Multiparticle correlations in pA collisions in the CGC

Pedro Agostini
Universidade de Santiago de Compostela and IGFAE
pedro.agostini@usc.es
with Néstor Armesto (IGFAE) and Tolga Altinoluk (NCBJ)

Based on arXiv:2103.08485

DIS 2021, April 14

Motivation

Collectivity effects in small systems

Our aim is to explain collectivity effects in pA collisions using the Color Glass Condensate effective theory as an alternative to hydrodynamics.

Altinoluk and Armesto: arXiv:2004.08185

4-particle correlation

Change of sign of the 4-particle cumulant, $c_2\{4\}$, seen by CMS (arXiv:1606.06198) and ALICE (arXiv:1406.2474).

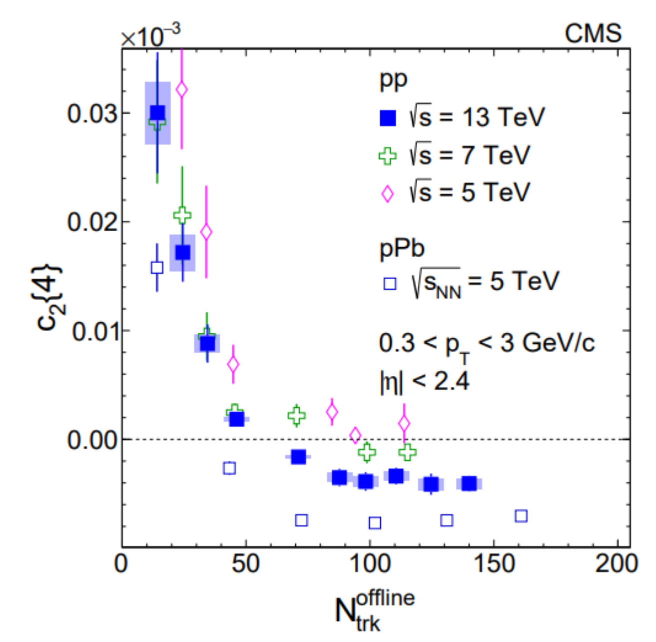
In the CGC:

We expect a positive 4-particle cumulant in the dilute-dilute regime

Dumitru, McLerran and Skokov: arXiv:1410.4844

and negative in the dilute-dense (multiple scattering) regime

Dusling, Mace and Venugopalan: arXiv:1706.06260



Theoretical background

n-gluon production spectrum in pA collisions at LO

$$\frac{d^n N}{\prod_{i=1}^n d^2\mathbf{k}_i} = \int \left(\prod_{i=1}^n \frac{d^2\mathbf{q}_{2i-1}}{(2\pi)^2}\frac{d^2\mathbf{q}_{2i}}{(2\pi)^2}\right) \underbrace{\left\langle \left(\prod_{i=1}^n g^2 \rho^{b_{2i-1}}(\mathbf{k}_i-\mathbf{q}_{2i-1})\rho^{b_{2i}*}(\mathbf{k}_i-\mathbf{q}_{2i})\right)\right\rangle_p}_{\text{Contribution from the projectile sources}} \underbrace{\left\langle \left(\prod_{i=1}^n \overline{\mathcal{M}}_{\lambda_i}^{a_i b_{2i-1}}(\mathbf{k}_i,\mathbf{q}_{2i-1})\overline{\mathcal{M}}_{\lambda_i}^{\dagger a_i b_{2i}}(\mathbf{k}_i,\mathbf{q}_{2i})\right)\right\rangle_T}_{\text{Contribution from the strong field of the target}}$$

