ln(Q)} = \gamma Z_Q, \text{ where } \gamma = -\frac{3 C_F \alpha_s(Q, \Lambda_{QCD})}{2\pi}$$

⇓

$$Z_{1/a} = \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right]^{\frac{3C_F}{b0}} Z_\mu$$

we can fix $\mu = 2 \text{ GeV}$

Resummation of the logarithmic divergence term to higher order

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a\Lambda_{QCD})} \right]$$

Solve the renormalization group equation to get the QCD rescaling factor:

$$\frac{dZ_Q}{d\ln(Q)} = \gamma Z_Q, \text{ where } \gamma = \frac{3 C_F \alpha_s(Q, \Lambda_{QCD})}{2\pi} + c2^* \alpha_s(Q, \Lambda_{QCD})^2$$

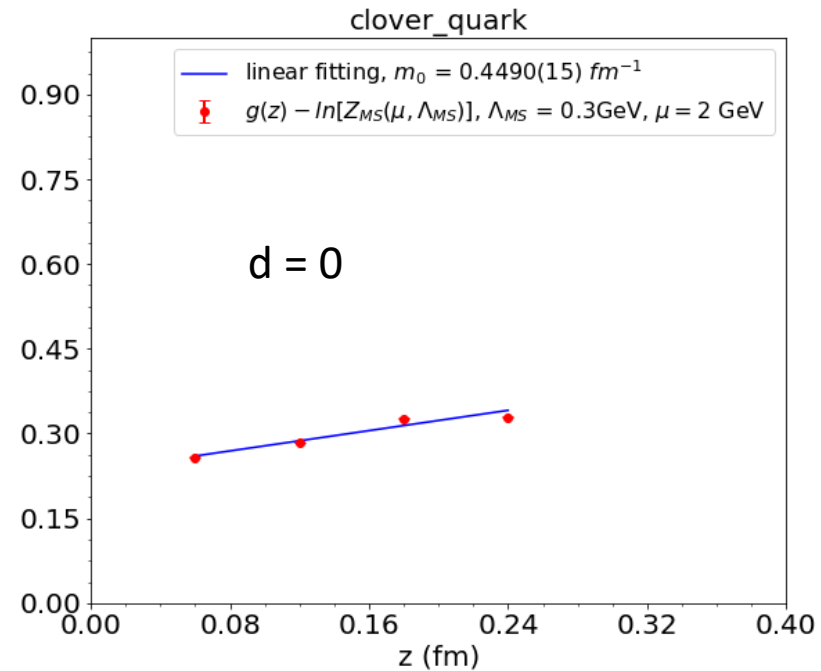
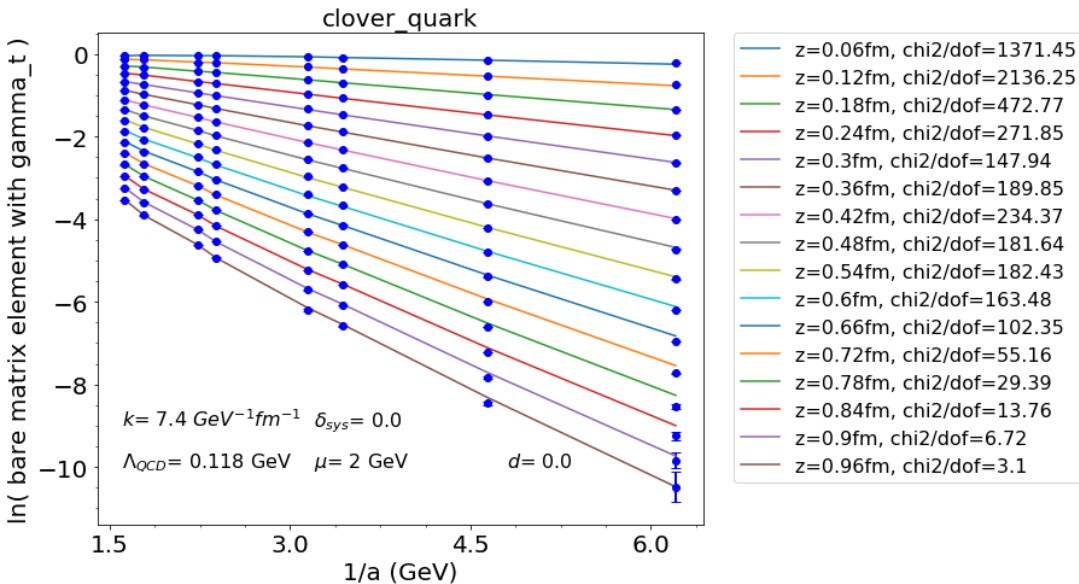
⇓

$$Z_{1/a} = \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right]^{\frac{3C_F}{b0}} \left[1 + \frac{d}{\ln(a\Lambda_{QCD})} \right] Z_\mu$$

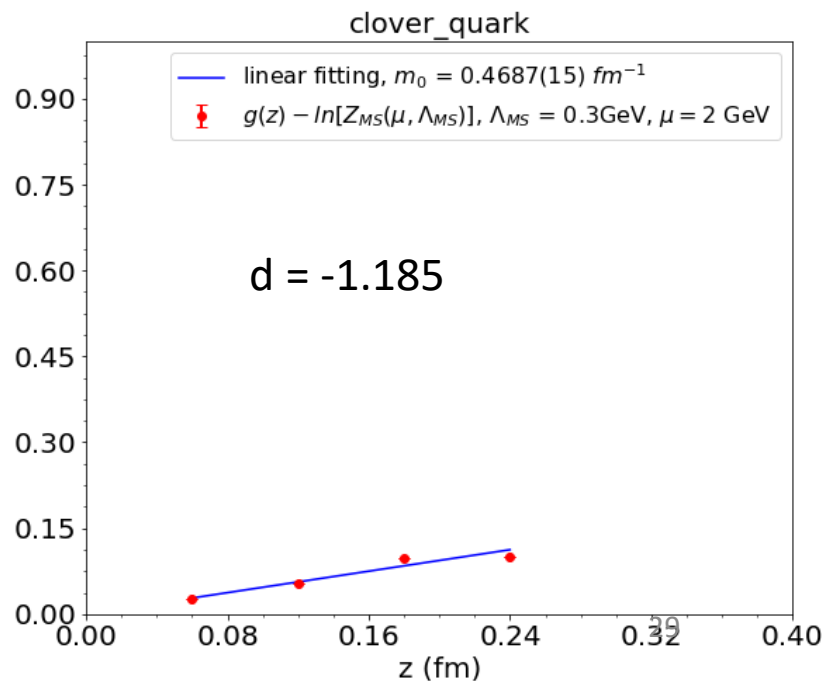
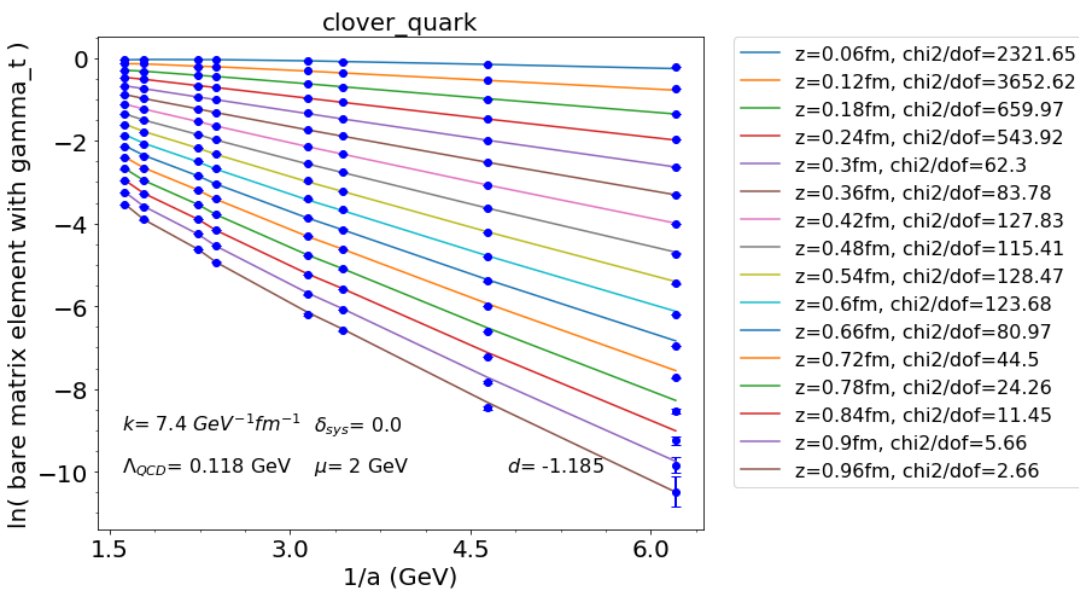
d is related to the actions and ensembles.

