Full next-to-eikonal quark propagator in the CGC and applications

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Outline of the talk

Eikonal approximation and beyond

High-energy scatterings:

$$\text{projectile : dilute proton} \; - \; \text{target : dense nucleus (CGC)}$$
$$\text{(anti)quark parton} \; - \; \text{large gluon background field } \mathcal{A}^{\mu}(x)$$

Eikonal approximation : Schockwave approximation for a x^--going target

* target gets Lorentz contracted to $x^+ = 0$ (zero width)
* independence on x^- due to Lorentz time dilation
* large hierarchy between components of the background field

Background field in the eikonal limit: $\mathcal{A}^{\mu}(x^-, x^+, \mathbf{x}) \approx \delta^{\mu -} \mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$

Strong hierarchy of components of $\mathcal{A}^{\mu}$ with respect to the boost factor γ:

$$\mathcal{A}^- = O(\gamma) \quad \gg \quad \mathcal{A}_{\perp} = O(1) \quad \gg \quad \mathcal{A}^+ = O(1/\gamma)$$

Eikonal approximation $\equiv$ leading power in γ or total energy of the collision
Subleading powers : potentially important corrections to CGC at moderate energies (EIC/RHIC)

Beyond eikonal approximation, at Next-to-Eikonal accuracy (NEik):

* target with finite width $\Rightarrow$ transverse motion of the parton within the medium
* interactions with $\mathcal{A}_{\perp}$ field cannot be neglected

High-energy expansion with small parameters $\sim \frac{L^+}{k^+} Q^2$ $\quad (L^+$: target width, k^+ : parton momentum, Q^2 : any transverse scale in the problem)

Quark propagator - basics

Full quark Feynman propagator in background field $\mathcal{A}^\mu(x)$

$$S_F(x,y)_{\alpha\beta} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}$$

free propagator + corrections due to interactions with the background field

Free quark Feynman propagator:

$$S_{0,F}(x,y)_{\alpha\beta} = (\mathbf{1})_{\alpha\beta} \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{i(\not{k}+m)}{[k^2-m^2+i\epsilon]}$$

Corrections:

- **at the eikonal order**

$$\delta S_F\Big|^{\text{Eik}} \equiv \delta S_F\Big|^{\text{Eik}}_{\text{pure } \mathcal{A}^-}$$

- **at the next-to-eikonal order**

$$\delta S_F\Big|^{\text{NEik}} \equiv \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

Quark propagator in the eikonal limit

$$S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}}_{\text{pure } \mathcal{A}^-}$$

In eikonal limit, the quark already interacts with arbitrarily many $\mathcal{A}^-$ fields
For generic x and y, with notations $\underline{k} \equiv (k^+, \mathbf{k})$, and $\check{k}$ on-shell version of k:

$$\begin{aligned}
S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} =\; & \mathbf{1}_{\alpha\beta}\, \delta^{(3)}(\underline{x}-\underline{y})\, \text{sgn}(x^- - y^-)\, \frac{\gamma^+}{4} \\
& + \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+) e^{-ix\cdot\check{q}+iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)}{(2k^+)^2} \\
& \times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Big\{ \theta(k^+)\theta(x^+ - y^+)\, \mathcal{U}_F(x^+, y^+; \mathbf{z})_{\alpha\beta} \\
& \qquad - \theta(-k^+)\theta(y^+ - x^+)\, \mathcal{U}_F^\dagger(y^+, x^+; \mathbf{z})_{\alpha\beta} \Big\}
\end{aligned}$$

Eikonal interactions with the medium resummed into the Wilson lines:

$$\mathcal{U}_F(x^+, y^+; \mathbf{z}) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\, \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z^+, \mathbf{z}) \right]^N$$

- background field $\mathcal{A}_a^-(z^+, \mathbf{z})$ has a finite support $[-L^+/2, L^+/2]$ - this is where the non-trivial medium contributions come from in the interval $[y^+, x^+]$
- if there is no support the propagator reduces to the Feynman propagator in vacuum

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = \underbrace{S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure }\mathcal{A}^-}}_{S_F\big|_{\text{pure }\mathcal{A}^-}} + \delta S_F\Big|^{\text{NEik}}_{\text{single }\mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp}$$

From now on, always $x^+ > L^+/2$ and $y^+ < -L^+/2$: quark propagating through the whole target

