

# **Factorized** approach to radiative corrections for inelastic **lepton-hadron** collisions

Nobuo Sato

In collaboration with :

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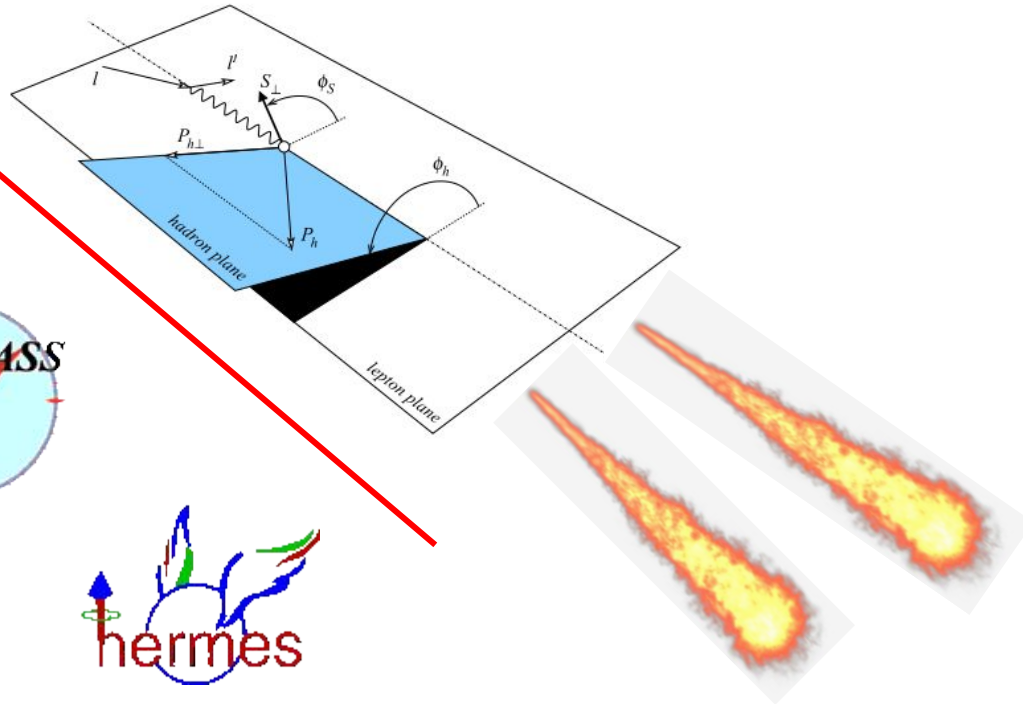
DIS21: Structure functions and pdfs



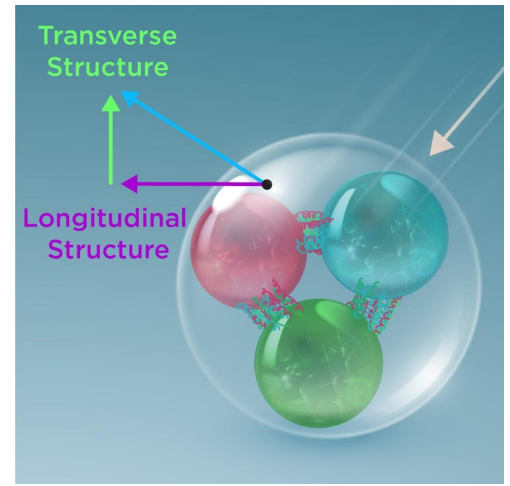
# SIDIS - the next frontier of hadron structure