However, we will treat d as a fine-tuning parameter.

*Xiangdong Ji, M.J. Musolf,
Sub-leading logarithmic
mass-dependence in
heavy-meson form-factors,
Physics Letters B, Volume
257, Issues 3–4, 1991*



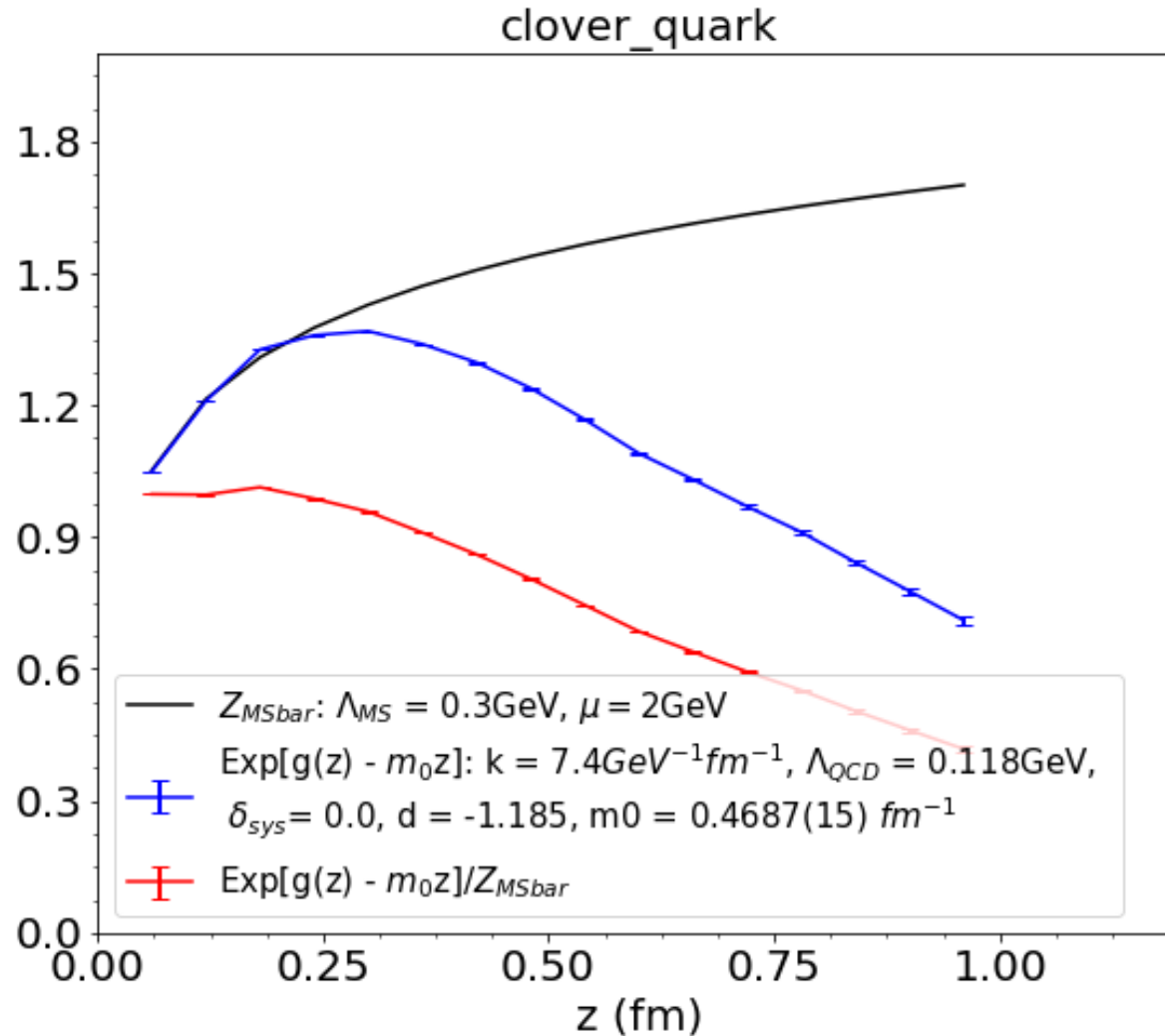
Tune d to make $g(z) - \ln(Z_{\overline{\text{MS}}}) \propto z$



$$kZ \frac{1}{a \ln(a \Lambda_{\text{QCD}})} + g(z) + \frac{3C_f}{b_0} \ln \left[\frac{\ln(1/(a \Lambda_{\text{QCD}}))}{\ln(\mu/\Lambda_{\text{QCD}})} \right] + \ln \left(1 + \frac{d}{\ln(a \Lambda_{\text{QCD}})} \right) + f_1(z)a \text{ for MILC}$$

$$kZ \frac{1}{a \ln(a \Lambda_{\text{QCD}})} + g(z) + \frac{3C_f}{b_0} \ln \left[\frac{\ln(1/(a \Lambda_{\text{QCD}}))}{\ln(\mu/\Lambda_{\text{QCD}})} \right] + \ln \left(1 + \frac{d}{\ln(a \Lambda_{\text{QCD}})} \right) + f_2(z)a \text{ for RBC}$$

After m_0 z subtraction, the results from lattice and the results from continuum perturbation theory can match in the region where $a < z < 0.2$ fm.



