A new unintegrated gluon distribution to probe saturation physics in DIS

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@ DIS 2021, April 13th, 2021

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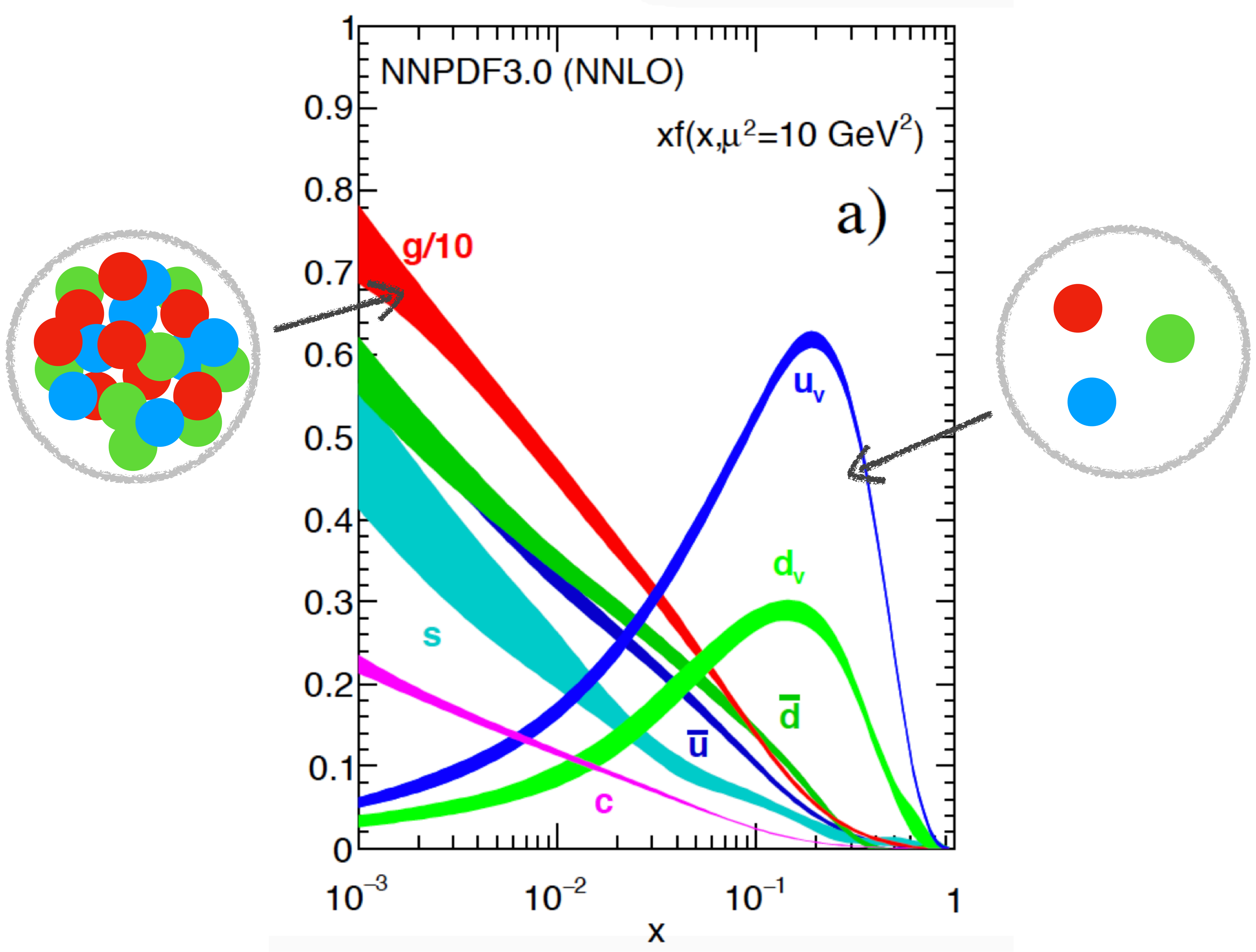
2001.06449 [hep-ph]

2006.14569 [hep-ph]

Gluon saturation at small x



Gluon density rises rapidly at small x: large occupation numbers →

Regime of strong classical fields: breakdown of the parton picture

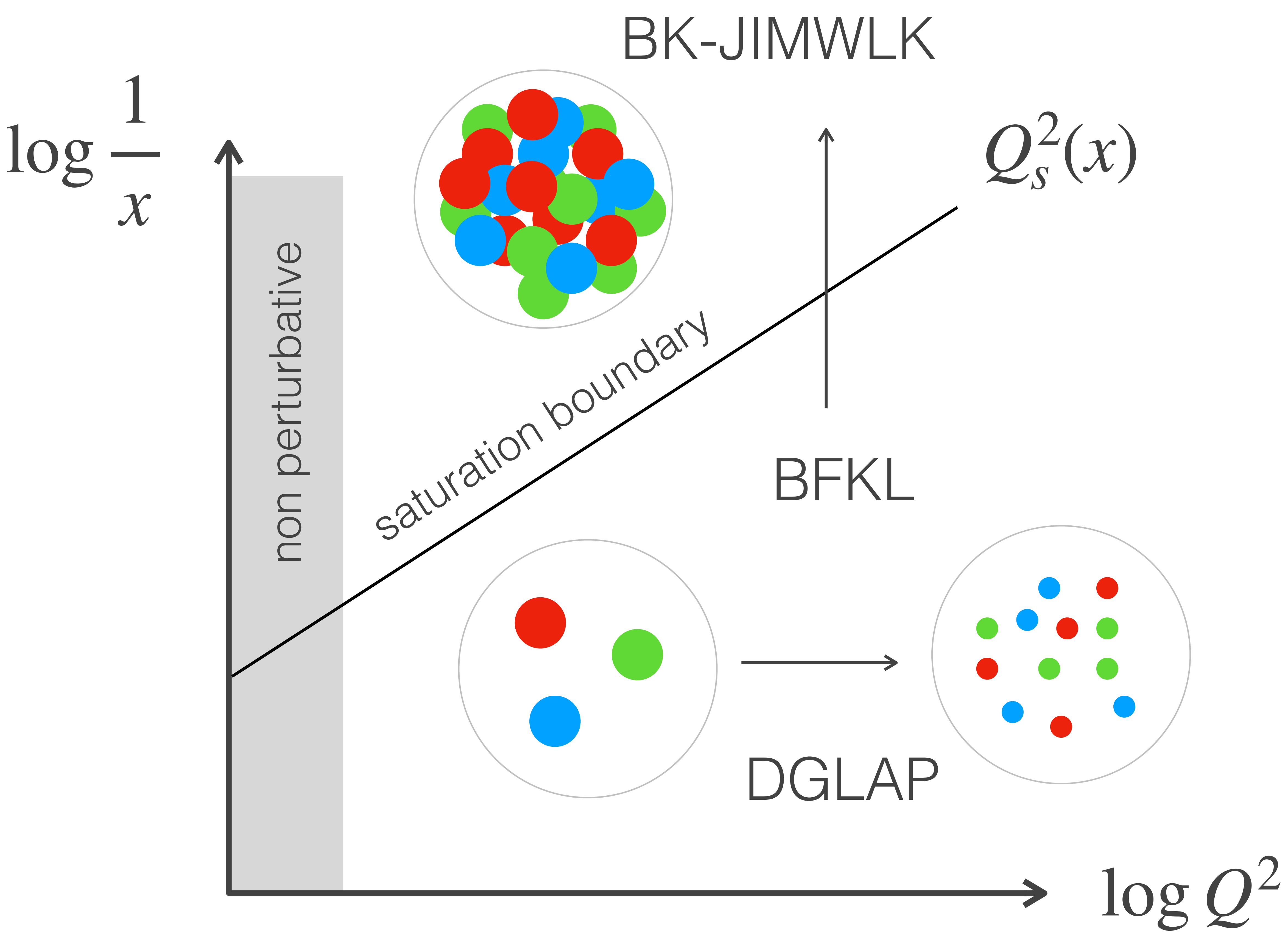
Gluon saturation at small x

- Gluon saturation criterion: large number of gluons populate the transverse extend of the proton leading to saturation when