Quark propagator in pure $\mathcal{A}^-$ background field up to next-to-eikonal order for positive energy:

$$\begin{aligned}
S_F(x,y)_{\alpha\beta}\Big|_{\text{pure }\mathcal{A}^-} = &\int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^2}\, e^{-ix\cdot\check{q}+iy\cdot\check{k}}\,(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)\\
&\times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\left\{\mathcal{U}_F\left(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\right)\right.\\
&-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z}\right)\overleftrightarrow{\partial_{\mathbf{z}^j}}\,\mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z}\right)\right]\\
&\left.-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z}\right)\overleftarrow{\partial_{\mathbf{z}^j}}\,\overrightarrow{\partial_{\mathbf{z}^j}}\,\mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z}\right)\right]\right\}
\end{aligned}$$

NEik corrections: transverse drift term + transverse Brownian motion term

Analog to earlier results on the gluon propagator with subeikonal corrections:
Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP **1407**, 068 (2014)
Altinoluk, Armesto, Beuf, Moscoso, JHEP **1601**, 114 (2016)

Subeikonal corrections: single $\mathcal{A}_\perp$ insertion

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

Replace one $\gamma^+ \mathcal{A}_a^-$ interaction by $\gamma^j \mathcal{A}_j^a$

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} = \int d^4z \; S_F(x,z)\Big|^{\text{Eik}} \left[-ig\,\gamma^j\, t^a\right] \mathcal{A}_j^a(\underline{z}) \quad S_F(z,y)\Big|^{\text{Eik}}$$

Subeikonal correction due to an interaction with $\mathcal{A}_\perp$:

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} = \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, \frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}$$
$$\times (\check{\not{q}} + m)\gamma^j\gamma^+\gamma^i\,(\check{\not{k}} + m) \int d^3\underline{z} \left[e^{-i\mathbf{z}\cdot\mathbf{q}}\, \mathcal{U}_F\!\left(\frac{L^+}{2}, z^+; \mathbf{z}\right)\right]$$
$$\times \left[\overleftarrow{\partial_{\mathbf{z}^j}}\left[gt\cdot\mathcal{A}_i(\underline{z})\right] - \left[gt\cdot\mathcal{A}_j(\underline{z})\right]\overrightarrow{\partial_{\mathbf{z}^i}}\right] \left[\mathcal{U}_F\!\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right) e^{i\mathbf{z}\cdot\mathbf{k}}\right]$$

Reminder: $x^+ > L^+/2$ and $y^+ < -L^+/2$:
quark propagating through the whole medium

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure }\mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single }\mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp}$$

Replace two $\gamma^+\mathcal{A}_a^-$ interactions by $\gamma^j\mathcal{A}_j^b$ and $\gamma^i\mathcal{A}_i^a$

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp} = \int d^4z \int d^4z'\; S_F(x,z')\Big|^{\text{Eik}} \left[-ig\,\gamma^j\, t^b\right] \mathcal{A}_j^b(\underline{z}') \times S_F(z',z)\Big|^{\text{Eik}} \left[-ig\,\gamma^i\, t^a\right] \mathcal{A}_i^a(\underline{z}) \quad S_F(z,y)\Big|^{\text{Eik}}$$

Contribution at NEik accuracy due to the instantaneous term in the middle eikonal propagator:

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp} = \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, \frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}} \times (\check{\not{q}} + m)\gamma^j\gamma^+\gamma^i\,(\check{\not{k}} + m) \int d^3\underline{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \times (-i)\;\mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}\right) \left[gt\cdot\mathcal{A}_j(\underline{z})\right] \left[gt\cdot\mathcal{A}_i(\underline{z})\right] \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right)$$

Next-to-eikonal quark propagator - spinor structure

Spinor structure:

- $\mathcal{A}^-$ field associated with $(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)$
- $\mathcal{A}_\perp$ field associated with $(\check{\not{q}}+m)\gamma^j\gamma^+\gamma^i\,(\check{\not{k}}+m)$

  - separate symmetric and anti-symmetric parts:

$$\gamma^j\gamma^+\gamma^i = \delta^{ij}\,\gamma^+ + \gamma^+\,\frac{[\gamma^i,\gamma^j]}{2}$$

(helicity independent + helicity dependent)

Helicity dependence:

$$[\gamma^i,\gamma^j] = -4i\epsilon^{ij}S^3$$

S^3 - helicity operator, acting on spinors as:

$$\begin{aligned} S^3 u(\check{k},h) &= h u(\check{k},h) \\ S^3 v(\check{k},h) &= -h v(\check{k},h) \end{aligned}$$

