Forward Higgs production at NLO using Lipatov's high energy effective action

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based on:

MH, Krzysztof Kutak, Andreas van Hameren, arXiv:2011.03193, *Eur.Phys.J.C* 81 (2021) 2, 112,
+ work in progress

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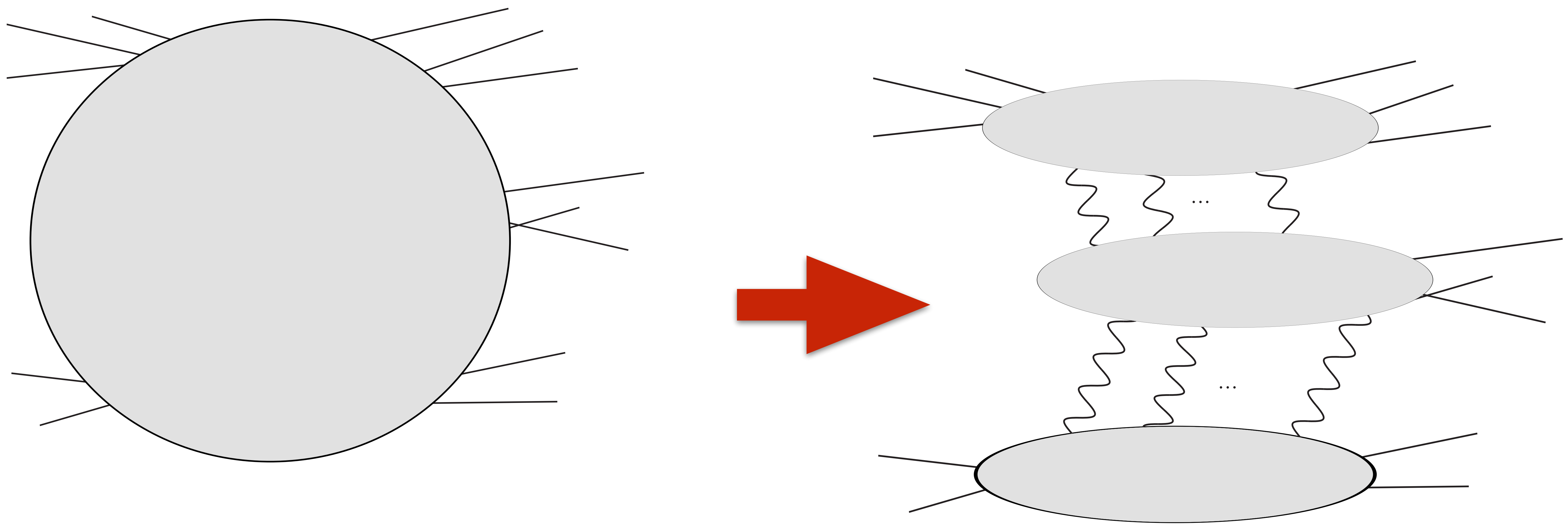
an action formalism for reggeized gluons: Lipatov's high energy effective action

- action for reggeized quarks: [Lipatov, Vyazovsky hep-ph/0009340]
- action for electroweak bosons: [Gomez Bock, MH, Sabio Vera, 2010.03621]

basic idea: [Lipatov; hep-ph/9502308]

relevant kinematics:
Multi-Regge-Kinematics
(separated in rapidity & transverse momenta of same order of magnitude)



correlator with regions localized in rapidity, significantly separated from each other

factorize using auxiliary degree of freedom = the reggeized gluon

- idea: factorize QCD amplitudes in the high energy limit through introducing a new kind of field: *the reggeized gluon $A_\pm$* (conventional QCD gluon: v_μ)

kinematics (strong ordering in light-cone momenta between different sectors): $$\partial_+ A_-(x) = 0 = \partial_- A_+(x).$$

underlying concept:

- reggeized gluon globally charged under SU(N_C) $$A_\pm(x) = -it^a A^a_\pm(x)$$

- but invariant under local gauge transformation

$$\delta_\mathrm{L} v\mu = \frac{1}{g}[D_\mu, \chi_L] \quad \text{vs.} \quad \delta_\mathrm{L} A_\pm = \frac{1}{g}[A_\pm, \chi_L] = 0$$

→ gauge invariant factorization of QCD correlators underlying idea:

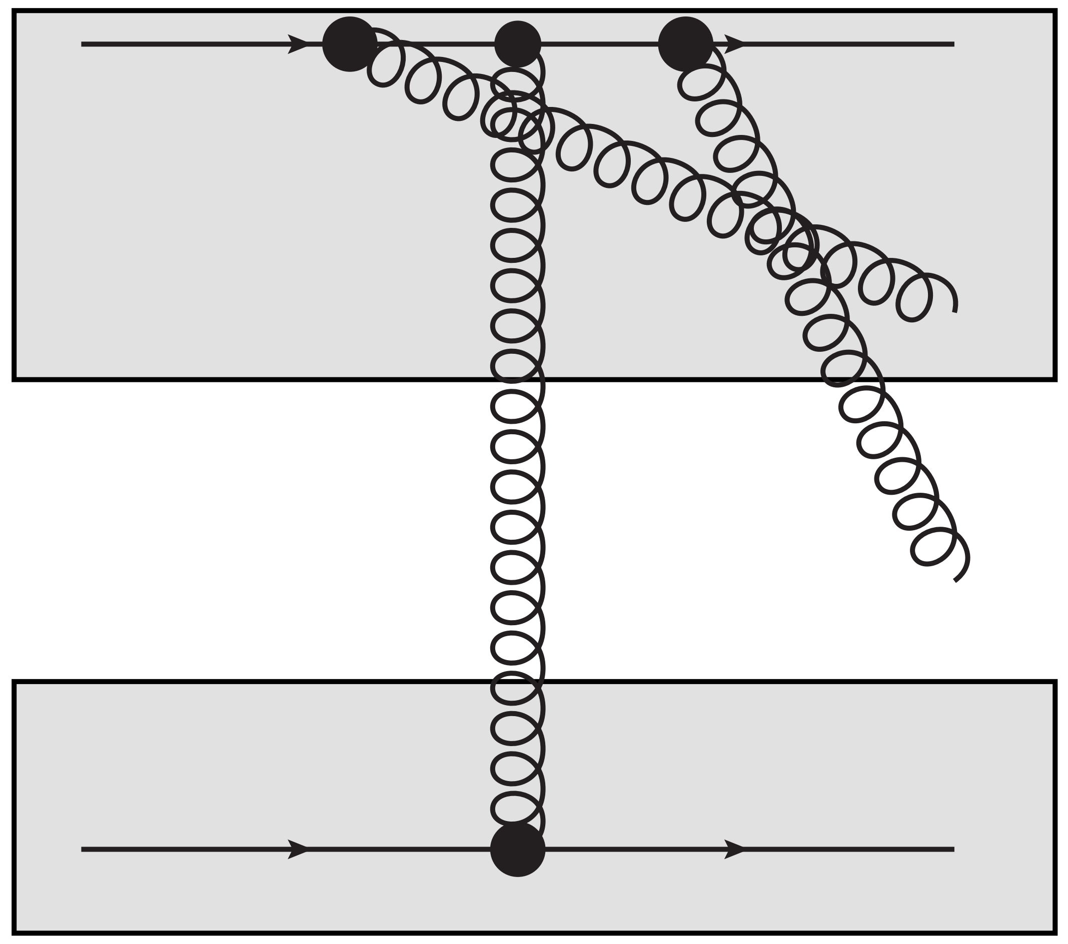
- ➜ integrate out specific details of (relatively) fast +/- fields
- ➜ description in sub-amplitude local in rapidity: QCD Lagrangian + universal eikonal factor