Contribution from the projectile sources

Contribution from the strong field of the target

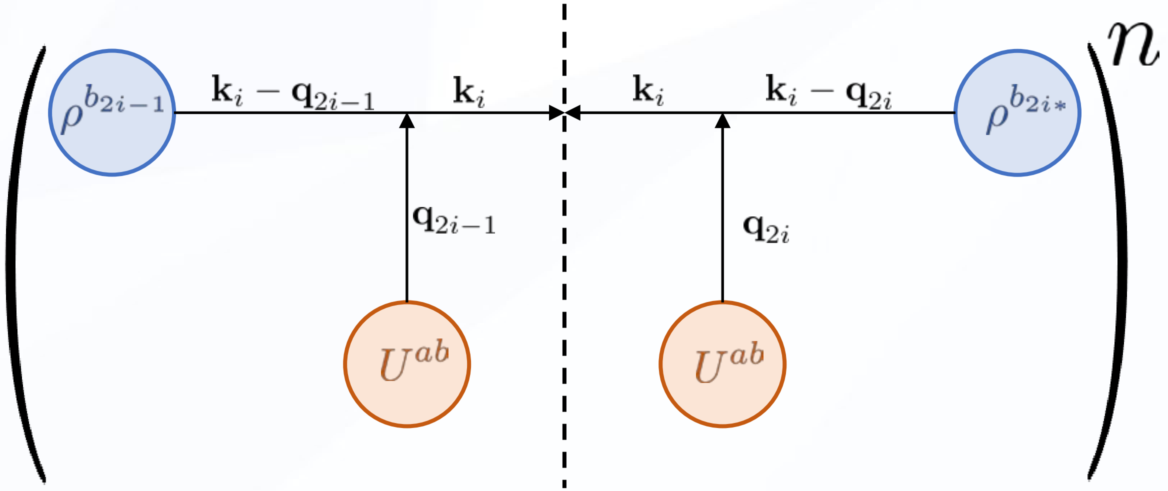
Reduced matrix amplitude

$$\overline{\mathcal{M}}_{\lambda}^{ab}(\mathbf{k},\mathbf{q}) = 2i\epsilon_{\lambda}^{i*}(\mathbf{k})L^i(\mathbf{k},\mathbf{q})\int_y e^{-i\mathbf{q}\cdot\mathbf{y}}U^{ab}(\mathbf{y})$$

$$L^i(\mathbf{k},\mathbf{q}) = \frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{(\mathbf{k}-\mathbf{q})^i}{(\mathbf{k}-\mathbf{q})^2}$$

Lipatov vertex

Wilson line



Theoretical background

Solving the projectile correlator

We use the McLerran-Venugopalan (MV) model for the sources, i.e. Gaussian statistics.

See the talk of A. Dumitru and the talk of F. Cougoulic for alternatives

Wick's Theorem:

$$\left\langle \rho^{b_1}(\mathbf{k}_1-\mathbf{q}_1)\rho^{b_2\dagger}(\mathbf{k}_1-\mathbf{q}_2)\cdots\rho^{b_{2n-1}}(\mathbf{k}_n-\mathbf{q}_{2n-1})\rho^{b_{2n}\dagger}(\mathbf{k}_n-\mathbf{q}_{2n})\right\rangle_p = \sum_{\omega\in\Pi(\chi)}\prod_{\{i,j\}\in\omega}\left\langle \rho^{b_i}(\mathbf{k}_i-\mathbf{q}_i)\rho^{b_j}(\mathbf{k}_j-\mathbf{q}_j)\right\rangle_p$$

$\Pi(\chi)$: Set of partitions of {1,2,…,2n} with disjoint pairs

Generalized MV:

$$g^2\left\langle \rho^{b_i}(\mathbf{k}_i-\mathbf{q}_i)\rho^{b_j}(\mathbf{k}_j-\mathbf{q}_j)\right\rangle_p = \frac{\delta^{b_ib_j}}{N_c^2-1}\mu^2\left[\mathbf{k}_i-\mathbf{q}_i,(-1)^{i+j}(\mathbf{k}_j-\mathbf{q}_j)\right]$$

$$\mu^2(\mathbf{k},\mathbf{q}) = e^{-\frac{(\mathbf{k}+\mathbf{q})^2}{4B_p^{-1}}}$$