$$S_\perp \sim \frac{\alpha_s}{Q^2} \times xg(x, Q^2)$$

- Defining the saturation scale

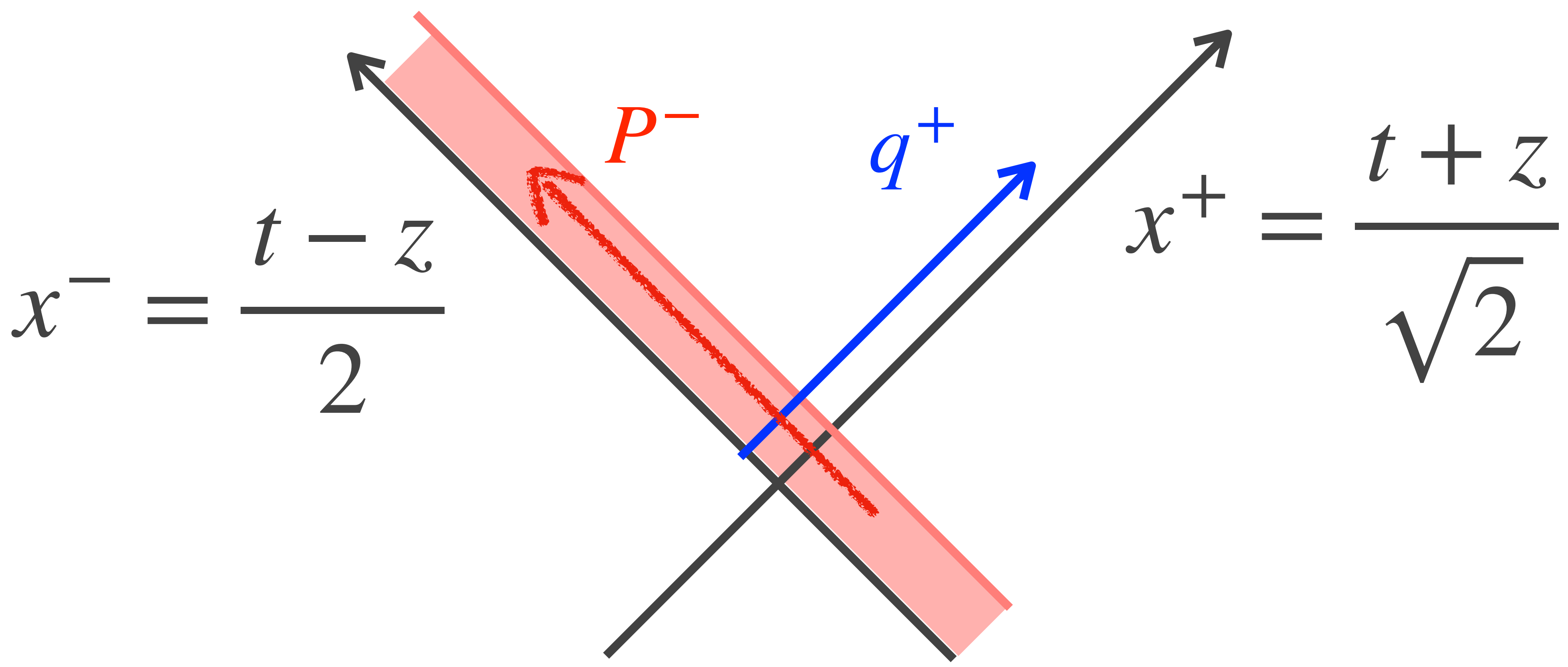
$$Q^2 \sim Q_s^2(x) \sim x^{-\lambda}$$

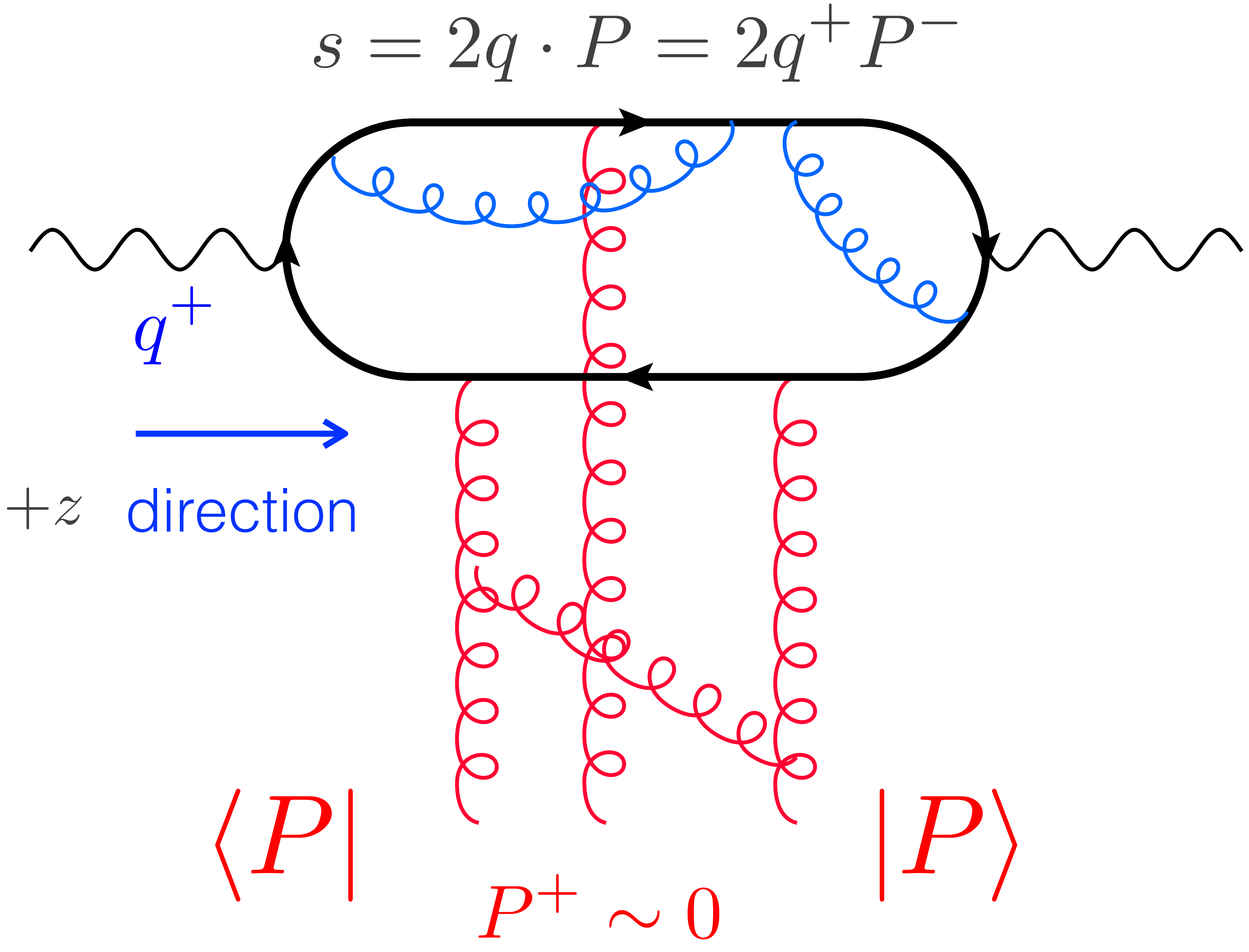


[Gribov, Levin, Ryskin, 1983- Mueller, Qiu, 1986, Venugopalan, McLerran (MV), Balitsky, Kovchegov (BK) Jallilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (JIMWLK) (1993-2001)]

Rapidity factorization at small x - coherent scattering

- **Regge limit:** $s \to \infty$ with Q^2 fixed
- Although gluons contribute only at NLO they dominate the cross section at small x. Dominant diagram in DIS: scattering of quark dipole moving in the +z direction off longitudinally polarized gluons in the target



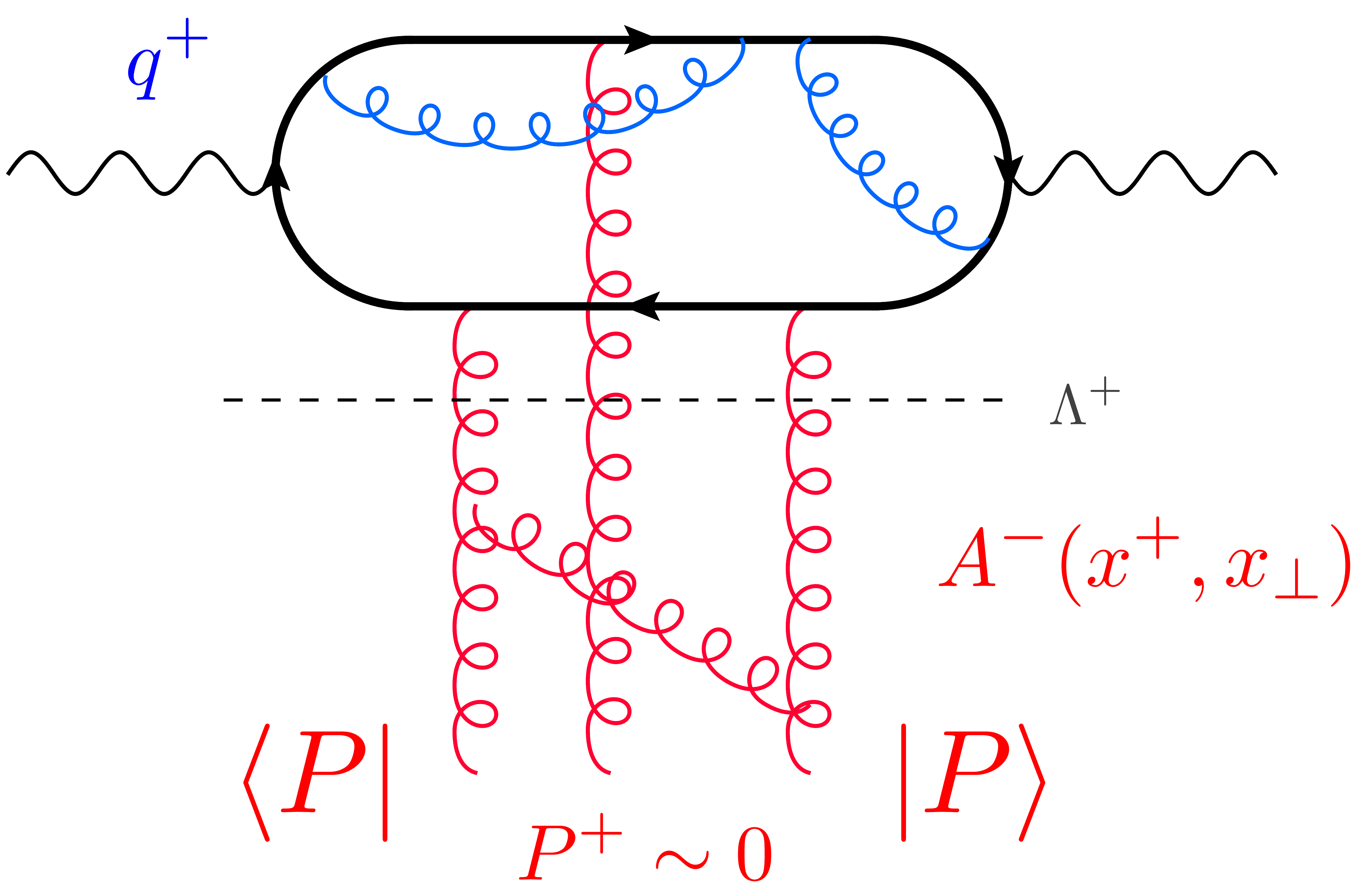


$x_{Bj} = Q^2/s \to 0$

Rapidity factorization at small x - coherent scattering

Step 1: Split the gluon fields into fast and slow gluons

$$A^\mu(k) \equiv A^\mu(k^+ < \Lambda^+) + a^\mu(k^+ > \Lambda^+)$$



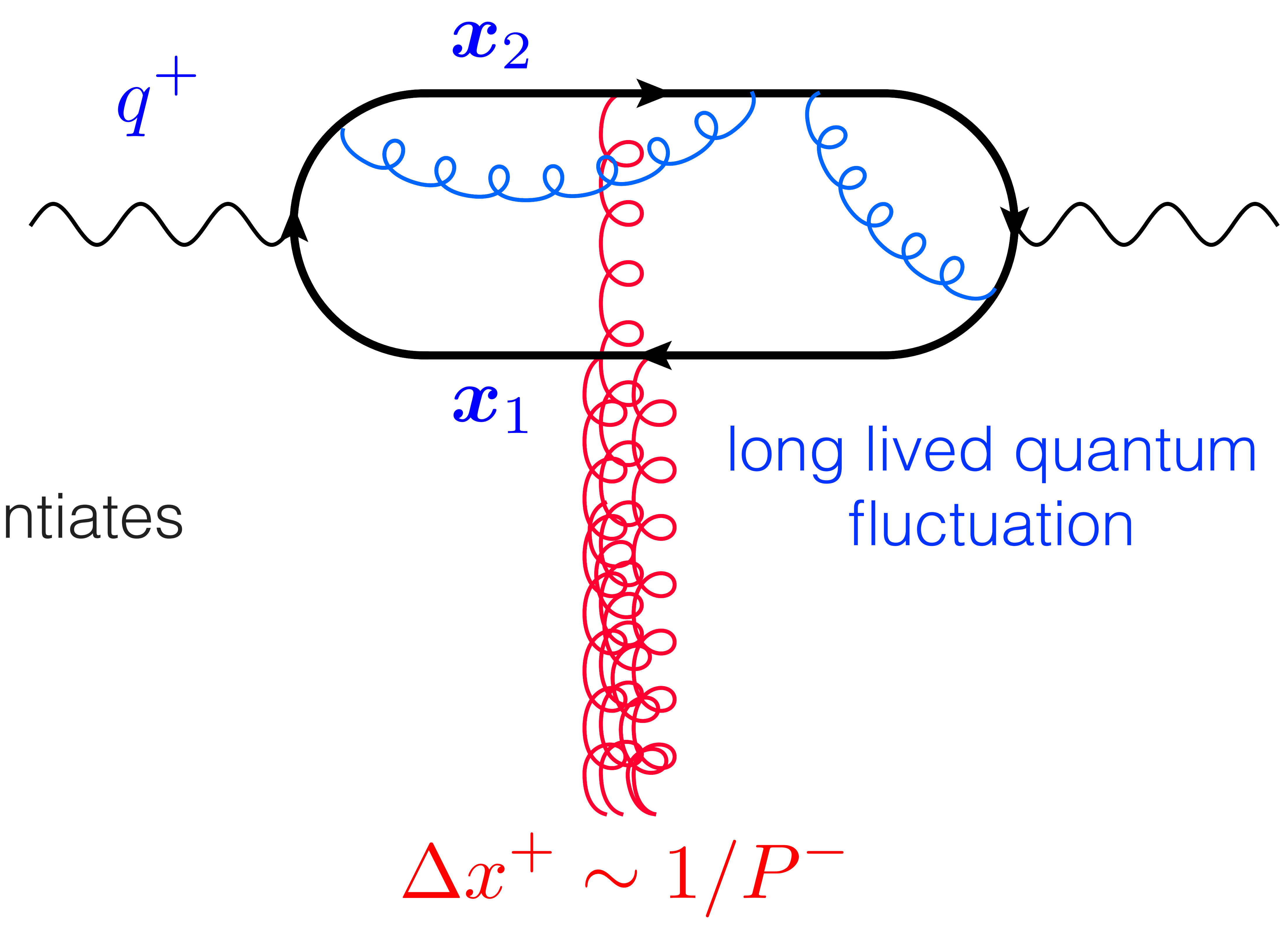
- The relevant d.o.f. in the saturation regime are strong classical fields $g\,A^- \sim 1$: boosted target field dominated by its - component

$$A^\mu(x) \quad \rightarrow \quad \gamma\, A^-(\gamma x^+, \frac{x^-}{\gamma}, \boldsymbol{x}) \quad A^+ \sim O(1/\gamma) \quad A_\perp \sim O(1)$$

Rapidity factorization at small x - coherent scattering

Step 2: shock wave approximation at small x by symmetry the - component of the momenta must also be strongly separated

$$q^- \equiv x_{\rm Bj} P^- \ll P^-$$



The interaction with the background field exponentiates and light cone time integrations decouple

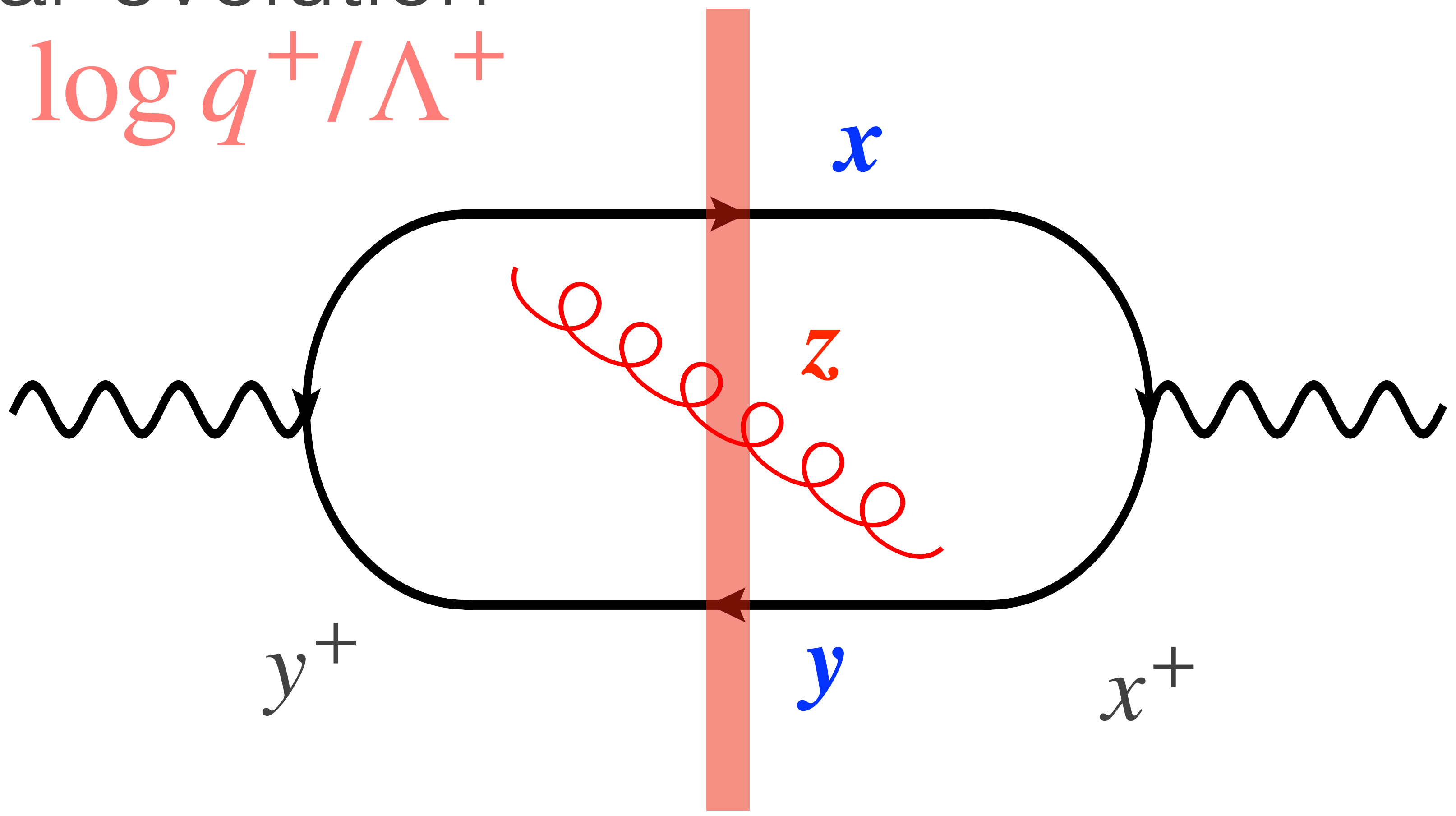
$$U_{\boldsymbol{x}} \equiv \mathcal{P}_+ \exp\left[ig \int_{-\infty}^{+\infty} {\rm d}z^+ A^-(z^+, \boldsymbol{x}_\perp)\right]$$