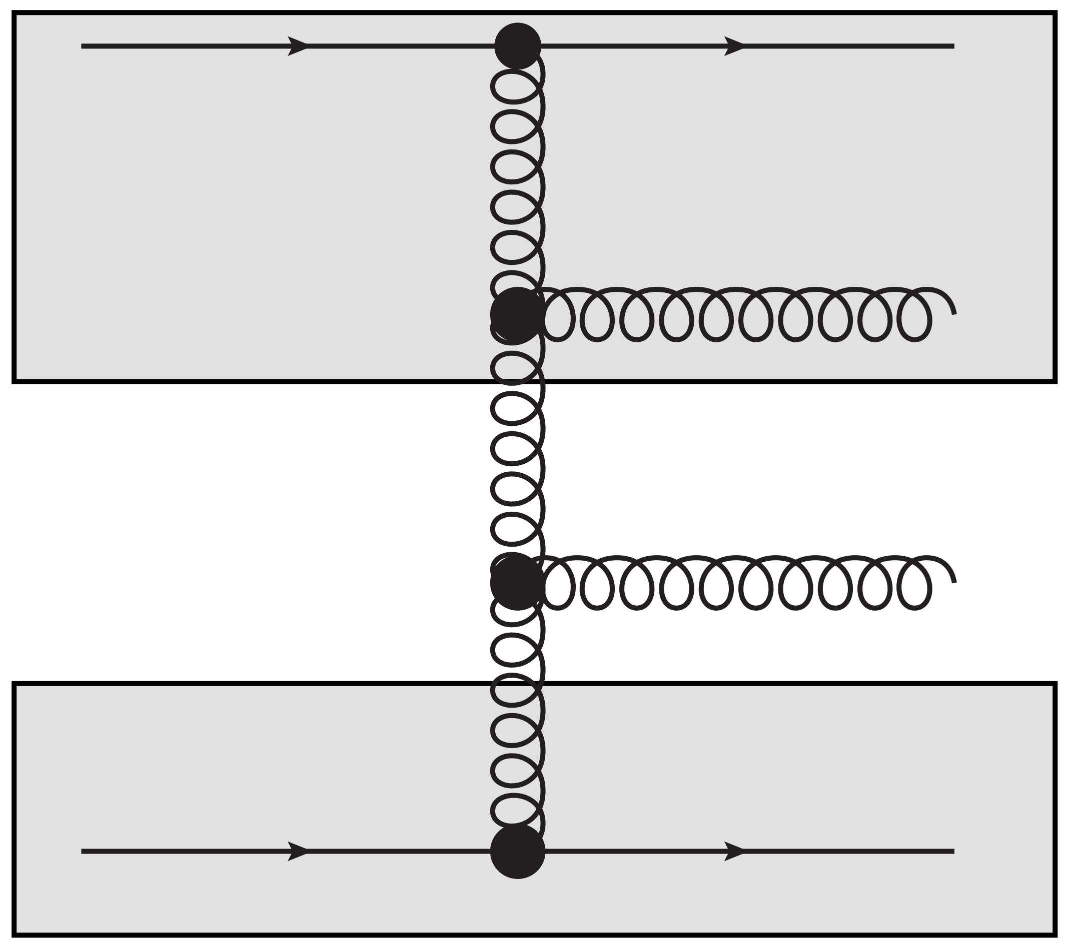
$$T_\pm[v_\pm] = -\frac{1}{g}\partial_\pm \mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^\pm} dx'^\pm v_\pm(x')\right)$$

- ➜ effective field theory for *each* local rapidity cluster

$$\boxed{S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind.}}}$$

$$\boxed{S_{\text{ind.}} = \int \mathrm{d}^4x \left\{ \mathrm{tr}\left[\left(T_-[v(x)] - A_-(x)\right)\partial_\perp^2 A_+(x)\right] + [\text{"}+\text{"} \leftrightarrow \text{"}-\text{"}]\right\}.}$$





non-trivial test:

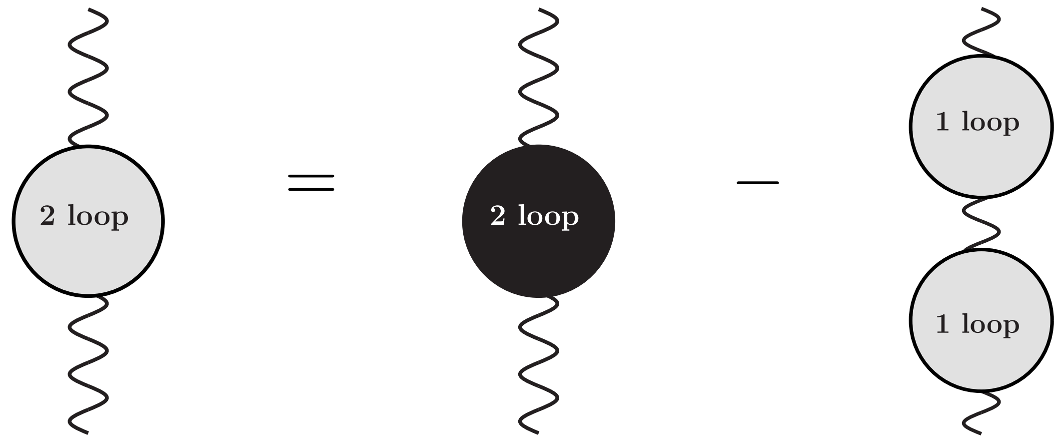
[Chachamis, MH, Madrigal, Sabio Vera;1202.0649, 1307.2591]

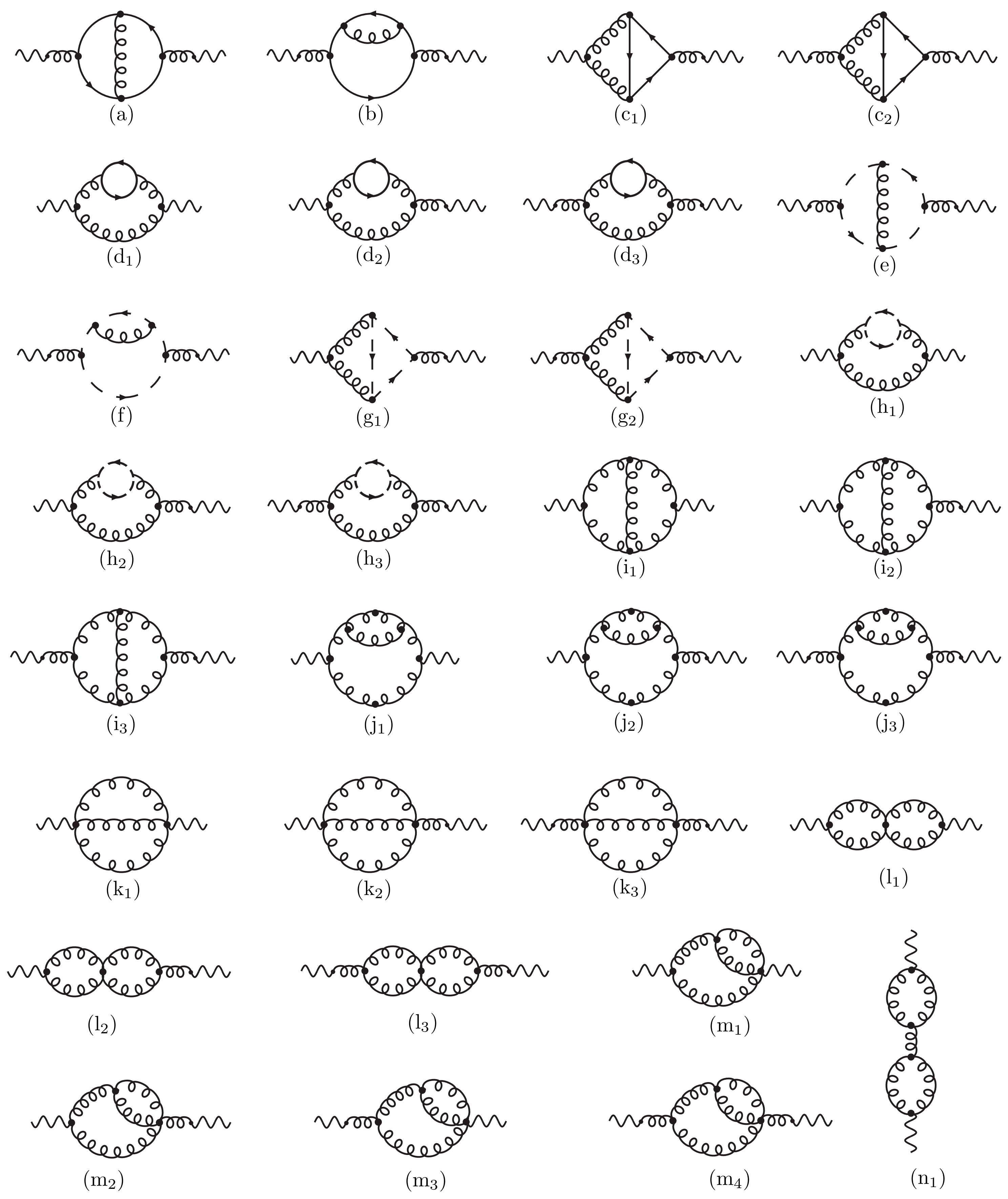
2 loop gluon trajectory from high energy singularity of the reggeized gluon propagator

$$G\left(\rho;\epsilon,\boldsymbol{q}^2,\mu^2\right) = \frac{i/2}{\boldsymbol{q}^2}\left\{1+\frac{i/2}{\boldsymbol{q}^2}\Sigma\left(\rho;\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right)+\left[\frac{i/2}{\boldsymbol{q}^2}\Sigma\left(\rho;\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right)\right]^2+\dots\right\}$$