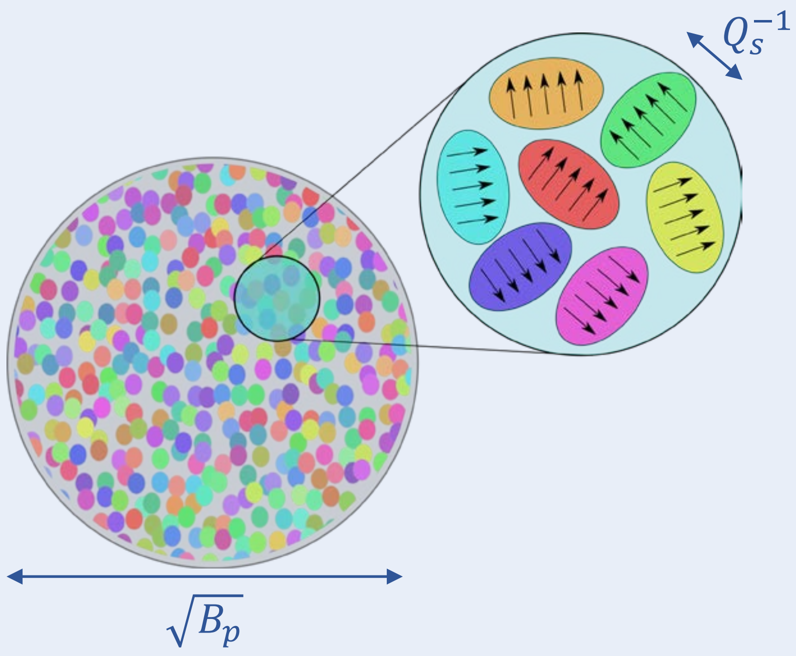
B_p is the transverse area of the projectile

Theoretical background

The area enhancement argument (Kovner and Rezaeian: 1707.06985)

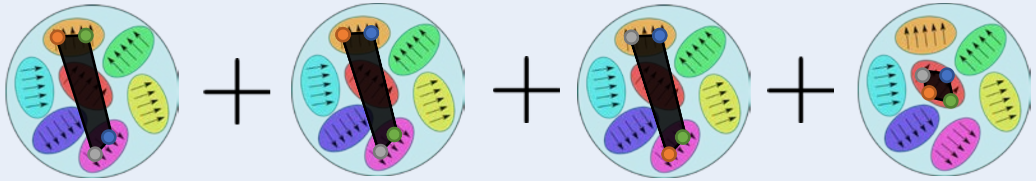
We want to evaluate (for example)

$$\int_{\mathbf{y}_1,\mathbf{y}_2,\mathbf{y}_3,\mathbf{y}_4} e^{-i\mathbf{q}_1\cdot\mathbf{y}_1+i\mathbf{q}_2\cdot\mathbf{y}_2-i\mathbf{q}_3\cdot\mathbf{y}_3+i\mathbf{q}_4\cdot\mathbf{y}_4}\left\langle Tr\left[U(\mathbf{y}_1)U^\dagger(\mathbf{y}_2)\right]Tr\left[U(\mathbf{y}_3)U^\dagger(\mathbf{y}_4)\right]\right\rangle_T$$



Domain model: The target is formed by chromoelectric domains with length of order Q_s^{-1}. The correlation between the particles comes from scatterings within the same domain:

The configuration that maximizes the correlator is such that the legs are grouped pairwise!



Area enhancement argument: These configurations in which more than two legs are close together are suppressed by the overlaping area after integration.

Theoretical background

The area enhancement argument (Kovner and Rezaeian: 1707.06985)

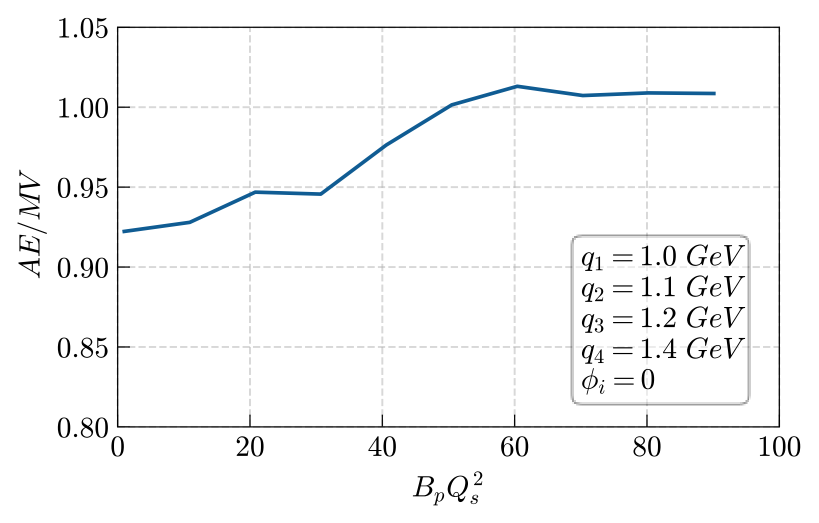
The area enhancement (AE) argument is analogous to assuming Gaussian statistics to the Wilson lines ➡ **Wick's Theorem**