EIC<sup>2</sup>

Jefferson Lab

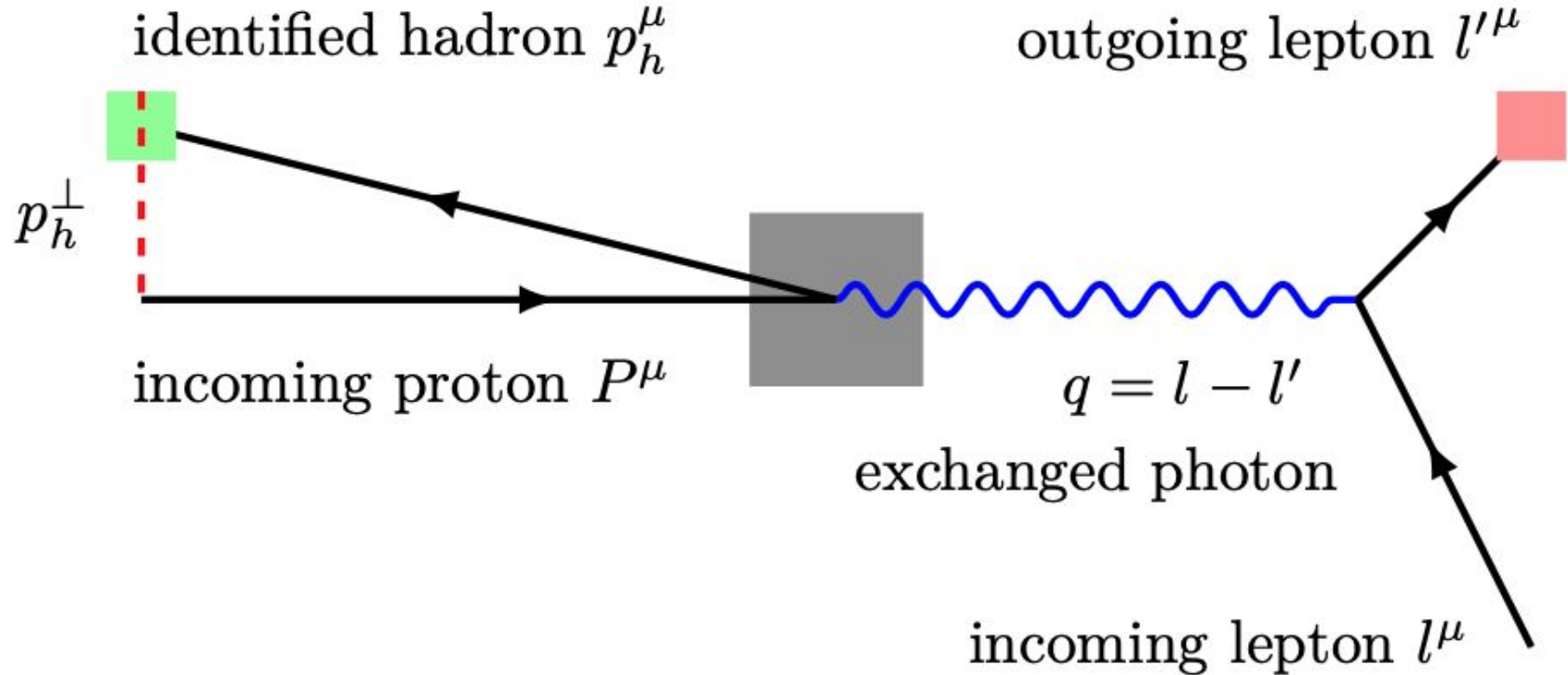


$$\begin{aligned}
& \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = \\
& \frac{\alpha^2}{xy Q^2} \frac{y^2}{2(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} \right. \\
& + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} \\
& + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
& + S_{\parallel} \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h F_{LL}^{\cos \phi_h} \right] \\
& + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
& + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
& + \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \left. \right] \\
& + |S_{\perp}| \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\
& + \left. \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\},
\end{aligned}$$

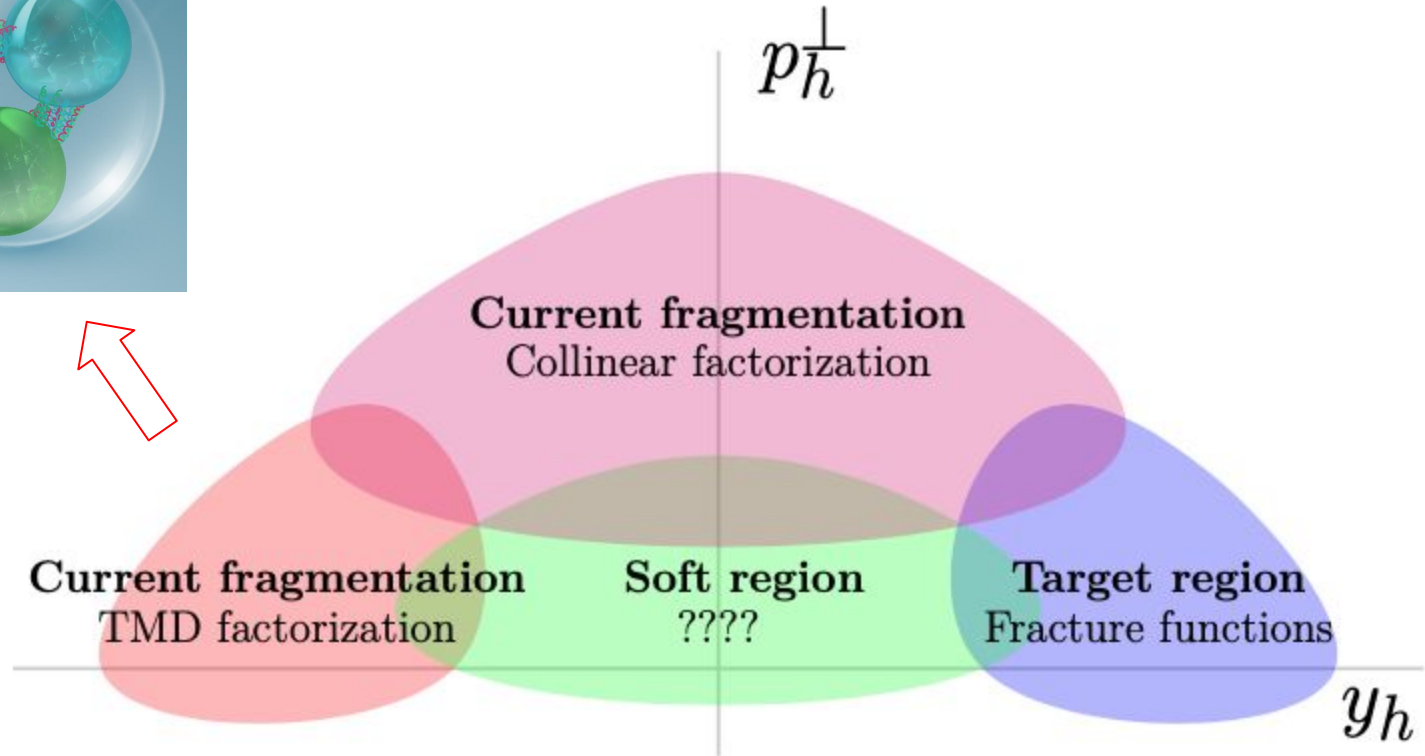
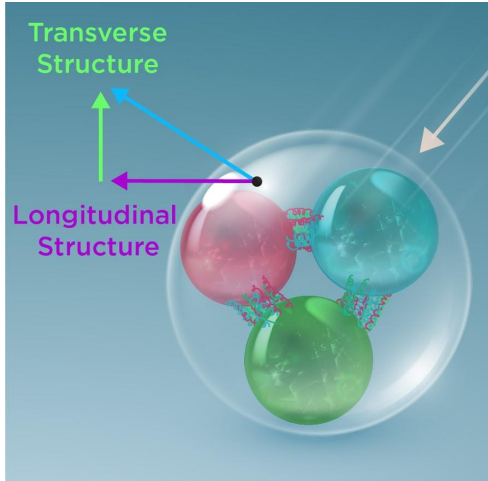


Name	Symbol	meaning
upol. PDF	$f_1^q$	U. pol. quarks in U. pol. nucleon
pol. PDF	$g_1^q$	L. pol. quarks in L. pol. nucleon
Transversity	$h_1^q$	T. pol. quarks in T. pol. nucleon
Sivers	$f_{1T}^{\perp(1)q}$	U. pol. quarks in T. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
⋮	⋮	⋮
FF	$D_1^q$	U. pol. quarks to U. pol. hadron
Collins	$H_1^{\perp(1)q}$	T. pol. quarks to U. pol. hadron
⋮	⋮	⋮

