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# Correlations between azimuthal asymmetries and multiplicity and mean transverse momentum in small collisions systems in the CGC

Néstor Armesto
*IGFAE, Universidade de Santiago de Compostela*
nestor.armesto@usc.es
with Tolga Altinoluk (NCBJ), Alex Kovner (UConn), Michael Lublinsky (BGU) and Vladimir Skokov (NCSU)

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FONDO EUROPEO DE DESENVOLVEMENTO REXIONAL "Unha maneira de facer Europa"

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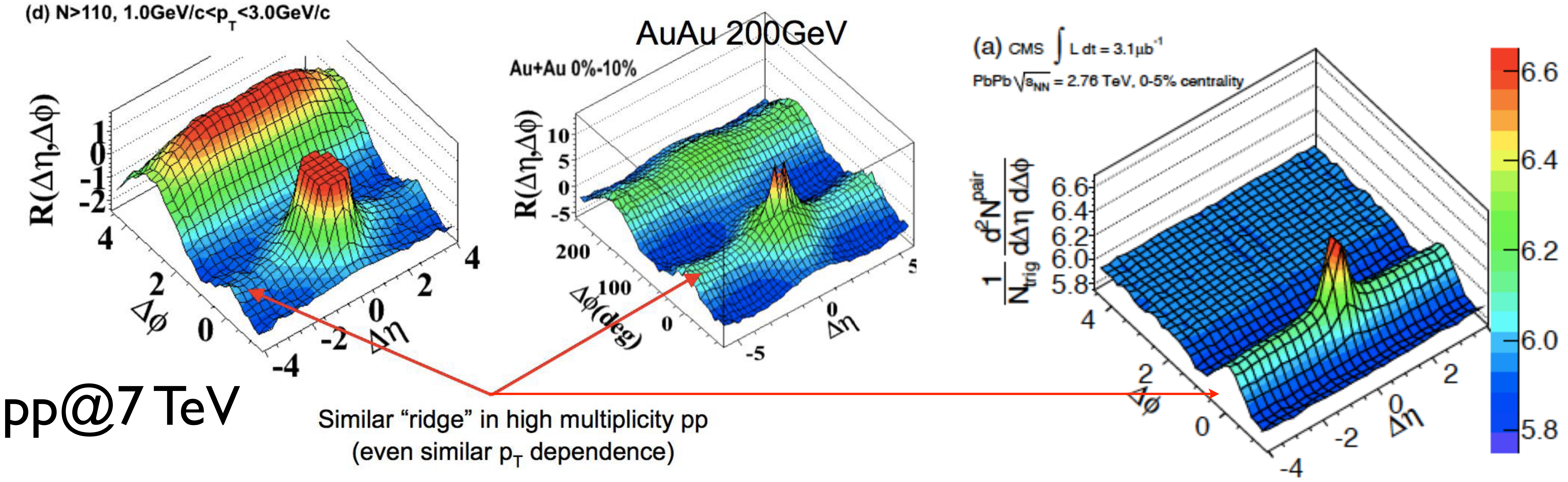
# Introduction:

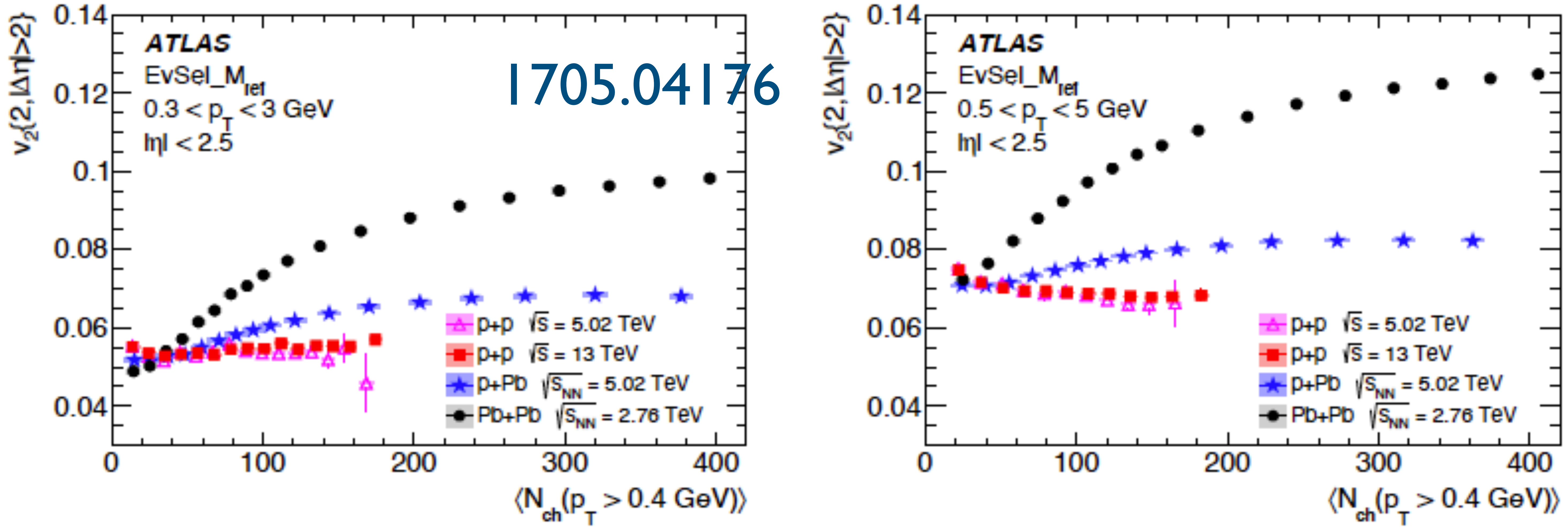



- Many QGP-like features observed in AA collisions at RHIC and the LHC are also observed in pp and pA (at large and not so large multiplicities): **small system puzzle**.
- **Ridge**: elongated structure in $\eta$ in two particle correlations, peaked at 0 and 180 degrees.
- Long range rapidity correlations give information about initial stages of the collision, and appear in several models: old string models, CGC, …
- Alternatives: imprint of the correlations in the wave functions of the incoming objects (initial state) versus effect of the strong final state interactions in a dense system (final state).

# Introduction:

- Motivation in this work:
  - ➜ Understanding the (weak) dependence of $v_2$ with multiplicity.
  - ➜ Examining the $v_2 - \langle p_T \rangle$ correlation proposed as a tool to determine the initial energy deposition and disentangle initial from final state effects in pp/pA (Bozek, 1601.04513; ATLAS, 1907.05176; Giacalone et al., 2006.17721; Lim et al., 2103.01348).

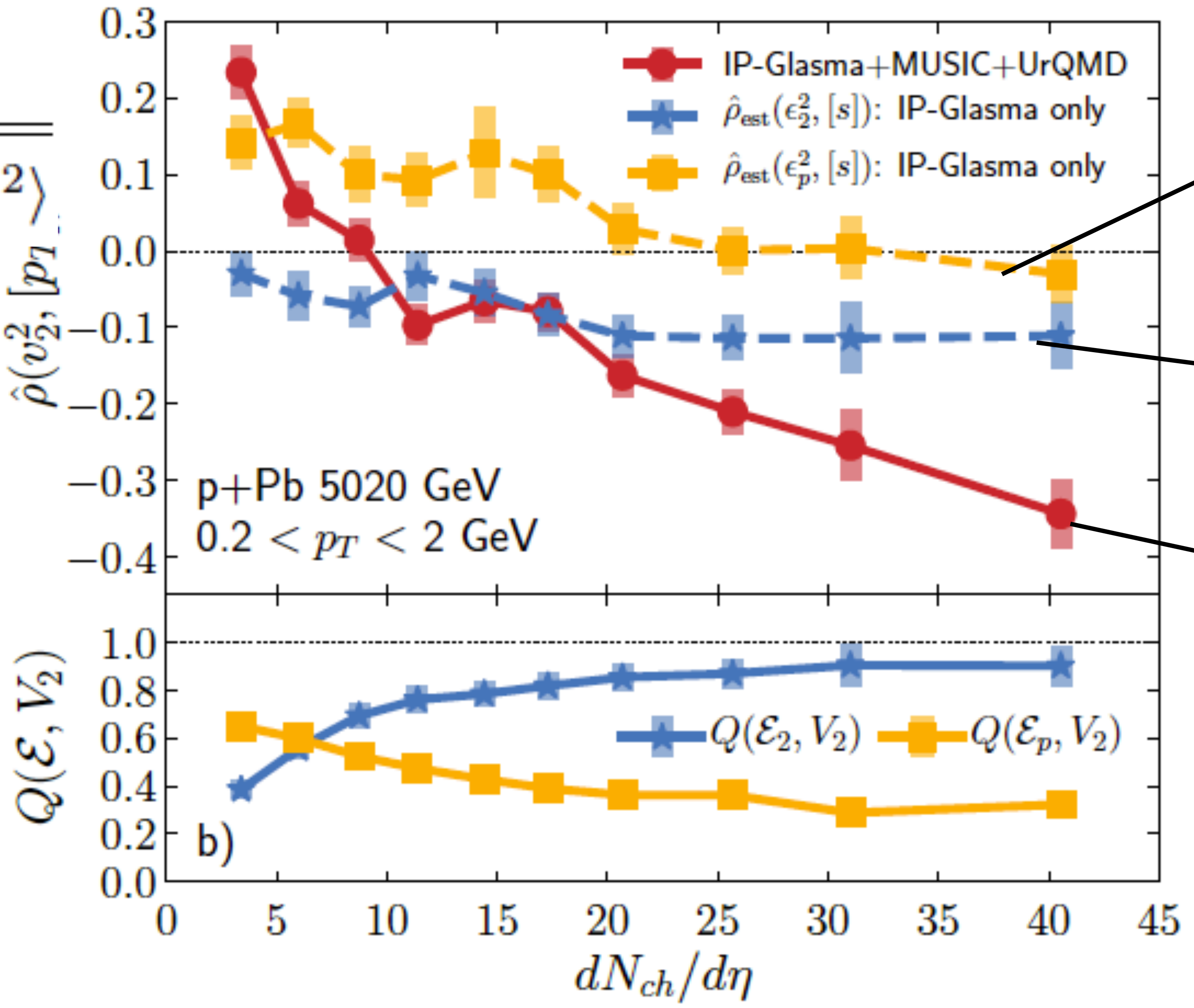
within the CGC initial state perspective.