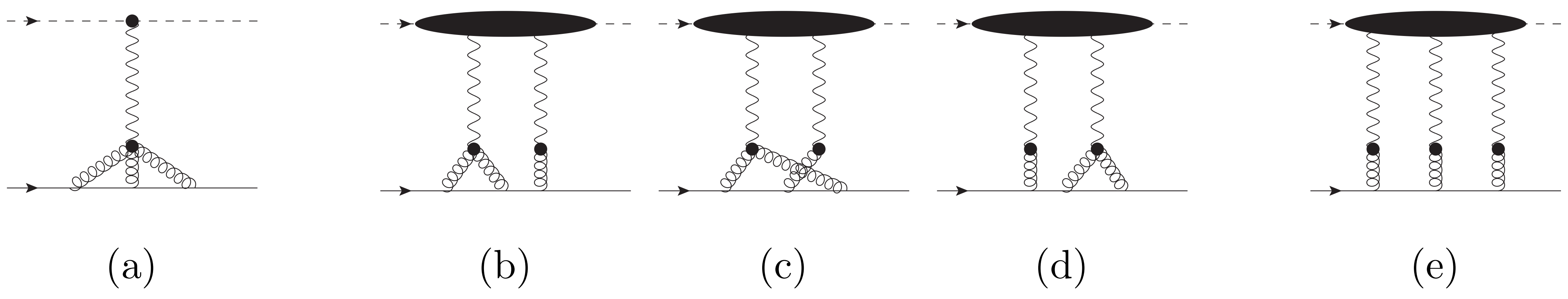
$$\Sigma\left(\rho;\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right) = \Sigma^{(1)}\left(\rho;\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right)+\Sigma^{(2)}\left(\rho;\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right)+\dots$$

$$\Sigma^{(2)}\left(\rho;\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right) =$$





A short appraisal of Lipatov's high energy effective action



- natural framework to address multi-reggeized gluon exchanges [MH, 0908.2576], [Braun *et.al.* hep-ph/0612323, 1103.3618, 1402.4786, 1702.04796], [Bondarenko, Lipatov, Prygarin, Pozdnyakov, 1708.05183]

- in particular: contains Balitsky-JIMWLK evolution (=Color Glass Condensate formalism) & background field propagators [MH, 1802.06755]

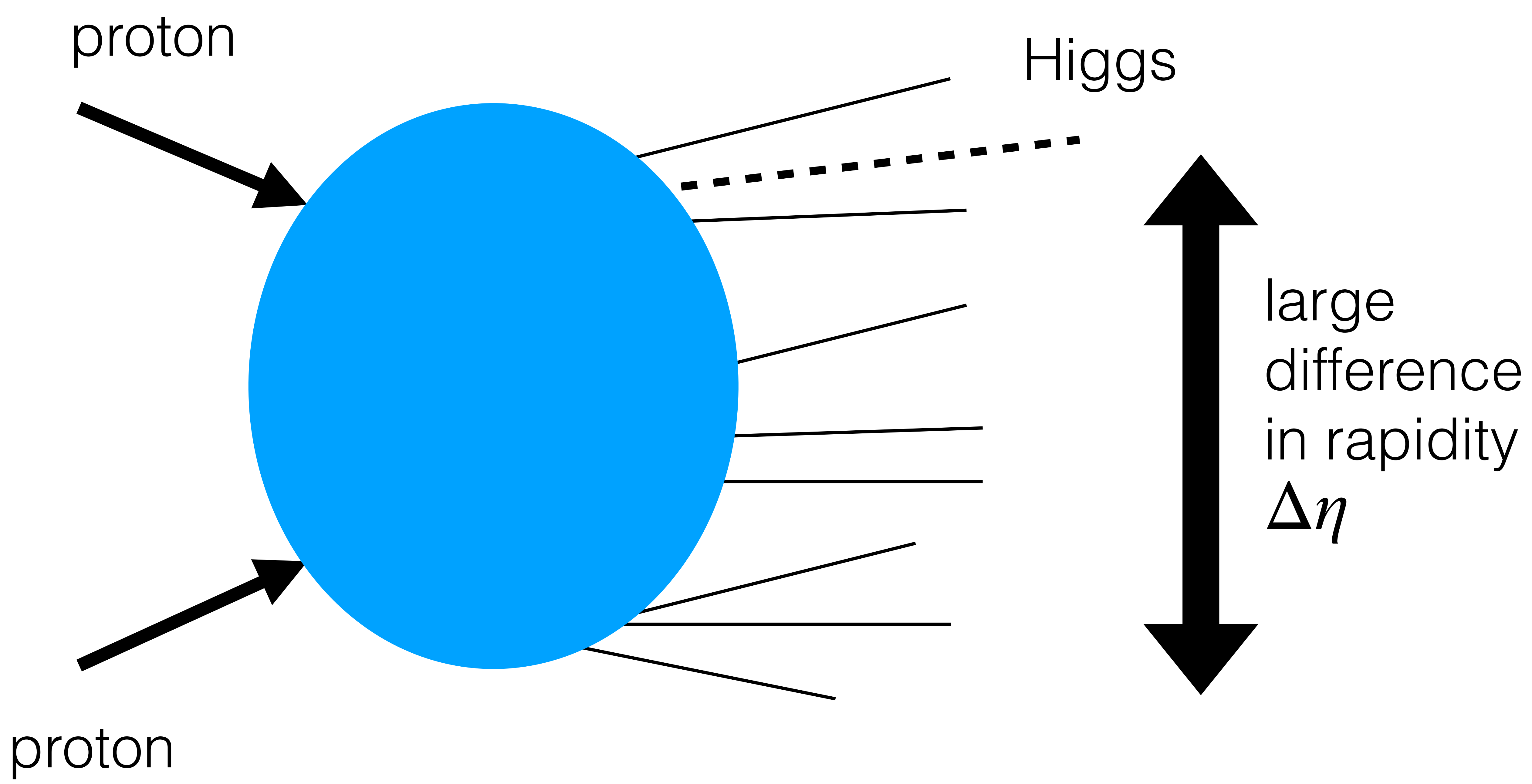
- NLO impact factors for jets without and with rapidity gap (2 Reggeon state) [MH, Madrigal, Murdaca, Sabio Vera, 1404.2937, 1406.5625, 1409.6704]

- 2 scale processes [Nefedov; 1902.11030]

- Complementary (dilute): spinor helicity amplitudes based formalism [van Hameren, Kotko, Kutak; 1211.0961], [van Hameren, Kutak, Salwa; 1308.2861]

→ well tested effective action formalism for high energy factorization

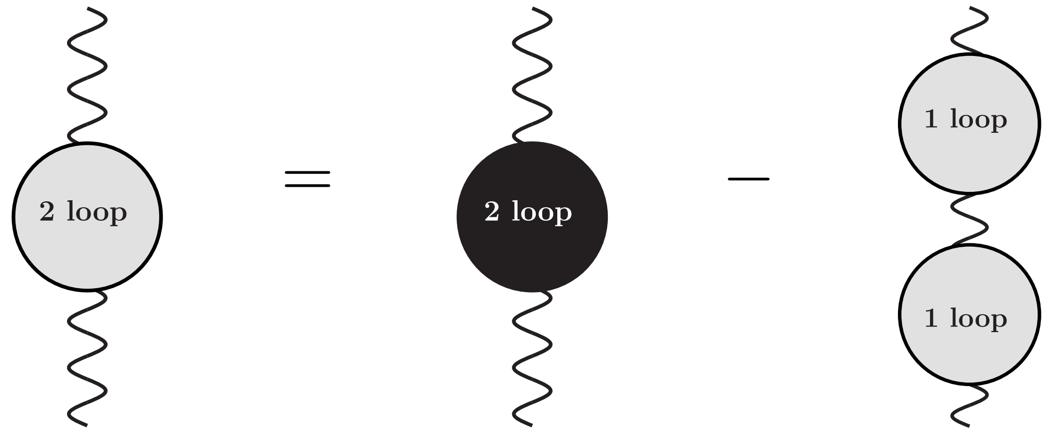
Forward Higgs production



Why is this of interest?