Quark propagator

$$S_F(x,y) = S_F(x,y)\Big|_{\text{unpol.}} + S_F(x,y)\Big|_{\text{h. dep.}}$$

Next-to-eikonal quark propagator - full result

Unpolarized part

$$S_F(x,y)\Big|_{\text{unpol.}} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^2}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\,(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)$$

$$\times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\,\bigg\{\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\Big)$$

$$-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]$$

$$-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]\bigg\}$$

Helicity-dependent part

$$S_F(x,y)\Big|_{\text{h. dep.}} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}$$

$$\times(\check{\not{q}}+m)\gamma^+\frac{[\gamma^i,\gamma^j]}{4}(\check{\not{k}}+m)\int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}$$

$$\times\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\, gt\cdot\mathcal{F}_{ij}(\underline{z})\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)$$

The final result fully gauge invariant (due to covariant derivatives):

$$\overrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\partial_{z^\mu}} + igt\cdot\mathcal{A}_\mu(\underline{z});\quad \overleftarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}}^{\dagger};\quad \overleftrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}} - \overleftarrow{\mathcal{D}_{z^\mu}}$$

Longitudinal chromo-magnetic field of the target associated with helicity:

$$\mathcal{F}^a_{ij}(\underline{z}) \equiv \partial_{\mathbf{z}^i}\mathcal{A}^a_j(\underline{z}) - \partial_{\mathbf{z}^j}\mathcal{A}^a_i(\underline{z}) - gf^{abc}\mathcal{A}^b_i(\underline{z})\mathcal{A}^c_j(\underline{z})$$

Scattering amplitude from quark propagator

- Simplest observable where NEik correction matter: quark-target cross section
- Need for the relevant scattering amplitude

S-matrix element

Formal definition:
$$S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = \langle 0|\hat{b}_{\text{out}}(\check{q},h,\beta)\hat{b}^{\dagger}_{\text{in}}(\check{k},h,\alpha)|0\rangle = (2k^+)2\pi\delta(q^+-k^+)i\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})$$

LSZ reduction:
$$S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = \lim_{x^+\to\infty}\lim_{y^+\to-\infty}\int d^2\mathbf{x}\int dx^-\int d^2\mathbf{y}\int dy^- \times e^{ix\cdot\check{q}-iy\cdot\check{k}}\,\bar{u}(\check{q},h')\gamma^+ S_F(x,y)_{\alpha\beta}\gamma^+ u(\check{k},h)$$

Quark-target scattering amplitude

$$\begin{aligned}
i\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q}) = \delta_{hh'}\int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Bigg\{&\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\Big)\\
&+\frac{\epsilon^{ij}h}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\Big(-igt\cdot\mathcal{F}_{ij}(\underline{z})\Big)\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]\\
&-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftarrow{\mathcal{D}_{\mathbf{z}j}}\,\overrightarrow{\mathcal{D}_{\mathbf{z}j}}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]\\
&-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{z}j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]\Bigg\}_{\alpha\beta}
\end{aligned}$$

Unpolarized cross section

Differential cross section for quark scattering on the target is

$$\frac{d^2\sigma^{qA\to q+X}}{d^2\mathbf{q}} = \frac{1}{(2\pi)^2}\,\frac{1}{2N_c}\sum_{h,h'}\sum_{\alpha,\beta}\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})^\dagger\,\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})\Big|_{q^+=k^+}$$

Cross section averaged over the target $\quad(\mathbf{z}-\mathbf{z}')\equiv\mathbf{r}$ and $(\mathbf{z}+\mathbf{z}')\equiv 2\mathbf{b}$

$$\left\langle\frac{d^2\sigma^{qA\to q+X}}{d^2\mathbf{q}}\right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\;e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\bigg\{1-\bar{P}(\mathbf{r})$$
$$+\bigg(\bar{O}(\mathbf{r})-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\Big[\mathcal{O}^j_{(1)}(\mathbf{r})-\mathcal{O}^{\dagger j}_{(1)}(\mathbf{r})\Big]-\frac{i}{2k^+}\Big[\mathcal{O}_{(2)}(\mathbf{r})-\mathcal{O}^\dagger_{(2)}(\mathbf{r})\Big]\bigg)\bigg\}$$

Dipole operator:

$$d_F(\mathbf{r}) = 1-\bar{P}(\mathbf{r})+\bar{O}(\mathbf{r})$$
$$d_F(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\left\langle \mathrm{tr}\Big[\mathcal{U}_F^\dagger\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{b}-\frac{\mathbf{r}}{2}\Big)\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\right\rangle_A$$

Decorated dipole operators:

$$\mathcal{O}^j_{(1)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+\left\langle\mathrm{tr}\Big[\mathcal{U}_F^\dagger\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big)\right.$$