$$\hat{\rho}(v_2^2, [p_T]) = \frac{\langle \hat{\delta} v_2^2 \, \hat{\delta}[p_T] \rangle}{\sqrt{\langle (\hat{\delta} v_2^2)^2 \rangle \langle (\hat{\delta}[p_T])^2 \rangle}}$$

$$\hat{\delta} O \equiv \delta O - \frac{\langle \delta O \delta N \rangle}{\sigma_N^2} \delta N$$

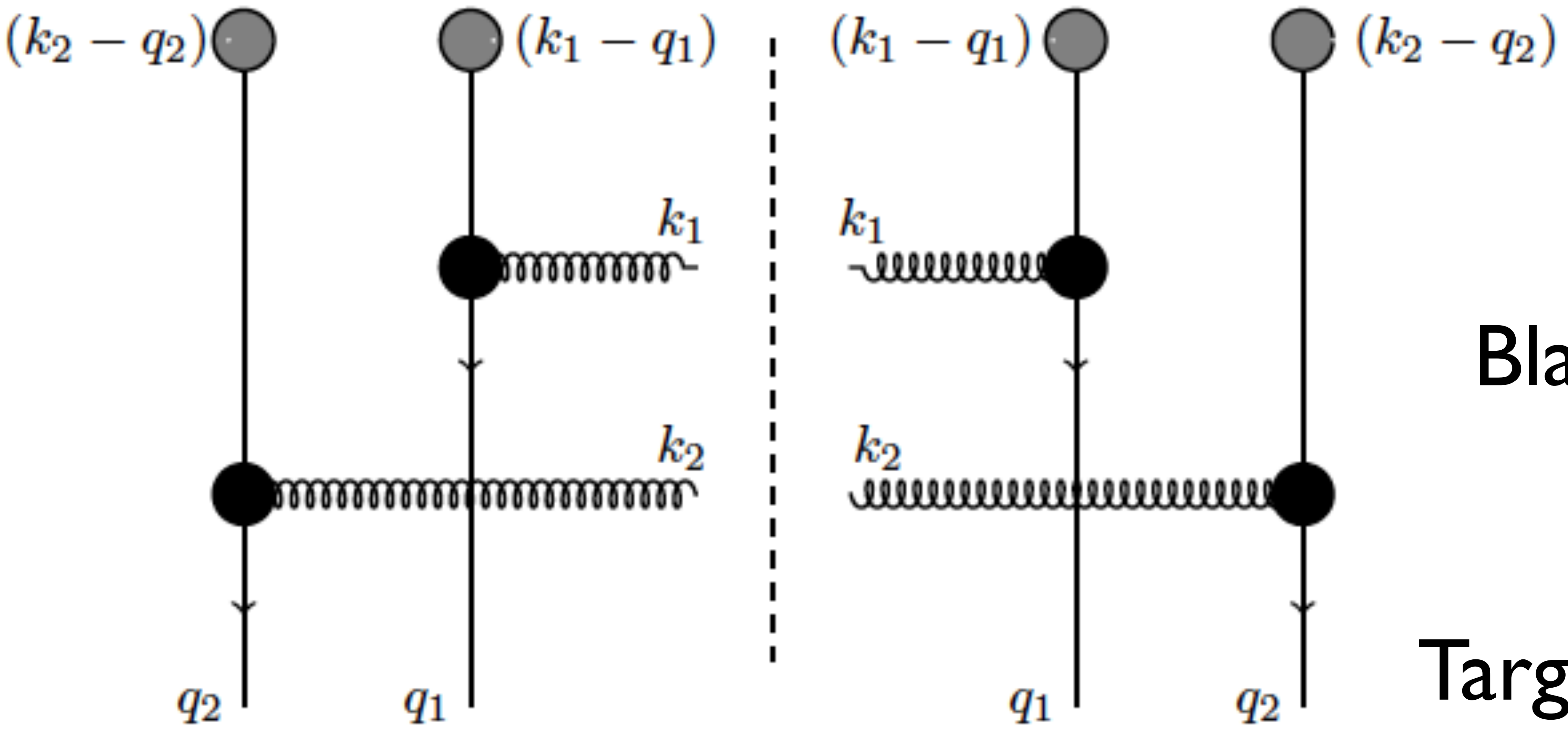
$$Q(\mathcal{E}, V_2) = \frac{\mathrm{Re}\langle \mathcal{E} V_2^* \rangle}{\sqrt{\langle |\mathcal{E}|^2 \rangle \langle |V_2|^2 \rangle}}$$




2006.15721
Initial state
Final state
Initial + final state

# Correlations in the CGC:

- Two physical effects in correlations in the CGC (see TA et al., 2004.08185):




Projectile: sources

Black blob: rescatterings

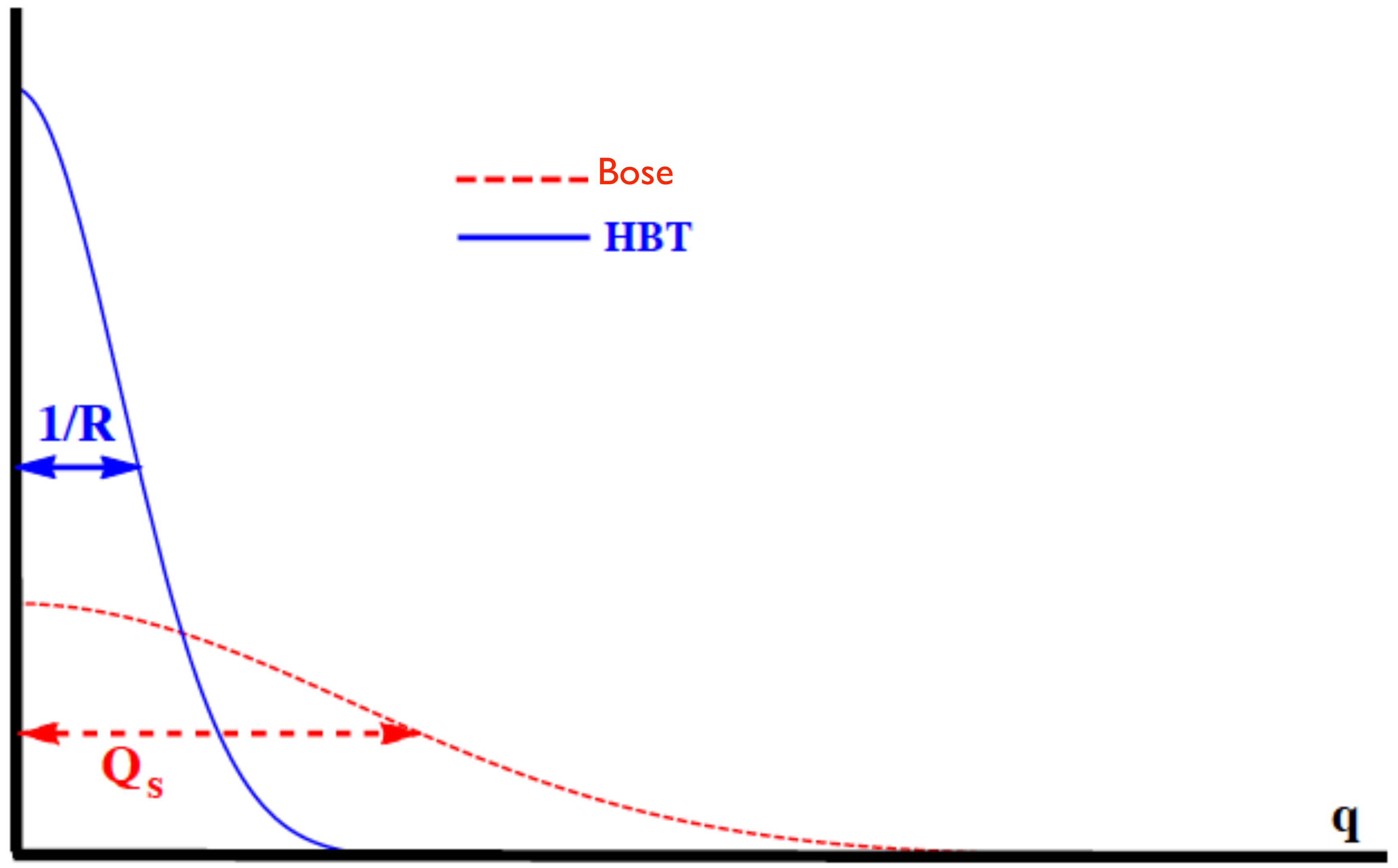
Target: classical colour field

1) Bose enhancement of gluons in the projectile wave function (TA et al.,1503.07126).