Factorization at small x:

$$\sigma \sim \int_{\boldsymbol{x}_1, \boldsymbol{x}_2} \mathcal{H}(\boldsymbol{x}_2 - \boldsymbol{x}_1) \otimes \langle P | \, {\rm Tr} \, U_{\boldsymbol{x}_1} U^\dagger_{\boldsymbol{x}_2} \, | P \rangle$$

BK and the NLO crisis

- Balitsky-Kovchegov (1996-1999) equation describes the non-linear evolution of the dipole scattering amplitude as function of the rapidity $Y \equiv \log q^+/\Lambda^+$

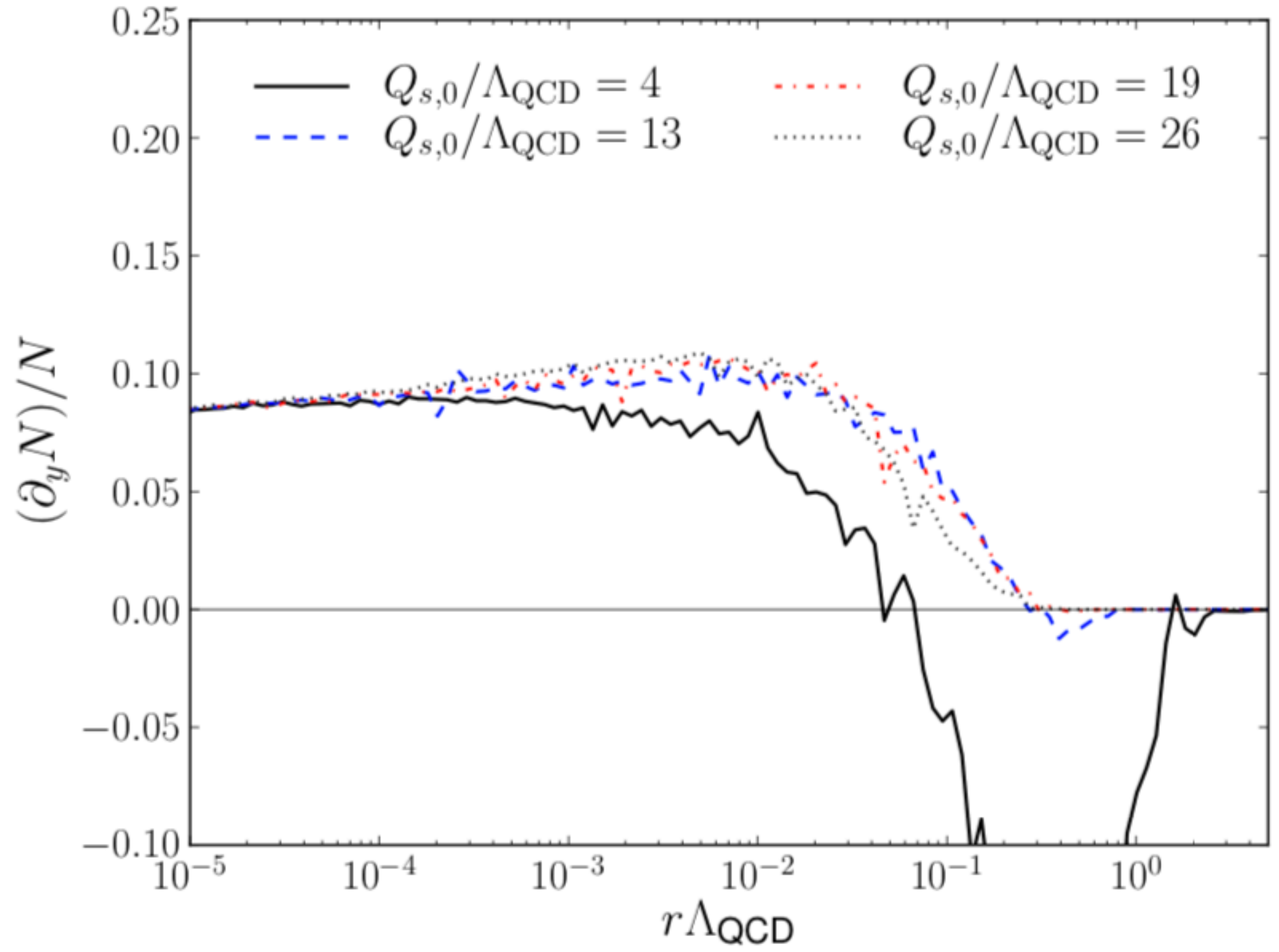


$$S_Y(\boldsymbol{x}-\boldsymbol{y}) \equiv \frac{1}{N_c} \langle \mathrm{Tr}\, U(\boldsymbol{x}) U^\dagger(\boldsymbol{y}) \rangle_Y$$

$$\frac{\partial}{\partial Y} S_Y(\boldsymbol{x}-\boldsymbol{y}) = \bar{\alpha}\mathcal{K}_{NLO} \otimes [S_Y(\boldsymbol{x}-\boldsymbol{z})S_Y(\boldsymbol{z}-\boldsymbol{y}) - S_Y(\boldsymbol{x}-\boldsymbol{y})] + \mathrm{non-dipole}$$

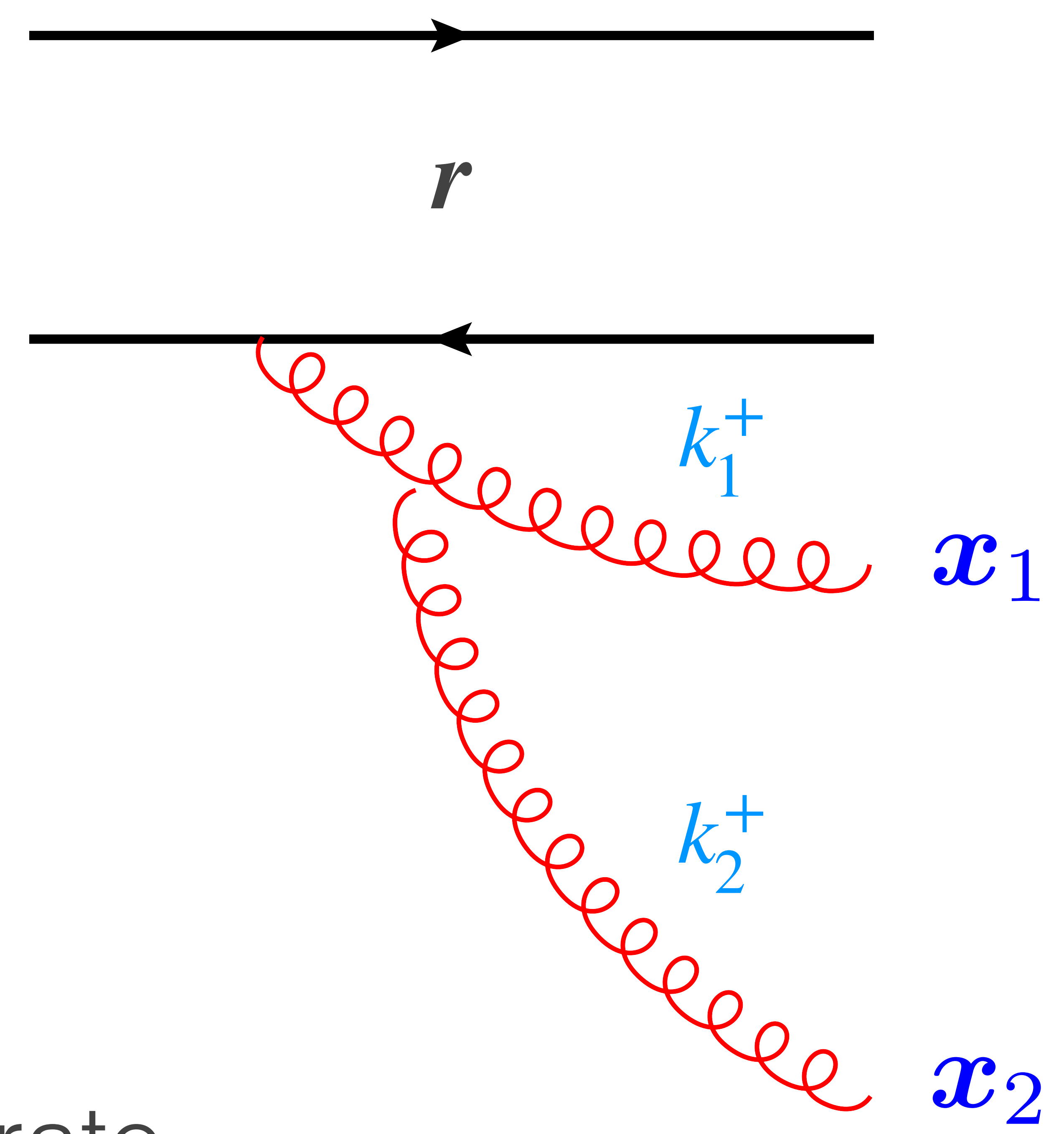
- At NLO BK equation [Balitsky and Chirilli (2008)] was found to be numerically unstable

[Lappi and Mäntysaari (2015)]



BK and the NLO crisis

- This is due to the fact that the rapidity variable $Y \equiv \log \frac{q^+}{\Lambda^+}$ evolves independently from $\boldsymbol{x}_\perp$ violating $k^- = xP^-$ ordering



formation time ordering implies

$$k_1^- = \boldsymbol{x}_1^2 k_1^+ \; < \; k_2^- = \boldsymbol{x}_2^2 k_2^+ \qquad \Rightarrow \qquad \frac{\boldsymbol{x}_1^2}{\boldsymbol{x}_2^2} < \frac{k_2^+}{k_1^+} \ll 1$$

$$k_1^+ \gg k_2^+$$

- Small dipoles that radiate larger dipoles $\boldsymbol{x}_1 \ll \boldsymbol{x}_2$ generate large collinear logarithms when $k_1^- \sim k_2^-$

$\Rightarrow$ potentially large double logs : NLO/ LO ~ $\alpha_s \log^2 \frac{1}{r_\perp^2} > 1$

[Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019)]

BK and the NLO crisis

[Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019)]

- Several solutions have been proposed: implementing kinematic bound, modification of the evolution kernel, better choice of the evolution variable, etc. However,
    - they spoil the renormalization picture established at LO
    - exhibit scheme dependence
    - no operator definition for systematic order by order calculations
    - not discussed at the level of observables: inclusive DIS

The elephant in the room: x dependence

- In the Regge limit distributions evaluated in the strict x=0 limit

$$f(k_\perp, x = 0)$$

- No x dependence at LO: quantum evolution generates rapidity dependence. Ambiguous connection to x.

