Momentum transfer dependence of heavy quarkonium electroproduction

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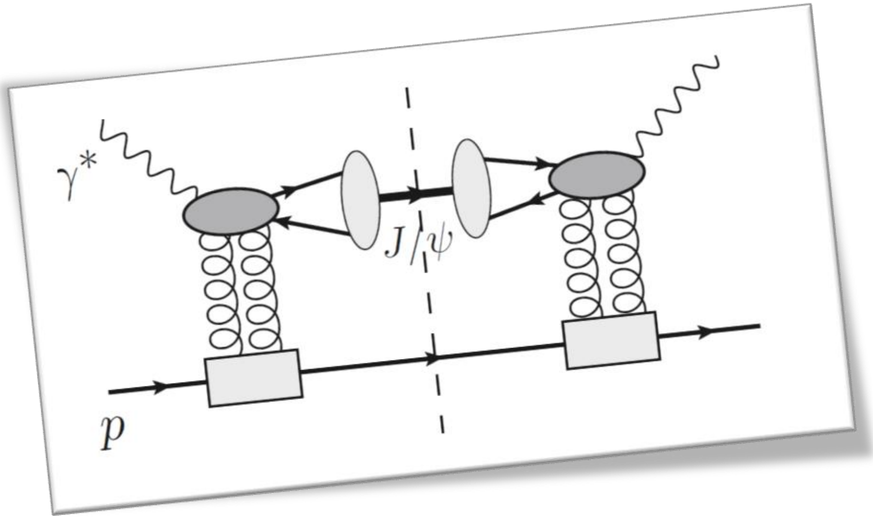
Based on arXiv:2102.06106

DIS 2021 | 15 April 2021

Outline

- Motivation & introduction
- Color dipole framework
- Dipole cross section
- $\vec{b}$-$\vec{r}$ correlation
- B slope
- Numerical results
- Conclusions



Based on:

- e-Print: 2102.06106 [hep-ph]

Why to be interested in VM?

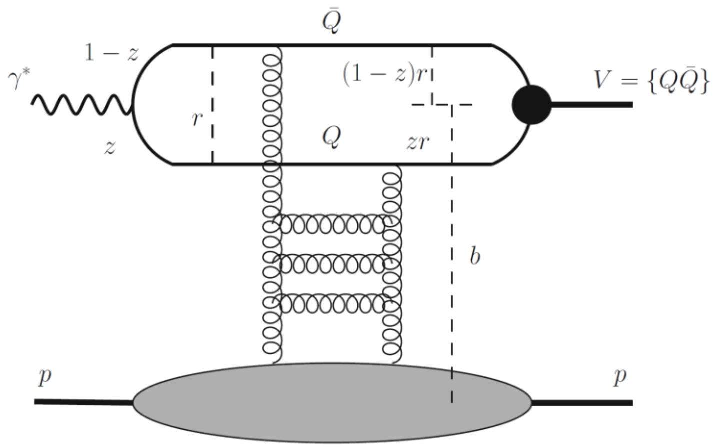
- **Vector Mesons (VM)** are used as a probe for example in heavy-ion collisions or saturation phenomena in ep, eA
- Mostly 1S states of heavy quarkonia are used J/ψ and Υ
- The size of heavy quarkonia is relatively small
- Natural way of calculation: **color dipole formalism**
- **Many theoretical quarkonia uncertainties, for example:**
  - Quarkonium vertex
  - Wave function vs $Q\bar{Q}$ potential

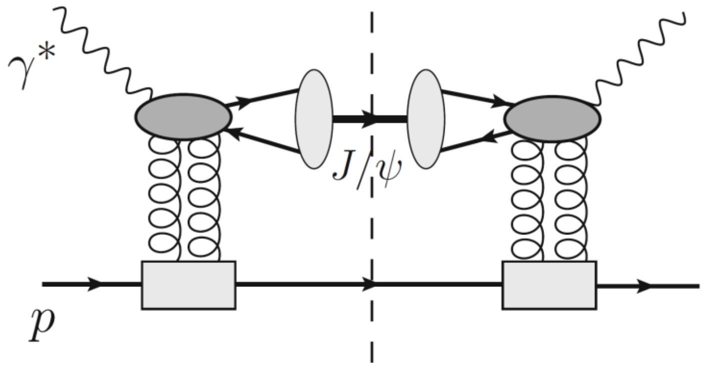
1^3S_1	1^{--}	$I=0,\ c\bar{c}$	0	$J/\psi(1S)$	3.0969
1^3S_1	1^{--}	$I=0,\ b\bar{b}$	0	$\Upsilon(1S)$	9.46030
1^3S_1	1^{--}	$I=1/2,\ u\bar{c},\bar{u}c$	0	D^*	2.00685
1^3S_1	1^{--}	$I=1/2,\ d\bar{c},\bar{d}c$	±1	D^*	2.01026
1^3S_1	1^{--}	$I=0,\ c\bar{s},\bar{c}s$	±1	$D_s^{*\pm}$	??
1^3S_1	1^{--}	$I=1/2,\ d\bar{b},\bar{d}b$	0	B^*	5.32465
1^3S_1	1^{--}	$I=1/2,\ u\bar{b},\bar{u}b$	±1	B^*	??
1^3S_1	1^{--}	$I=0,\ b\bar{s},\bar{b}s$	0	B_s^*	5.4154
1^3D_1	1^{--}	$I=0,\ c\bar{c}$	0	$\psi(3770)$	3.77313
1^3D_1	1^{--}	$I=0,\ c\bar{s},\bar{c}s$	±1	$D_{s1}^*(2700)^\pm$	2.7083
2^3S_1	1^{--}	$I=0,\ c\bar{c}$	0	$\psi(2S)$	3.686097
2^3S_1	1^{--}	$I=0,\ b\bar{b}$	0	$\Upsilon(2S)$	10.02326
3^3S_1	1^{--}	$I=0,\ b\bar{b}$	0	$\Upsilon(3S)$	10.3552
4^3S_1	1^{--}	$I=0,\ b\bar{b}$	0	$\Upsilon(4S)$	10.5794