$$\propto \delta^{(2)}[k_1 - q_1 - (k_2 - q_2)] + \delta^{(2)}[k_1 - q_1 + (k_2 - q_2)]$$

2) HBT of gluons separated in rapidity (TA et al., 1509.03223; Kovchegov et al., 1212.1195,1310.6701).

$$\propto \delta^{(2)}(k_1 - k_2) + \delta^{(2)}(k_1 + k_2)$$



# Two particle correlations: $v_2$

To $\mathcal{O}(g\rho)$ (see Ming Li's flash talk):

$$\frac{dN^{(2)}}{d^2k_1 d^2k_2} \propto \int_{z_i\bar{z}_i} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)} \int_{x_i y_i} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)$$

$$\times\left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\rho^{b_1}(y_1)\rho^{b_2}(y_2)\right\rangle_P$$

$$\times\left\langle [U(z_1)-U(x_1)]^{a_1c}[U^\dagger(\bar{z}_1)-U^\dagger(y_1)]^{cb_1}[U(z_2)-U(x_2)]^{a_2d}[U^\dagger(\bar{z}_2)-U^\dagger(y_2)]^{db_2}\right\rangle_T$$

- $A^i(z)=z^i/z^2$: standard WW fields.
- Projectile averages: MV model (see Adrian Dumitru's talk):

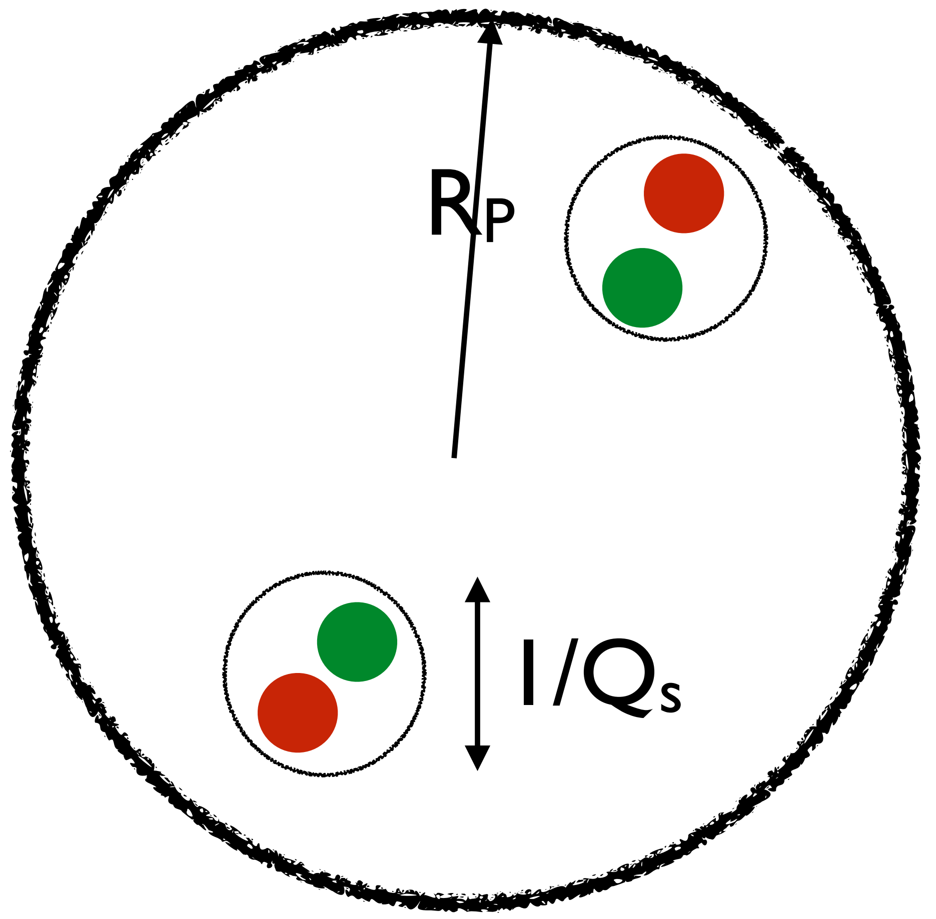
$$\langle\rho^{a_1}\rho^{a_2}\rho^{b_1}\rho^{b_2}\rangle = \langle\rho^{a_1}\rho^{b_1}\rangle\langle\rho^{a_2}\rho^{b_2}\rangle + \langle\rho^{a_1}\rho^{a_2}\rangle\langle\rho^{b_1}\rho^{b_2}\rangle + \langle\rho^{a_1}\rho^{b_2}\rangle\langle\rho^{a_2}\rho^{b_1}\rangle$$

$$\langle\rho^a(x)\rho^b(y)\rangle = \delta^{ab}\mu^2(x,y)$$

- Target averages: leading contributions in $R_pQ_s$ to the transverse integrals (TA et al., 1805.07739; see Pedro Agostini's talk):

$$\langle Q(x,y,z,v)\rangle_T \longrightarrow d(x,y)d(z,v)+d(x,v)d(z,y)+\frac{1}{N_c^2-1}d(x,z)d(y,v),$$

$$\langle D(x,y)D(z,v)\rangle_T \longrightarrow d(x,y)d(z,v)+\frac{1}{(N_c^2-1)^2}\left[d(x,v)d(y,z)+d(x,z)d(v,y)\right]$$



# Two particle correlations: $v_2$

- Assuming translational invariance and keeping the leading terms that produce correlations:

$$\frac{dN^{(2)}}{d^2k_1d^2k_2} = \left.\frac{dN^{(2)}}{d^2k_1d^2k_2}\right|_{dd} + \left.\frac{dN^{(2)}}{d^2k_1d^2k_2}\right|_{Q} \qquad \left.\frac{dN^{(2)}}{d^2k_1d^2k_2}\right|_{Q} \propto \int_{q_1q_2} d(q_1)d(q_2)\left[I_{Q,1}+I_{Q,2}\right]$$

$$\begin{aligned} I_{Q,1} &= \mu^2(k_1-q_1,q_2-k_2)\,\mu^2(k_2-q_2,q_1-k_1)\,L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)+(k_2\to -k_2)\\ I_{Q,2} &= \mu^2(k_1-q_1,q_1-k_2)\,\mu^2(k_2-q_2,q_2-k_1)\,L^i(k_1,q_1)L^i(k_1,q_2)\,L^j(k_2,q_1)L^j(k_2,q_2)+(k_2\to -k_2) \end{aligned}$$

$$L^i(k,q) = \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right]$$ : Lipatov vertex

- To go ahead:
  - ➜ MV model: $\mu^2(k,q) = (2\pi)^2\delta^{(2)}(k+q)$, and GBW model: $d(q) = \frac{4\pi}{Q_s^2}e^{-q^2/Q_s^2}$.
  - ➜ We assume $k_2^2, k_3^2 \gg Q_s^2$ and neglect exponentially suppressed terms.
  - ➜ We take only leading $N_c$ terms.

# Two particle correlations: $v_2$

- Both HBT and Bose enhancement terms:

$$Q_2 = \alpha_s^2 (4\pi)^2 (N_c^2-1)\ \mu^4 S_\perp (2\pi)^2 \left[\delta^{(2)}(k_2+k_3)+\delta^{(2)}(k_2-k_3)\right] \frac{1}{2}\frac{Q_s^4}{k_2^8}$$

$$Q_1 = \alpha_s^2 (4\pi)^2 (N_c^2-1)\ \mu^4 S_\perp \frac{1}{\pi Q_s^2} e^{-(k_2-k_3)^2/2Q_s^2} \left\{ \left[\frac{1}{2}+\frac{2^2 Q_s^2}{(k_2+k_3)^2}+\frac{2^4 Q_s^4}{(k_2+k_3)^4}\right] \frac{1}{k_2^2 k_3^2}\frac{(k_2-k_3)^4}{(k_2+k_3)^4} + Q_s^4 \frac{2^6}{(k_2+k_3)^8}\left[1+(k_2^i-k_3^i)\left(\frac{k_2^i}{k_2^2}-\frac{k_3^i}{k_3^2}\right)\right]\right\} + (k_3 \to -k_3)$$

$$v_2^2(k,k',\Delta) = \frac{\int_{k-\Delta/2}^{k+\Delta/2} k_2 dk_2 \int_{k'-\Delta/2}^{k'+\Delta/2} k_3 dk_3 \int d\phi_2 d\phi_3 e^{i2(\phi_2-\phi_3)} \frac{d^2N^{(2)}}{d^2k_2 d^2k_3}}{\int_{k-\Delta/2}^{k+\Delta/2} k_2 dk_2 \int_{k'-\Delta/2}^{k'+\Delta/2} k_3 dk_3 \int d\phi_2 d\phi_3 \frac{d^2N^{(2)}}{d^2k_2 d^2k_3}}$$

- We keep only the correlated part in numerator and everything in the denominator.
- $S_\perp$: transverse area of the projectile; $\lambda \sim [S_\perp Q_s^2]^{-1}$ IR cutoff.
- We integrate over transverse momentum bins: $k, k' \gg \Delta \sim Q_s$.
- For pA: $Q_s \simeq 1\ GeV,\ \ S_\perp \simeq 1/\ \Lambda_{QCD}^2 \Rightarrow \lambda \simeq 25; v_2^2$ scaled by $(N_c^2-1)S_\perp Q_s^2 \simeq 200$.