Large differences between perturbative and non-perturbative RI/MOM factors at large z

B. Different Fitting functions

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda)} + g(z)$$

Not accurate enough

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda)} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases}$$

Our current function

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]} \right] + \ln \left[1 + \frac{d}{\ln(a\Lambda_{QCD})} \right]$$

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda)} b0 \ln \left(\frac{1}{a^2 \Lambda^2} \right) \alpha_s (1 + \lambda \alpha_s) + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases}$$

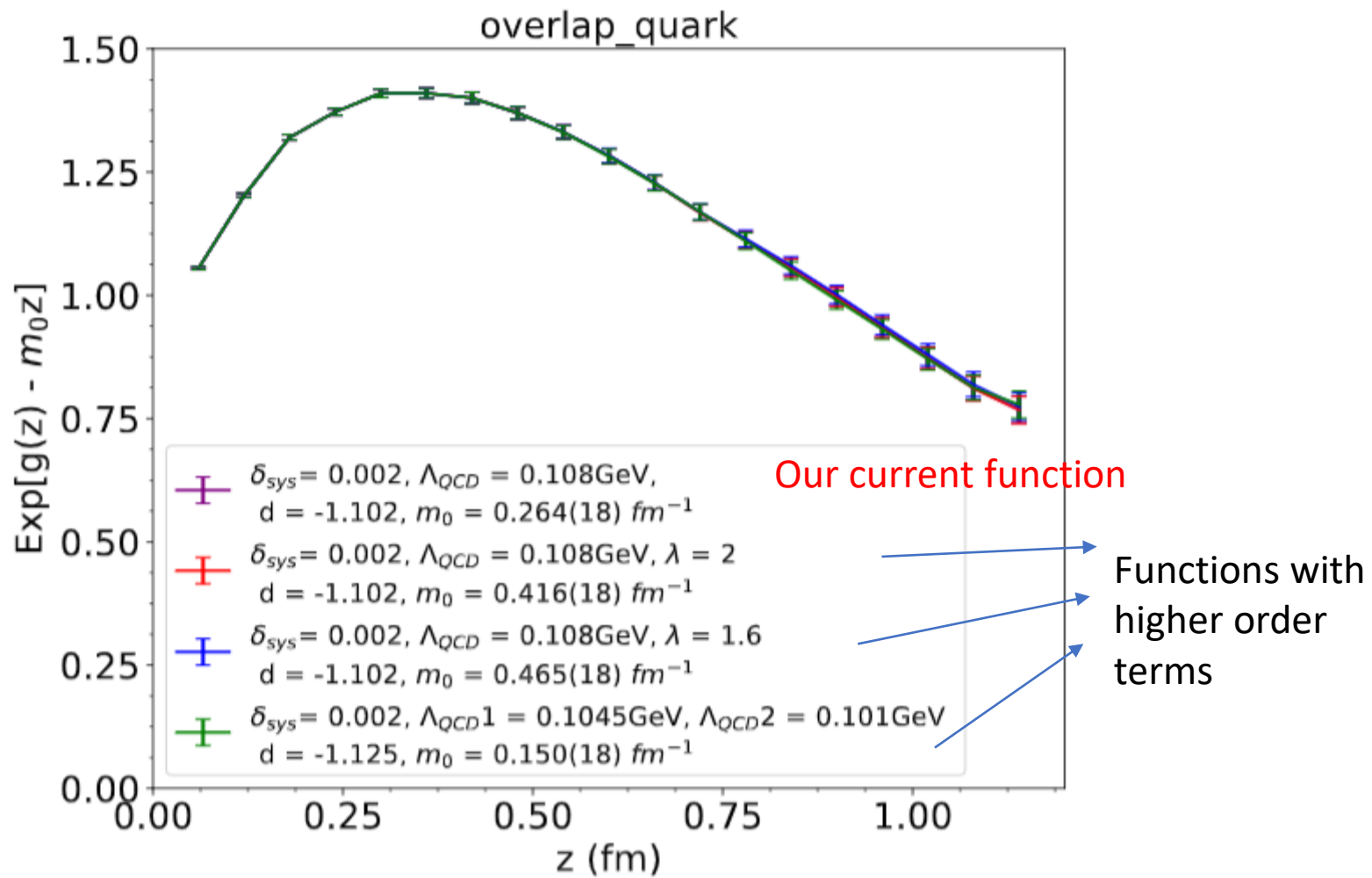
Same physical results as our current one

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda)} b0 \ln \left(\frac{1}{a^2 \Lambda^2} \right) \frac{\alpha_s}{1 - \lambda \alpha_s} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases}$$

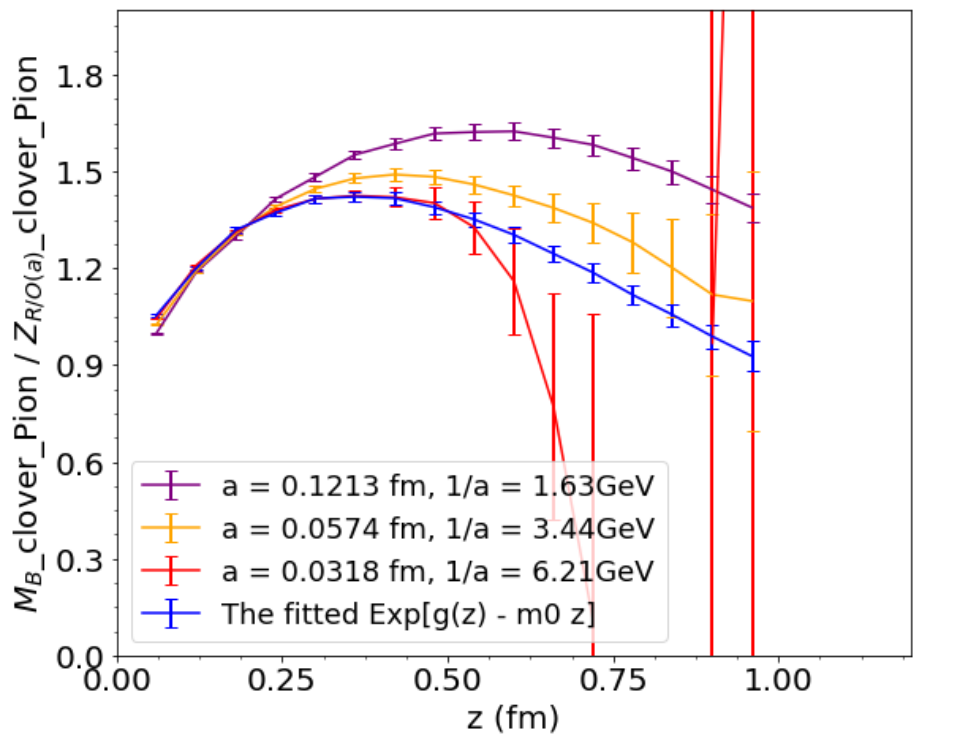
$$\ln(Z) = g(z) + \begin{cases} \frac{kz}{a \ln(a\Lambda_{QCD1})} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD1})]}{\ln[\mu/\Lambda_{QCD1}]} \right] + \ln \left[1 + \frac{d}{\ln(a\Lambda_{QCD1})} \right] + f1(z)a, MILC \\ \frac{kz}{a \ln(a\Lambda_{QCD2})} + \frac{3C_F}{b0} \ln \left[\frac{\ln[1/(a\Lambda_{QCD2})]}{\ln[\mu/\Lambda_{QCD2}]} \right] + \ln \left[1 + \frac{d}{\ln(a\Lambda_{QCD2})} \right] + f2(z)a, RBC \end{cases}$$

$$\ln(Z) = \frac{kz}{a \ln(a\Lambda)} b0 \ln \left(\frac{1}{a^2 \Lambda^2} \right) \alpha_s (1 + \lambda \alpha_s) + g(z) + \begin{cases} f11(z)a + f12(z)a^2, MILC \\ f21(z)a + f22(z)a^2, RBC \end{cases}$$