$$\int \left\langle Tr\left[U(\mathbf{y}_1)U^\dagger(\mathbf{y}_2)\right] Tr\left[U(\mathbf{y}_3)U^\dagger(\mathbf{y}_4)\right]\right\rangle_T \approx \int \left\langle Tr\left[U(\mathbf{y}_1)U^\dagger(\mathbf{y}_2)\right]\right\rangle_T \left\langle Tr\left[U(\mathbf{y}_3)U^\dagger(\mathbf{y}_4)\right]\right\rangle_T$$
$$+\frac{1}{N_c^2-1}\left[\left\langle Tr\left[U(\mathbf{y}_1)U(\mathbf{y}_3)\right]\right\rangle_T \left\langle Tr\left[U(\mathbf{y}_2)^\dagger U^\dagger(\mathbf{y}_4)\right]\right\rangle_T + \left\langle Tr\left[U(\mathbf{y}_1)U^\dagger(\mathbf{y}_4)\right]\right\rangle_T \left\langle Tr\left[U(\mathbf{y}_3)U^\dagger(\mathbf{y}_2)\right]\right\rangle_T\right]$$

Altinoluk, Armesto, Kovner and Lublinsky: arXiv:1805.07739

- GBW model: $\frac{1}{N_c^2-1}\left\langle Tr\left[U(\mathbf{y}_1)U^\dagger(\mathbf{y}_2)\right]\right\rangle_T = e^{-\frac{Q_s^2}{4}(\mathbf{y}_1-\mathbf{y}_2)^2}$

- The difference of the Fourier transform of the quadrupole using the AE argument with respect to the MV model is of order 10 % when $B_pQ_s^2 \sim 10$



Models

Regularization of the IR divergences

We have to deal with the IR divergences of $L^i(\mathbf{k},\mathbf{q}_1)L^i(\mathbf{k},\mathbf{q}_2) = \left(\frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{(\mathbf{k}-\mathbf{q}_1)^i}{(\mathbf{k}-\mathbf{q}_1)^2}\right)\left(\frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{(\mathbf{k}-\mathbf{q}_2)^i}{(\mathbf{k}-\mathbf{q}_2)^2}\right)$

We assume:

- Collinear limit: $(\mathbf{k}-\mathbf{q})^2 \sim 0$
- Gaussian regularization

$$L^i(\mathbf{k},\mathbf{q}_1)L^i(\mathbf{k},\mathbf{q}_2) \to \frac{(2\pi)^2}{\xi^2}\exp\left\{-\frac{[\mathbf{k}-(\mathbf{q}_1+\mathbf{q}_2)/2]^2}{\xi^2}\right\}$$

This is a simplification with the cost of losing kinematical range (not valid for $\boldsymbol{k}^2 \gg Q_s^2$)

This asumption can be related with the Wigner function approach: Lappi: arXiv:1501.05505

Lappi, Schenke, Schlichting and Venugopalan: arXiv:1509.03499

Models

Relation with the Wigner function approach

- Collinear limit and Gaussian regularization for the Lipatov vertices
- Gaussian statistics for the projectile sources

Is equivalent to the Wigner function approach but including correlations in the projectile

$$W^{b_1b_2b_3b_4}(\mathbf{b}_1,\mathbf{p}_1,\mathbf{b}_2,\mathbf{p}_2) = \frac{1}{(N_c^2-1)^2}\frac{1}{\pi^4\xi^4B_p^2}e^{-(\mathbf{p}_1^2+\mathbf{p}_2^2)/\xi^2}e^{-(\mathbf{b}_1^2+\mathbf{b}_2^2)/B_p}\Big[\delta^{b_1b_2}\delta^{b_3b_4} + \delta^{b_1b_3}\delta^{b_2b_4}2\pi B_p\delta^{(2)}(\mathbf{b}_1-\mathbf{b}_2)e^{-(\mathbf{p}_1+\mathbf{p}_2)^2/(2B_p^{-1})}$$
$$+\delta^{b_1b_4}\delta^{b_2b_3}2\pi B_p\delta^{(2)}(\mathbf{b}_1-\mathbf{b}_2)e^{-(\mathbf{p}_1-\mathbf{p}_2)^2/(2B_p^{-1})}\Big]$$