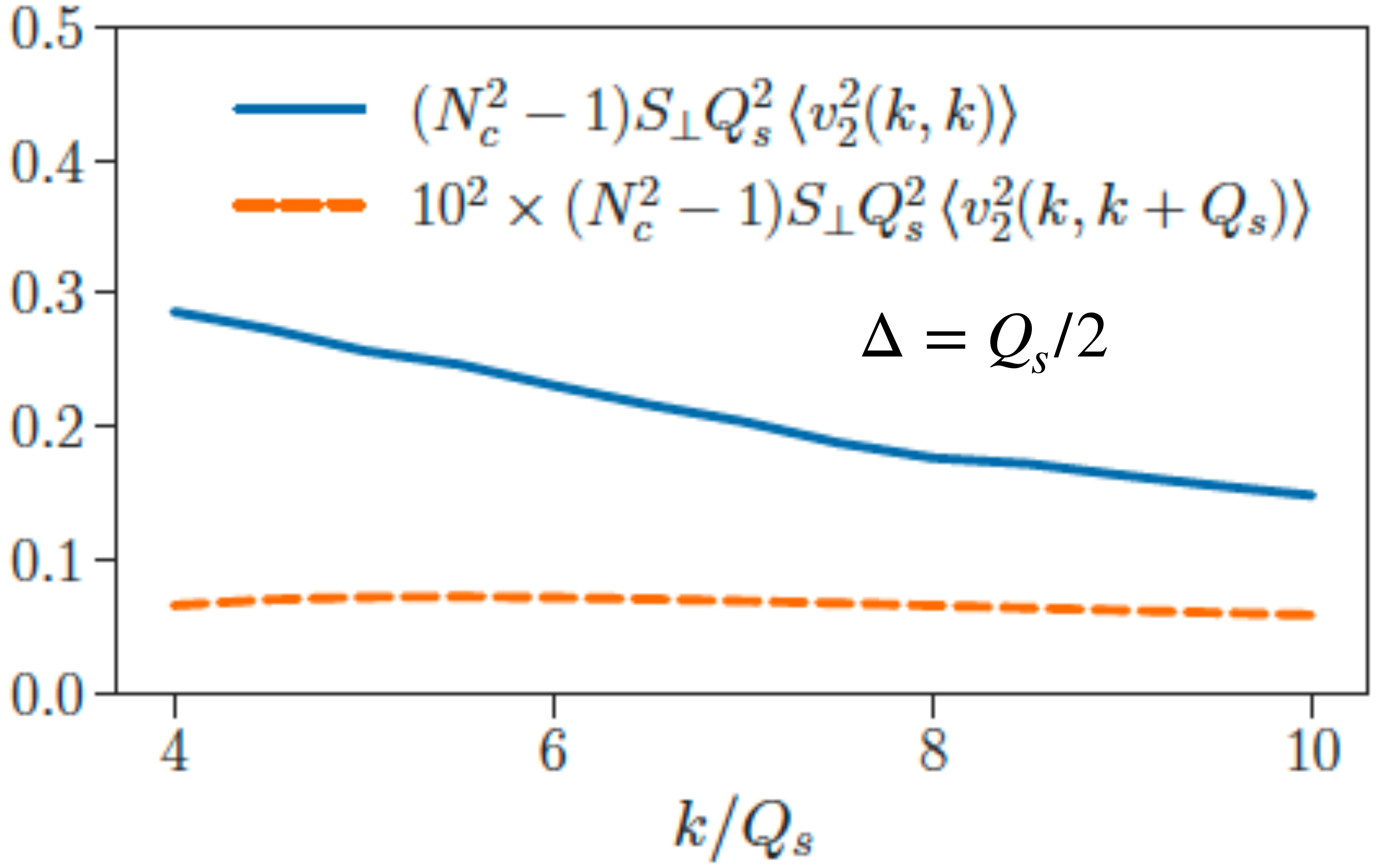
# Two particle correlations: $v_2$

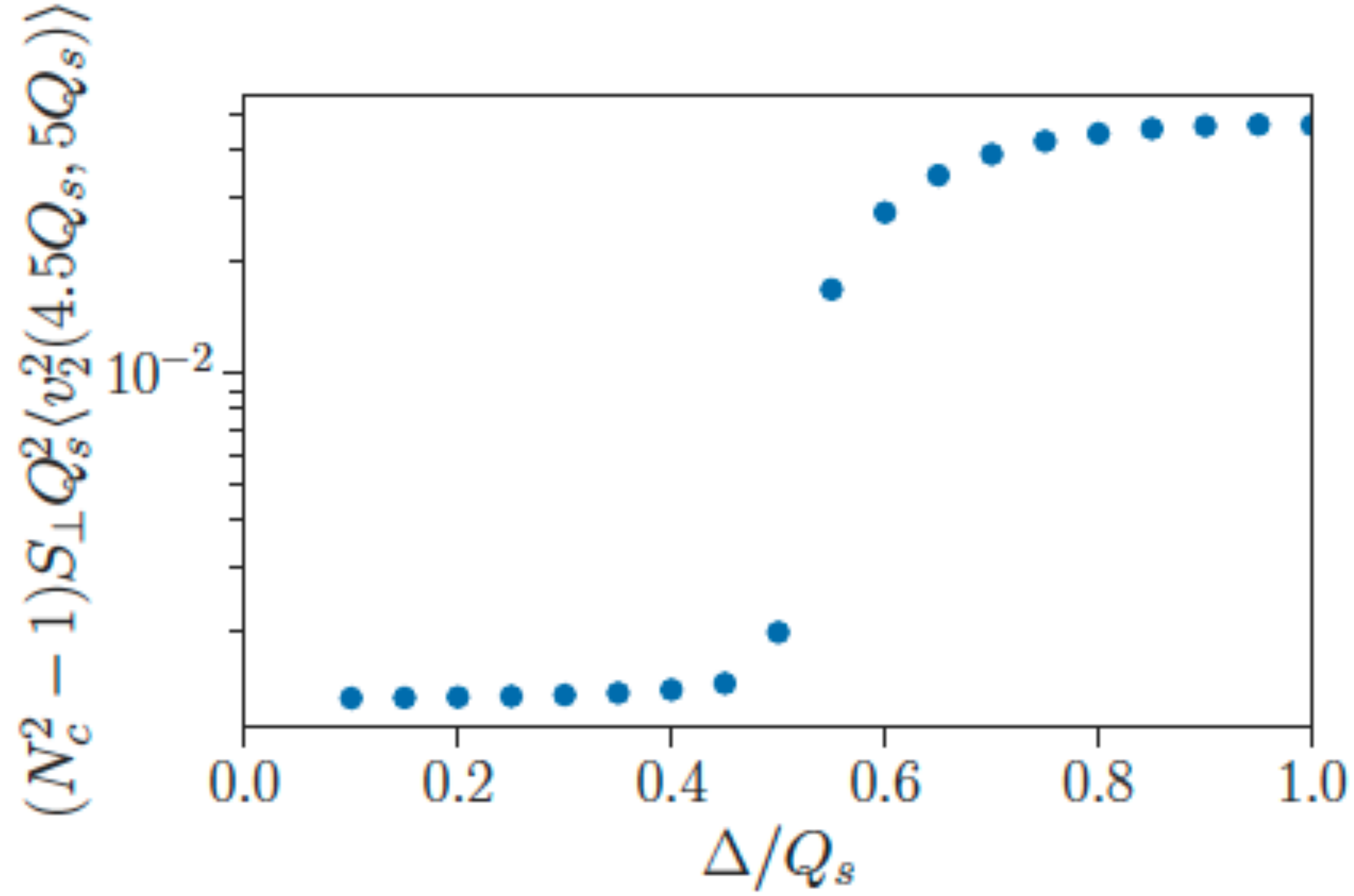
- Transverse momentum width of Bose enhancement $\sim Q_s \gg$ width of HBT:

non-overlapping bins
$\Delta < |k - k'|$
only BE contribution

$\Delta \approx |k - k'|$
HBT starts to contribute

overlapping bins
$\Delta > |k - k'|$
BE+HBT contribution




- BE dominated regime: weak $k$ dependence (very similar for BE and single inclusive squared).
- BE+HBT: HBT largely dominates, steeper $k$ dependence.
- Transition from BE to BE+HBT controlled by the bin width.

# Three particle correlations: $v_2 - \langle N \rangle$ and $v_2 - \langle p_T \rangle$

- In the CGC, the multiplicity dependence of any observable demands a projector of projectile & target averages on multiplicity (Dumitru et al., 1704.05917, 1802.06111): not yet available.

$$\mathcal{O}_{N,v_2} = \frac{\int d\phi_2 d\phi_3 \; e^{i2(\phi_2-\phi_3)} \int d^2k_1 \frac{dN^{(3)}}{d^2k_1 d^2k_2 d^2k_3}\Big|_X}{\int d\phi_2 d\phi_3 \; e^{i2(\phi_2-\phi_3)} \frac{dN^{(2)}}{d^2k_2 d^2k_3}\Big|_Q \int d^2k_1 \frac{dN^{(1)}}{d^2k_1}}$$

$$\frac{dN^{(3)}}{d^2k_1 d^2k_2 d^2k_3} = \frac{dN^{(3)}}{d^2k_1 d^2k_2 d^2k_3}\Big|_{ddd} + \frac{dN^{(3)}}{d^2k_1 d^2k_2 d^2k_3}\Big|_{dQ} + \frac{dN^{(3)}}{d^2k_1 d^2k_2 d^2k_3}\Big|_X$$

*ddd*: 3 gluons uncorrelated
*dQ*: 2 gluons correlated, 1 not
*X*: 3 gluons correlated

- Single inclusive:

$$\frac{dN^{(1)}}{d^2k_1} \propto \int_{q_1} d(q_1)\, \mu^2(k_1 - q_1, q_1 - k_1)\, L^i(k_1, q_1) L^i(k_1, q_1)$$

$$\frac{dN^{(1)}}{d^2k_1} = \alpha_s(4\pi)(N_c^2 - 1)\, \mu^2\, S_\perp\, e^{-k_1^2/Q_s^2} \left\{ \frac{2}{k_1^2} - \frac{1}{k_1^2} e^{k_1^2/Q_s^2} + \frac{1}{Q_s^2}\left[ \mathrm{Ei}\left(\frac{k_1^2}{Q_s^2}\right) - \mathrm{Ei}\left(\frac{k_1^2 \lambda}{Q_s^2}\right)\right]\right\}$$

- Weak dependence on IR regulator $\lambda$.