# The Breit Frame

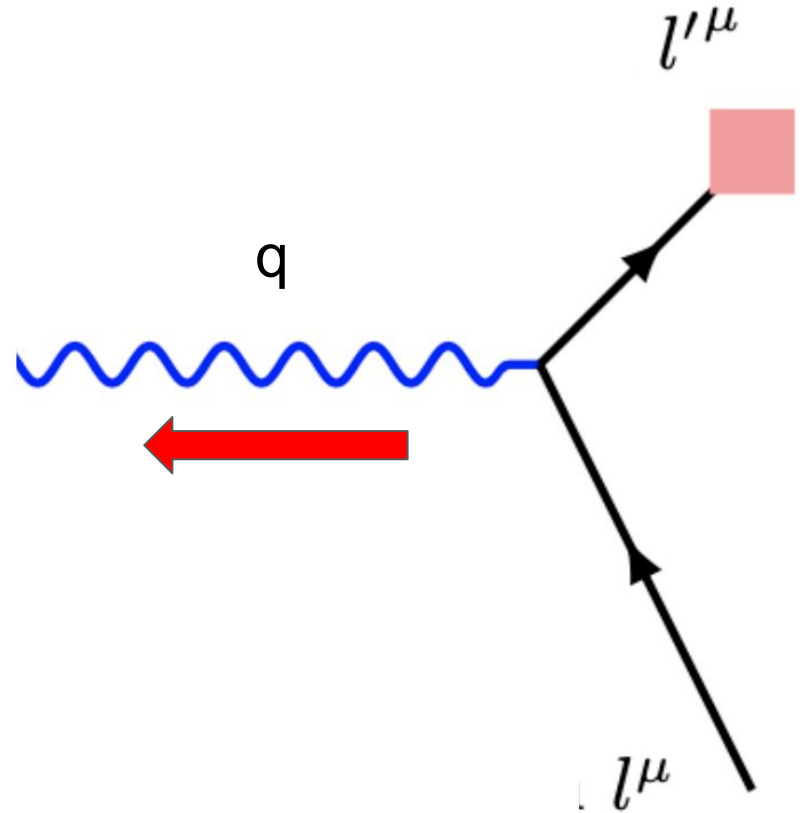


# SIDIS regions

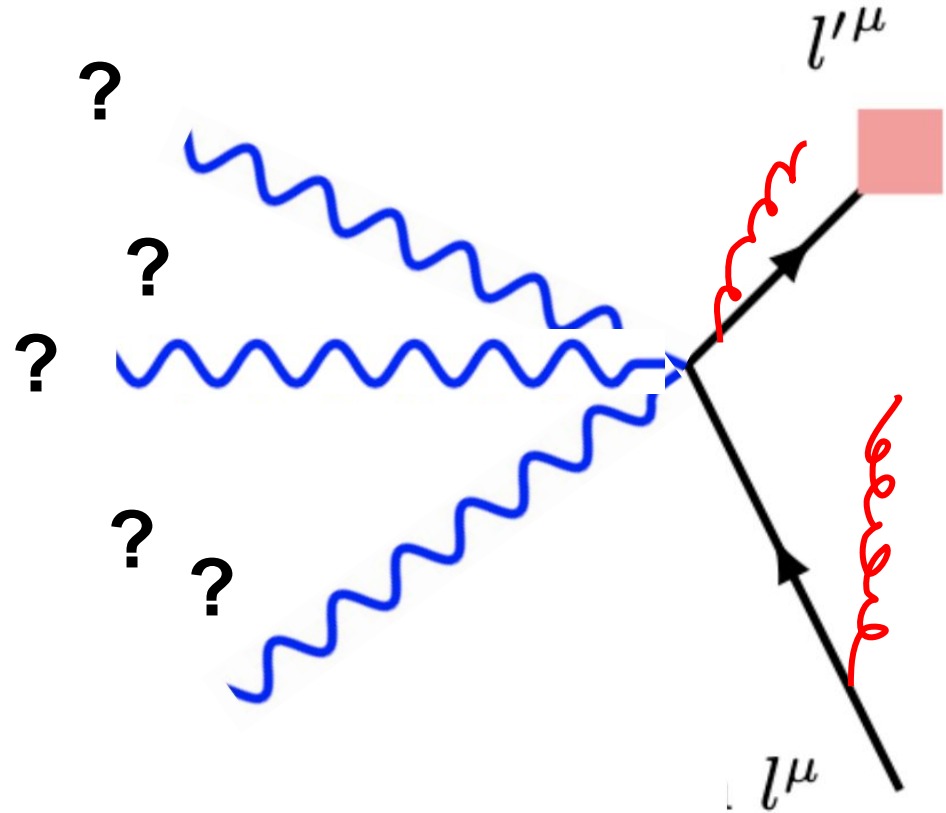


# But ....

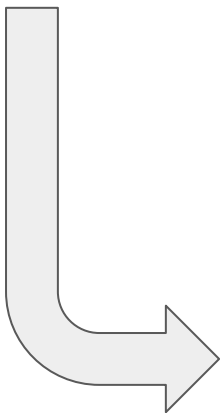
- Factorization theorem relies on  $q$  moving along  $-z$
- How do we know that  $q$  is exactly along  $-z$  even-by-event?
- Role of QED radiation?



- In the presence of QED radiation, **the  $q$  direction is not fixed**
- The experimental Breit Frame **does not need to coincide with the actual Breit-frame** needed in QCD factorization



# How to proceed ?



arXiv.org > hep-ph > arXiv:2008.02895

**High Energy Physics – Phenomenology**

*[Submitted on 6 Aug 2020 ([v1](#)), last revised 17 Mar 2021 (this version, v3)]*

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[Tianbo Liu](#), [W. Melnitchouk](#), [Jian-Wei Qiu](#), [N. Sato](#)