In contrast with the factorization assumption:

$$W^{b_1b_2b_3b_4}(\mathbf{b}_1,\mathbf{p}_1,\mathbf{b}_2,\mathbf{p}_2) = \frac{\delta^{b_1b_2}\delta^{b_3b_4}}{(N_c^2-1)^2}\frac{1}{\pi^4\xi^4B_p^2}e^{-(\mathbf{p}_1^2+\mathbf{p}_2^2)/\xi^2}e^{-(\mathbf{b}_1^2+\mathbf{b}_2^2)/B_p}$$

Lappi, Schenke, Schlichting and Venugopalan: arXiv:1509.03499
Davy, Marquet, Xiao and Zhang: arXiv:1808.09851

In summary

n-gluon production in pA scattering

$$\frac{d^n N}{\prod_{i=1}^n d^2\mathbf{k}_i} = \int \left(\prod_{i=1}^n \frac{d^2\mathbf{q}_{2i-1}}{(2\pi)^2} \frac{d^2\mathbf{q}_{2i}}{(2\pi)^2} \right) \left\langle \left(\prod_{i=1}^n g^2 \rho^{b_{2i-1}}(\mathbf{k}_i - \mathbf{q}_{2i-1}) \rho^{b_{2i}*}(\mathbf{k}_i - \mathbf{q}_{2i}) \right) \right\rangle_p \left\langle \left(\prod_{i=1}^n \overline{\mathcal{M}}_{\lambda_i}^{a_i b_{2i-1}}(\mathbf{k}_i, \mathbf{q}_{2i-1}) \overline{\mathcal{M}}_{\lambda_i}^{\dagger a_i b_{2i}}(\mathbf{k}_i, \mathbf{q}_{2i}) \right) \right\rangle_T$$

- We asume Gaussian statistics for the projectile sources → Wick's theorem → $(2n-1)!!$ terms
- We use the area enhancement argument for the target average → Wick's theorem → $(2n-1)!!$ terms
- Since the integrand are Gaussians the integral is trivial

$$\frac{d^n N}{\prod_{i=1}^n d^2\mathbf{k}_i} = \text{Sum of } (2n-1)!!^2 \text{ terms}$$

$(2n-1)!!^2 = 9, 225, 11025, 893025, \ldots$ for $n = 2, 3, 4, 5, \ldots$

But the number of "non-trivial" terms can be reduced drastically by realizing the symmetries between the momenta even at all N_c.

Diagrammatic method given in "PA, Altinoluk and Armesto: *arXiv:2103.08485*"

Correlation

The cumulant method

- Is a method for extracting the “non-flow” contribution to the correlation.

- Cumulants: $$c_n\{2\} = \left\langle e^{in(\phi_1-\phi_2)} \right\rangle$$
$$c_n\{4\} = \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle - 2\left\langle e^{in(\phi_1-\phi_2)} \right\rangle^2$$

- Event average: $$\left\langle e^{in(\phi_1+\cdots+\phi_{m/2}-\phi_{m/2+1}-\cdots-\phi_m)} \right\rangle = \frac{\int \left(\prod_{i=1}^m \frac{d^2\mathbf{k}_i}{(2\pi)^2}\right) \frac{d^m N}{\prod_{i=1}^m d^2\mathbf{k}_i} e^{in(\phi_1+\cdots+\phi_{m/2}-\phi_{m/2+1}-\cdots-\phi_m)}}{\int \left(\prod_{i=1}^m \frac{d^2\mathbf{k}_i}{(2\pi)^2}\right) \frac{d^m N}{\prod_{i=1}^m d^2\mathbf{k}_i}}$$