# Three particle correlations: $v_2 - \langle N \rangle$ and $v_2 - \langle p_T \rangle$

$$\left.\frac{dN^{(3)}}{d^2k_1 d^2k_2 d^2k_3}\right|_X \propto \int_{q_1 q_2 q_3} d(q_1)d(q_2)d(q_3)\left[I_{X,1}+I_{X,2}+I_{X,3}+I_{X,4}+I_{X,5}\right]$$

- $I_{X,i}$: 3 $\mu^2$ functions and 6 Lipatov vertices, 8 terms each.

- $X_1$, $X_3$, $X_4$ are BE-type contributions:

$$\begin{aligned} X_1 &= \frac{1}{2}\alpha_s^3(4\pi)^6(N_c^2-1)\,\mu^6\,S_\perp\,e^{-(k_2-k_3)^2/2Q_s^2}\,\frac{1}{k_2^4} \\ &\times\left\{\left(\frac{1}{2}+Q_s^2\left[\frac{1}{k_2^2}+\frac{2^2}{(k_2+k_3)^2}\right]+Q_s^4\left[\frac{3}{k_2^4}+\frac{2!}{k_2^2}\frac{2^2}{(k_2+k_3)^2}+\frac{2^4}{(k_2+k_3)^4}\right]\right)\frac{1}{k_2^2k_3^2}\frac{(k_2-k_3)^4}{(k_2+k_3)^4}\right. \\ &\left.+\,Q_s^4\frac{2^6}{(k_2+k_3)^8}\left[1+(k_2^i-k_3^i)\left(\frac{k_2^i}{k_2^2}-\frac{k_3^i}{k_3^2}\right)\right]\right\}. \end{aligned}$$

- $X_2$, $X_5$ are HBT-type contributions:

$$X_2 = \alpha_s^3\frac{1}{2}(4\pi)^7(N_c^2-1)\,\mu^6\,S_\perp\left[\delta^{(2)}(k_2+k_3)+\delta^{(2)}(k_2-k_3)\right]\frac{1}{4}\frac{Q_s^6}{k_2^{12}}$$

- Again we integrate over transverse momentum bins:

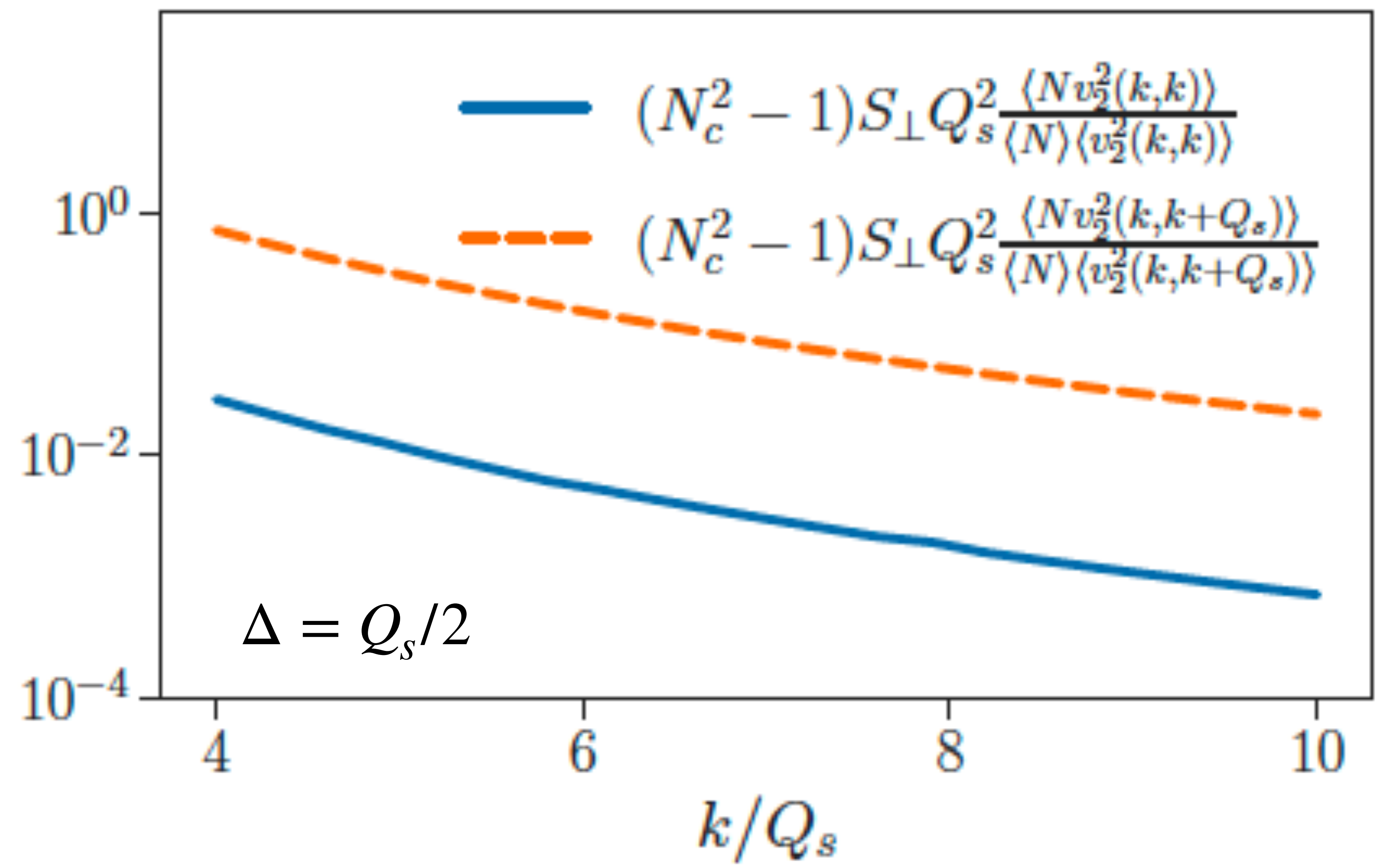
$$\left.\frac{dN^{(3)}}{d^2k_1d^2k_2d^2k_3}\right|_X \to \int_{k-\Delta/2}^{k+\Delta/2} k_2dk_2 \int_{k'-\Delta/2}^{k'+\Delta/2} k_3dk_3 \left.\frac{dN^{(3)}}{d^2k_1d^2k_2d^2k_3}\right|_X$$

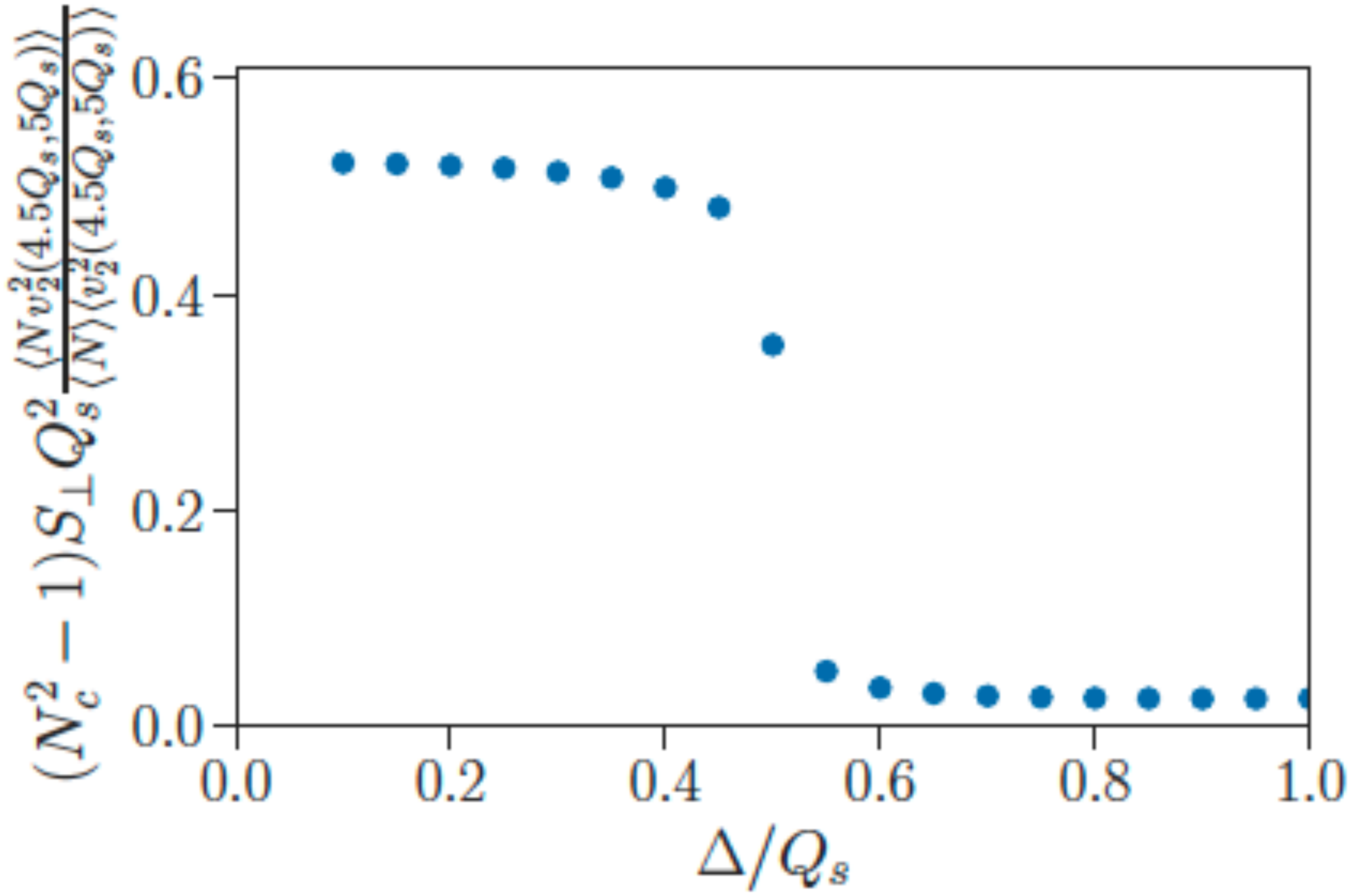
# Three particle correlations: $v_2 - \langle N \rangle$ and $v_2 - \langle p_T \rangle$