MK, J.Nemchik, Phys.Rev.D 102 (2020) 11, 114033
MK, J.Nemchik, R.Pasechnik, Eur.Phys.J.C 80 (2020) 2, 92
J.Cepila, MK, J.Nemchik, R.Pasechnik, Eur.Phys.J.C 79 (2019) 6, 495
J.Cepila, MK, J.Nemchik, R.Pasechnik, Eur.Phys.J.C 79 (2019) 2, 154

Or replay the talk by T. Lappi

Color dipole formalism for VM





Amplitude:

$$\mathcal{A}^{\gamma^* p \to V p}(x, Q^2, \vec{q}) = \langle V|\tilde{A}|\gamma^*\rangle = \int d^2r \int_0^1 d\alpha\, \Psi_V^*(\vec{r}, \alpha)\, \mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q})\, \Psi_{\gamma^*}(\vec{r}, \alpha, Q^2)$$

t-dependent differential cross section:

$$\frac{d\sigma^{\gamma^* p \to V p}(s, Q^2, t = -q^2)}{dt} = \frac{1}{16\,\pi} \left| \mathcal{A}^{\gamma^* p \to V p}(s, Q^2, \vec{q}) \right|^2$$

For more details, replay the summary talk by T. Lappi

Dipole cross section I

- Describes the interaction of $Q\bar{Q}$ with a proton
- Nonperturbative effects, **no theoretically calculable**
- Just models
  - Nevertheless, qualitatively we have ideas what is going inside
  - In perturbative area described by gluon distribution function
- Various models on the market: GBW, KST, IP-Sat, BGBK, BK, …
- Usually, they are fitted from DIS data, mostly from HERA
- However, such a fit is integrated over impact parameter $\vec{b}$
- For *t*-dependence **we need *b*-dependent dipole cross section (amplitude)**

Dipole cross section II

- Dipole cross section usually represented in terms of gluon density

$$\sigma_{Q\bar{Q}}(r,x) = \frac{4\pi}{3}\int \frac{d^2k}{k^4}\left[1 - e^{-i\vec{k}.\vec{r}}\right]\alpha_s(k^2)\mathcal{F}(x,k^2)$$

  - Where $\mathcal{F}(x,k^2)$ is the unintegrated gluon structure function of the nucleon

- In practice, the usual phenomenological approach is using the Fourier transform as

$$\mathcal{A}_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{q}) = \int d^2b\, e^{-i\vec{b}\cdot\vec{q}}\,\mathcal{A}_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{b})$$

where apply

$$\sigma_{Q\bar{Q}}(r,x) = \mathrm{Im}\mathcal{A}_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{q}=0) = 2\int d^2b\,\mathrm{Im}\mathcal{A}^N_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{b})$$