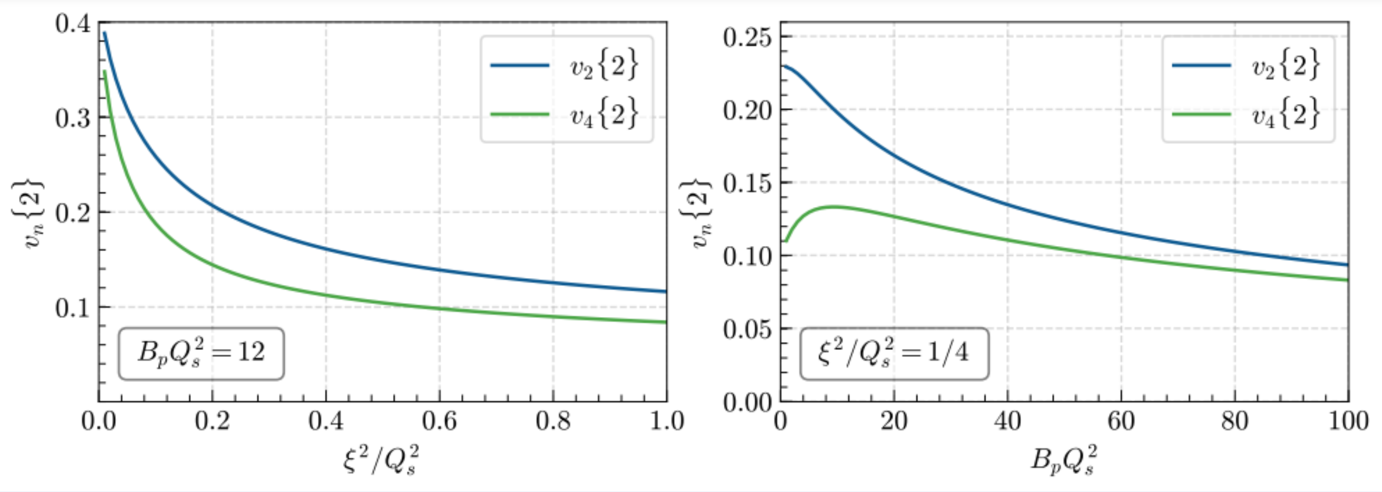
- Azimuthal harmonics: $$v_n\{2\} = (c_n\{2\})^{1/2}$$
$$v_n\{4\} = (-c_n\{4\})^{1/4}$$

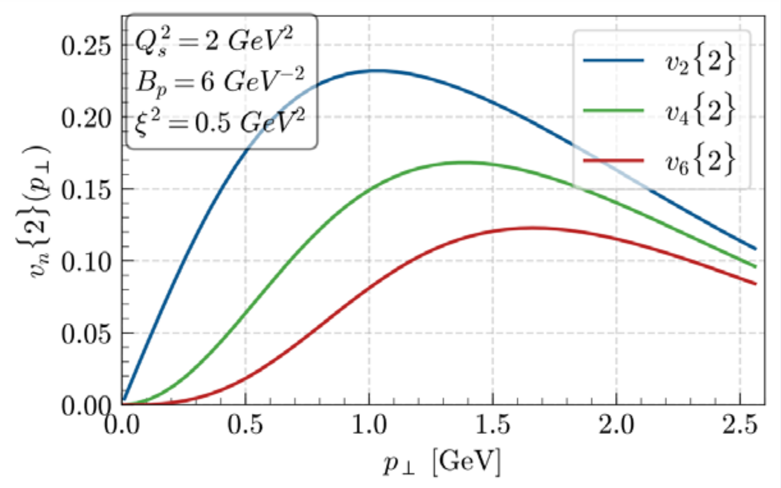
- Analogous expressions for the differential cumulants, i.e. with p_T dependence.