]

$$\left.\times\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\right\rangle_A$$

$$\mathcal{O}_{(2)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-L^+/2}^{L^+/2}dz^+\left\langle\mathrm{tr}\Big[\mathcal{U}_F^\dagger\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big)\right.$$
$$\left.\times\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\overleftarrow{\mathcal{D}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}}\;\overrightarrow{\mathcal{D}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\right\rangle_A.$$

Unpolarized cross section : antiquark and symmetries

Anti-quark–target differential cross section:

$$\left\langle \frac{d^2\sigma^{\bar{q}A\to\bar{q}+X}}{d^2\mathbf{q}} \right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\; e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\Big\{1-\bar{P}(\mathbf{r}) - \Big(\bar{O}(\mathbf{r}) - \frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\Big[\mathcal{O}^j_{(1)}(\mathbf{r}) - \mathcal{O}^{\dagger j}_{(1)}(\mathbf{r})\Big] - \frac{i}{2k^+}\Big[\mathcal{O}_{(2)}(\mathbf{r}) - \mathcal{O}^{\dagger}_{(2)}(\mathbf{r})\Big]\Big)\Big\}$$

- By definition : $d_F(\mathbf{r})^\dagger = d_F(-\mathbf{r})$

  $\Rightarrow$ Decomposition $d_F(\mathbf{r}) = 1 - \bar{P}(\mathbf{r}) + \bar{O}(\mathbf{r})$ with
  - Real part $1 - \bar{P}(\mathbf{r})$ even in $\mathbf{r}$
  - Imaginary term $\bar{O}(\mathbf{r})$ odd in $\mathbf{r}$

- **Signature transformation ($\mathcal{U} \to \mathcal{U}^\dagger$) and charge conjugation ($q \to \bar{q}$):**

  $\bar{P}(\mathbf{r})$: Pomeron is **even** under both transformations
  $\bar{O}(\mathbf{r})$: Odderon is **odd** under both transformations

  * **Eikonal terms contain both Pomeron and Odderon**
  * **Next-to-eikonal corrections are of Odderon-type**

Quark helicity asymmetry

The difference between the cross sections for a quark of positive and negative helicity scattering on the nucleus target is:

$$\frac{d^2\Delta\sigma^{qA\to q+X}}{d^2\mathbf{q}} \equiv \frac{1}{(2\pi)^2}\,\frac{1}{2N_c}\sum_{h,h'}\sum_{\alpha,\beta}(2h)\,\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})^\dagger\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})\Big|_{q^+=k^+}$$

Quark helicity asymmetry averaged over the target

$$\left\langle\frac{d^2\Delta\sigma^{qA\to q+X}}{d^2\mathbf{q}}\right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\,e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\,\frac{(-i)}{4k^+}\Big[O_{(3)}(\mathbf{r}) - O^\dagger_{(3)}(\mathbf{r})\Big]$$

New decorated dipole operator:

$$O_{(3)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-L^+/2}^{L^+/2}dz^+\Big\langle \mathrm{Tr}\Big[\mathcal{U}_F^\dagger\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big)$$
$$\times\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big\{\epsilon^{ij}\Big[gt\cdot\mathcal{F}_{ij}\Big(z^+,\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A$$

The anti-quark helicity asymmetry is identical!

Remarks:

- $O_{(3)}(\mathbf{r})$ behaves neither like Pomeron nor like Odderon (odd under signature transformation but even under charge conjugation)
- $\mathcal{F}_{ij}$ and $O_{(3)}(\mathbf{r})$ vanish for unpolarized target, but not for longitudinally polarized target $\Rightarrow$ Double longitudinal spin asymmetry A_{LL}

Summary and final remarks

- Full next-to-eikonal (NEik) expression for the quark propagator through the background field derived
  - Corrections due to transverse motion (drift and Brownian) of the quark while crossing the Lorentz contracted target
  - Corrections due to interaction with the $\mathcal{A}_\perp$ components of the background field
  - However, effects of x^- dependence of the $\mathcal{A}_\mu$ and of the $\mathcal{A}^+$ component would appear only at NNEik accuracy
- Gauge covariant expression: covariant derivatives and field strength insertions in Wilson lines
- Eikonal limit blind to spin, whereas one NEik term is proportional to helicity
- Helicity piece consistent with results of Kovchegov *et al.* (2016-2020)
- Detailed comparison with results of Chirilli (2019-2021) underway, but mostly in agreement, except maybe the term $\mathcal{O}_j$ which do not arise in our calculation
- qA (and $\bar{q}A$) cross section (unpolarized and helicity asymmetry) studied at NEik
- **The derived quark propagator is of a general form and therefore of a general use: applicable for different scattering processes** (calculation of DIS di-jet production in progress, and other observables in project)