- The dipole model (with locality in transverse space) is inconsistent with x dependence

Bialas, Navelet and Peschanski (2000)

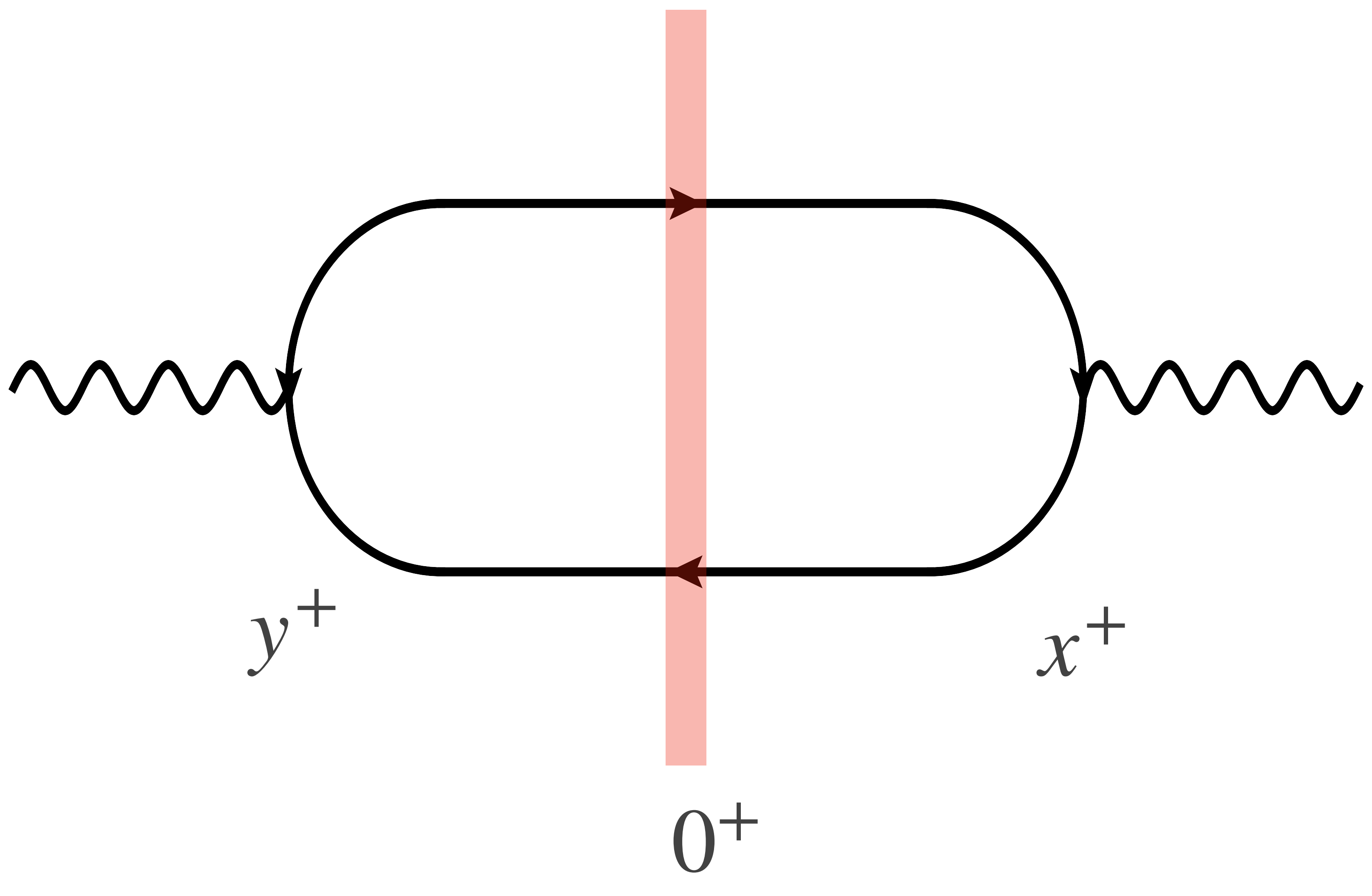
Neglecting x in the distribution is the origin of the improper account of the collinear logs at NLO

What (else) can be done to solve these issues?

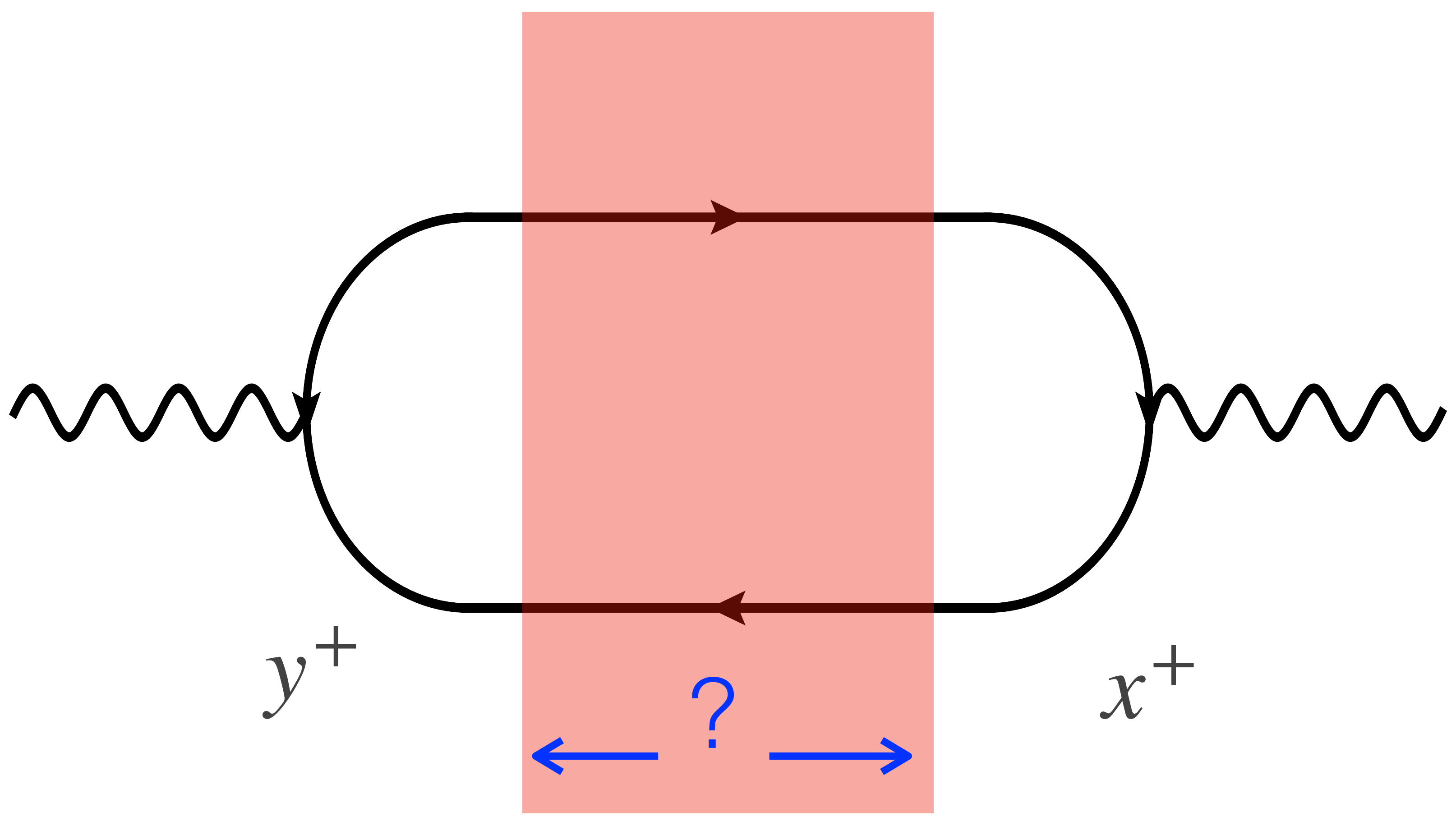
- revisit the shock wave factorization scheme
- restore the x dependence of the gluon distribution to connect Regge and Bjorken limits for proper and systematic account of collinear logs

Inclusive DIS beyond shock wave

- **Decoupling of time integrals:** In the shock wave approximation the times of the photon splitting into quark antiquark pair are integrated form $0 < x^+ < +\infty$ and $-\infty < y^+ < 0$ which yields the photon wave functions



- What are the integration limits of the vertices if one relaxes the shock wave approximation?
- What is the longitudinal extent of the shock wave ?

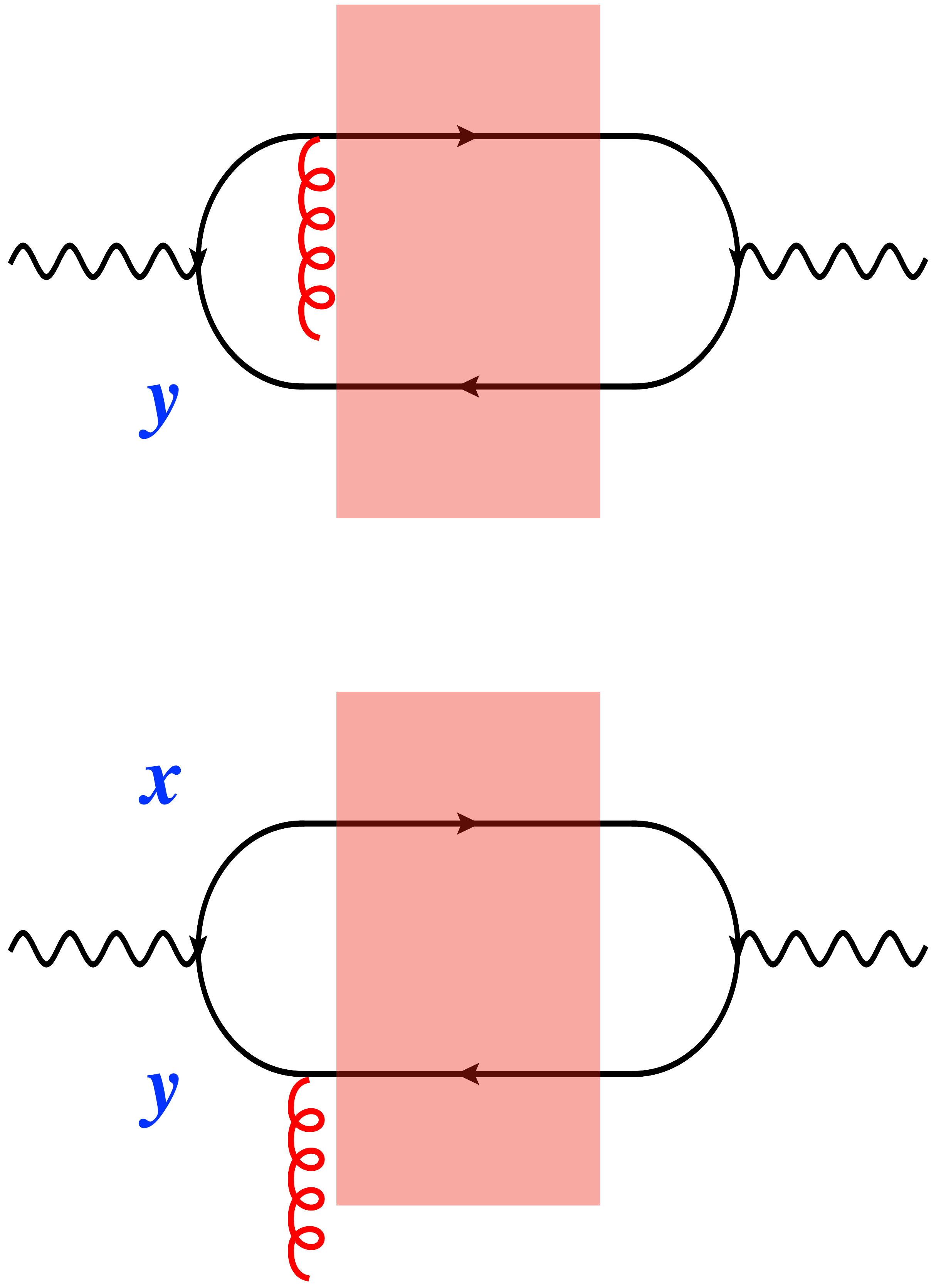


Inclusive DIS beyond shock wave

- **Solution**: extracting the first and last interactions provides a physical boundary to the shock wave
- 4 contributions that combines into to one
- Consider the left side of the diagram first
- The two gluon fields combine to generate field strength tensor

$$A^-(\boldsymbol{x}) - A^-(\boldsymbol{y}) = \int_0^1 \mathrm{d}s\, r^i \partial^i A^-(\boldsymbol{y} + s\boldsymbol{r}) = \int_0^1 \mathrm{d}z^i\, F^{i-}(\boldsymbol{z})$$