- Higgs phenomenology: only for events which identify a forward Higgs
- Higgs + jet configurations with resummation of $(\alpha_s \Delta\eta)^n$
  e.g. [Celiberto, Ivanov, Mohammed, Papa; 2008.00501]
- program to define combined DGLAP & low x evolution with TMD splitting kernels [MH, Kusina, Kutak, Serino, 1711.04587, 1607.01507], see also poster by Lissa Keersmaekers
- Higgs = a colorless final state & gives access to the gluon distribution

How to organize an NLO (and beyond) calculation using the high energy effective action?



- fully worked for virtual corrections → determination of the 2 loop Regge trajectory
- cross-checked & works

$$\omega^{(2)}(\mathbf{q}^2) = \frac{(\omega^{(1)}(\mathbf{q}^2))^2}{4}\left[\frac{11}{3} - \frac{2n_f}{3N_c} + \left(\frac{\pi^2}{3} - \frac{67}{9}\right)\epsilon + \left(\frac{404}{27} - 2\zeta(3)\right)\epsilon^2\right]$$

real corrections:

- essentially the same
- but deal with Multi-Regge-Kinematics means:
  - strong ordering in rapidity
  - in general <u>arbitrary</u> transverse momenta
- need to work with convolution integrals instead of products in general not a problem & well known from *e.g.* conventional collinear factorization

Starting point: hybrid factorization

Higgs production in fragmentation/forward region of proton 1

$$p = x_H p_A + \frac{M_H^2 + \boldsymbol{p}^2}{x_H s} p_B + p_T$$

$$\eta_H = \ln \frac{x_H \sqrt{s}}{\sqrt{\mathbf{p}^2 + M_H^2}}$$

collinear parton distribution of proton 1: large x

unintegrated gluon distribution of proton 2: low x

- off-shell → high energy factorization
- defined through 2 reggeized gluon state

$$\frac{d^3\sigma}{d^2\boldsymbol{p}dx_H} = \int_{x_H}^1 \frac{dz}{z} \sum_{a=q,g} f_a\left(\frac{x_H}{z}, \mu_F^2\right) \int \frac{d^2\boldsymbol{k}}{\pi} \frac{d\hat{C}_{ag^*\to H}(\mu_F^2, \eta_a; z, \boldsymbol{k})}{d^2\boldsymbol{p}dx_H} \mathcal{G}(\eta_a, \boldsymbol{k}),$$

object of interest: coefficient

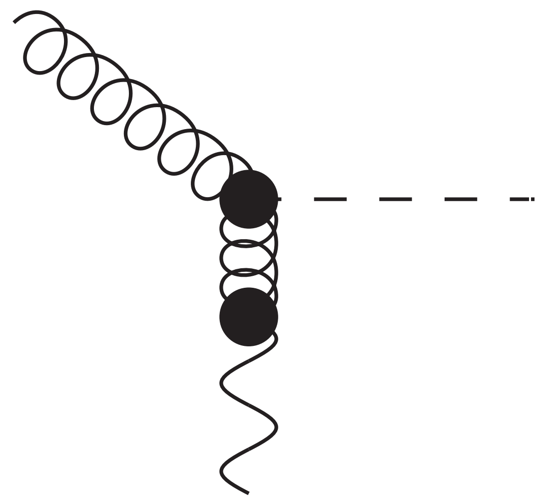
$$\frac{d^3\hat{C}^{NLO}_{ag^*\to H}}{dx_H d^2\boldsymbol{p}} = \sigma_0 \left(\frac{d^3\hat{C}^{(0)}_{ag^*\to H}}{dx_H d^2\boldsymbol{p}} + \frac{\alpha_s}{2\pi} \cdot \frac{d^3\hat{C}^{(1)}_{pg^*\to H}}{dx_H d^2\boldsymbol{p}} + \ldots \right).$$

<u>not included:</u>

- multiple reggeized gluon exchange (Glauber gluons)
- no high density effects
- possible, but beyond this work

Tree level:

conventional gluon



reggeized = high energy factorized gluon

coupling Higgs gluon field through effective Lagrangian ($m_t \to \infty$)

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} g_H H F^a_{\mu\nu} F^{\mu\nu}_a$$

$$g_H = -\frac{\alpha_s}{3\pi v}\left(1 + \frac{\alpha_s}{4\pi} 11\right) + \mathcal{O}(\alpha_s^3)$$

[Shifman, Vainshtein, Voloshin, Zakharov, 1979],
[Ellis, Gaillard, Nanopoulos, 1976],
[Ravindran, Smith, Van Neerven, 2002]

$$\frac{d\hat{C}^{(0)}_{gg^* \to H}(\mu_F^2, \eta_a; z, \boldsymbol{k})}{d^2\boldsymbol{p}\, dx_H} = \delta^{(2)}(\boldsymbol{p} - \boldsymbol{k})\delta(1 - z)$$

$$\sigma_0 = \frac{g_H^2 \pi}{8(N_c^2 - 1)}$$

$$\frac{d^3\hat{C}^{NLO}_{ag^* \to H}}{dx_H d^2\boldsymbol{p}} = \sigma_0 \left(\frac{d^3\hat{C}^{(0)}_{ag^* \to H}}{dx_H d^2\boldsymbol{p}} + \frac{\alpha_s}{2\pi} \cdot \frac{d^3\hat{C}^{(1)}_{pg^* \to H}}{dx_H d^2\boldsymbol{p}} + \dots \right).$$

Virtual corrections

- determined in [Nefedov, 1902.11030]
- rapidity divergences regulated through tilting light-cone directions of eikonal propagators $n^\pm \to n^\pm + e^{-\rho} n^\mp$ with $\rho \to \infty$ parametrizing the singularities
- collinear, UV and soft singularity regulated through dimensional regularization in $d = 4 + 2\epsilon$ dimensions

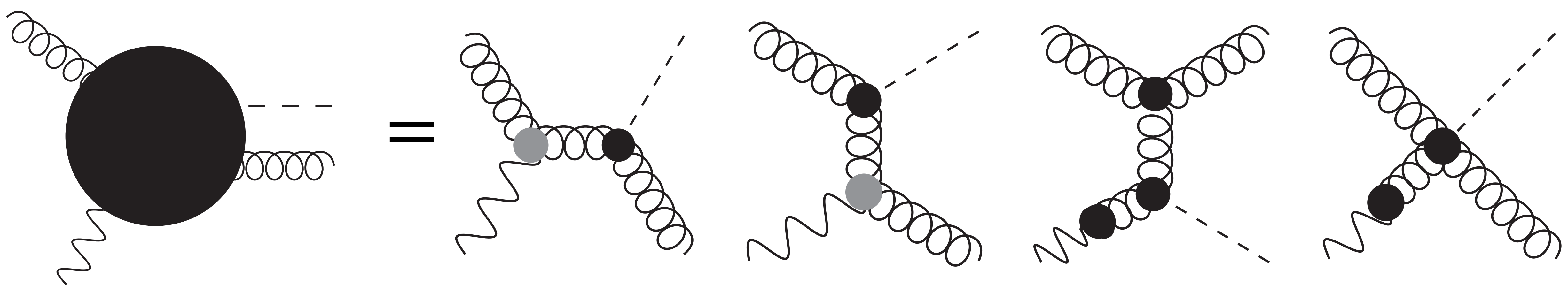
$$\frac{dh^{(1)}_{gg^*\to H}(z,\boldsymbol{k})}{d^2\boldsymbol{p}dx_H} = \frac{dh^{(0)}_{gg^*\to H}(z,\boldsymbol{k})}{d^2\boldsymbol{p}dx_H}\frac{\alpha_s}{2\pi}\cdot\left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^\epsilon \Bigg\{-\frac{C_A}{\epsilon^2} - \frac{1}{\epsilon}\left(\frac{8C_A}{3} - \frac{2n_f}{3}\right)$$
$$+\frac{C_A}{\epsilon}\left[-\rho + \ln\frac{\boldsymbol{k}^2}{(p_a^+)^2}\right] + C_A\left[2\mathrm{Li}_2\left(1+\frac{M_H^2}{\boldsymbol{k}^2}\right) + \frac{\pi^2}{6} + \frac{49}{9}\right] + 11 - \frac{10}{9}n_f\Bigg\}$$