Dipole cross section III

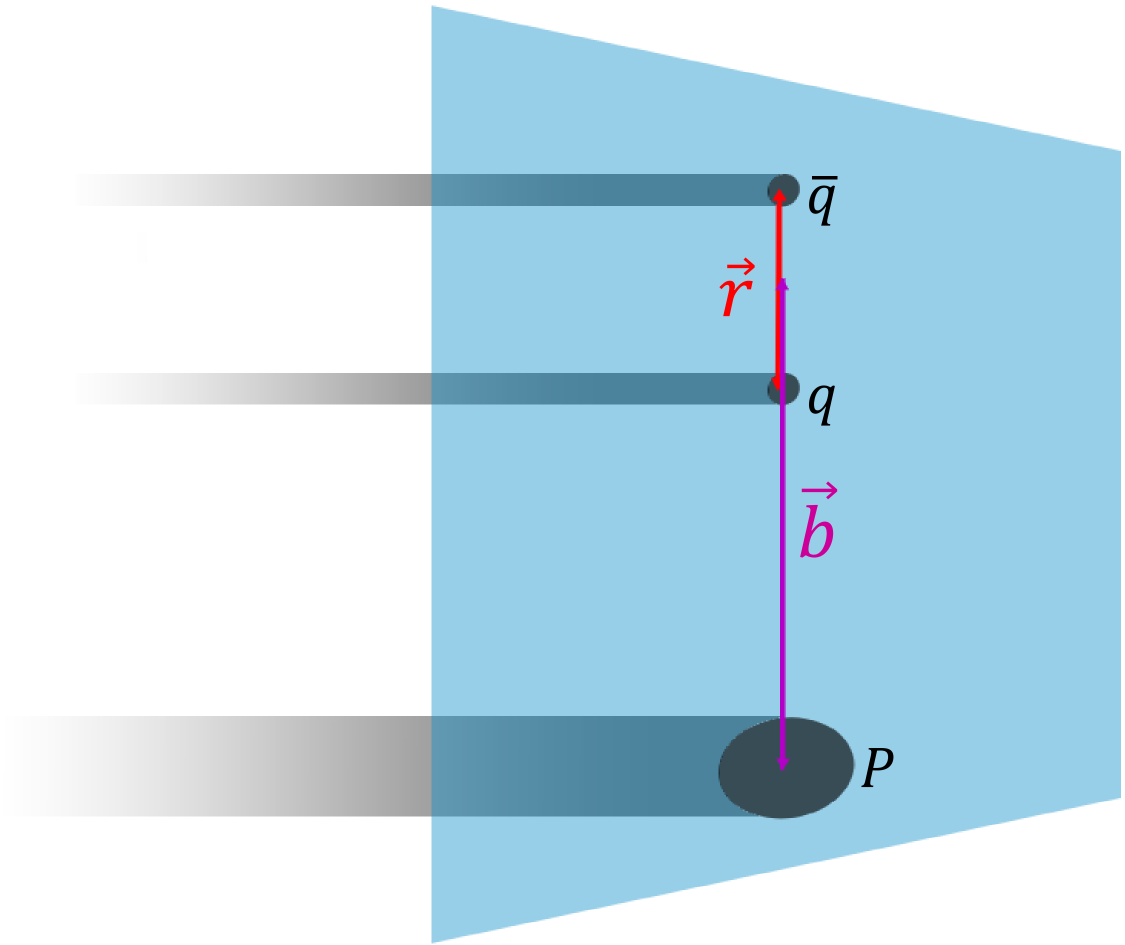
- **Question:** how to determine the partial amplitude $\mathrm{Im}\,\mathcal{A}^{N}_{Q\bar{Q}}\left(\vec{r}, x, \alpha, \vec{b}\right)$??
  - Answer: for the **first approximation** we can consider Gaussian distribution, e.g. IP-sat

$$\sigma^{N}_{q\bar{q}}(\rho, x) = 2\int d^2b\,\mathcal{N}(x, \rho, b), \qquad T_G(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

$$\mathcal{N}(x, \rho, b) = \left(1 - \exp\left[-\frac{\pi^2\rho^2}{2N_c}\alpha_S(\mu^2)xg(x, \mu^2)T_G(b)\right]\right)$$

- But what about the proper $\vec{b}$-$\vec{r}$ correlation?