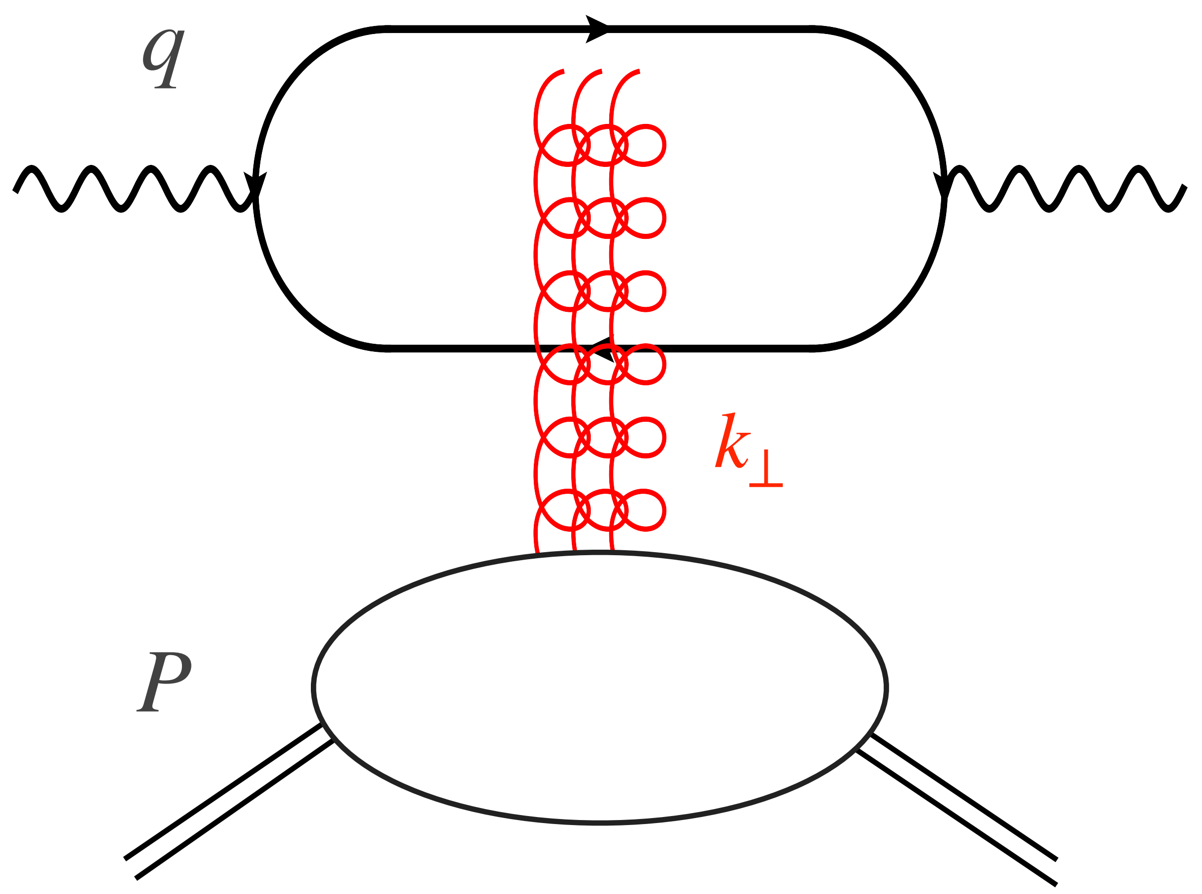
- Where $\boldsymbol{z}(s) \equiv s\,\boldsymbol{x} + (1-s)\,\boldsymbol{y}$ is a straight line trajectory in the transverse plane



$\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y}$

Inclusive DIS beyond shock wave

- DIS cross-section takes a similar form to that of the shock wave: identical wave functions



Shock wave factorization:

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}\, z\, P(z) \int_{\boldsymbol{r},\boldsymbol{b}} \mathrm{d}\,\boldsymbol{r}\ |\varphi(z(1-z)|\boldsymbol{r}|^2 Q^2)|^2\ \langle \mathrm{Tr} U(\boldsymbol{r}) U^\dagger(0) \rangle_Y$$

Beyond shock wave:

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}\, z \int_{\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{b}_1,\boldsymbol{b}_2} \varphi(z(1-z)|\boldsymbol{r}_1|^2 Q^2)\ \varphi^*(z(1-z)|\boldsymbol{r}_2|^2 Q^2)$$
$$\times\, \mathrm{e}^{i\, x_{Bj} P^-(x_2^+ - x_2^+)}\ \langle P | O_{\mathrm{dipole}}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{b}_1, \boldsymbol{b}_2, x_1^+, x_2^+ | z) | P \rangle$$

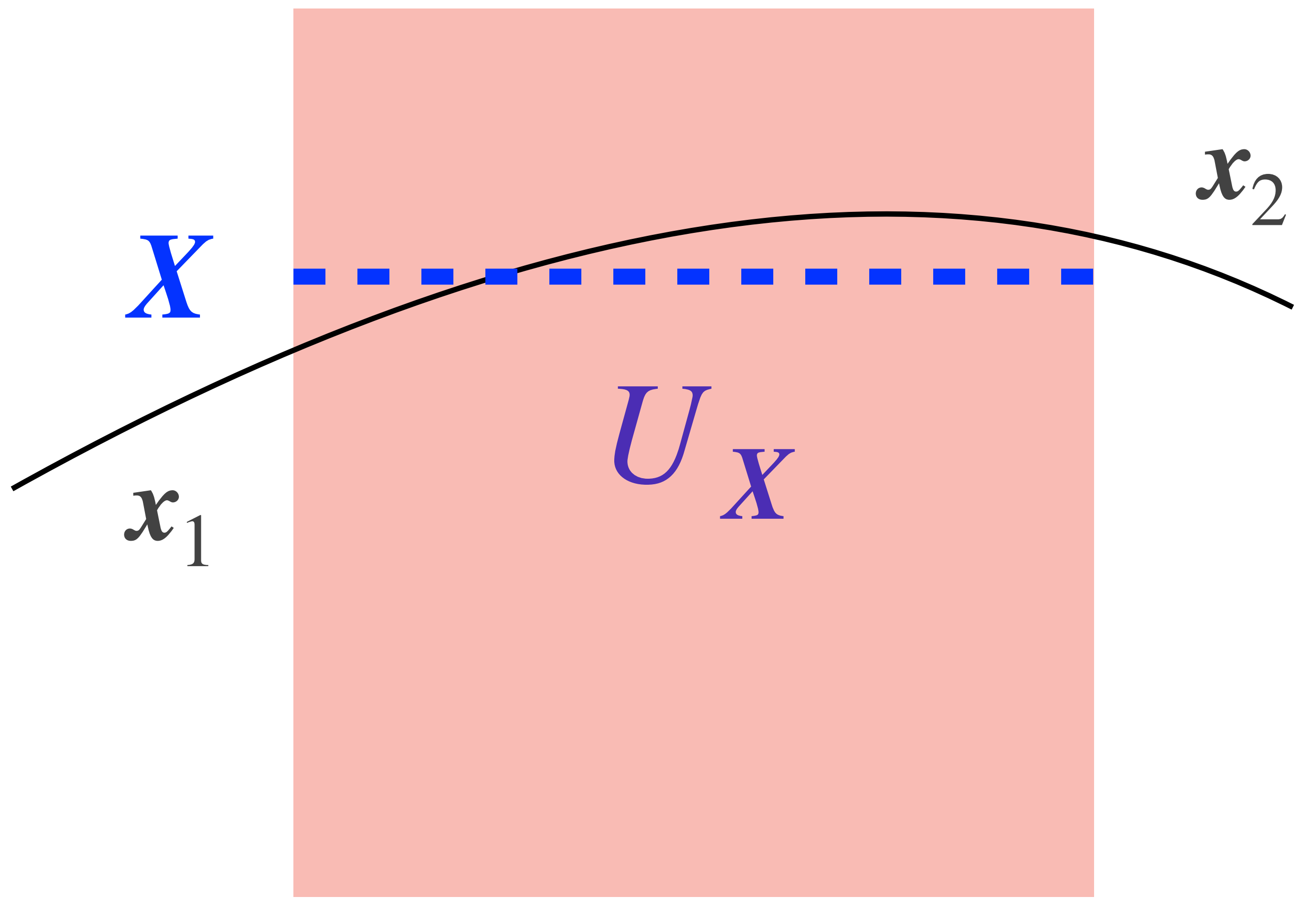
x-dependent Fourier phase

gauge invariant dipole operator

Classical expansion

- Leading power in s and Q^2 can be obtained by neglecting transverse recoil of fast partons

$$\Delta \boldsymbol{x}^2 \sim x_{\mathrm{Bj}}/Q^2$$



x_{Bj} -suppressed in the Regge limit

Q^2 -suppressed in the Bjorken limit

$$\mathcal{G}_{p^+}(x^+, \boldsymbol{x}_2; y^+, \boldsymbol{x}_1) = \mathcal{G}_0(\boldsymbol{x}_2 - \boldsymbol{x}_1, x_2^+ - y_1^+)\, U_X(x_2^+, x_1^+) + \dots$$

Altinoluk, Armesto, Beuf, Martinez, Salgado (2015)

shock wave (eikonal limit)

$$\lim_{p^+ \to +\infty} \mathcal{G}_{p^+}(x^+, \boldsymbol{x}; y^+, \boldsymbol{y}) = \delta(\boldsymbol{x} - \boldsymbol{y})\, U_{\boldsymbol{x}}(x^+, y^+)$$

- x encoded in quantum diffusion: FT w.r.t. $\boldsymbol{u} = \boldsymbol{x}_2 - \boldsymbol{x}_1$

$$\mathcal{G}_{p^+}(x_2, x_1^+, \boldsymbol{X}; \boldsymbol{\ell}) = \mathrm{e}^{i \frac{\ell^2}{2zq^+} \Delta x^+}\, U_X(x_2^+, x_1^+) + \dots \qquad x_F \equiv \frac{\ell^2 + z\bar{z}Q^2}{2z\bar{z}\, s}$$

A Novel Unintegrated Gluon Distribution

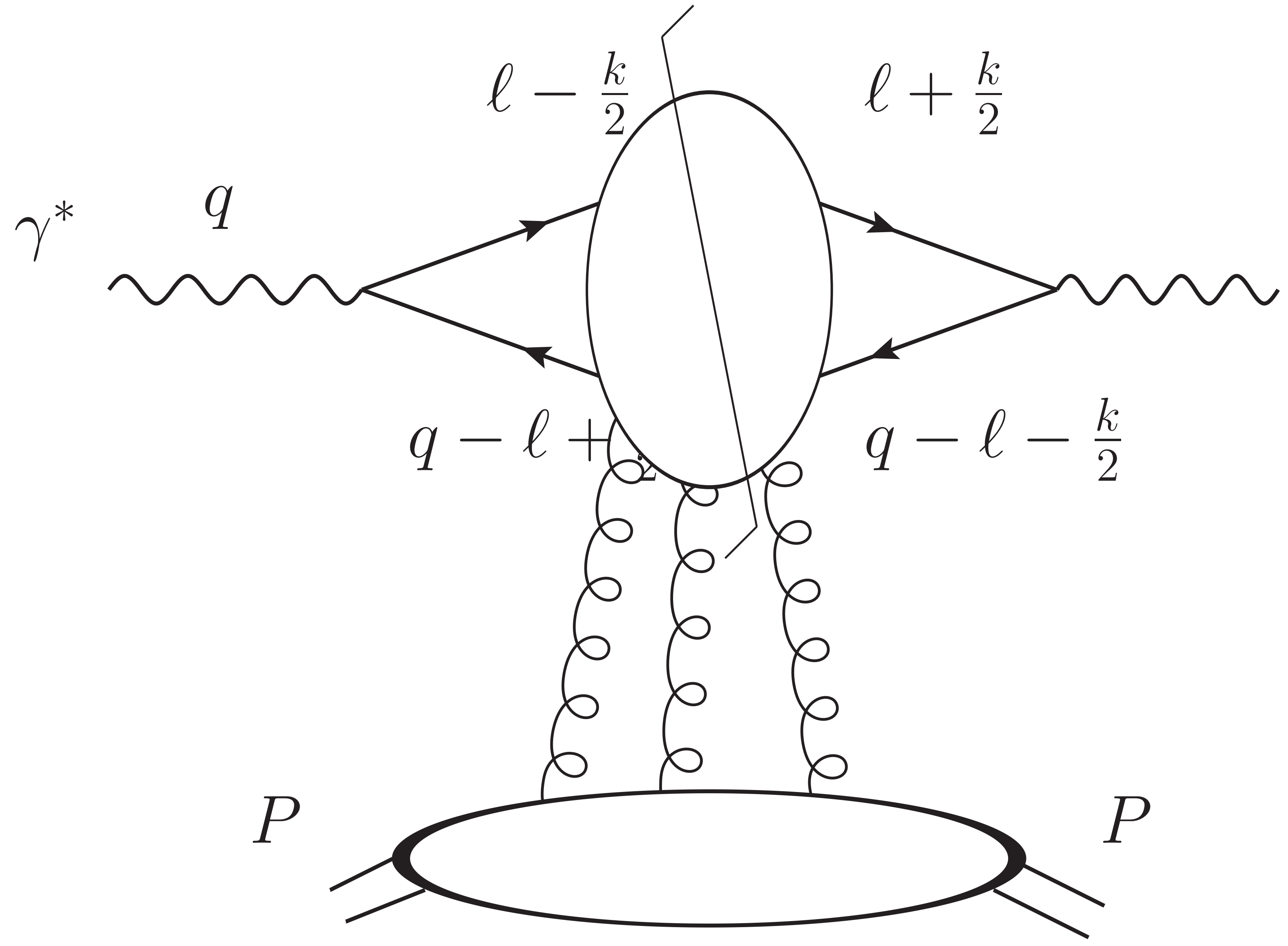
2006.14569 [hep-ph]

Factorization formula for DIS at arbitrary x

- After integration over the factorized free propagators that lead to the Feynman x phase, we obtain the factorization formula (for the transverse photon cross-section), in momentum space,

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}\,z \int_0^1 \mathrm{d}x \int_{\boldsymbol{\ell},\boldsymbol{k}} \partial^i \varphi\left(\boldsymbol{\ell} - \frac{\boldsymbol{k}}{2}\right) \partial^j \varphi^* \left(\boldsymbol{\ell} + \frac{\boldsymbol{k}}{2}\right) \delta\left(x - x_{Bj} - \frac{\boldsymbol{\ell}^2}{2z\bar{z}q^+}\right)$$