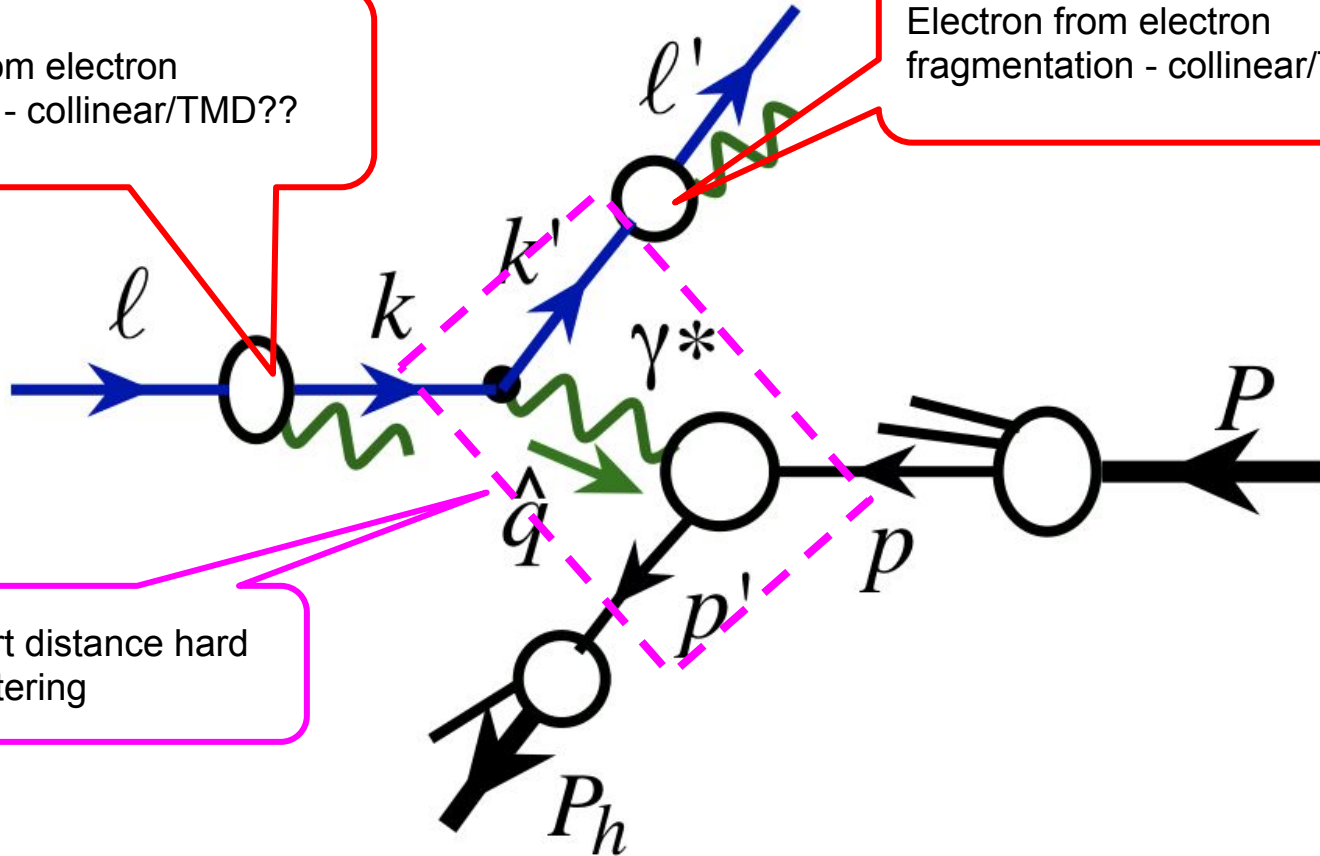
Lepton Distribution Function (**LDF**)

Electron from electron  
distribution - collinear/TMD??

Lepton Fragmentation Function (**LFF**)

Electron from electron  
fragmentation - collinear/TMD??