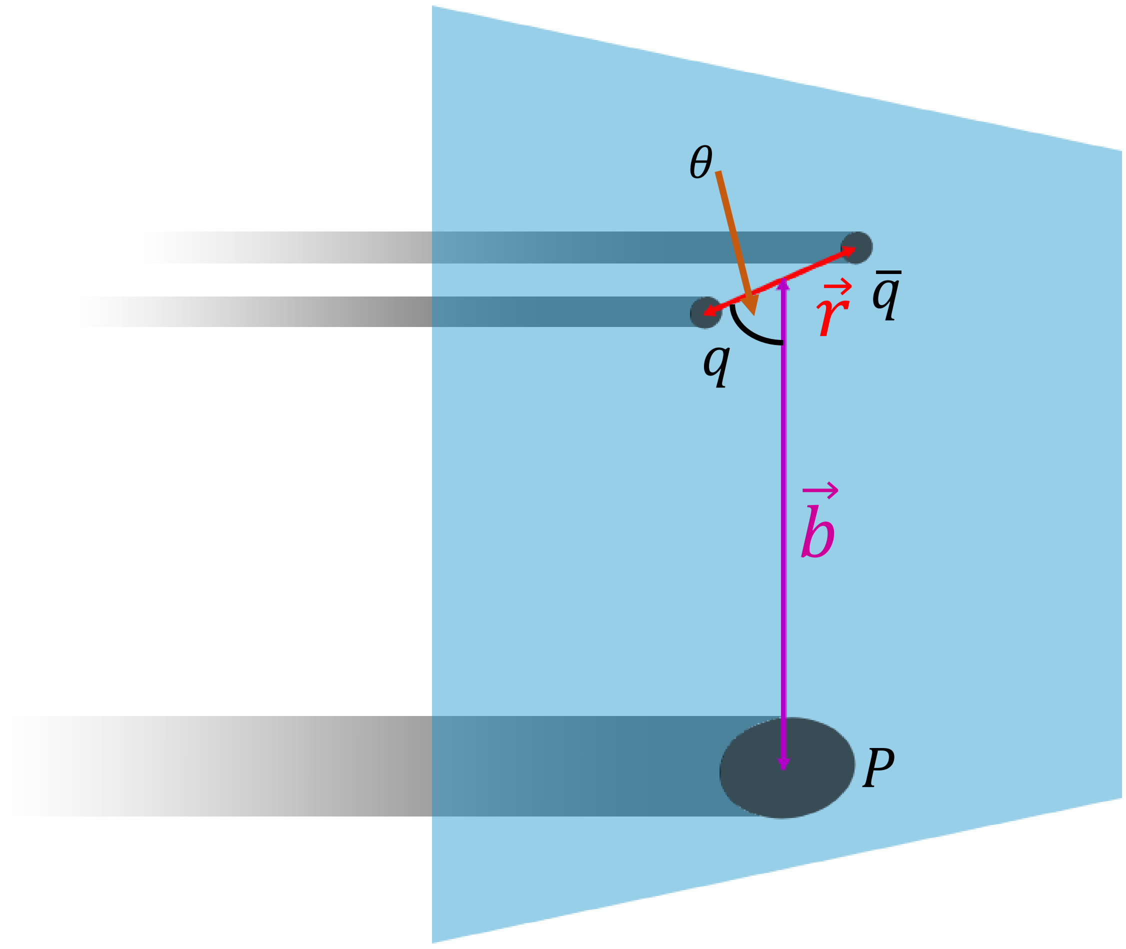
$\vec{b}$-$\vec{r}$ correlation



This is the case of maximal contribution.

$\vec{b} \parallel \vec{r}$ is simpler to calculate, no angle dependence.

$\vec{b}$-$\vec{r}$ correlation



In reality, the angle between $\vec{b}$-$\vec{r}$ can be arbitrary.

One should integrate over all possibilities.

This is a challenge e.g. for BK. So far, only the $\vec{b} \parallel \vec{r}$ approximation was used.

Partial dipole amplitude in the saturation model

- Inspired by the Born approximation with two-gluon exchange

$$\mathrm{Im}\mathcal{A}^N_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{b}) = \frac{1}{12\pi}\int\frac{d^2k\,d^2k'}{k^2\,k'^2}\sqrt{\alpha_s(k^2)\,\alpha_s(k'^2)}\,\mathcal{F}(x,\vec{k},\vec{k}')\,e^{i\,\vec{b}\cdot(\vec{k}-\vec{k}')} \times \left(e^{-i\,\vec{k}\cdot\vec{r}\alpha}-e^{i\,\vec{k}\cdot\vec{r}\,(1-\alpha)}\right)\left(e^{i\,\vec{k}'\cdot\vec{r}\,\alpha}-e^{-i\,\vec{k}'\cdot\vec{r}\,(1-\alpha)}\right),$$

B. Z. Kopeliovich, H. J. Pirner, A. H. Rezaeian and I. Schmidt, Phys. Rev. D 77, 034011 (2008)

Fulfils amplitude is zero for $\vec{b}\perp\vec{r}$ and maximal for $\vec{b}\parallel\vec{r}$

The off-diagonal unintegrated gluon density:

$$\mathcal{F}(x,\vec{k},\vec{k}') = \frac{3\,\sigma_0}{16\,\pi^2\sqrt{\alpha_s(k^2)\,\alpha_s(k'^2)}}\,k^2\,k'^2\,R_0^2(x)\exp\left[-\frac{1}{8}\,R_0^2(x)\,(k^2+k'^2)\right]\exp\left[-\frac{1}{2}\,R_N^2(x)(\vec{k}-\vec{k}')^2\right]$$

fulfilling

$$\sigma_{Q\bar{Q}}(r,x) = \sigma_0\left(1-\exp\left[-\frac{r^2}{R_0^2(x)}\right]\right)$$

Partial dipole amplitude in the saturation model

- Then one can **derive** the form:

$$\mathrm{Im}\mathcal{A}^{N}_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{b}) = \frac{\sigma_0}{8\pi\mathcal{B}(x)}\left\{\exp\left[-\frac{\left[\vec{b}+\vec{r}(1-\alpha)\right]^2}{2\mathcal{B}(x)}\right] + \exp\left[-\frac{(\vec{b}-\vec{r}\alpha)^2}{2\mathcal{B}(x)}\right] - 2\exp\left[-\frac{r^2}{R_0^2(x)} - \frac{\left[\vec{b}+(1/2-\alpha)\vec{r}\right]^2}{2\mathcal{B}(x)}\right]\right\},$$

where $\mathcal{B}(x) = R_N^2(x) + R_0^2(x)/8$

- This prescription is universal for all exponential-like dipole cross section
  - R_0^2 can be obtained from GBW, BGBK, …