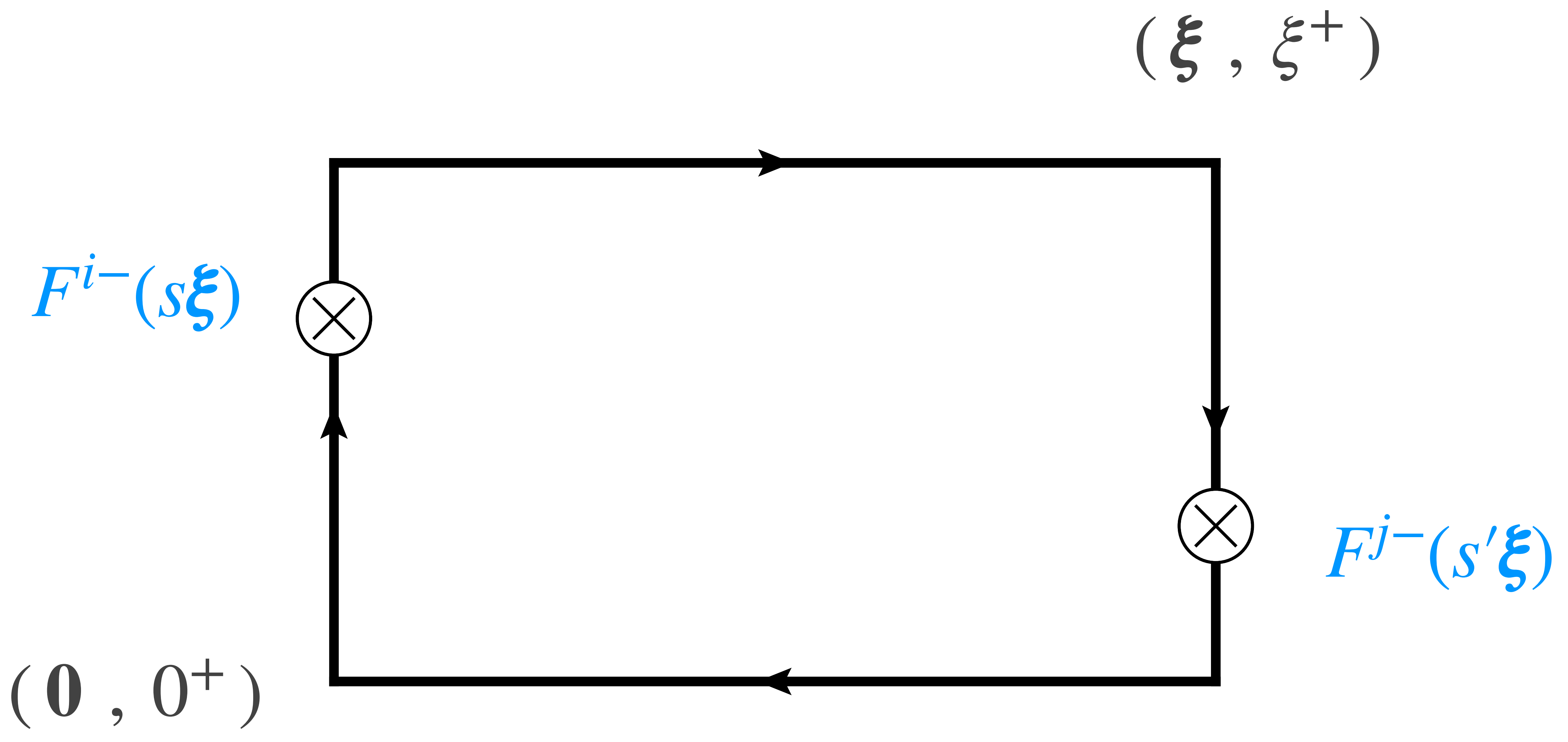
$$\times\, xG^{ij}(x, \boldsymbol{k}) \; + \; O\left(k_\perp^2/s\right)$$



- Same wave functions as in small x
- The delta function relates x in the gluon distribution to x_{Bj}
- Gluon distribution different form small x

x-dependent and gauge invariant unintegrated gluon distribution

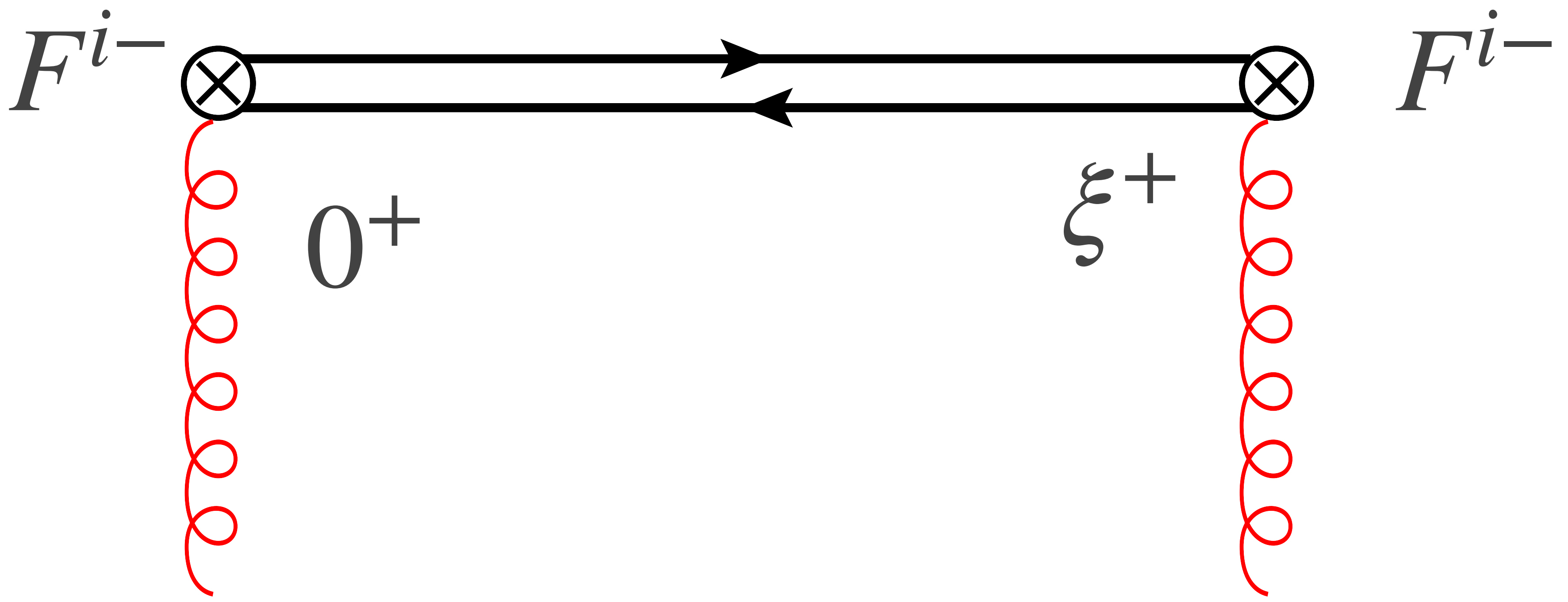
$$xG^{ij}(x,k_\perp) \equiv 2\int \frac{d\xi^+ d\boldsymbol{\xi}}{(2\pi)^3 P^-} e^{ixP^-\xi^+ - ik\cdot\boldsymbol{\xi}} \langle P|\,\mathrm{Tr}\,[0,\xi^+]_{\boldsymbol{\xi}}\, F^{j-}(\xi^+, s'\boldsymbol{\xi})\,[\xi^+,0]_{\boldsymbol{0}}\, F^{i-}(0, s\boldsymbol{\xi})\,|P\rangle$$



Bjorken and Regge limits of the uPDF

- Integrating over $k_\perp$ yields $\xi_\perp = 0$ and we recover the gluon PDF

$$xg(x, \mu^2) = 2\int \frac{\mathrm{d}\xi^+}{(2\pi)P^-} e^{ixP^-\xi^+} \langle P | \mathrm{Tr}\,[0, \xi^+]\, F^{i-}(\xi^+)\, [\xi^+, 0]\, F^{i-}(0) | P \rangle$$



- At $x = 0$ we recover the small x dipole operator $\quad \xi^i \xi^j G^{ij}(x = 0, \boldsymbol{\xi}) \to \langle P | \mathrm{Tr}\; U_{\boldsymbol{\xi}}\, U_{\mathbf{0}}^\dagger | P \rangle$

Provides the interpolation between the leading twist term in the Bjorken limit and the eikonal term in the Regge limit

Summary

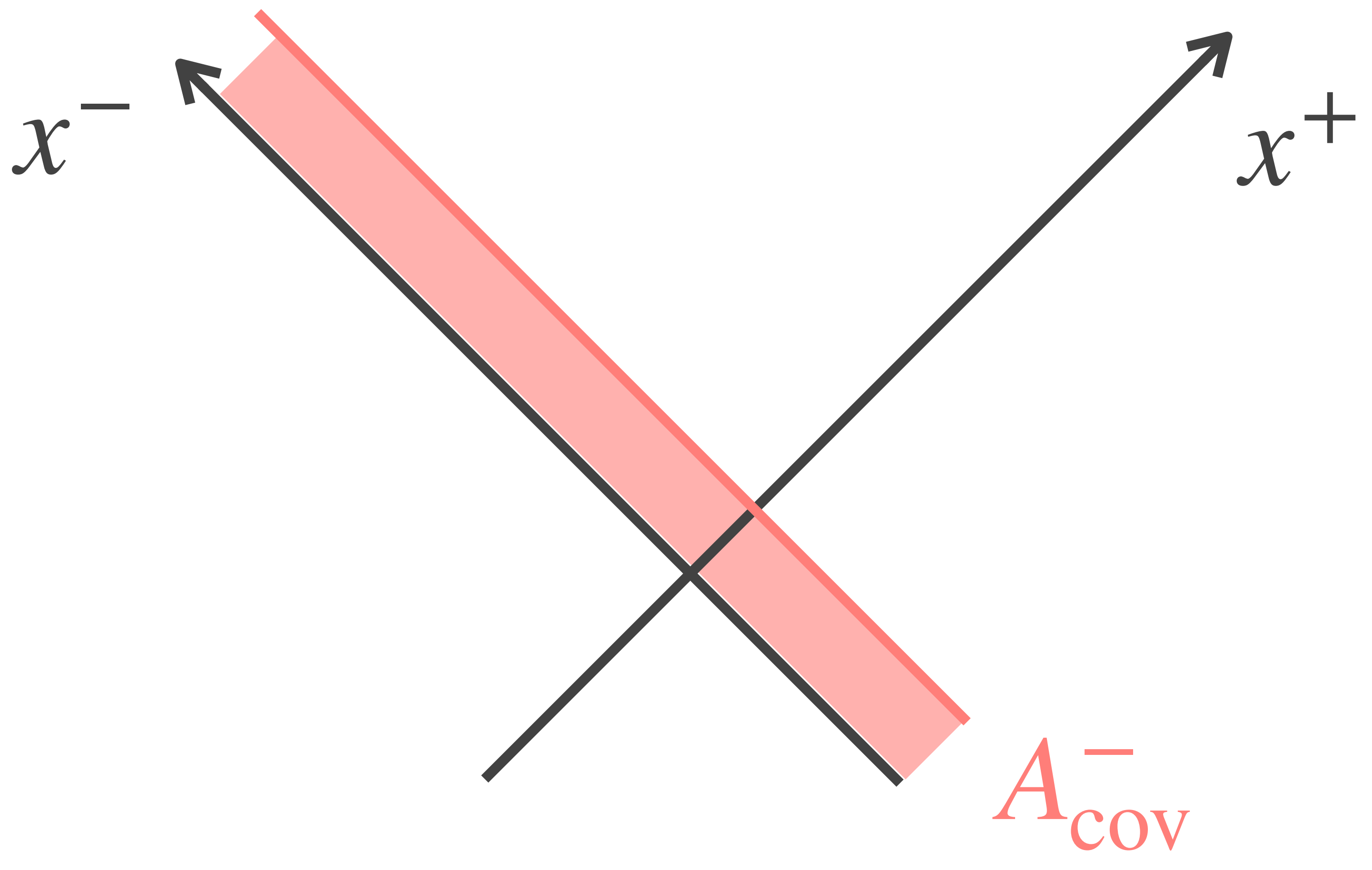
- There are issues with collinear logs at small x that can be traced back to the shock wave prescription
- Minimal correction of the semi-classical approach to small x solves the problem from first principles
- In the case of inclusive DIS: while the hard part is unchanged a new (gauge invariant) unintegrated gluon distribution compatible with x dependence emerges that interpolates between the dipole operator at small x and the gluon PDF at leading twist

Outlook

- Compute the evolution equation for the x-dependent uPDF
- Investigate other processes (eg. DVCS)
- Potential probe of gluon saturation on the lattice?