high energy divergence & log in energy:
proportional to 1-loop gluon trajectory

Real corrections

regulator on gluon rapidity
general: unobserved final state particle

$$\frac{d^3 h^{(0)}_{gg^*\to Hg}(z,\boldsymbol{k})}{dx_H d^2\boldsymbol{p}} = \frac{\alpha_s C_A \sigma_0}{2\pi_\epsilon \boldsymbol{k}^2} H_{ggH}(z,\boldsymbol{p},\boldsymbol{k})\theta\left(\eta_g + \frac{\rho}{2}\right)$$



$$H_{ggH}(z,\boldsymbol{p},\boldsymbol{k}) = \frac{2}{z(1-z)}\Bigg\{2z^2 + \frac{(1-z)zM_H^2(\boldsymbol{k}\cdot\boldsymbol{r})[z^2+(1-z)\cdot 2\epsilon] - 2z^3(\boldsymbol{p}\cdot\boldsymbol{r})(\boldsymbol{p}\cdot\boldsymbol{k})}{\boldsymbol{r}^2(\boldsymbol{p}^2+(1-z)M_H^2)}$$
$$+\frac{(1+\epsilon)(1-z)^2z^2M_H^4}{2}\left(\frac{1}{\boldsymbol{\Delta}^2+(1-z)M_H^2}+\frac{1}{\boldsymbol{p}^2+(1-z)M_H^2}\right)^2$$
$$-\frac{2z^2(\boldsymbol{p}\cdot\boldsymbol{\Delta})^2+2\epsilon\cdot(1-z)^2z^2M_H^4}{(\boldsymbol{p}^2+(1-z)M_H^2)(\boldsymbol{\Delta}^2+(1-z)M_H^2)} - \frac{2z(1-z)^2M_H^2}{\boldsymbol{\Delta}^2+(1-z)M_H^2} - \frac{2z(1-z)^2M_H^2}{\boldsymbol{p}^2+(1-z)M_H^2}$$
$$-\frac{(1-z)zM_H^2(\boldsymbol{k}\cdot\boldsymbol{r})[z^2+(1-z)\cdot 2\epsilon] - 2z^3(\boldsymbol{\Delta}\cdot\boldsymbol{r})(\boldsymbol{\Delta}\cdot\boldsymbol{k})}{\boldsymbol{r}^2(\boldsymbol{\Delta}^2+(1-z)M_H^2)}\Bigg\}$$
$$+\frac{2\boldsymbol{k}^2}{\boldsymbol{r}^2}\left\{\frac{z}{1-z}+z(1-z)+2(1+\epsilon)\frac{(1-z)}{z}\frac{(\boldsymbol{k}\cdot\boldsymbol{r})^2}{\boldsymbol{k}^2\boldsymbol{r}^2}\right\};$$

$$H_{qqH}(z,\boldsymbol{p},\boldsymbol{k}) = \frac{1+\epsilon}{z}\left[z^2+4(1-z)\frac{(\boldsymbol{k}\cdot\boldsymbol{r})^2}{\boldsymbol{k}^2\boldsymbol{r}^2}\right]$$

using 3 different methods

- high energy effective action
- kT-factorization (in light-cone gauge)
- KaTie Monte Carlo
- cross-checked against collinear results for $\mathbf{k}\to 0$ (off-shell gluon TM) [Dawson, 1991]

How to obtain a coefficient?

- UV as for collinear calculation

$$\alpha_s = \alpha_s(\mu_R)\left[1 + \frac{\alpha_s(\mu_R)\beta_0}{(4\pi)}\left(\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{\mu^2}\right)\right], \quad \beta_0 = \frac{11C_A}{3} - \frac{2n_f}{3},$$

using that $g_H \sim \alpha_s(\mu_R)$

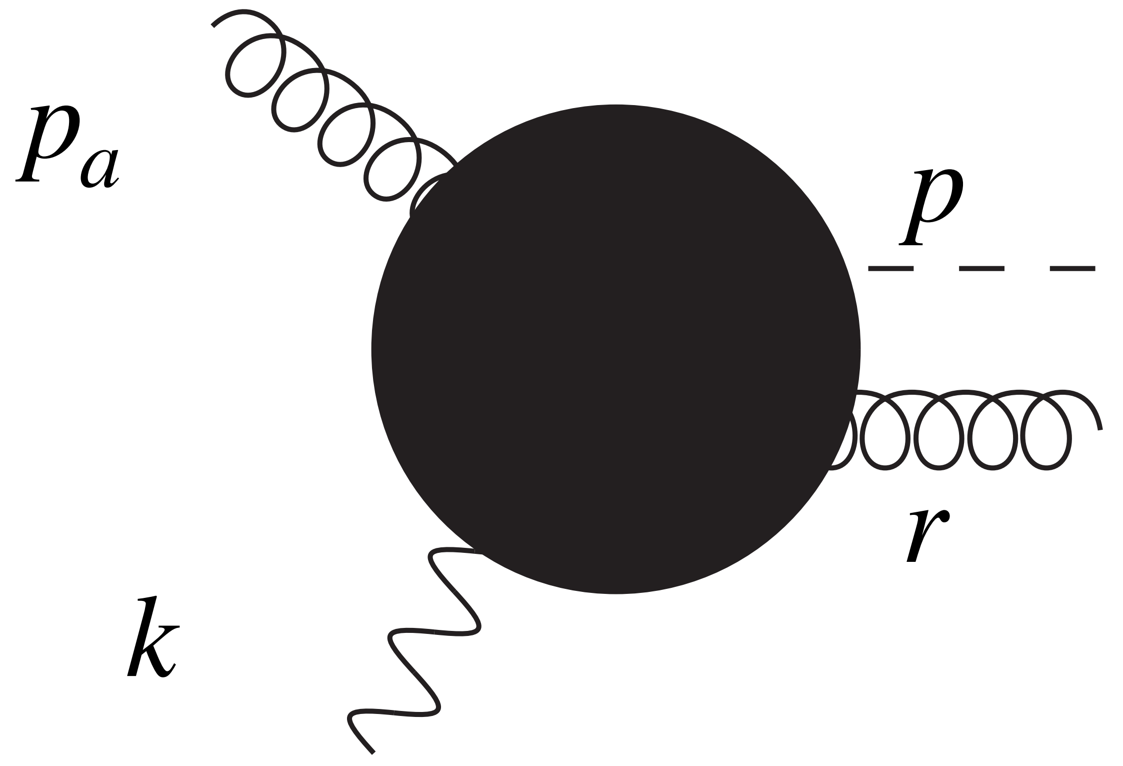
off-shell partonic result contains

- **high energy divergencies (parametrized by $\rho \to \infty$)**
- **collinear, soft and UV divergencies**