Short distance hard  
scattering



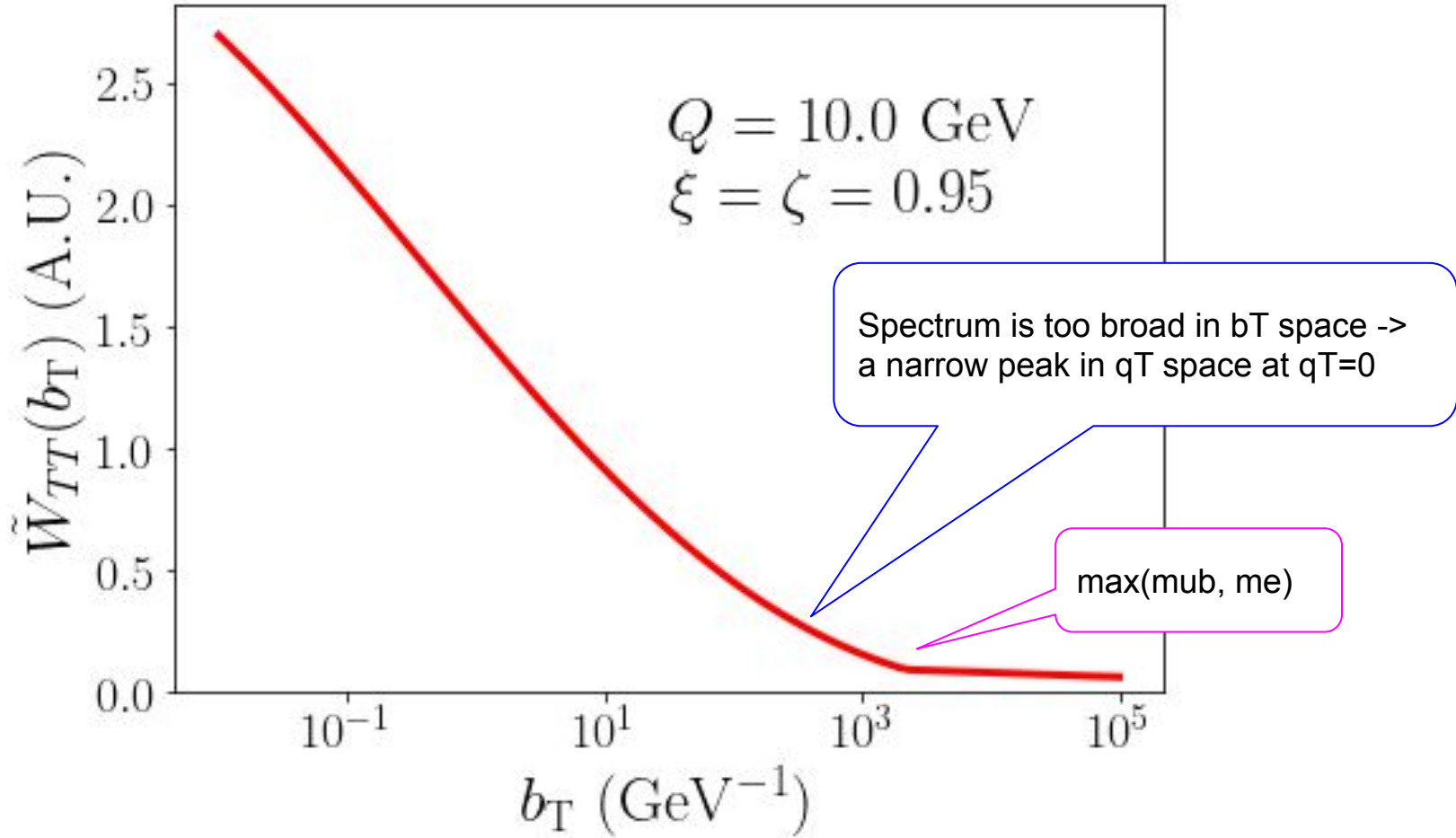
# Key observation

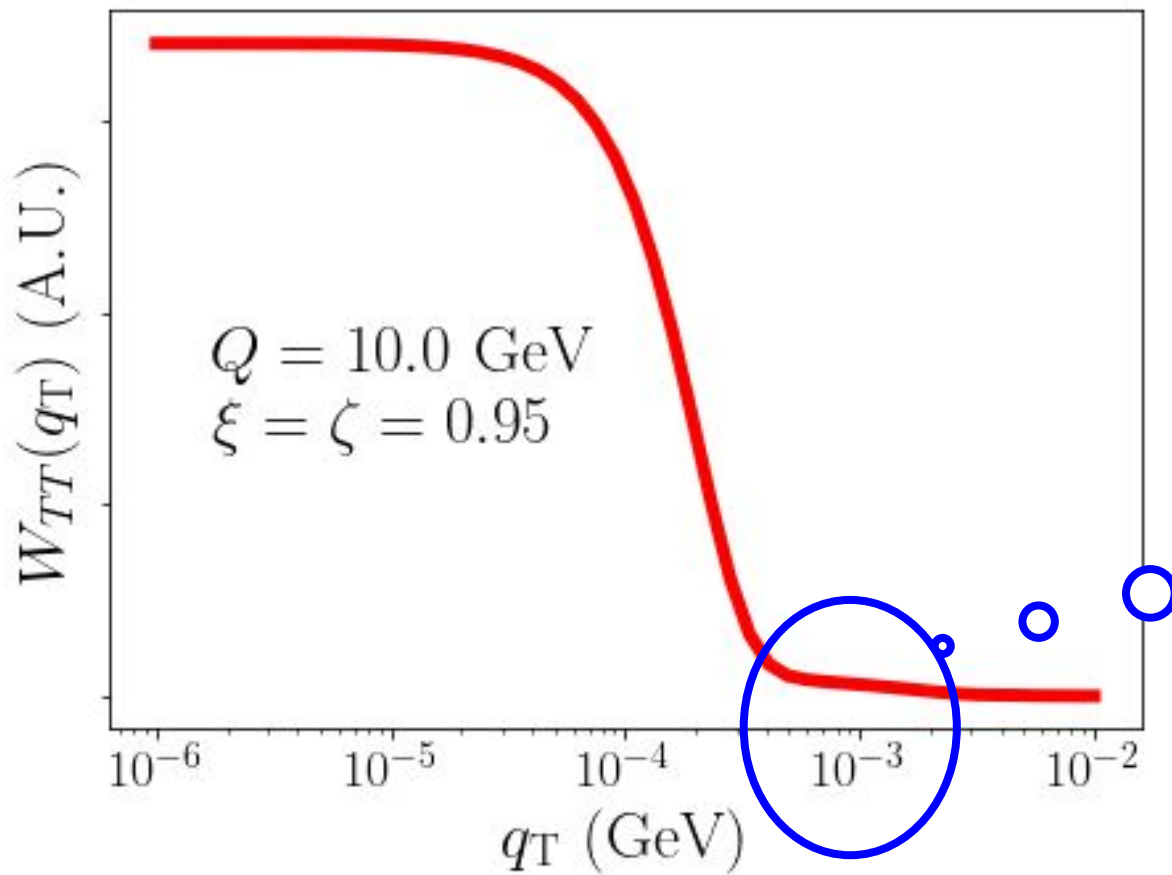
$$L_{\rho\sigma}(\xi_B, \zeta_B, Q^2, \hat{\mathbf{q}}_T^2) = \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{q}_T \cdot \mathbf{b}} \widetilde{W}_{\rho\sigma}(\xi_B, \zeta_B, Q^2, b) + Y_{\rho\sigma}(\xi_B, \zeta_B, Q^2, \hat{\mathbf{q}}_T^2),$$



Collinear LDF and  
LFFs

$$\begin{aligned} \widetilde{W}_{TT}(\xi_B, \zeta_B, Q^2, b) = & 2 \int_{\xi_B}^1 \frac{d\xi}{\xi} \int_{\zeta_B}^1 \frac{d\zeta}{\zeta^2} \boxed{f(\xi) D(\zeta)} C_f(\lambda) C_D(\eta) \\ & \times \exp \left\{ - \int_{\mu_b^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left[ A(\alpha(\mu')) \ln \frac{\mu_Q^2}{\mu'^2} + B(\alpha(\mu')) \right] \right\} \end{aligned}$$





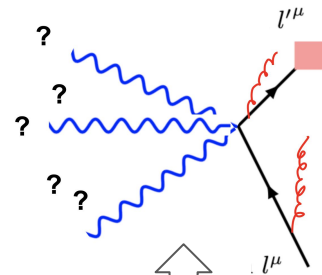
We can ignore  
TMD LDF and  
LFFs

$$\begin{aligned}
& \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = \\
& \frac{\alpha^2}{xy Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} \right. \\
& + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} \\
& + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
& + S_{\parallel} \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h F_{LL}^{\cos \phi_h} \right] \\
& + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
& + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
& + \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \left. \right] \\
& + |S_{\perp}| \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\
& + \left. \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}, \quad (
\end{aligned}$$



**+ QED  
???**

# SIDIS with QED



Collinear LDF and LFFs

$$\frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h,T}^2} = \int_{\zeta_{\min}}^1 d\zeta \int_{\xi_{\min}(\zeta)}^1 d\xi f(\xi) d(\zeta) \frac{\hat{x}}{x \xi \zeta}$$

QED rotational effects

$$\times \left[ \frac{\alpha_{\text{EM}}^2}{\hat{x} \hat{y} \hat{Q}^2} \frac{\hat{y}^2}{2(1 - \hat{\epsilon})} \left( 1 + \frac{\hat{\gamma}}{2\hat{x}} \right) F_{UU}(\hat{x}, \hat{Q}^2, \hat{z}, \hat{P}_{h,T}) \right]$$