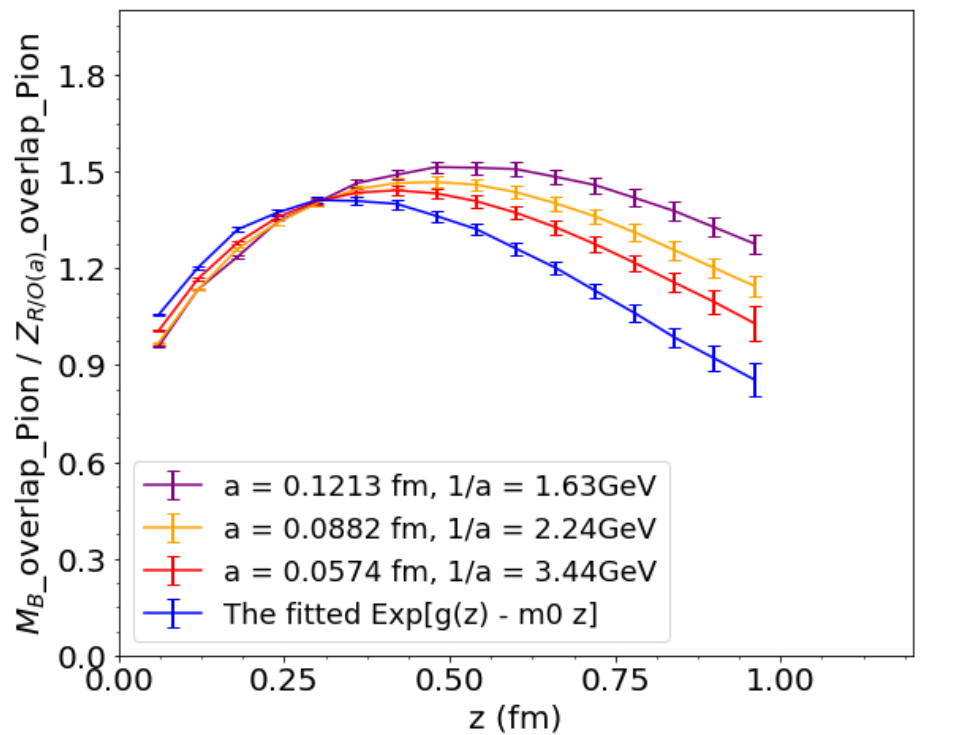
Unstable



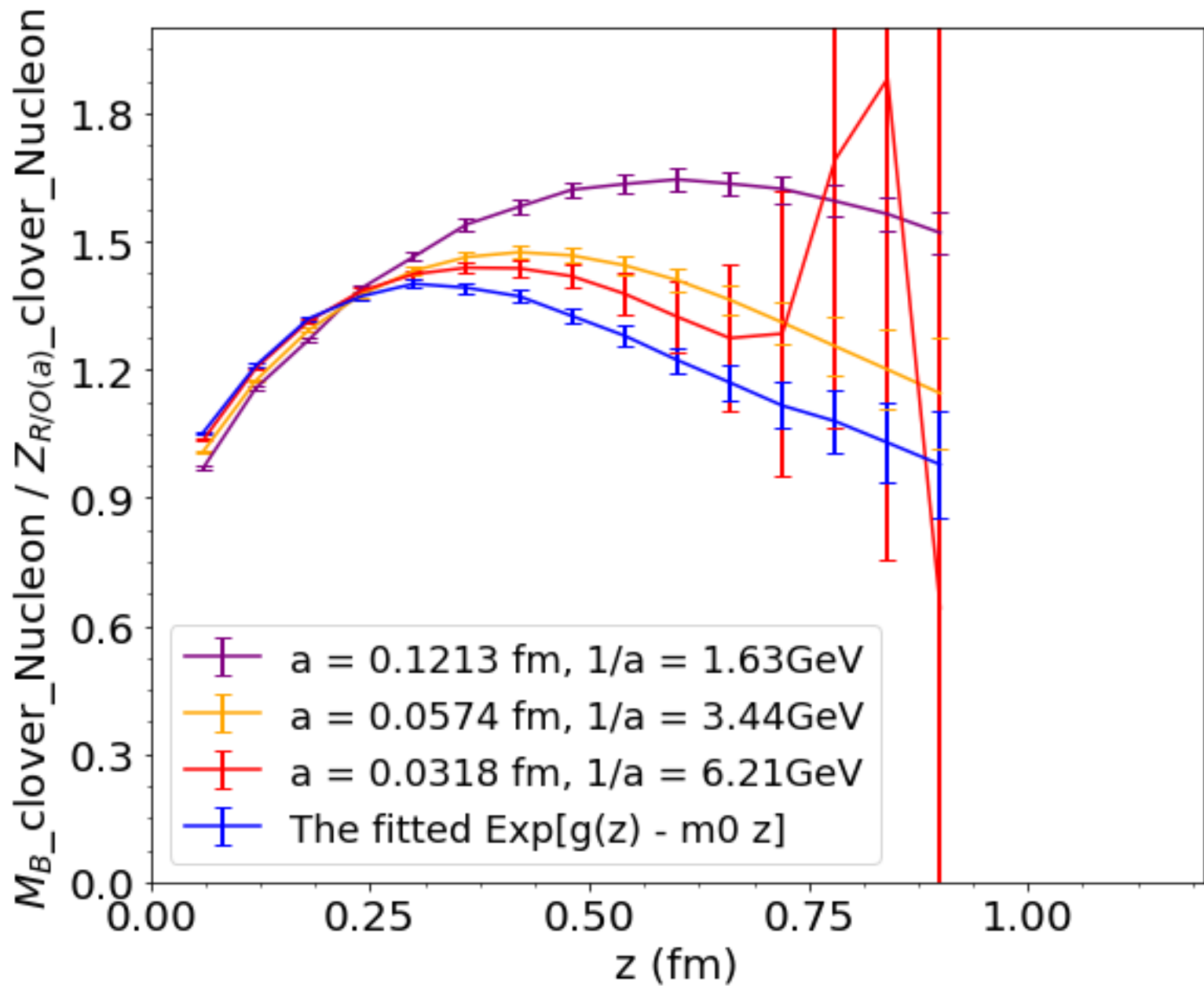
C. Discretization Effect



$$Z_{R/O(a)} = \text{Exp}\left[kz \frac{1}{a \ln(a \Lambda_{QCD})} + \frac{3C_f}{b_0} \ln\left[\frac{\ln(1/(a \Lambda_{QCD}))}{\ln(\mu/\Lambda_{QCD})}\right] + \ln\left(1 + \frac{d}{\ln(a \Lambda_{QCD})}\right) + m_0 z\right]$$