1 loop partonic parton distribution

$$\Gamma_{ba}(z) = \delta_{ba}\delta(1-z) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\epsilon} + \ln\frac{\mu_F^2}{\mu^2}\right)P_{ga}(z) + \mathcal{O}(\alpha_s^2)$$

coll. gluon/quark



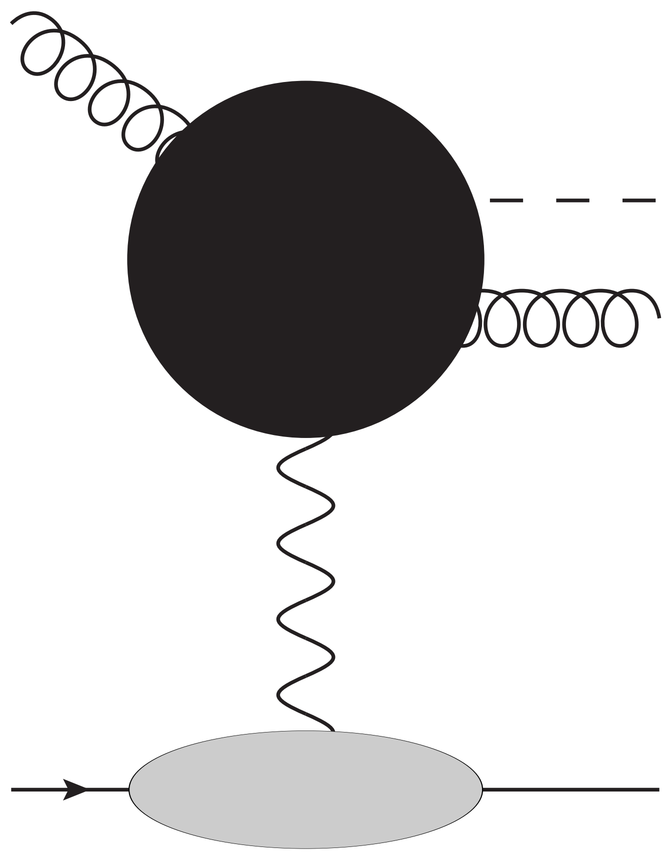
$$\frac{d^3 h^{(0)}_{ag^*\to Ha}}{dx_H d^2\boldsymbol{p}} = \sum_{b=q,g}\int_{x_H}^{1} d\xi \int d^{2+2\epsilon}\tilde{\boldsymbol{k}}\, \frac{d^3\hat{C}_{bg^*\to Hb}(z,\tilde{\boldsymbol{k}},\boldsymbol{p})}{dx_H d^2\boldsymbol{p}}\, \boxed{\Gamma_{ba}\left(\xi, \frac{\mu_F^2}{\mu^2}\right)}\, \boxed{\tilde{\Gamma}_{g^*g^*}(\xi,\tilde{\boldsymbol{k}},\boldsymbol{k})}.$$

1-loop unintegrated gluon distribution

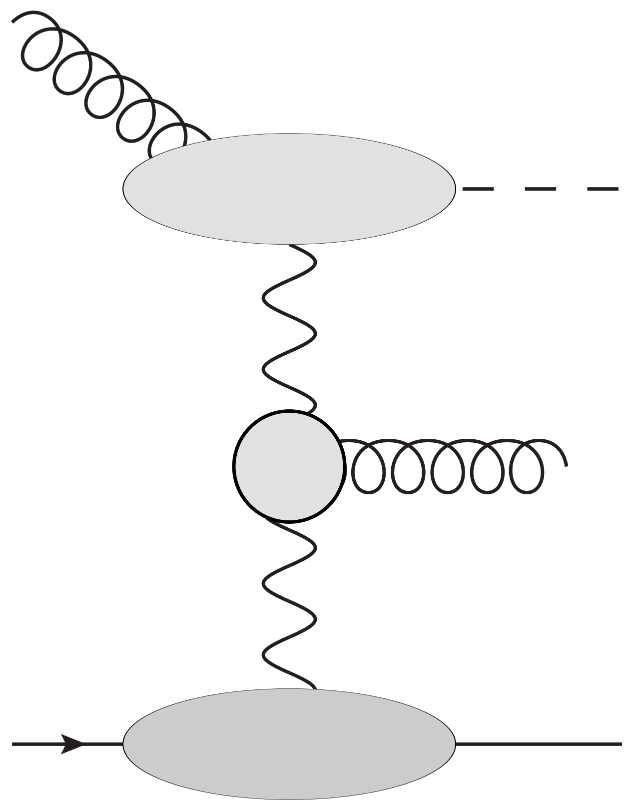
$$\tilde{\Gamma}_{g^*g^*}(\xi,\tilde{\boldsymbol{k}},\boldsymbol{k}) = \delta(1-\xi)\,\delta^{(2)}(\boldsymbol{k}-\tilde{\boldsymbol{k}})\left[1 - \frac{\alpha_s}{2\pi}\left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^{\epsilon}\left(\frac{5C_A - 2n_f}{6\epsilon} - \frac{31C_A - 10n_f}{18}\right)\right.$$
$$\left. + \,\delta(1-\xi)\,(\rho + 2\eta_a)\left[\frac{\alpha_s C_A}{2\pi_\epsilon(\tilde{\boldsymbol{k}}-\boldsymbol{k})^2} - \delta^{(2)}(\boldsymbol{k}-\tilde{\boldsymbol{k}})\frac{\alpha_s}{2\pi\epsilon}\left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^{\epsilon}\right]\right.,$$

- more interesting: <u>external legs</u>

Origin of the 1-loop unintegrated gluon distribution: subtraction & cancellation of rapidity divergencies



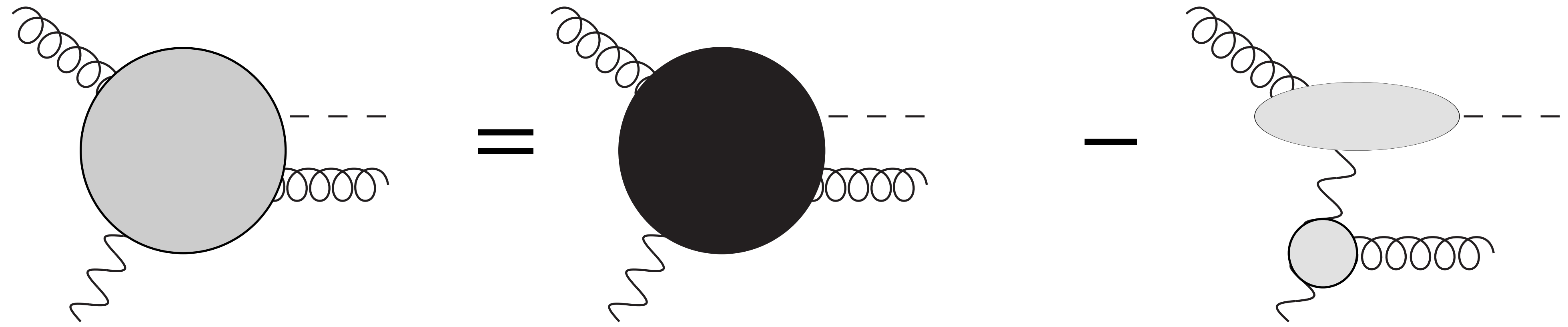
fwd Higgs + gluon (quasi-elastic): our result