Hatted variables all depend on xi and zeta

Hadron pT in the true Breit frame

# Collinear LDFs and LFFs

$$f_{i/e}(\xi) = \int \frac{dz^-}{4\pi} e^{i\xi\ell^+ z^-} \langle e | \bar{\psi}_i(0) \gamma^+ \Phi_{[0,z^-]} \psi_i(z^-) | e \rangle$$

$$D_{e/j}(\zeta) = \frac{\zeta}{2} \sum_X \int \frac{dz^-}{4\pi} e^{i\ell'^+ z^- / \zeta} \text{Tr} [\gamma^+ \langle 0 | \bar{\psi}_j(0) \Phi_{[0,\infty]} | e, X \rangle \langle e, X | \psi_j(z^-) \Phi_{[z^-,\infty]} | 0 \rangle].$$

**Attention:** These objects are perturbatively calculable

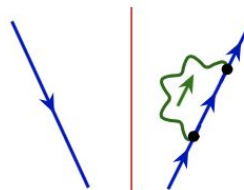
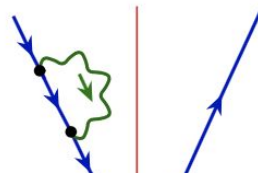
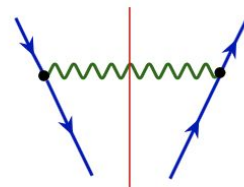
# Input scale LDFs/LFFs

$$f_{e/e}^{(0)}(\xi) = \delta(\xi - 1)$$

$$f_{e/e}^{(1)}(\xi, \mu^2) = \frac{\alpha}{2\pi} \left[ \frac{1 + \xi^2}{1 - \xi} \ln \frac{\mu^2}{(1 - \xi)^2 m_e^2} \right]_+$$

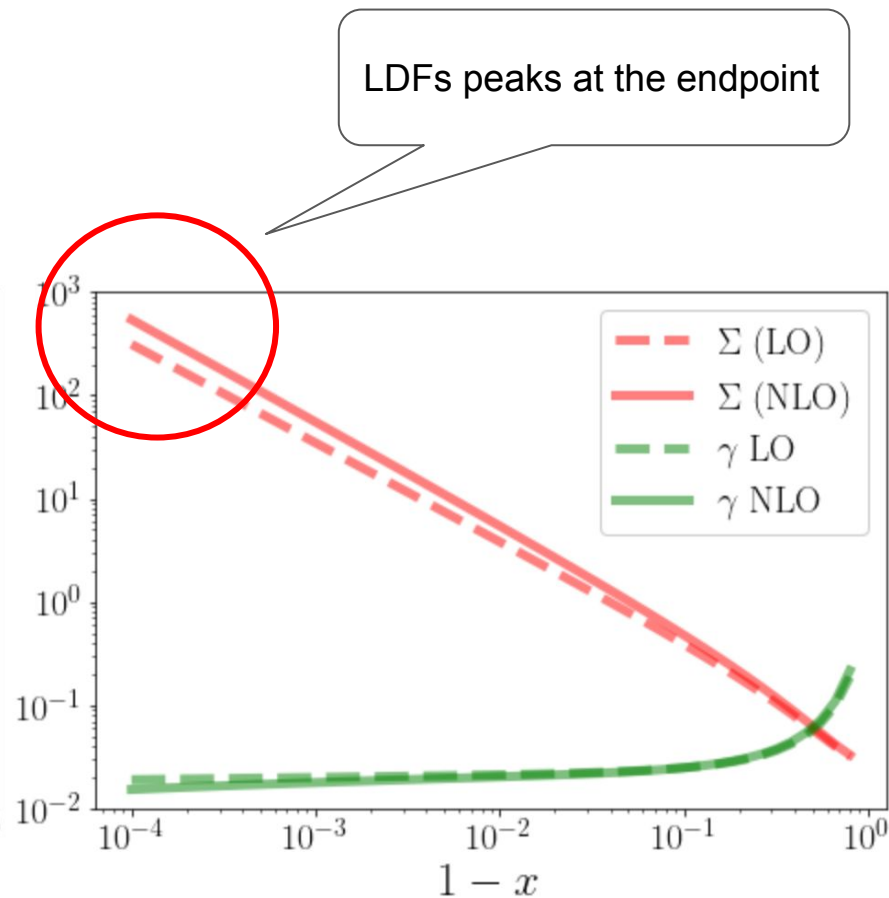
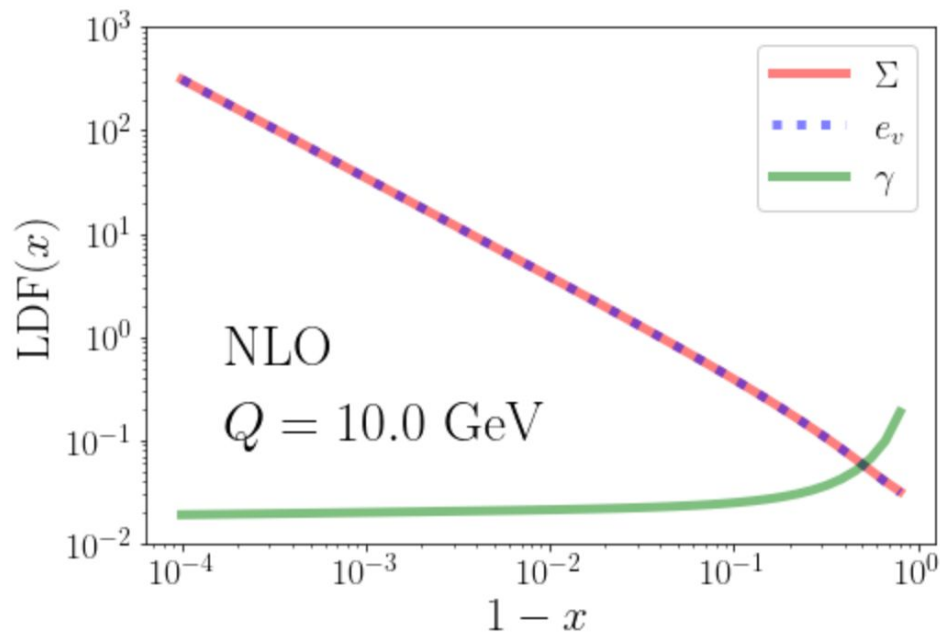
$$D_{e/e}^{(0)}(\zeta) = \delta(\zeta - 1)$$

$$D_{e/e}^{(1)}(\zeta, \mu) = \frac{\alpha}{2\pi} \left[ \frac{1 + \zeta^2}{1 - \zeta} \ln \frac{\zeta^2 \mu^2}{(1 - \zeta)^2 m_e^2} \right]_+$$