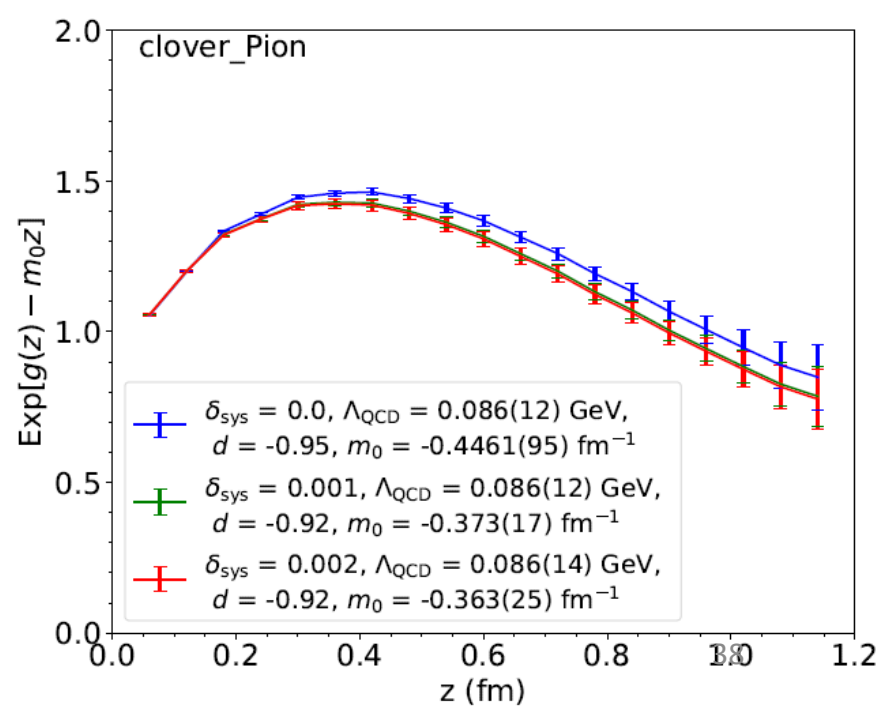
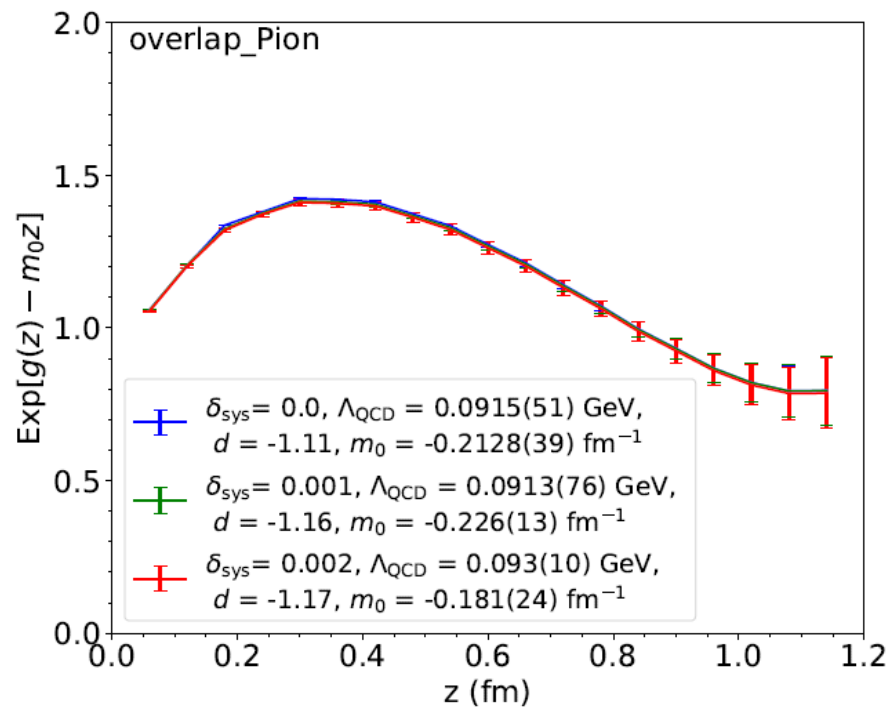
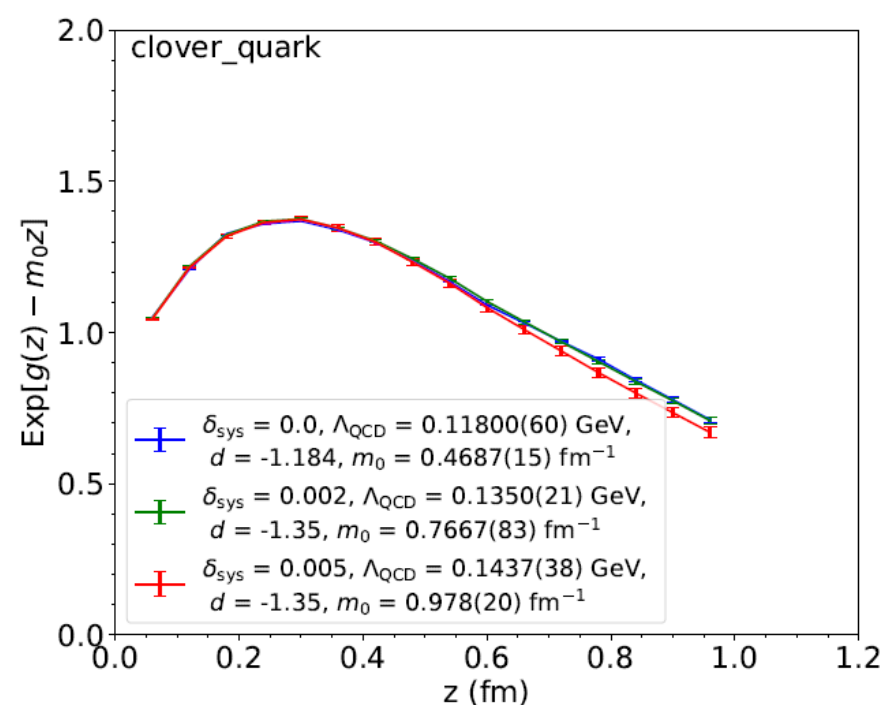
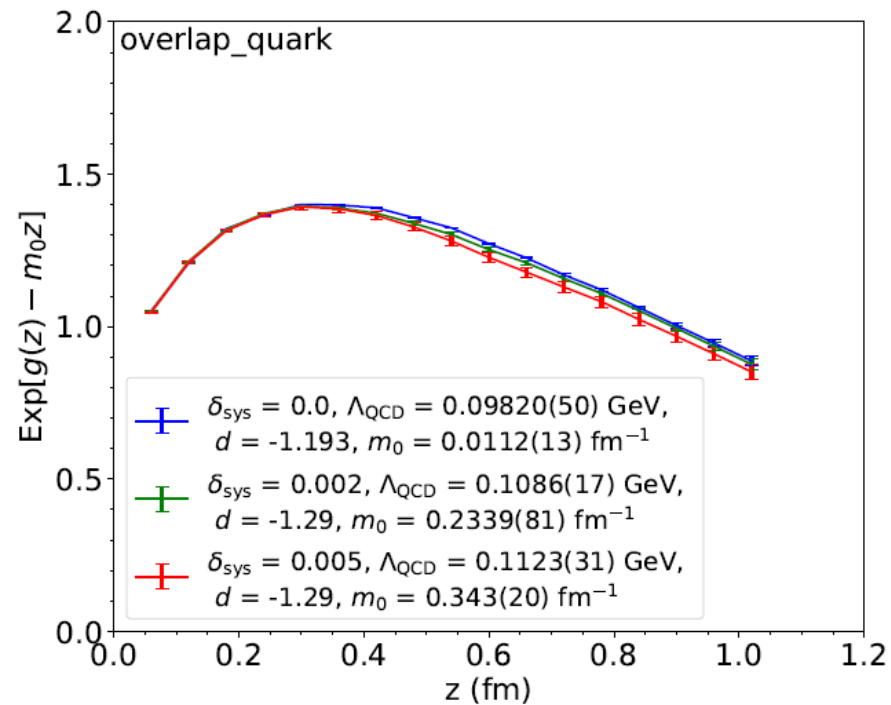


$$Z_{R/O(a)} = \text{Exp}\left[kz \frac{1}{a \ln(a \Lambda_{QCD})} + \frac{3C_f}{b_0} \ln\left[\frac{\ln(1/(a \Lambda_{QCD}))}{\ln(\mu/\Lambda_{QCD})}\right] + \ln\left(1 + \frac{d}{\ln(a \Lambda_{QCD})}\right) + m_0 z\right]$$



$$Z_{R/O(a)} = \text{Exp}\left[k z \frac{1}{a \ln(a \Lambda_{QCD})} + \frac{3C_f}{b_0} \ln\left[\frac{\ln(1/(a \Lambda_{QCD}))}{\ln(\mu/\Lambda_{QCD})}\right] + \ln\left(1 + \frac{d}{\ln(a \Lambda_{QCD})}\right) + m_0 z\right]_{36}$$