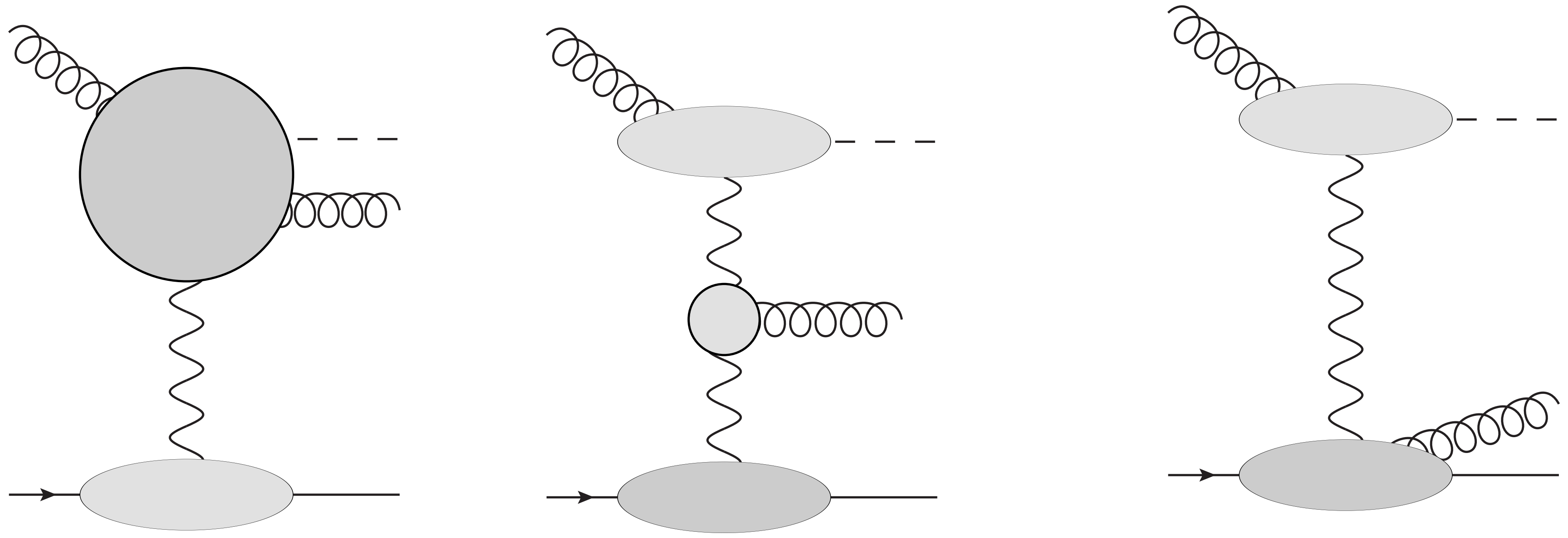


central gluon production (high energy factorized)→the BFKL kernel!

both: + virtual corrections

both diagrams have an overlap region → need to remove this overlap [MH, Sabio Vera; 1110.6741]



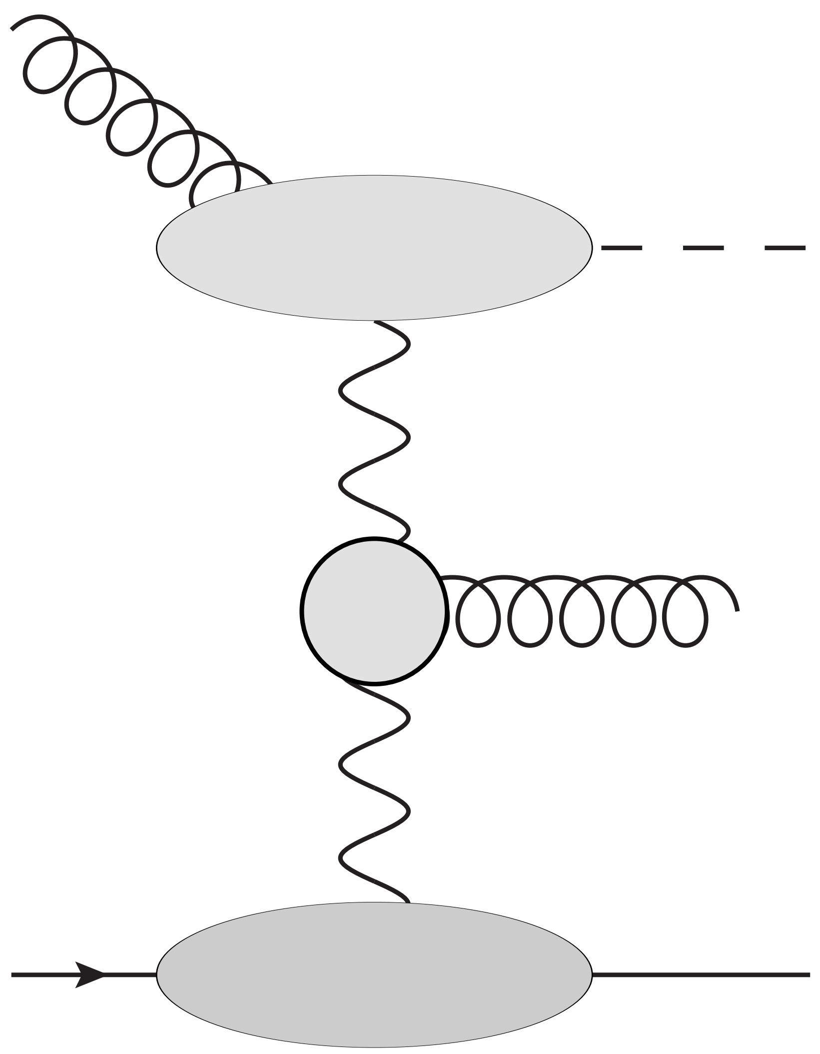


- no overlap any more
- finite for $\rho \to \infty$
- correct

Transition function

next step: introduce transition function $Z^{\pm}$ to make cancellation of divergencies between individual contributions explicit

$$d\sigma_{ab}^{\mathrm{NLO}} = \left[C_{a,B}(\rho) \otimes G_B(\rho) \otimes C_{b,B}(\rho)\right]$$



renormalized reggeized gluon Green's function

$$G_B(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho) = \left[Z^+\left(\frac{\rho}{2} - \eta_a\right) \otimes G_R(\eta_a, \eta_b) \otimes Z^-\left(\frac{\rho}{2} + \eta_b\right)\right](\boldsymbol{k}_1, \boldsymbol{k}_2)$$

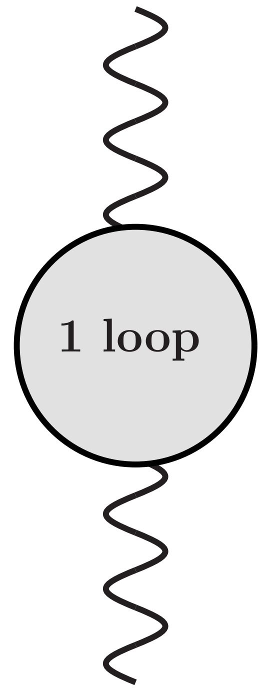
$$\frac{d}{d\hat{\rho}} Z^+(\hat{\rho}; \boldsymbol{k}, \boldsymbol{q}) = \left[Z^+(\hat{\rho}) \otimes K_{\mathrm{BFKL}}\right](\boldsymbol{k}, \boldsymbol{q}),$$
$$\frac{d}{d\hat{\rho}} Z^-(\hat{\rho}; \boldsymbol{k}, \boldsymbol{q}) = \left[K_{\mathrm{BFKL}} \otimes Z^-(\hat{\rho})\right](\boldsymbol{k}, \boldsymbol{q}),$$

transition function subject to BFKL kernel → possibility to define it within effective action

renormalized subtracted coefficient

$$C_{a,R}(\eta_a; \boldsymbol{k}_1) \equiv \left[C_a(\rho) \otimes Z^+\left(\frac{\rho}{2} - \eta_a\right)\right](\boldsymbol{k}_1)$$

Note: reggeized gluon self energy contains also finite terms
→ those terms are moved to impact factors
→ universal, but ρ independent terms



$\eta_{a,b}$: factorization parameter
→ BFKL equation for G_R as RG equation

Real-virtual cancellations:

Our final result still contains $\frac{1}{\epsilon}, \frac{1}{\epsilon^2}$ poles and divergent convolution integrals

can be shown relatively easily that those poles cancel (phase space slicing techniques)

numerics: something like dipole subtraction [Catani, Seymour; hep-ph/9605323] would be desirable …..