Backup

Background field and transverse gauge links



target sources

$$J^\nu(x) \to J^-(x^+, x_\perp)$$

and $J^+ = J_\perp = 0$

- consider a target boosted along the $-z$ direction close to the light cone. Due to time dilation the target color sources are "frozen" in the - direction

- Yang-Mills equations $[D_\mu, F^{\mu\nu}] = J^\nu$ can be solved exactly (together with the continuity equation $[D_\mu, J^\mu] = 0$) in covariant gauge $\partial \cdot A = 0$ (or light-cone gauge $A^+ = 0$)

$$A^-_{\rm cov} = -\frac{1}{\partial_\perp^2} J^- \quad \text{and} \quad A^+ = A_\perp = 0$$

Background field and transverse gauge links

- under an arbitrary gauge rotation $\Omega(x^+, x_\perp)$ the target field transforms as

$$A^- \;\to\; \Omega_{\boldsymbol{x}}(x^+)\; A^-_{\rm cov}\;(x^+, \boldsymbol{x})\; \Omega^{-1}_{\boldsymbol{x}}(x^+) \;-\; \frac{1}{ig}\Omega_{\boldsymbol{x}}(x^+)\;\partial^-\;\Omega^{-1}_{\boldsymbol{x}}(x^+)$$

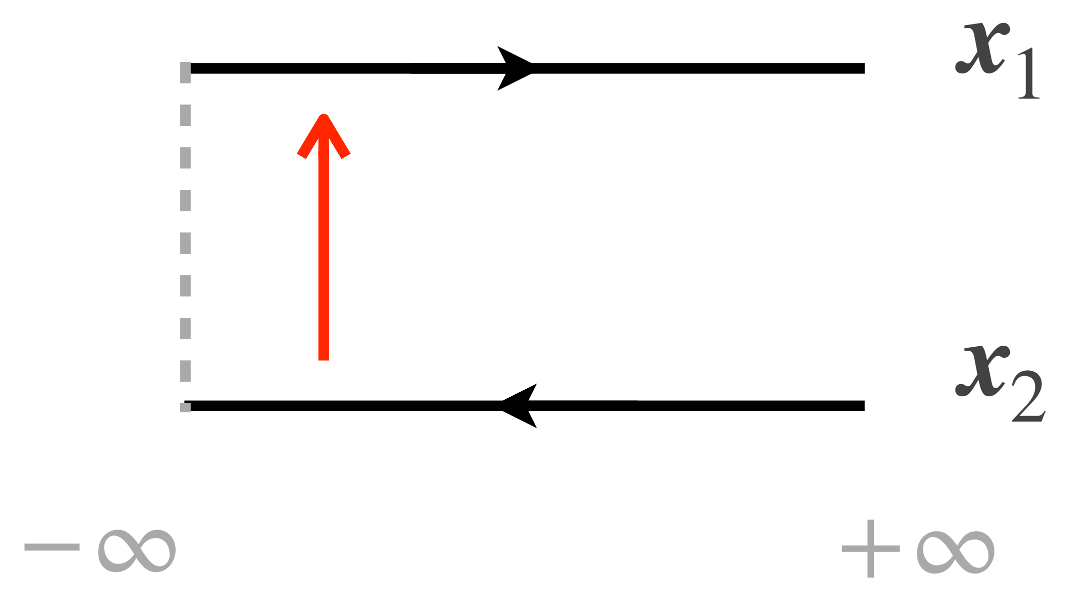
$$\boldsymbol{A}^i \;\to -\frac{1}{ig}\;\Omega_{\boldsymbol{x}}(x^+)\;\partial^i\;\Omega^{-1}_{\boldsymbol{x}}(x^+)$$

- exploiting the residual gauge freedom we can generate a transverse pure gauge
- N.B.: the partonic picture is manifest in the LC-gauge $A^- = 0$ (with $\boldsymbol{A}_\perp \neq 0$)
- small x observables are (in the dilute/dense limit) more naturally expressed in the wrong LC-gauge $A^- \neq 0$ (with $\boldsymbol{A}_\perp = 0$).
- in order to connect to the partonic interpretation one needs to deal with transverse fields

Background field and transverse gauge links

- geometric interpretation of the all twist resummation

$$U_{\boldsymbol{x}_1} = U_{\boldsymbol{x}_2} - \boldsymbol{r}^i \int_0^1 \mathrm{d}s\ (\partial^i U_{\boldsymbol{x}_2 + s\boldsymbol{r}})$$



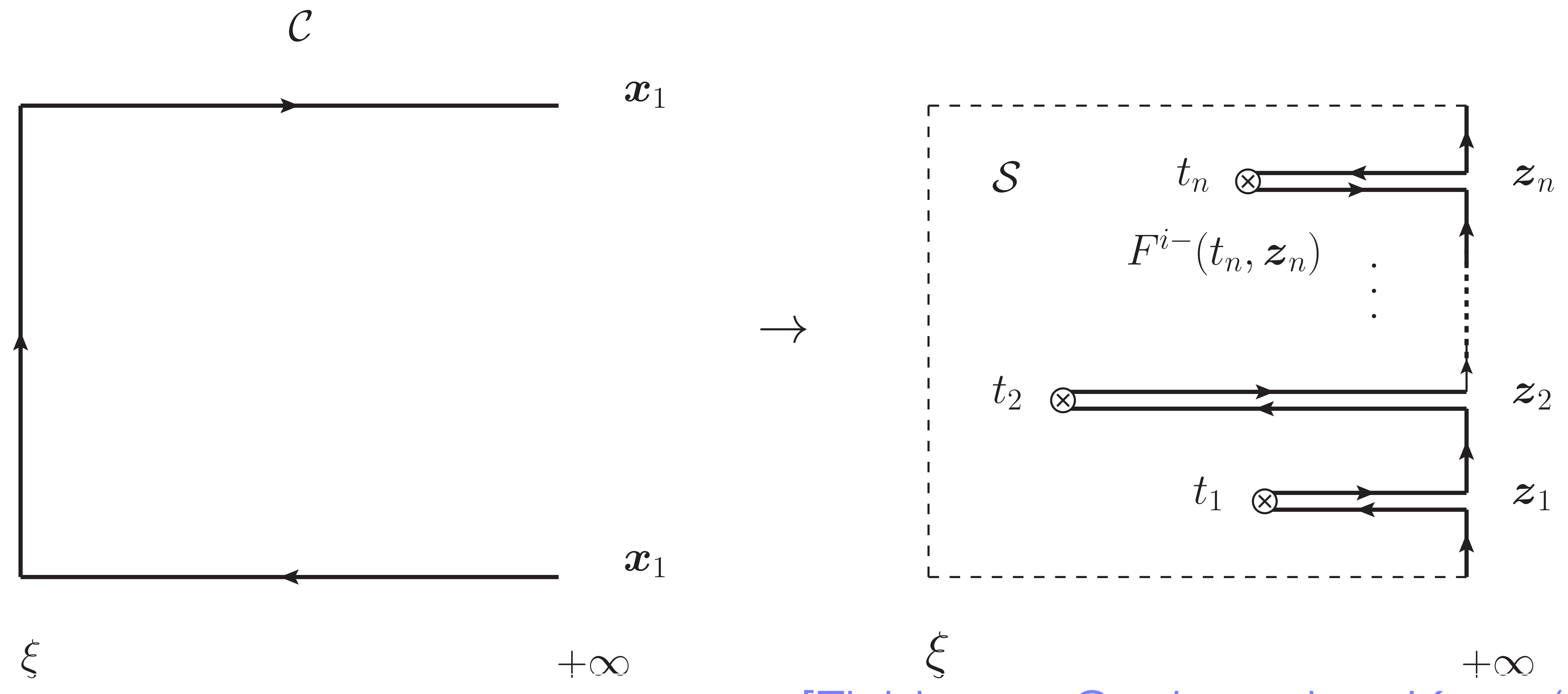
- and noticing that $\frac{1}{ig}(\partial^i U_{\boldsymbol{x}}) U_{\boldsymbol{x}} \equiv A^i(\boldsymbol{x})$, one can express the dipole operator (in the background field A^-) as a transverse gauge link:

$$U_{\boldsymbol{x}_1} U^\dagger_{\boldsymbol{x}_2} = [\boldsymbol{x}_1, \boldsymbol{x}_2] = 1 - ig \int_{\boldsymbol{x}_2}^{\boldsymbol{x}_1} \mathrm{d}\boldsymbol{z}\, A^i(\boldsymbol{z})\ [\boldsymbol{z}, \boldsymbol{x}_2]$$

- dipole operator can be expessed in terms of tranverse link operators

Background field and transverse gauge links

- non-Abelian Stokes' theorem: more generally, the dipole operator can be written as a path ordered tower of "twisted" field strength tensor (i.e. dressed with future pointing Wilson lines)



[Fishbane, Gasiorowicz, Kaus (1981) Wiedemann (2000)
YMT, Boussarie (2020)]

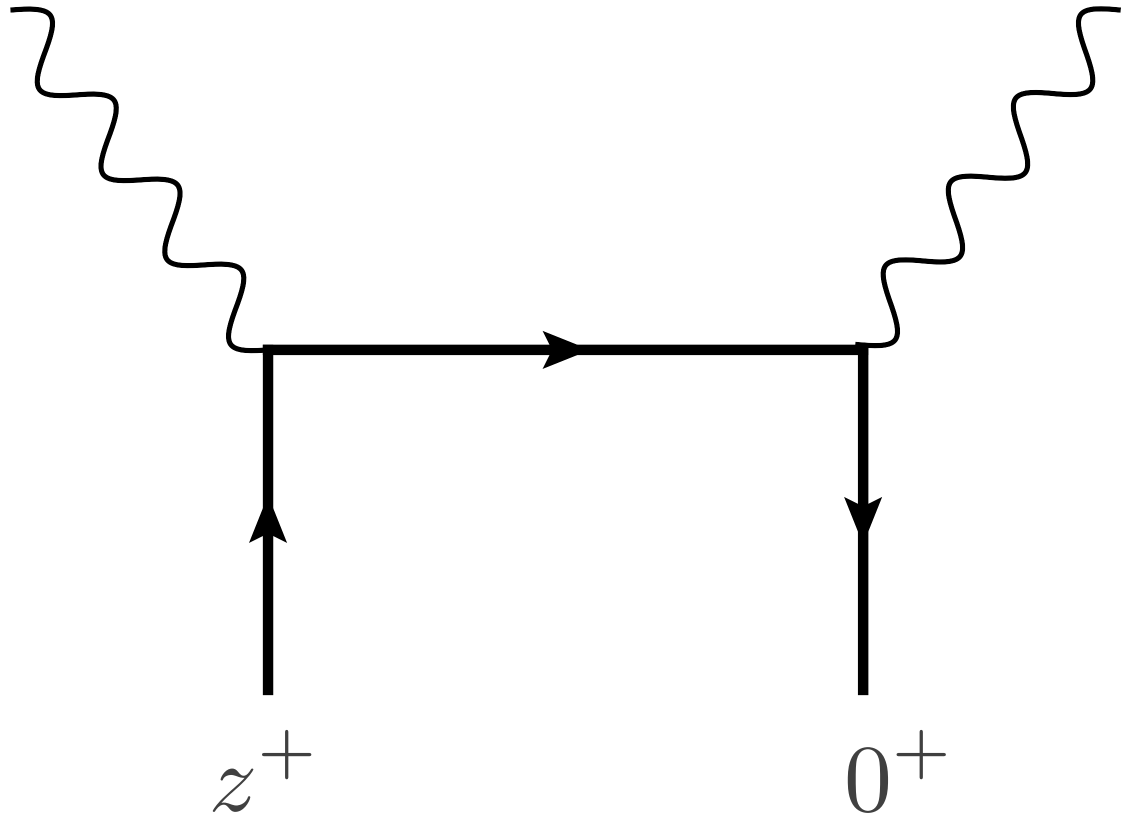
$$U_{x_2}U^{\dagger}_{x_1} \equiv P\exp\left[-ig\int_S \mathrm{d}t\mathrm{d}z\,[+\infty, x^+]_x\, F^{i-}(x^+, x)\,[x^+, +\infty]_x\right]$$