non-overlapping bins
$\Delta < |k - k'|$
only BE contribution

$\Delta \approx |k - k'|$
HBT starts to contribute

overlapping bins
$\Delta > |k - k'|$
BE+HBT contribution

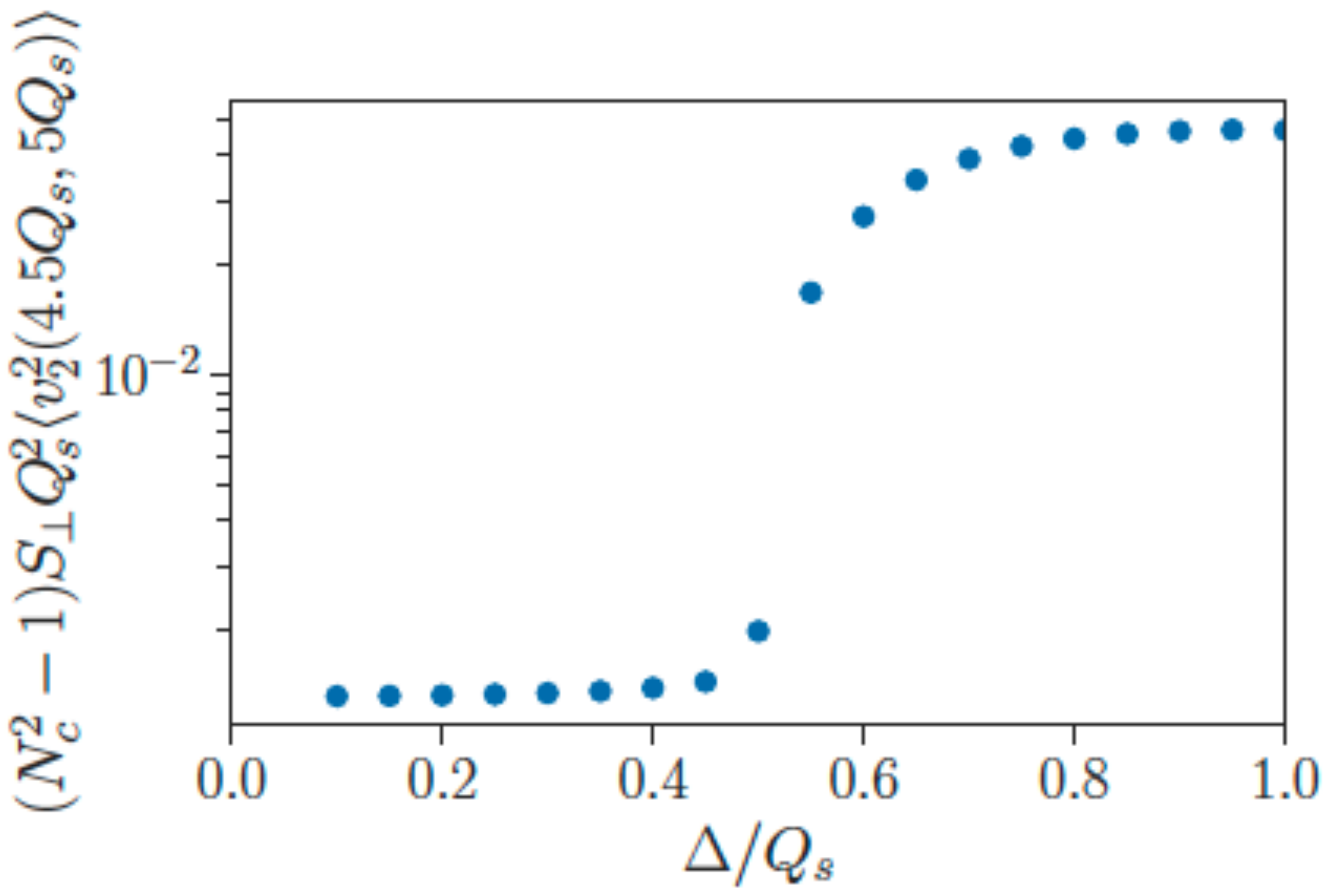


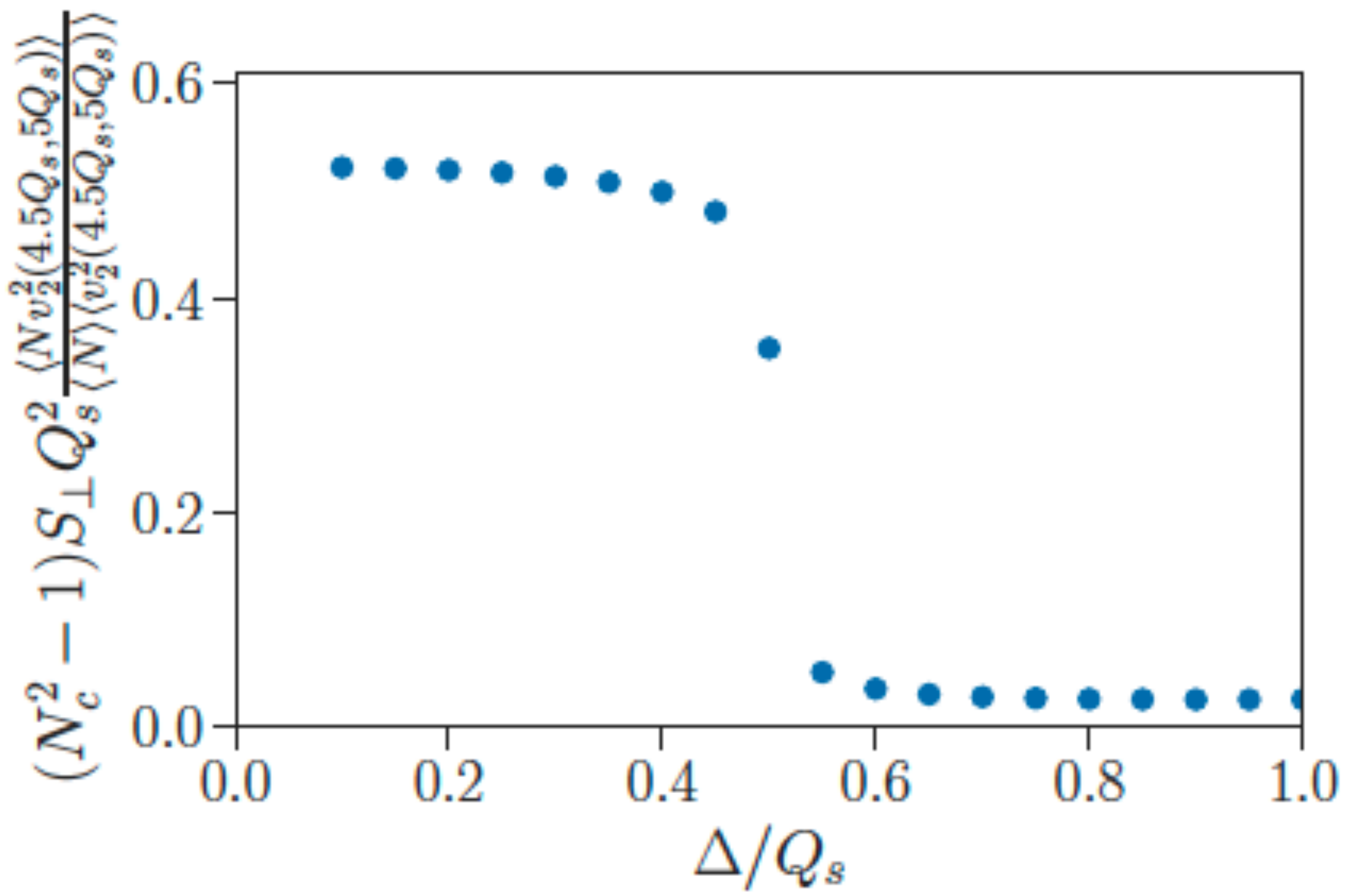




- BE dominated regime: no HBT contribution to $v_2$; BE+HBT: HBT largely dominates $v_2$.
- $\mathcal{O}_{N,v_2}$ decreasing function of $k$ (multiplicity dominated by soft gluons).
- $\mathcal{O}_{N,v_2}$ correlations goes from sizeable to negligible when going from the BE to the BE+HBT regime: opposite transition to that in $v_2$.