B slope

- **Common mischief:**
  - People tend to fit VM *t*-dep data to get proper parameters and reproduce the B slope

- B slope can be calculated as

$$B_{Q\bar{Q}}(r,x) = \frac{1}{2}\langle b^2 \rangle = \frac{1}{\sigma_{Q\bar{Q}}(r,x)} \int d^2b\, b^2\, \mathrm{Im}\mathcal{A}^N_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{b})$$

then, for $\alpha = 1/2$, we can express $\boldsymbol{\mathcal{B}(x)}$ as

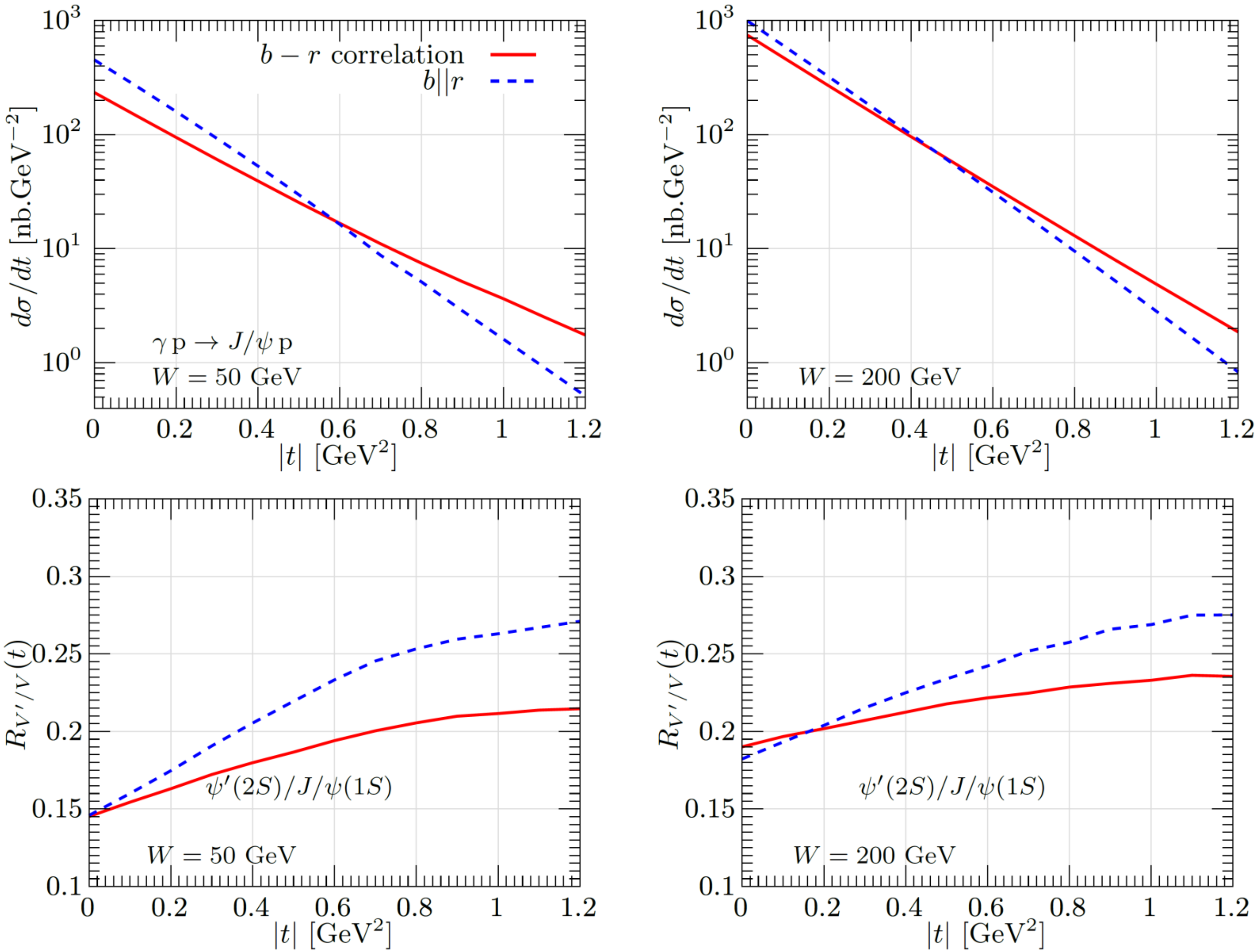
$$B_{Q\bar{Q}}(r,x) = \frac{1}{\sigma_{Q\bar{Q}}(r,x)} \int d^2b\, b^2\, \mathrm{Im}\mathcal{A}^N_{Q\bar{Q}}(\vec{r},x,\alpha=1/2,\vec{b}) = \mathcal{B}(x) + \frac{r^2}{8\left[1 - e^{-r^2/R_0^2(x)}\right]}$$

where

J. Nemchik, N. N. Nikolaev, E. Predazzi and B. Zakharov, Z. Phys. C 75, 71 (1997)
MK, J. Nemchik, R. Pasechnik and J. Cepila, Eur. Phys. J. C79, 154 (2019)