Results

2 gluon production

- Analytical result for the 2-particle azimuthal harmonics
- Numerical values lie in the bulk of experimental data





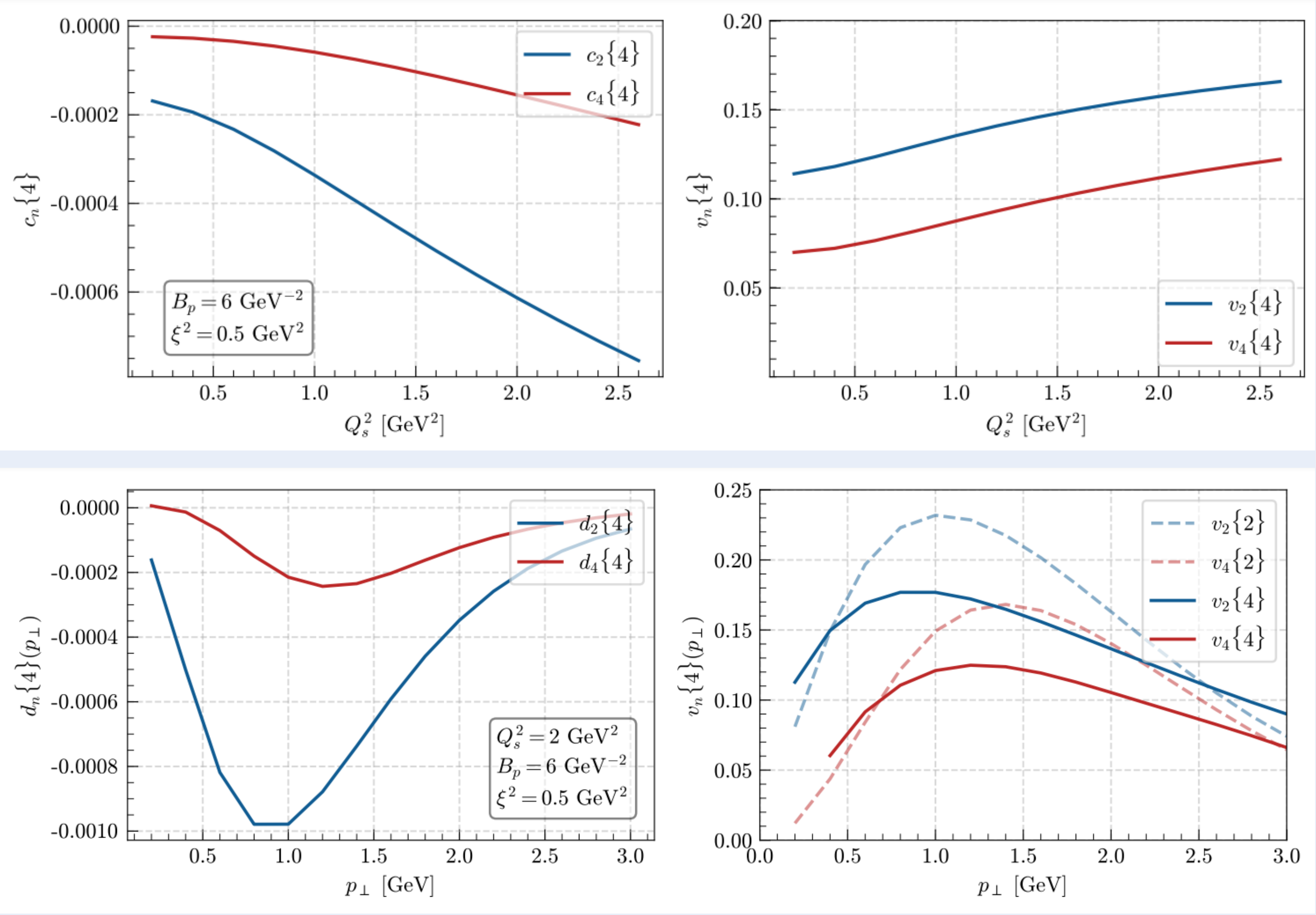
Results

4 gluon production

- Calculation performed at all orders in the number of colours, N_c
- Negative 4-particle cumulants and differential cumulants

  ⇓

  Well defined azimuthal harmonics
- Numerical values lie in the bulk of experimental data
- If we do not include correlations in the projectile the cumulants are positive



Conclusions

We have evaluated the multi-gluon spectrum up to 4-gluon production by using the area enhancement argument.

The collinear limit of our approach is the same as the Wigner function approach but including momentum correlations in the projectile WF.

We have seen that the 4-particle cumulant is negative for gluon production in the pA regime

Some improvements can be made:

- Perform the calculation without the collinear limit
- Explore the dilute-dilute (pp) limit. Is $c_2\{4\}$ positive?

Thank you!