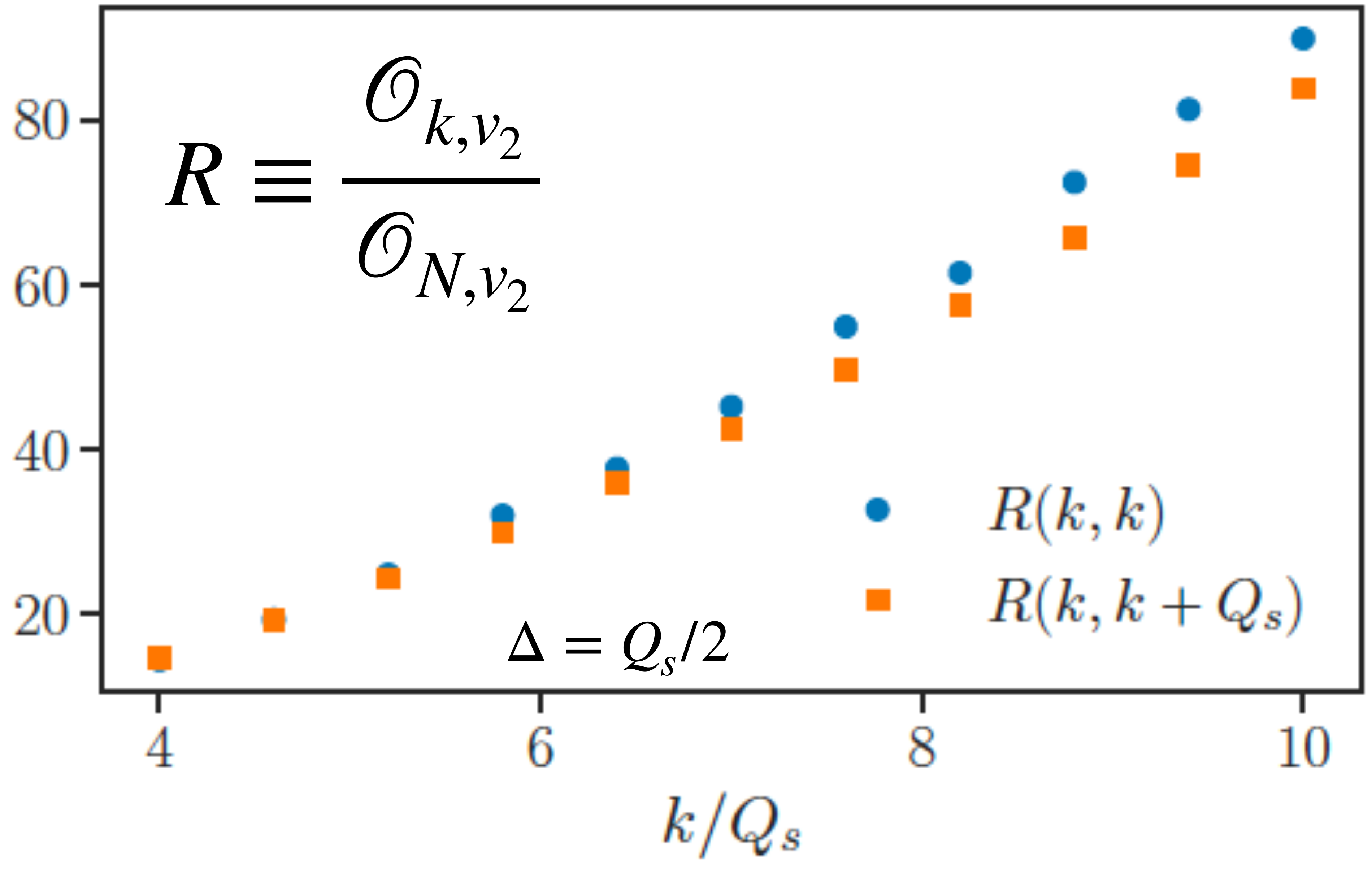
# Three particle correlations: $v_2 - \langle N \rangle$ and $v_2 - \langle p_T \rangle$

- BE dominated regime ($\Delta < 0.5Q_s$): $v_2$ small and sizeable $v_2 - \langle N \rangle$ correlations.
- BE+HBT ($\Delta > 0.5Q_s$): $v_2$ large and negligible $v_2 - \langle N \rangle$ correlations.
- Drop in $v_2 - \langle N \rangle$ driven by the sharp increase in $v_2$.




- $\mathcal{O}_{N,v_2}$ and $\mathcal{O}_{k,v_2}$ show similar behaviours but $\mathcal{O}_{k,v_2}$ falls slower with $k$.



# Conclusions:

- We have computed $v_2$ and its correlations with average multiplicity and transverse momentum in pA (dilute-dense) collisions in the CGC.

- We assume translational invariance and consider leading terms in density and in area, the lowest order in $N_c$ and large transverse momentum $k \gg Q_s$; we do not attempt to describe data but to see the effects of quantum statistics, both in the wave functions of the colliding hadrons and in the production process, on correlations.

- $v_2$ correlations with average multiplicity and transverse momentum are very small and show similar behaviours.

- Due to the onset of the HBT contribution, $v_2 - \langle N \rangle$ and $v_2 - \langle p_T \rangle$ correlations show a characteristic pattern with the transverse momentum bin width opposite to that found in $v_2$.

- We are examining the size and influence of subleading terms in $N_c$ (corrections to BE of the projectile and BE of the target).

# Conclusions:

***Thank you very much for your attention!!!***

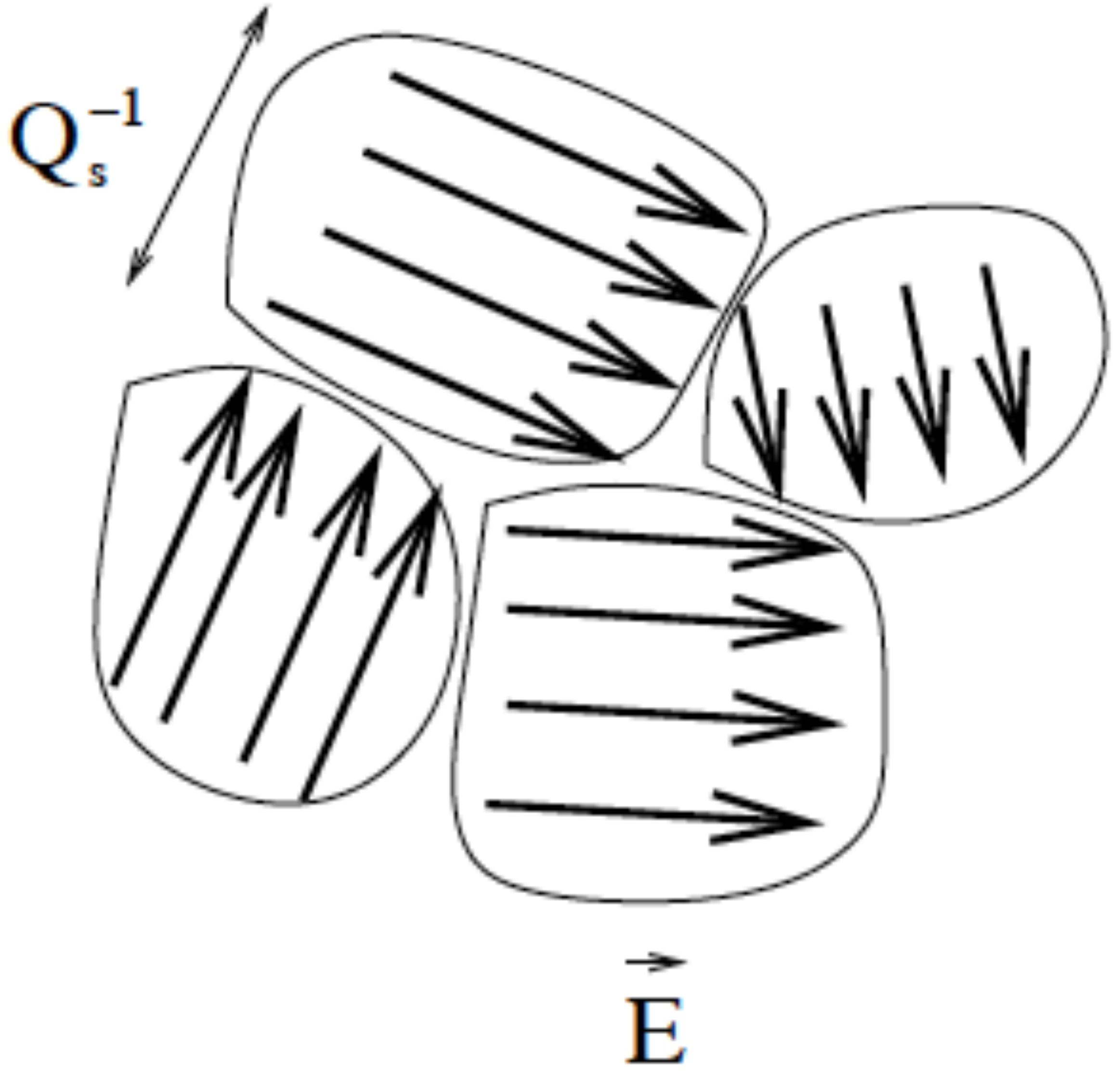
# Contents:

***Backup***

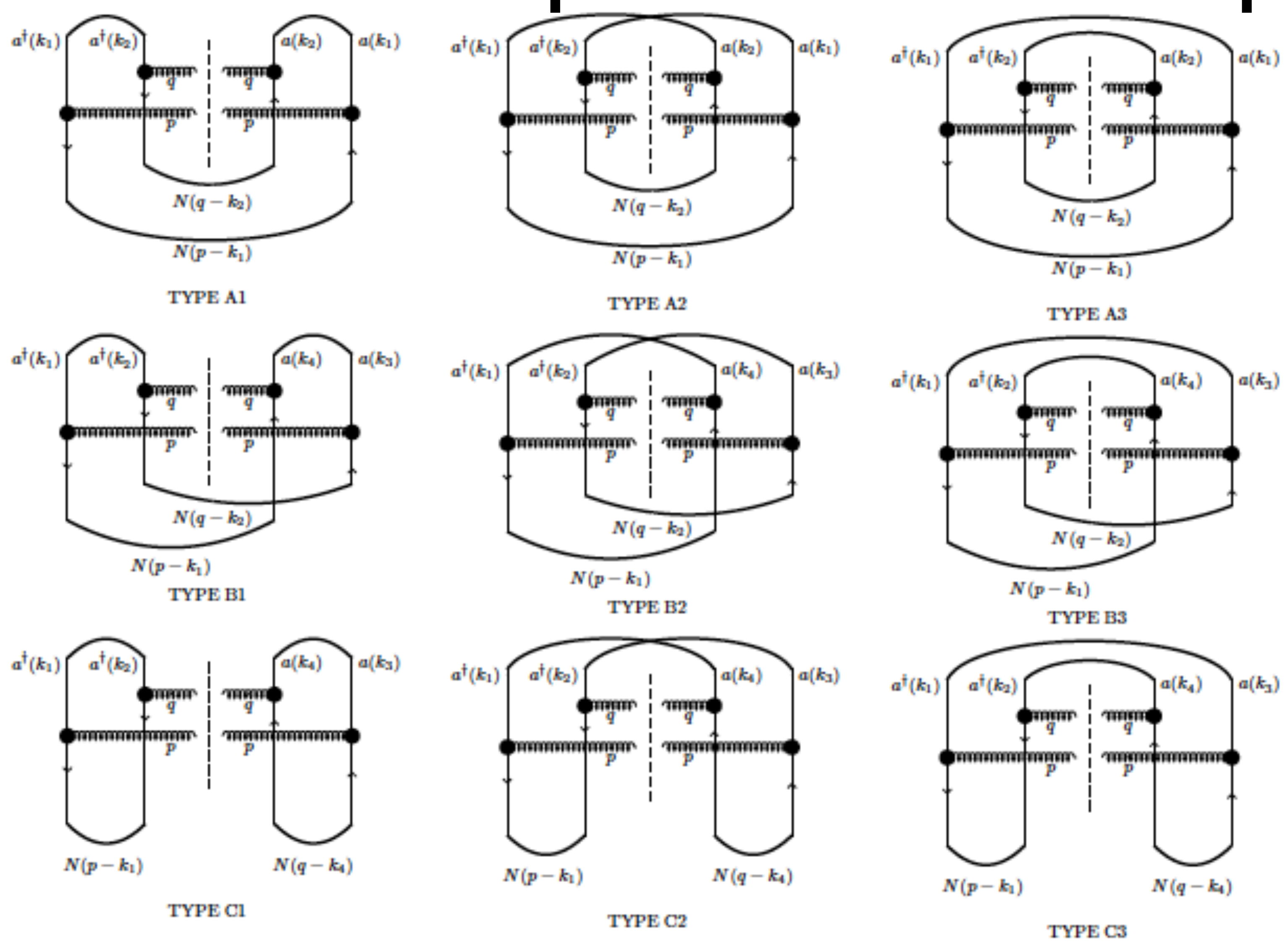
# Correlations in the CGC:

- Several explanations in the CGC, that use/assume that:
  - ➜ the final state carries the imprint of initial-state correlations,
  - ➜ the CGC wave function is rapidity invariant over $Y \propto 1/\alpha_s$,
  - ➜ the projectile is a dilute object (proton).

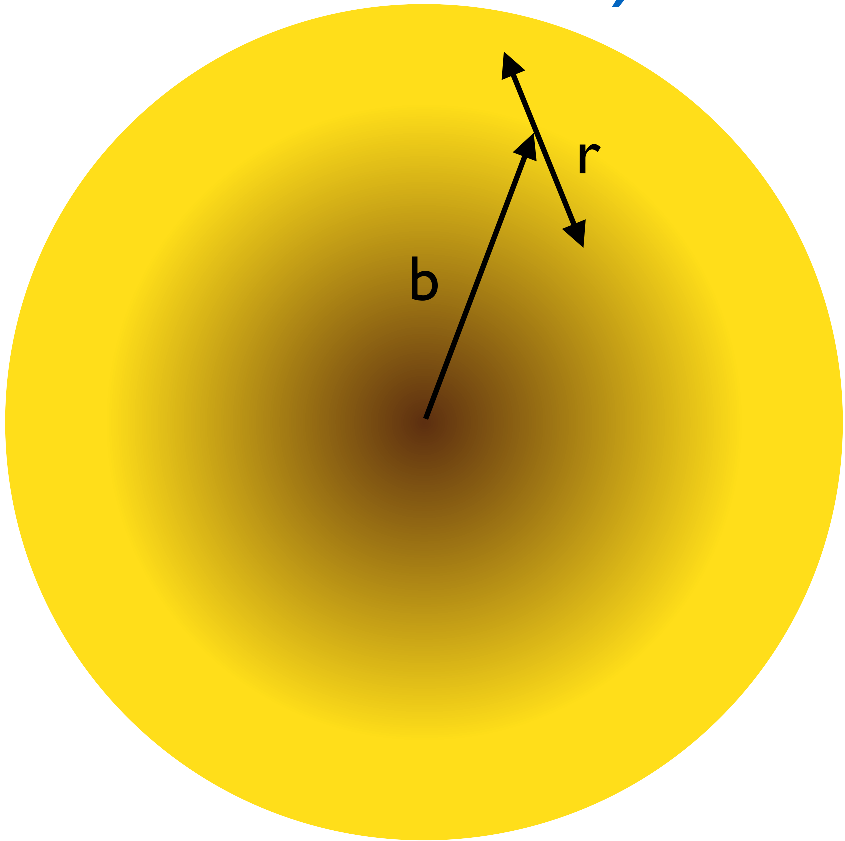
- Local anisotropy of target fields (Kovner-Lublinsky, Dumitru-McLerran-Skokov).




- “Glasma graphs” (Dusling-Gelis-Jalilian-Marian-Lappi-McLerran-Venugopalan, Kovchegov-Werpteny): successful description of data in pp, pPb.



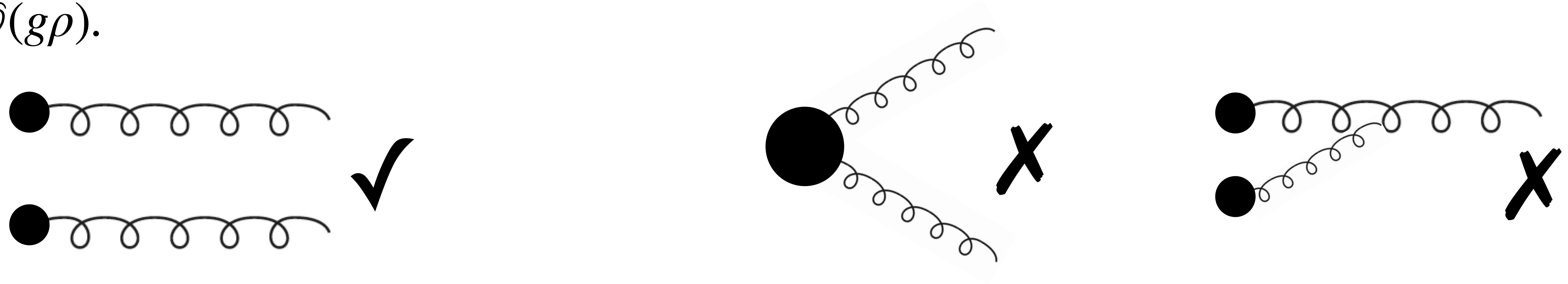

- Spatial variation of partonic density (Levin-Rezaeian-Iancu).




- Also interference (Blok-Jakel-Strikman-Wiedemann).

# Correlations in the CGC:

• CGC calculations for the central rapidity region resum terms in which each source emits one gluon, $\mathcal{O}(g\rho)$.



➜ Odd harmonics require additional terms $\mathcal{O}(g^2\rho)$ (Skokov et al., 1611.09870, 1612.07790, 1802.08166, see Ming Li's flash talk for effects on single inclusive particle production).

• Glasma graph calculations are valid for a dilute target (pp) and usually performed for two particles (up to 4 in Ozonder, 1409.6347, 1712.05571):

➜ Extension to dilute-dense (pA) numerically (Lappi et al., 1509.03499; Mace et al., 1705.00745, 1706.06260) or analytically (TA et al., 1804.02910, 1808.04896).

➜ Many gluons in pA (see Pedro Agostini's talk).

• Correlations are subleading in $N_c$ in the MV model: new ones including anisotropies (Dumitru-Skokov; see Adrian Dumitru's talk).