here: divergencies related to convolution integral → can't directly apply those techniques

$$\int \frac{d^2\boldsymbol{k}}{\pi} \frac{d\hat{C}_{ag^*\to H}(\mu_F^2, \eta_a; z, \boldsymbol{k})}{d^2\boldsymbol{p} dx_H} \mathcal{G}(\eta_a, \boldsymbol{k})$$

instead propose:

$$\int \frac{d^{2+2\epsilon}\boldsymbol{r}}{\pi^{1+\epsilon}} \frac{\kappa(\boldsymbol{r})}{\boldsymbol{r}^2} G((\boldsymbol{p}+\boldsymbol{r})^2) = \int \frac{d^2\boldsymbol{r}}{\pi} \left[\frac{\kappa(\boldsymbol{r})}{\boldsymbol{r}^2}\right]_+ G(\boldsymbol{p}+\boldsymbol{r})^2) + \boxed{\int \frac{d^{2+2\epsilon}\boldsymbol{r}}{\pi^{1+\epsilon}} \frac{\kappa(\boldsymbol{r})}{\boldsymbol{r}^2} \frac{\boldsymbol{p}^2 G(\boldsymbol{p}^2)}{\boldsymbol{r}^2 + (\boldsymbol{p}+\boldsymbol{r})^2}}$$

calculated analytically & added to virtual corrections →combined expression finite

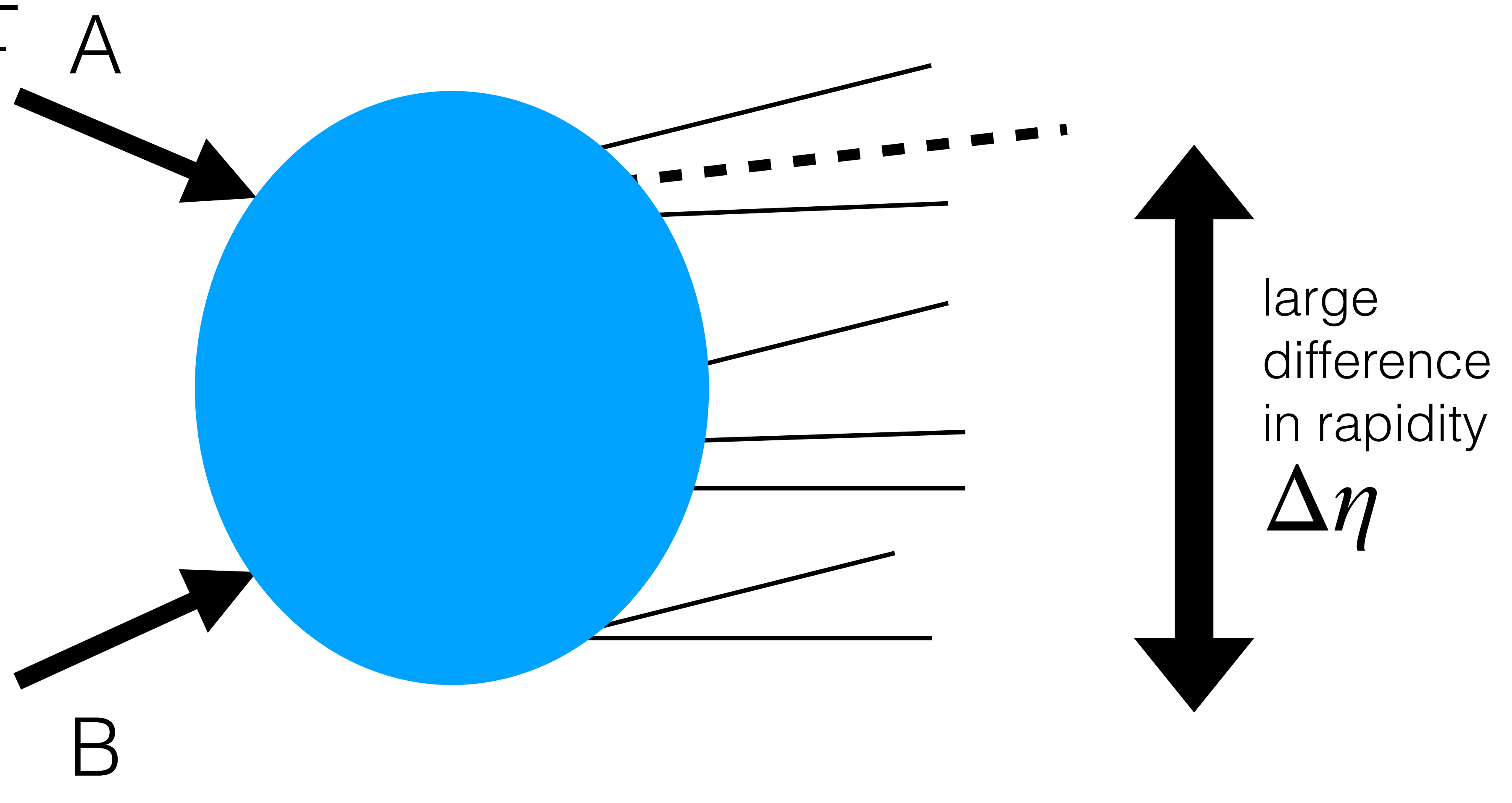
$$\int \frac{d^2\boldsymbol{r}}{\pi} \left[\frac{\kappa(\boldsymbol{r})}{\boldsymbol{r}^2}\right]_+ G((\boldsymbol{p}+\boldsymbol{r})^2) \equiv \int \frac{d^2\boldsymbol{r}}{\pi} \frac{\kappa(\boldsymbol{r})}{\boldsymbol{r}^2} \left[G(\boldsymbol{p}+\boldsymbol{r})^2) - \frac{\boldsymbol{p}^2 G(\boldsymbol{p}^2)}{\boldsymbol{r}^2 + (\boldsymbol{p}+\boldsymbol{r})^2}\right]$$

- in general: numerically for real corrections
- finiteness checked for Mellin representation of $G(\mathbf{q}^2) = \int \frac{d\gamma}{2\pi i} \left(\frac{\mathbf{q}^2}{Q_0^2}\right)^\gamma \tilde{G}(\gamma)$

From rapidity to proton momentum fraction x

- high energy factorization: naturally formulated in *rapidity* $\eta = \frac{1}{2} \ln \frac{p^+}{p^-}$

  $\rightarrow$ resummation of $\Delta\eta = \frac{1}{2} \ln \frac{p_A^+ p_B^-}{p_A^- p_B^+}$
- phenomenology: want to resum $\ln \frac{1}{x} = \ln \frac{p_B^-}{p_A^-}$ $\rightarrow$ unintegrated gluon fits given in this variable
- change of variable trivial at LL, non-trivial at NLL and beyond
- problem: $\Delta\eta = \ln \frac{1}{x} + \frac{1}{2} \ln \frac{M_H^2 + \mathbf{p_A}^2}{\mathbf{p_B}^2}$ $\rightarrow$ large extra collinear log
- MRK diagrams: worked out in principle in [Bartels, Sabio Vera, Schwennsen; hep-ph/0608154]



effective action: need to consider most general transition function (with finite contribution)

- was already needed for 2-loop trajectory and 1-loop virtual impact factors [Chachamis, MH, Madrigal, Sabio Vera;1202.0649, 1307.2591]
- now: with k_T dependence to eliminate large logs from hadronic impact factor

$$Z^+(\hat\rho; \boldsymbol{k}, \boldsymbol{q}) = \delta^{(2+2\epsilon)}(\boldsymbol{k} - \boldsymbol{q}) + \hat\rho K_{\mathrm{BFKL}}(\boldsymbol{k}, \boldsymbol{q}) + \frac{\hat\rho^2}{2} K_{\mathrm{BFKL}} \otimes K_{\mathrm{BFKL}}(\boldsymbol{k}, \boldsymbol{q})$$
$$+ f^+ \otimes \hat\rho K_{\mathrm{BFKL}}(\boldsymbol{k}, \boldsymbol{q}) + f^+(\boldsymbol{k}, \boldsymbol{q}) + \frac{f^+ \otimes f^+(\boldsymbol{k}, \boldsymbol{q})}{2} \ldots,$$

work in progress …

Conclusions

- Derived coefficient for forward Higgs production within the $m_t \to \infty$ effective gluon-Higgs coupling
- Introduced transition function to cancel high energy divergencies
- Presented subtraction mechanism ($\rightarrow$mimics dipole subtraction)
  achieves elegant cancelation of soft-collinear cancelation between real & virtual corrections

Outlook:

- Numerical studies + most general reggeization scale setting
- generalized TMD factorization along the lines of [MH, Kusina, Kutak, Serino, 1711.04587, 1607.01507]
- phenomenology?

picture: www.gob.mx

Thanks a lot for attention!