- $B_{Q\bar{Q}}$ is probed at the well known scanning radius $r \approx r_S$
- $B_{Q\bar{Q}}(r = r_S, x, Q^2)$ can be associated with the diffraction slope $B(\gamma^* \to V, x, Q^2)$

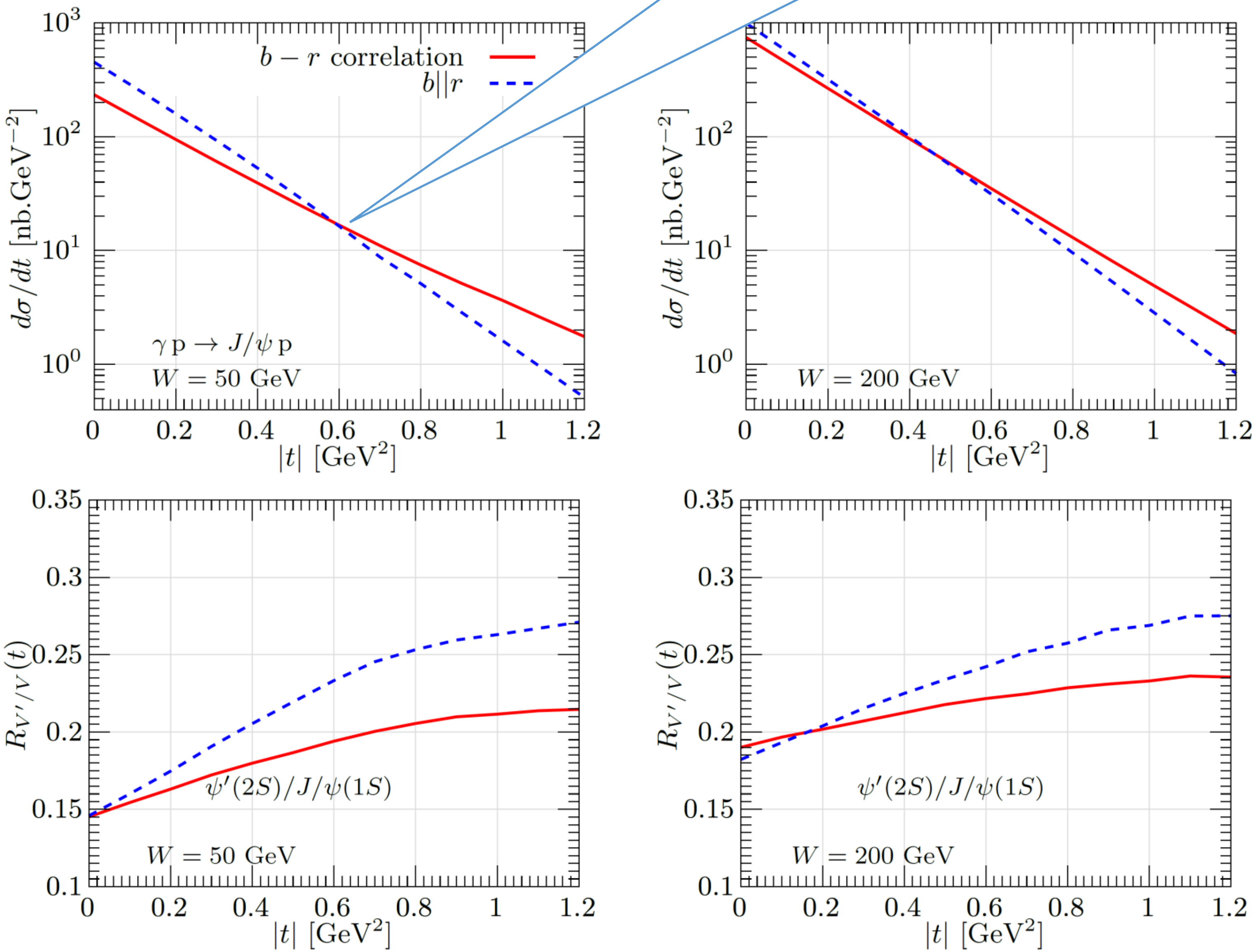
Comparison



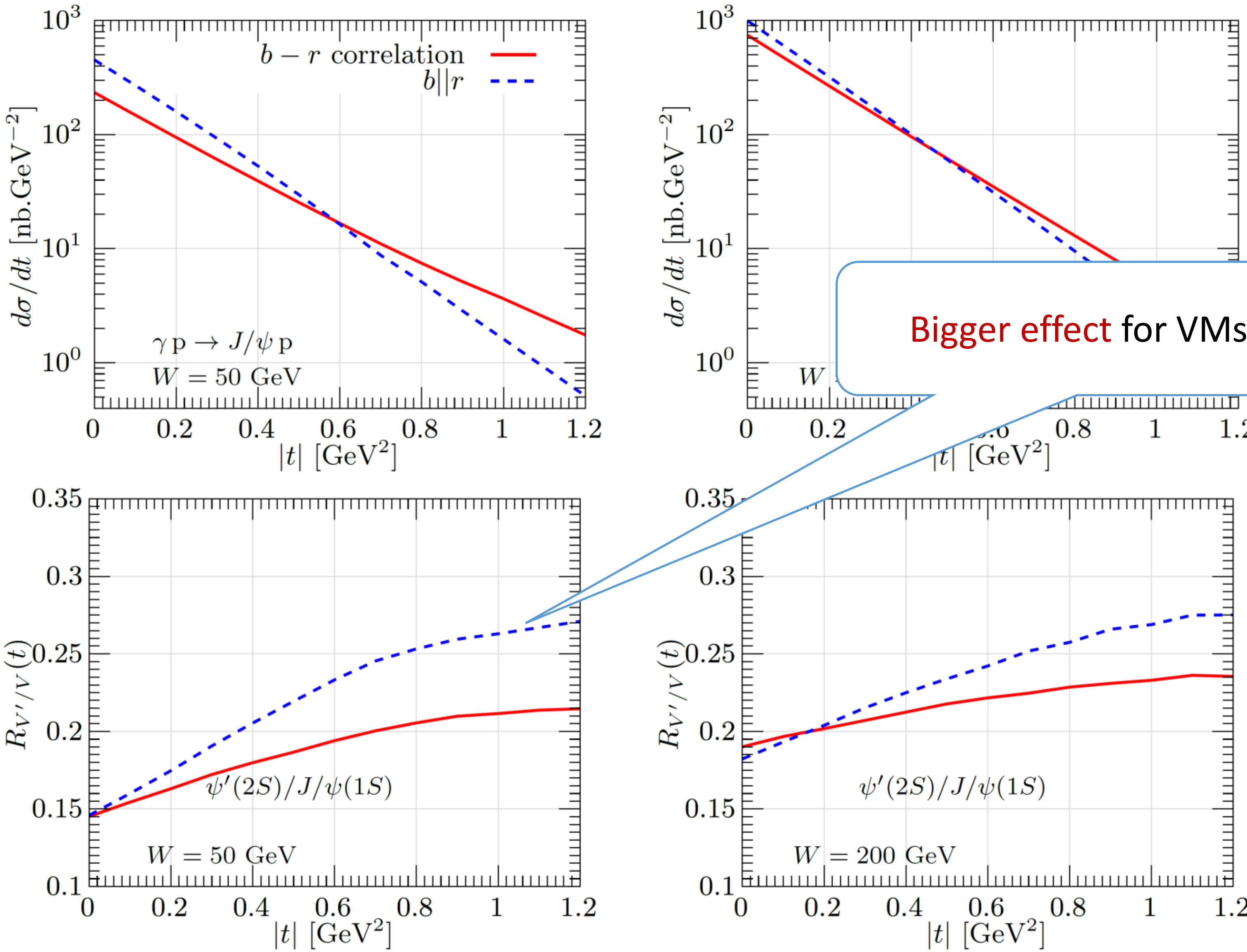
$b-r$ correlation
$b||r$
$d\sigma/dt$ [nb.GeV^{-2}]
$|t|$ [GeV^2]
$\gamma\,\mathrm{p} \to J/\psi\,\mathrm{p}$
$W = 50$ GeV
$W = 200$ GeV
$R_{V'/V}(t)$
$\psi'(2S)/J/\psi(1S)$
$W = 50$ GeV
$\psi'(2S)/J/\psi(1S)$
$W = 200$ GeV

Comparison

$\vec{b} \parallel \vec{r}$ gives maximal slope. The real is smaller.