D. Dummy Systematic Errors



E. Some vacuum matrix elements

Analysis of Wilson Loop, Wilson Link, VEV

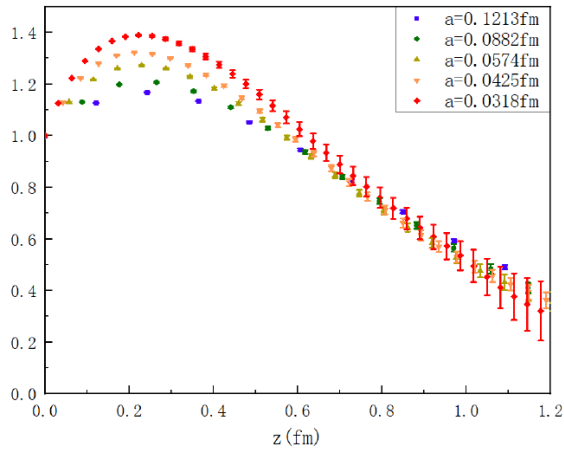


FIG. 6. The pion quasi-PDF matrix element renormalized by the linear divergence part of Wilson Loop.

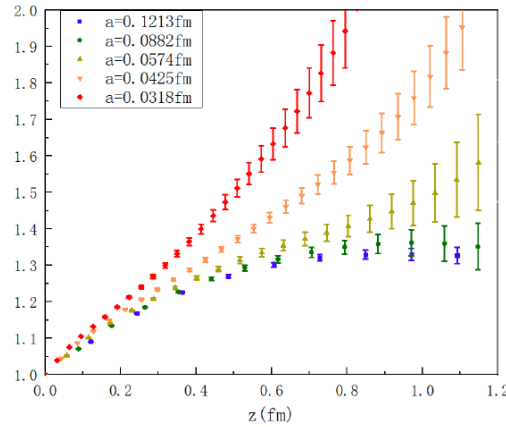


FIG. 7. The effective decay rate of Landau gauge fixed Wilson link (upper panel), and pion matrix element renormalized by the Wilson link.

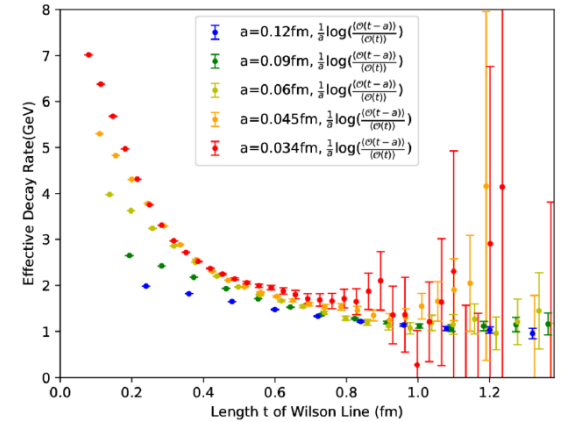


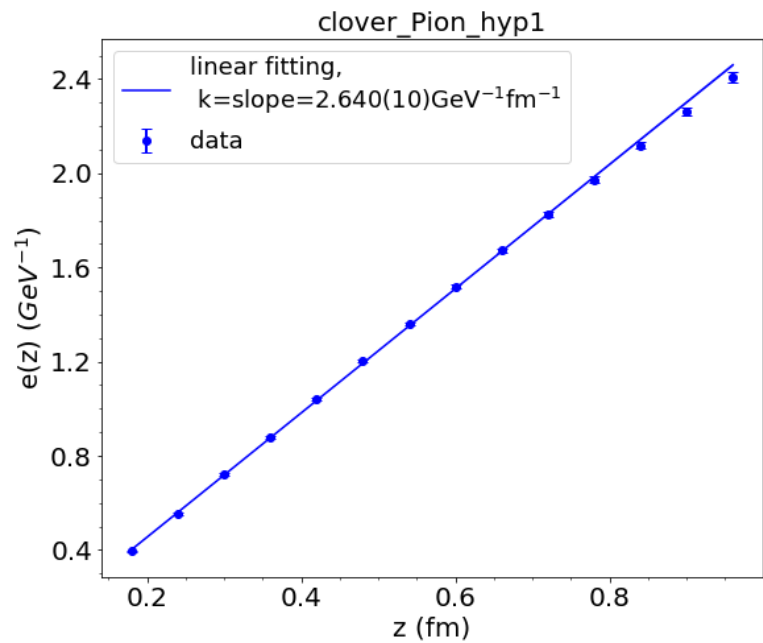
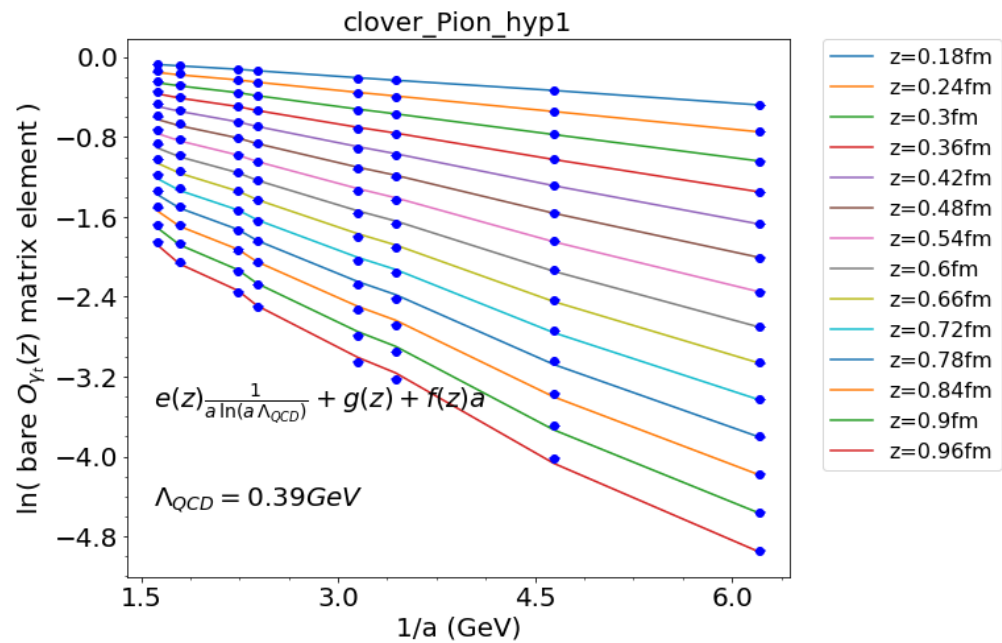
FIG. 4. Effective decay rate $\frac{1}{a} \log \left(\frac{O_{\gamma_t}(t-a)}{O_{\gamma_t}} \right)$ of the O_{γ_t} VEV. The gaps between the results at different lattice spacings decreases on t .

$$\ln(M) = \frac{e(z)}{a \ln(a \Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases}$$

Step1: for any given z , fit dependence on a to find $e(z)$

step2: Plot $e(z)$ as a function of z .

step3: Compare the slopes of $e(z)$ between different matrix elements.