# Evolved LDFs



# Subtraction trick

$$\sigma = \int_{\zeta_{\min}}^1 d\zeta \int_{\xi_{\min}(\zeta)}^1 d\xi f(\xi) d(\zeta) H(\xi, \zeta)$$



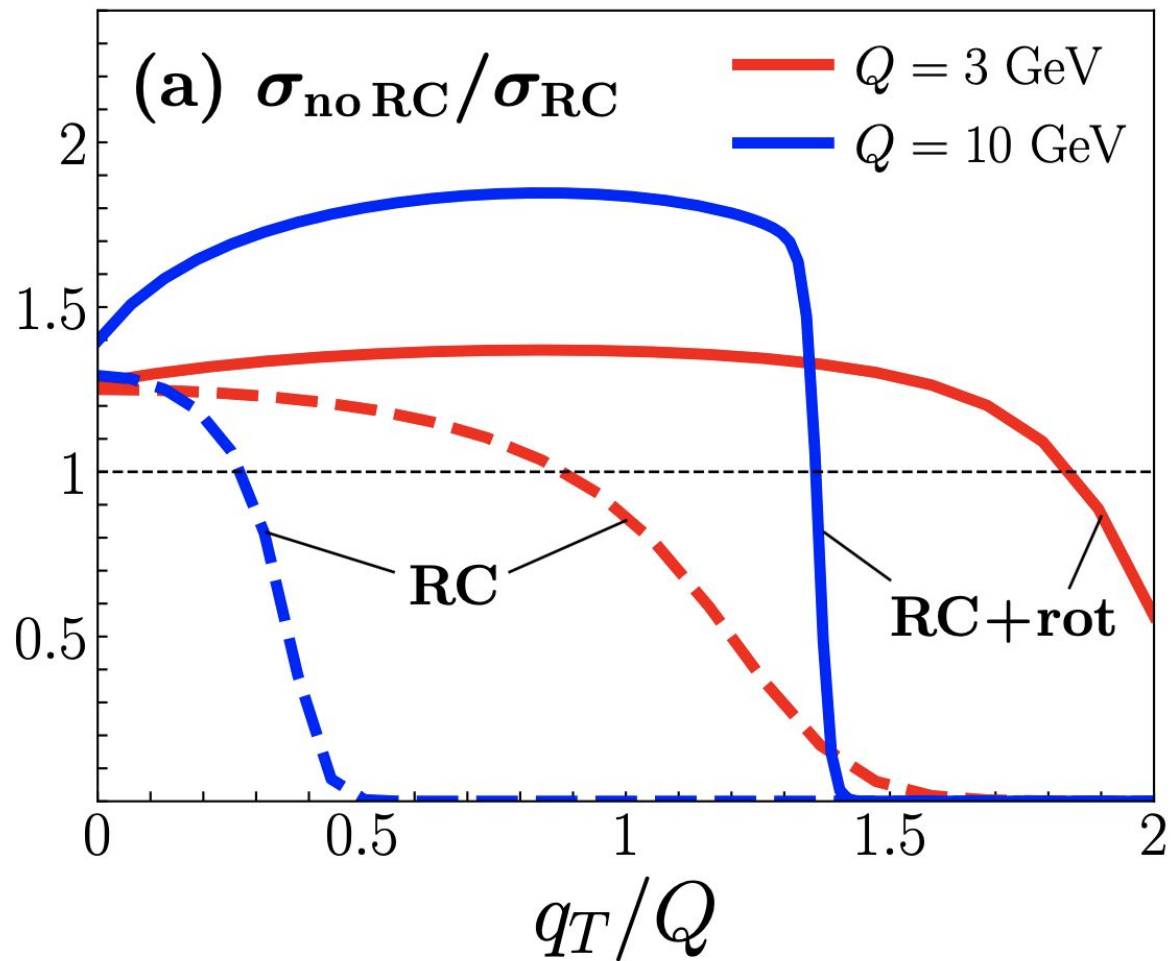
$$\sigma = \int_{\zeta_{\min}}^1 d\zeta d(\zeta) [g(\zeta) - g(1)] + g(1) \frac{\zeta_{\min}}{2\pi i} \int dN \zeta_{\min}^{-N} \frac{D_N}{N-1}$$

$$D_N = \int_0^1 d\zeta \zeta^{N-1} d(\zeta)$$

$$F_N = \int_0^1 d\xi \xi^{N-1} f(\xi)$$

$$g(\zeta) = \int_{\xi_{\min}(\zeta)}^1 d\xi f(\xi) [H(\xi, \zeta) - H(1, \zeta)] + H(1, \zeta) \frac{\xi_{\min}(\zeta)}{2\pi i} \int dN \xi_{\min}(\zeta)^{-N} \frac{F_N}{N-1}$$

We remove the numerically problematic region and compute the difference accurately in Mellin space



$$\sqrt{s} = 140 \text{ GeV}$$

$$y = 0.4$$

$$z = 0.5$$

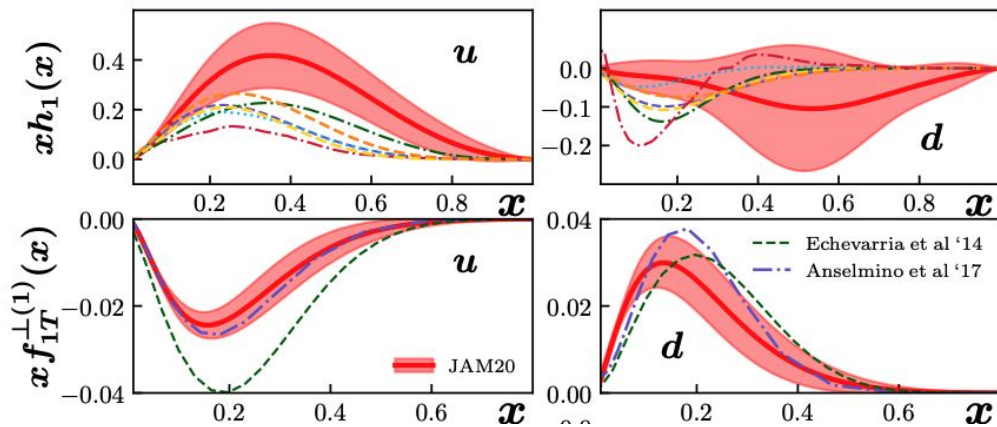
# What about spin structures?

arXiv:2002.08384 (hep-ph)