PDF's and gauge invariance

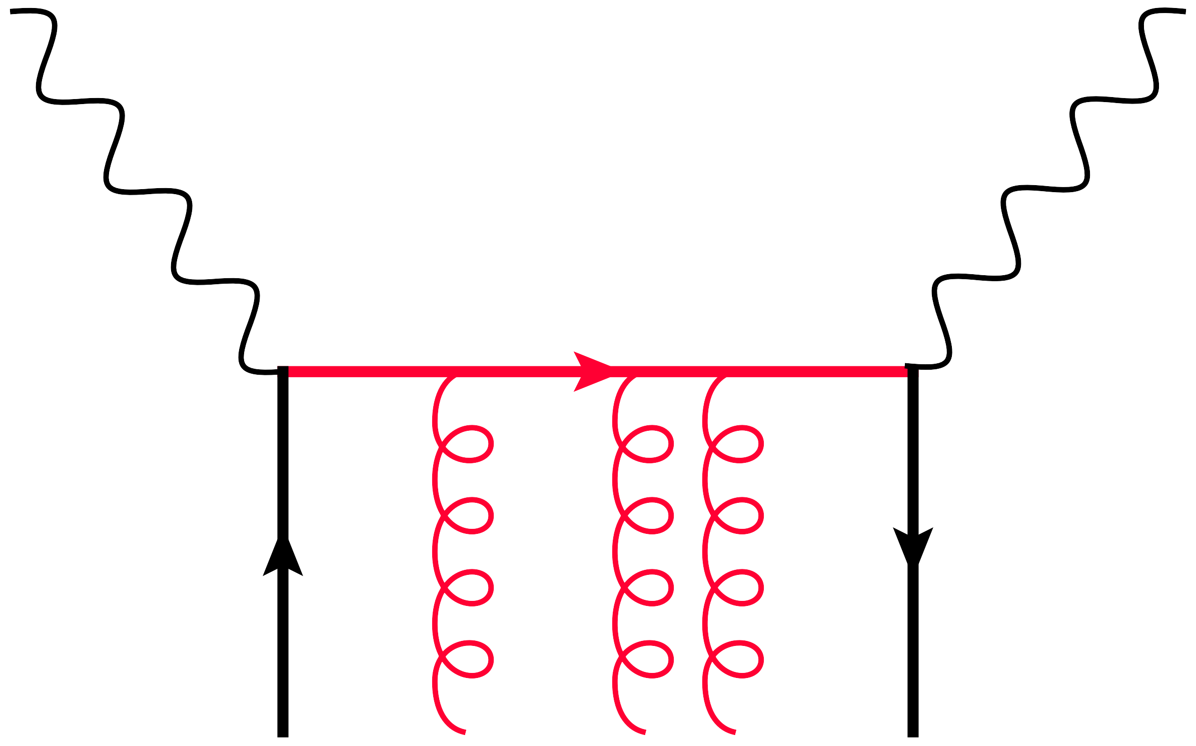
- Number density interpretation possible in light-cone gauge $A^- = A^0 - A^3 = 0$

$$xq(x) \propto \int \mathrm{d}z^+ \, \mathrm{e}^{ixP^- z^+} \, \langle P|\bar{\psi}(z^+)\, \gamma^- \, \psi(0)|P\rangle$$

$$P^- = \frac{E - P_z}{\sqrt{2}} \qquad\qquad x^+ = \frac{t + z}{\sqrt{2}}$$



- In an arbitrary gauge a path ordered Wilson line (gauge link) is required. Resums interactions with longitudinally polarized gluons in the target



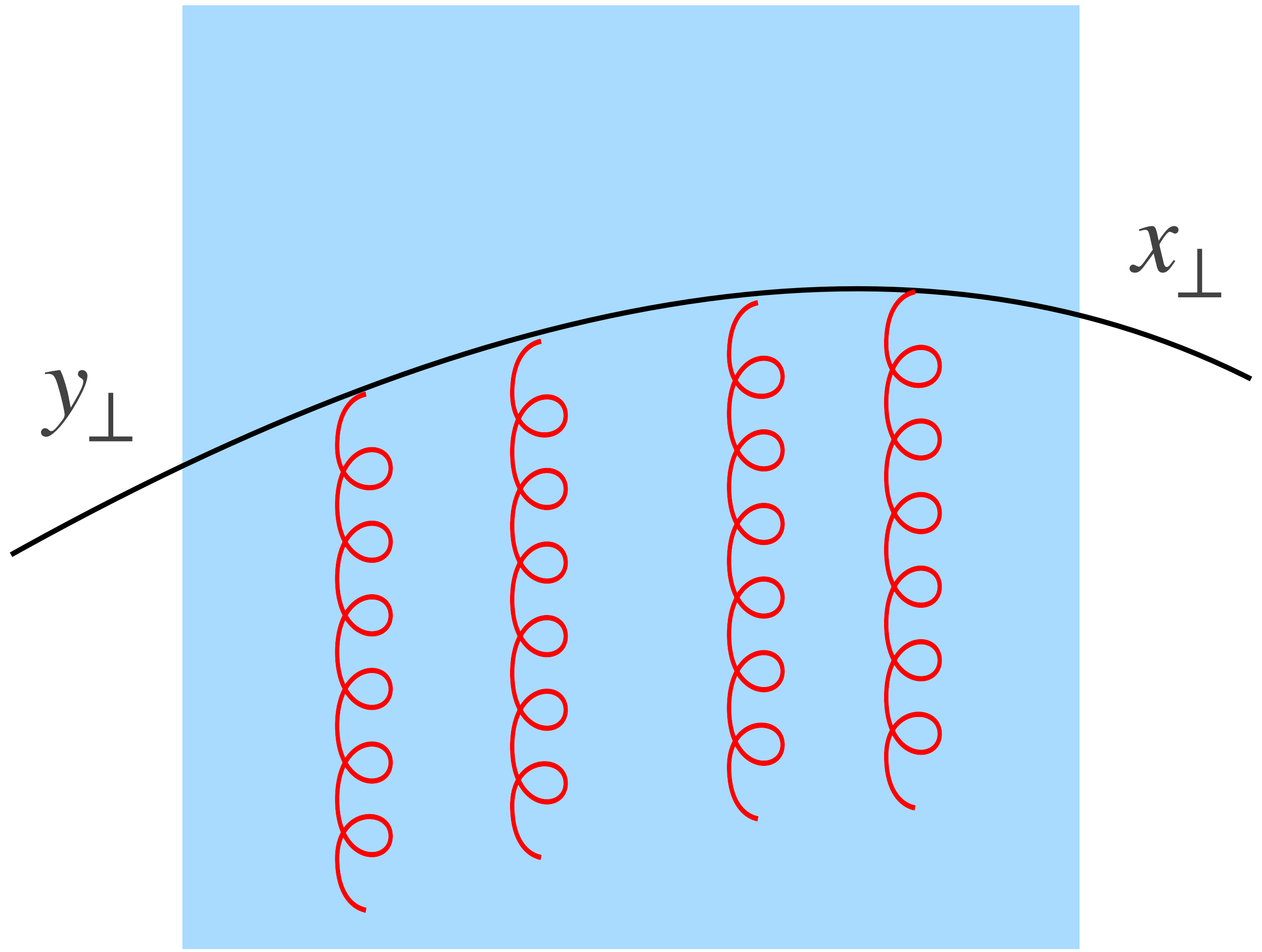
$$xq(x) \propto \int \mathrm{d}z^+ \, \mathrm{e}^{ixP^- z^+} \, \langle P|\bar{\psi}(z^+)\, [z^+, 0^+] \, \psi(0)|P\rangle$$

$$[z^+, 0^+] \equiv \mathcal{P}_+ \exp\left[ig \int_0^{z^+} \mathrm{d}z'^+ A^-(z'^+, z_\perp = 0) \right]$$

$$A^- \equiv A^-_a t^a$$

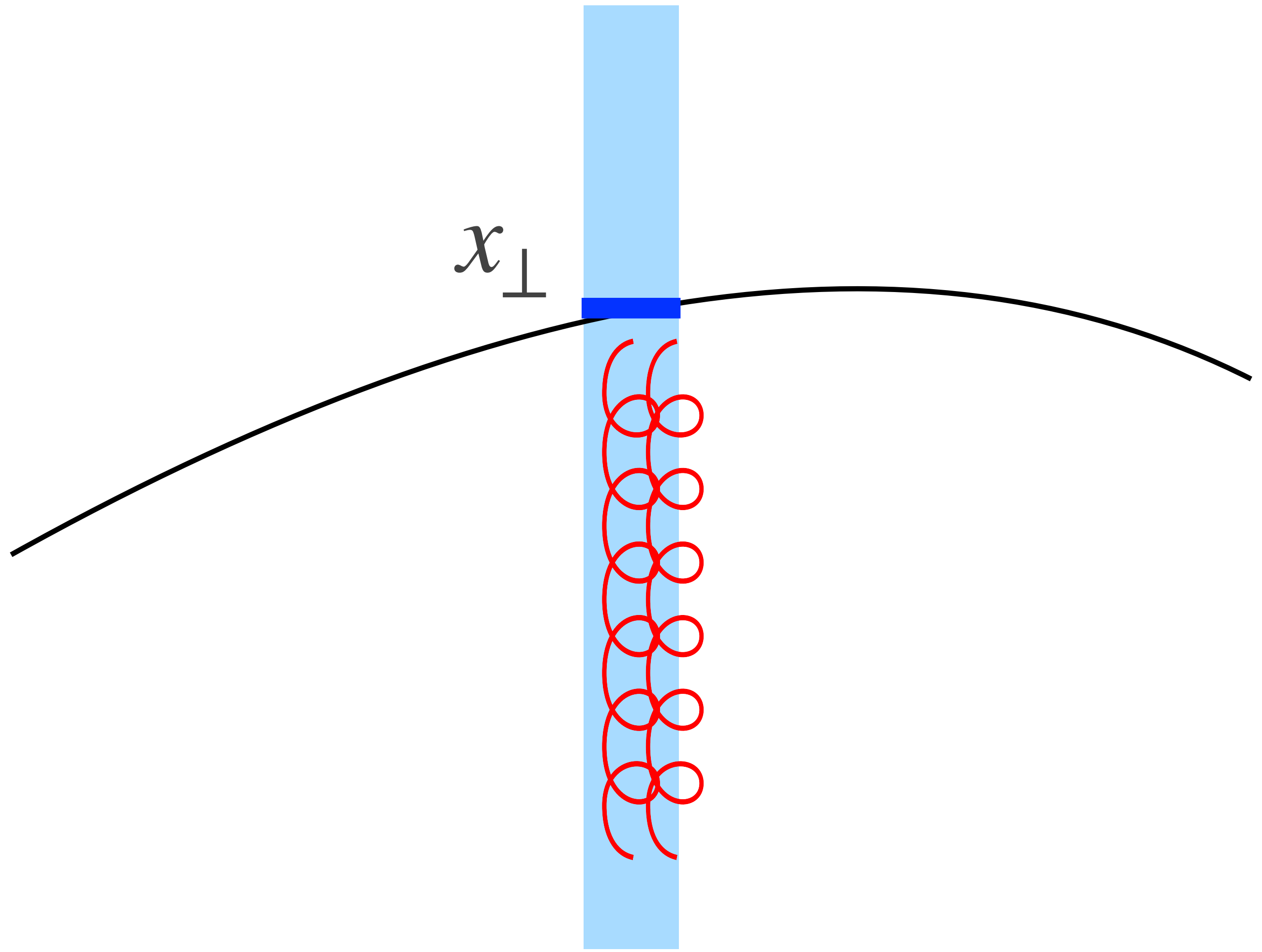
Revisiting the Shock Wave Approximation

- Under the assumption that all transverse momenta are of same order along the ladder, the leading power $1/s$ is obtained letting $p^+ \to \infty$ for any particle propagating inside the shock wave



Non-eikonal propagator

$$\mathcal{G}_{p^+}(x^+, y^+) = \left[i\frac{\partial}{\partial x^+} - \frac{\hat{p}_\perp^2}{2p^+} - gA^- \right]^{-1}$$



shock wave (eikonal limit)

$$\lim_{p^+ \to +\infty} \mathcal{G}_{p^+}(x^+, \boldsymbol{x}; y^+, \boldsymbol{y}) = \delta(\boldsymbol{x} - \boldsymbol{y})\, U_{\boldsymbol{x}}(x^+, y^+)$$

Revisiting the Shock Wave Approximation

- This limit neglects quantum diffusion

$$\mathcal{G}^0_{p^+}(x^+, \boldsymbol{x}; y^+, \boldsymbol{y}) = \frac{p^+}{2i\pi\,\Delta x^+}\, \mathrm{e}^{\,i\frac{(\boldsymbol{x}-\boldsymbol{y})^2\,p^+}{\Delta x^+}}$$

- it is important when $(\Delta\,\boldsymbol{x})^2 \sim \Delta x^+/p^+ \sim s^{-1}$
- In effect, the phase relates the transverse dynamics to longitudinal dynamics, this is the phase that appears in the definition of PDF's
- It encodes the information about k^-'s in the target. It is expected to be non-negligible away from the strongly ordered region in k^-.

Factorization formula for DIS at arbitrary x

- Combining all phases we obtain

$$ik^-\Delta x^+ \equiv i\,\frac{\ell^2 + z\bar{z}Q^2}{2z\bar{z}\,q^+}\Delta x^+$$

- This is nothing but Feynman x that we encounter when deriving the DGLAP limit

$$x_F \equiv \frac{\ell^2 + z\bar{z}Q^2}{2z\bar{z}\,s}$$

Inclusive DIS beyond shock wave

- applying the same trick to the r.h.s. we obtain the following hadronic operator

$$O_{\text{dipole}}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{y}_1, \boldsymbol{y}_2, x_1^+, x_2^+) \equiv \int_{\boldsymbol{y}_1}^{\boldsymbol{x}_1} \mathrm{d}z_1^i \int_{\boldsymbol{y}_2}^{\boldsymbol{x}_2} \mathrm{d}z_2^j$$

$$\langle P | \operatorname{Tr} \mathscr{G}(\boldsymbol{y}_2, x_2^+; \boldsymbol{y}_1 x_1^+ | (1-z)q^+) F^{j-}(x_1^+, \boldsymbol{z}_1) \mathscr{G}(\boldsymbol{x}_1, x_1^+; \boldsymbol{x}_2, x_2^+ | zq^+) F^{i-}(x_2^+, \boldsymbol{z}_2) | P \rangle$$

- performing a gauge rotation leads to transverse gauge links: explicit gauge invariance
- dependence on + momenta of the dipole zq^+ and $(1-z)q^+$ can be factorized with further approximations