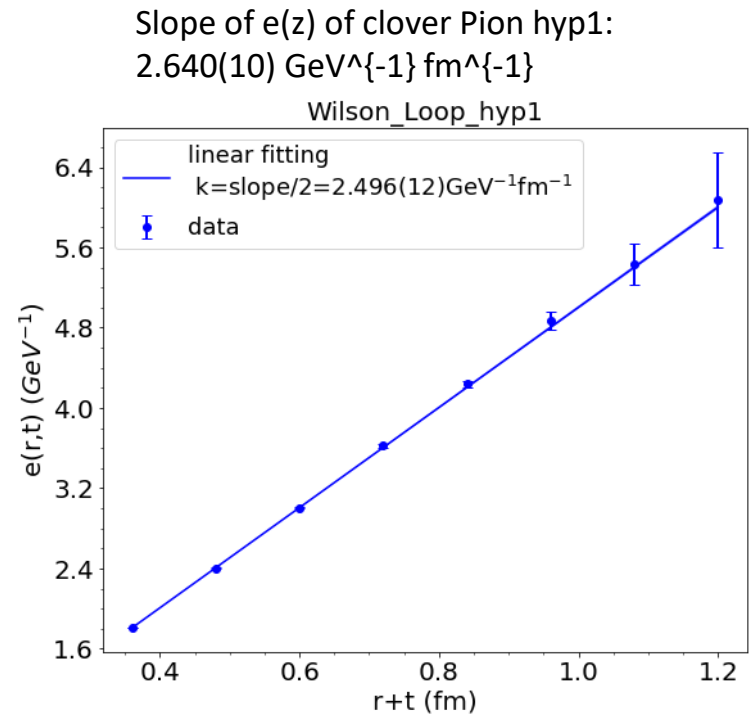
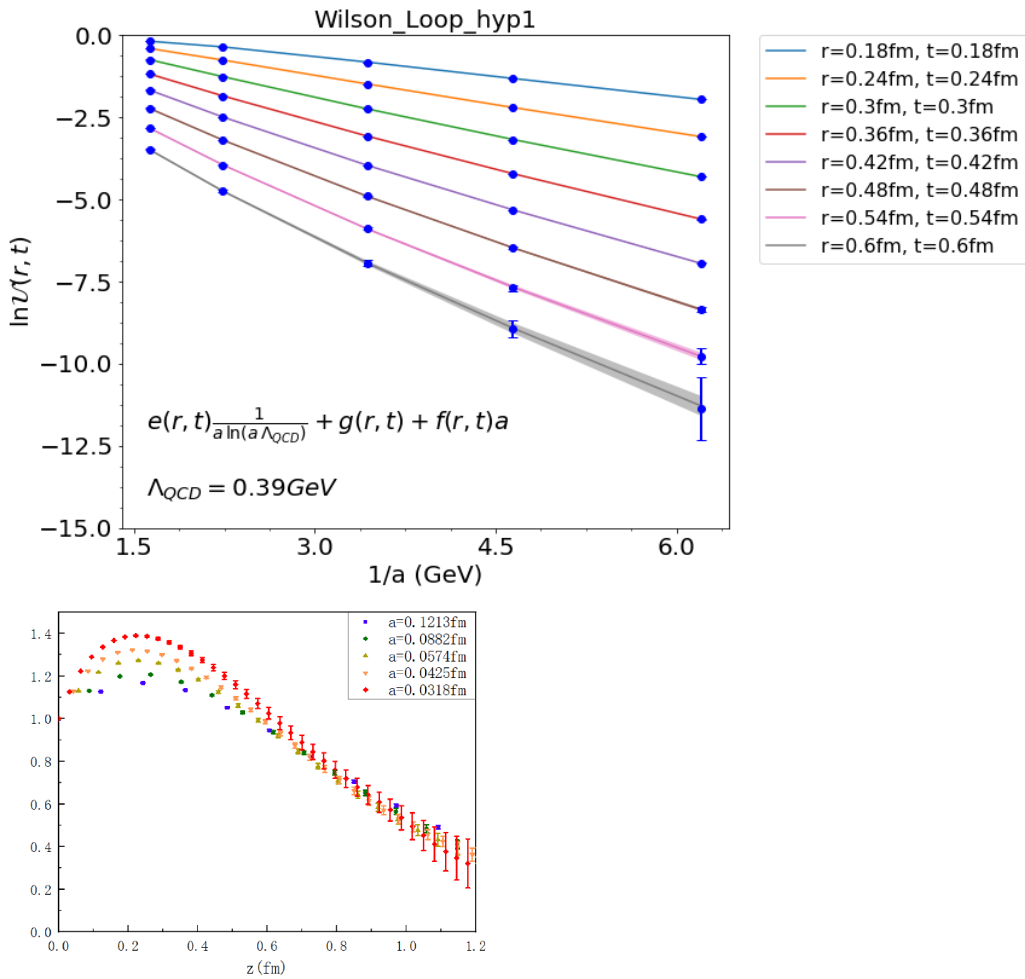


FIG. 6. The pion quasi-PDF matrix element renormalized by the linear divergence part of Wilson Loop.