[Submitted on 19 Feb 2020 (v1), last revised 2 Sep 2020 (this version, v2)]

## Origin of single transverse-spin asymmetries in high-energy collisions

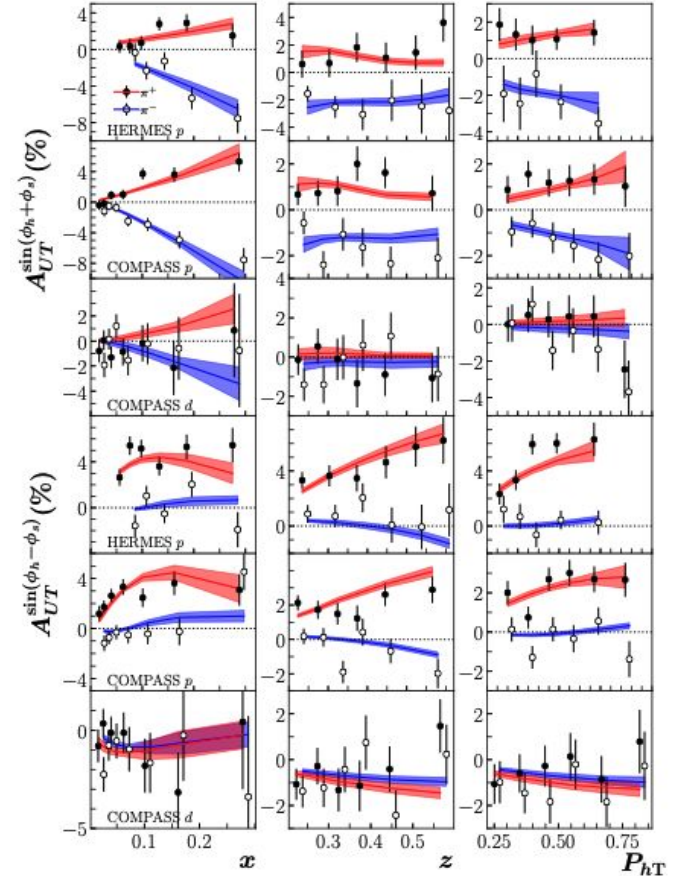
Justin Cammarota, Leonard Gamberg, Zhong-Bo Kang, Joshua A. Miller, Daniel Pitonyak, Alexei Prokudin, Ted C. Rogers, Nobuo Sato

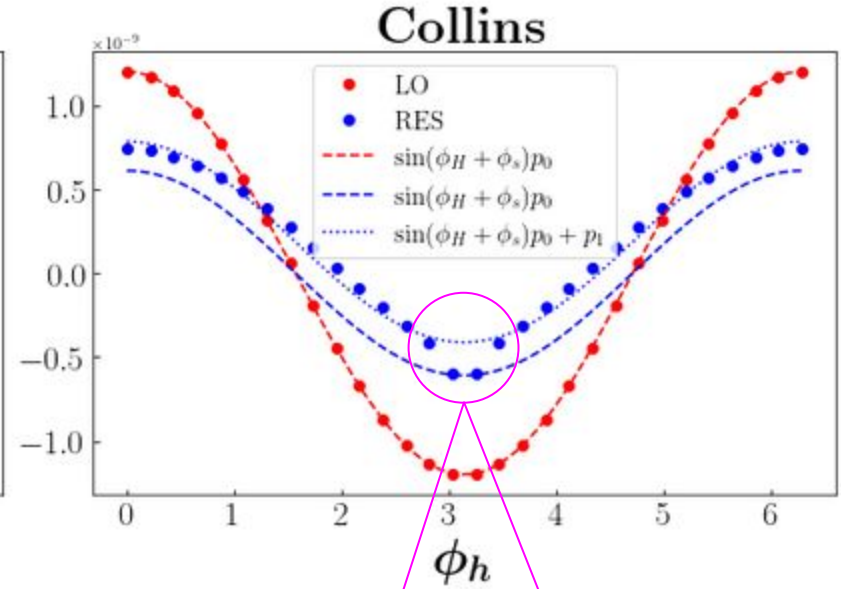
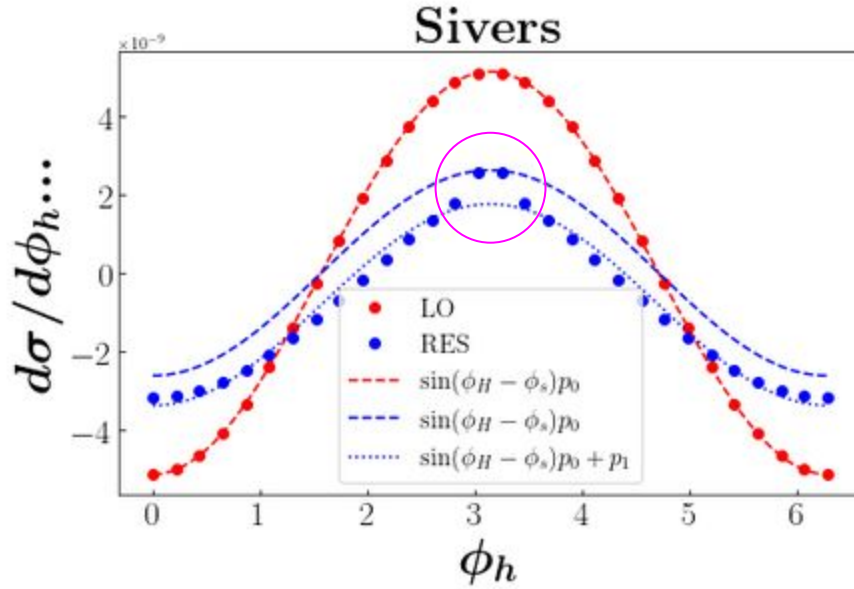


Transversity

Sivers

$$\begin{aligned}
& \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = \\
& \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\
& + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \\
& + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
& + S_{\parallel} \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
& + |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
& + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
& + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \\
& + |S_{\perp}| \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\
& \left. \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}, \quad (
\end{aligned}$$





- Curves are a fits
- Dots are full theory calculations

QED corrections generates non trivial signal around  $\phi_h = \pi$

# Summary/Outlook

- In the presence of QED radiation, **the  $q$  direction is not fixed**
- The experimental Breit Frame **does not need to coincide with the actual Breit-frame** needed in QCD factorization
- QED effects **needs** to take into account for the next frontier

EIC<sup>2</sup>