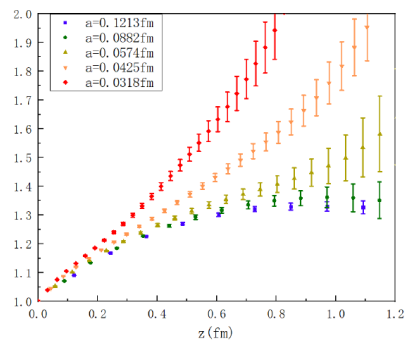
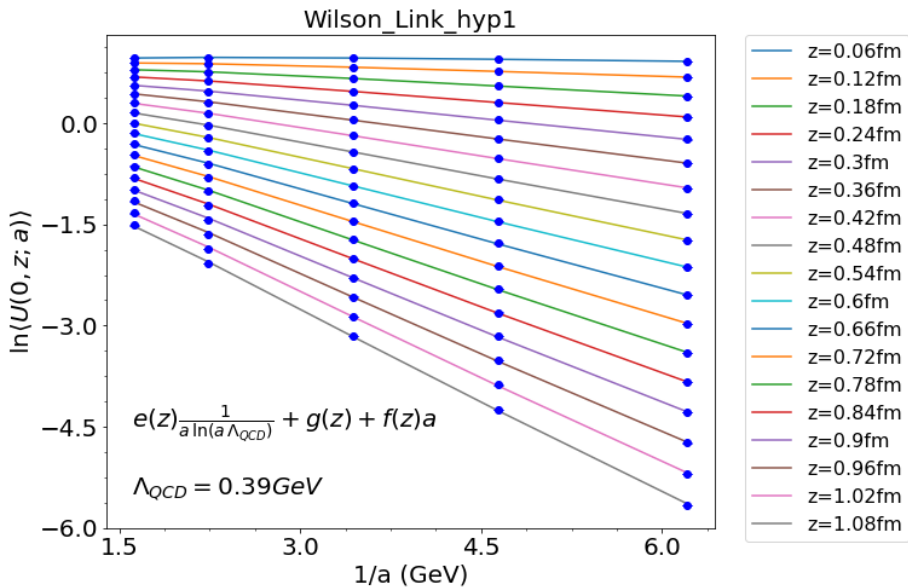
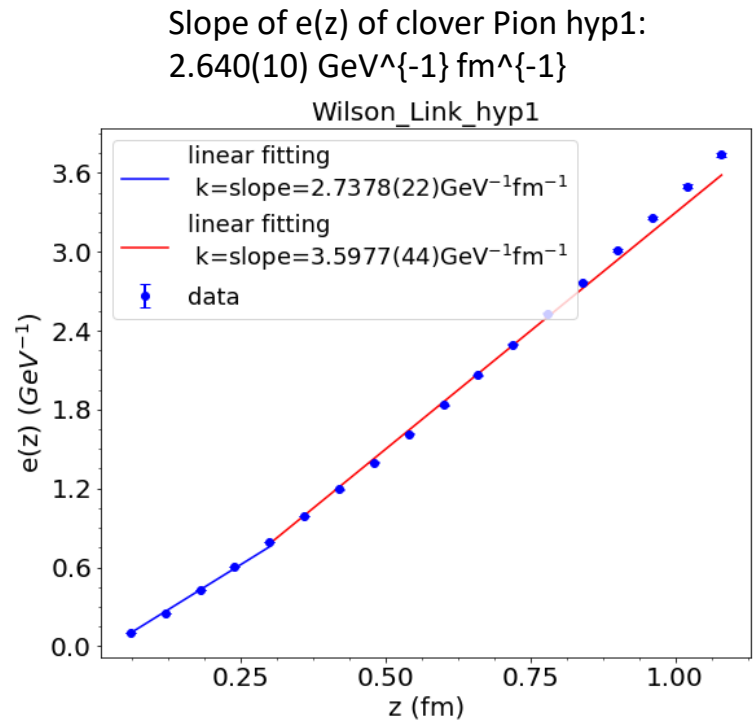
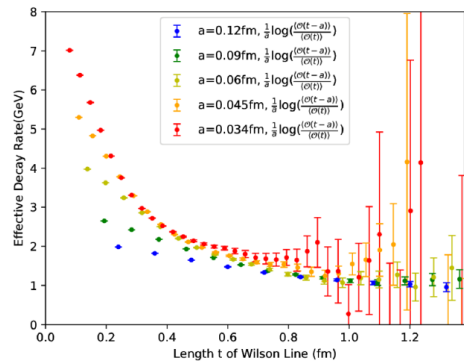
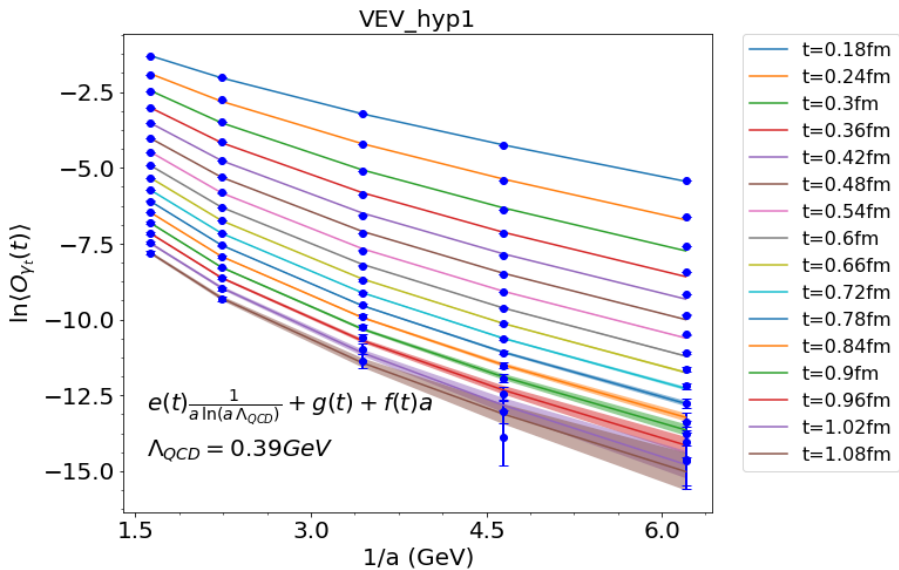


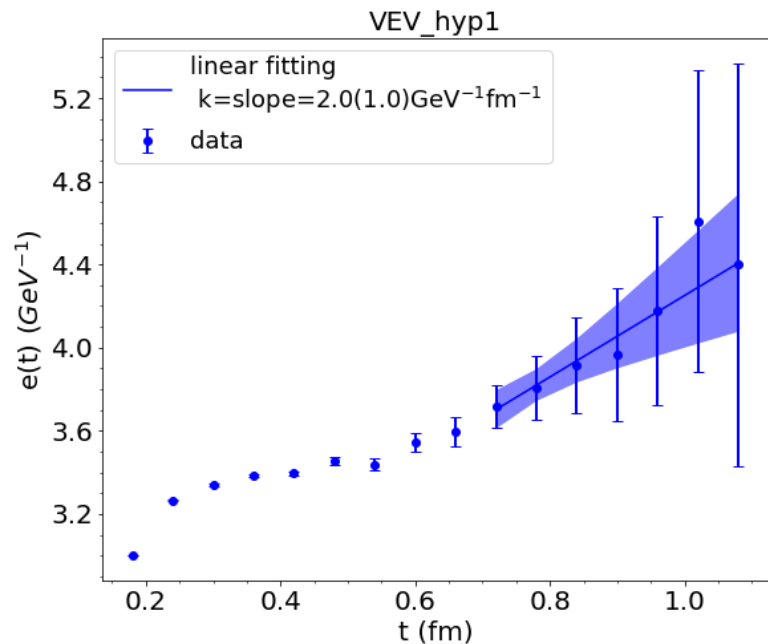
FIG. 7. The effective decay rate of Landau gauge fixed Wilson link (upper panel), and pion matrix element renormalized by the Wilson link.



At small- z , it can be calculated in perturbation theory where the linear divergence works well. For large- z , the matrix element does not have a transfer-matrix interpretation in this gauge.



Slope of e(z) of clover Pion hyp1:
2.640(10) $\text{GeV}^{-1} \text{ fm}^{-1}$



$$O_{\gamma_t}(z) = c_0 * \bar{\psi} \gamma^t \psi + \dots$$

$$\langle \bar{\psi} \gamma^t \psi \rangle = 0$$

At small z, $\langle O_{\gamma_t}(z) \rangle$ only has higher twist terms. Small residual $\text{Exp}[g(z)]$, large discretization error $O(a)$.

Scalar matrix elements may avoid these problems

FIG. 4. Effective decay rate $\frac{1}{a} \log(\frac{O_{\gamma_t}(t-a)}{O_{\gamma_t}(t)})$ of the O_{γ_t} VEV. The gaps between the results at different lattice spacings decrease on t.

F. Renormalization and PDF

$$\alpha_s(Q, \Lambda) = \frac{2\pi}{b_0 * \text{Log}[Q/\Lambda]}$$

Solve $\alpha_s(M_Z, \Lambda)=0.1179$ (PDG2020 world average) to get $\Lambda= 0.245$ GeV

Then $\alpha_s(\mu, \Lambda) = 0.332$

Fix $\mu = 2\text{GeV}$