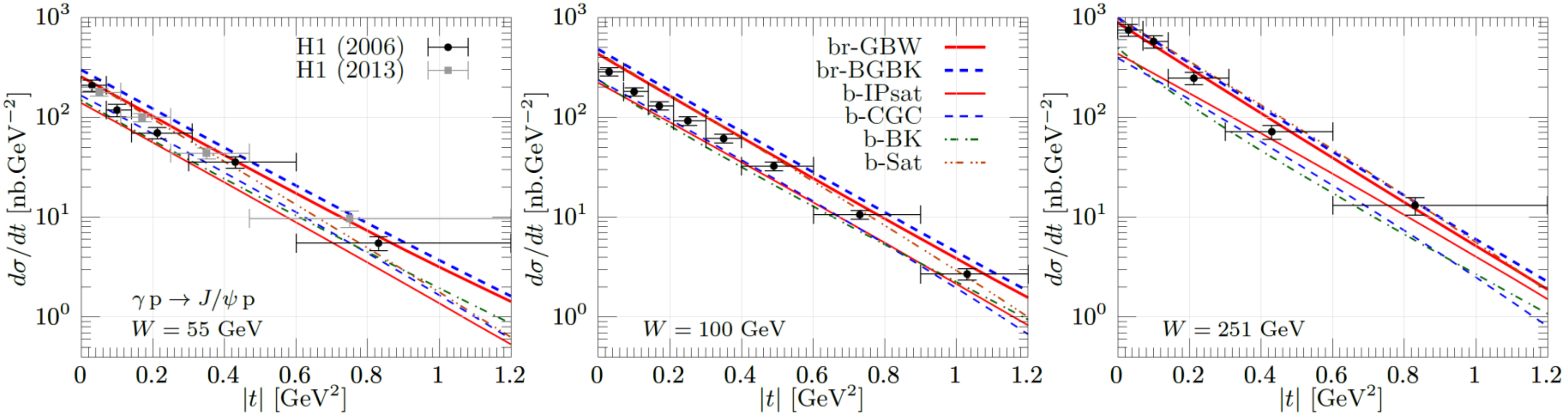


Comparison

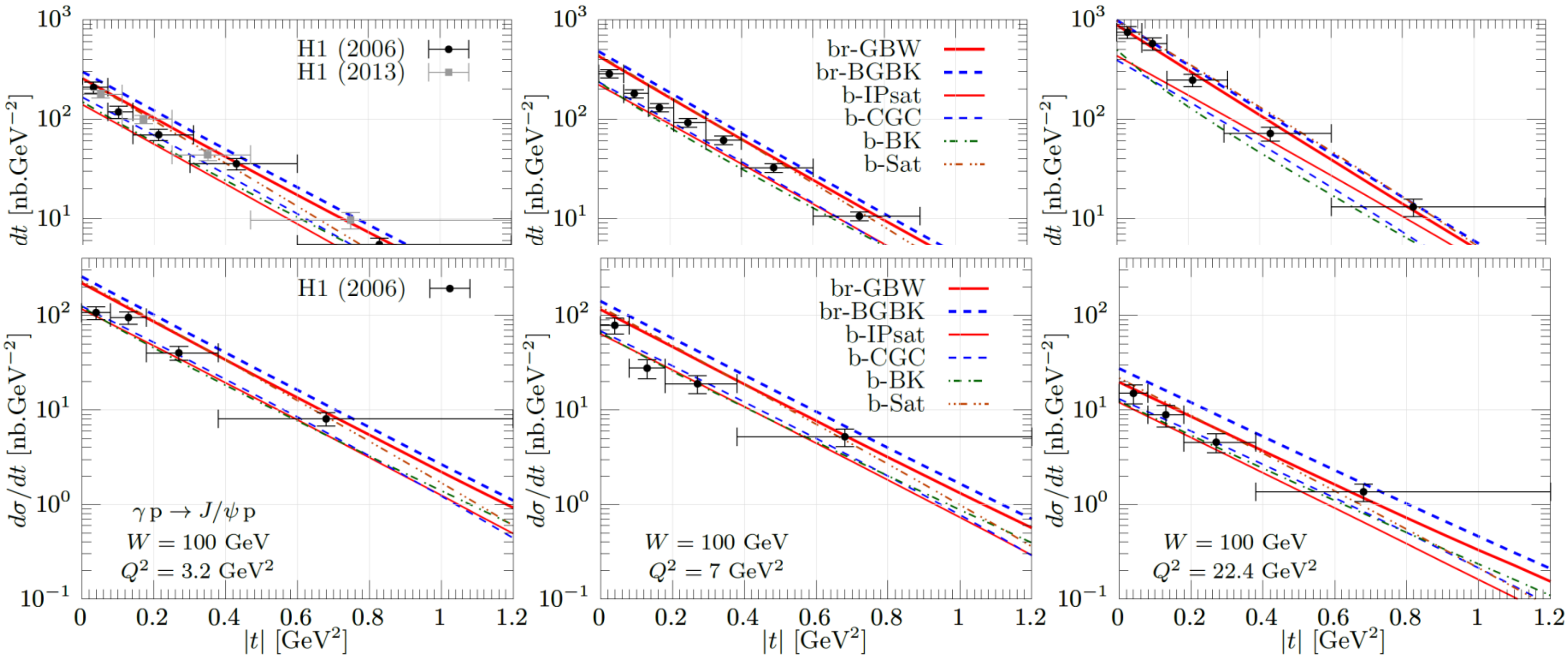


Bigger effect for VMs ratio.

Results

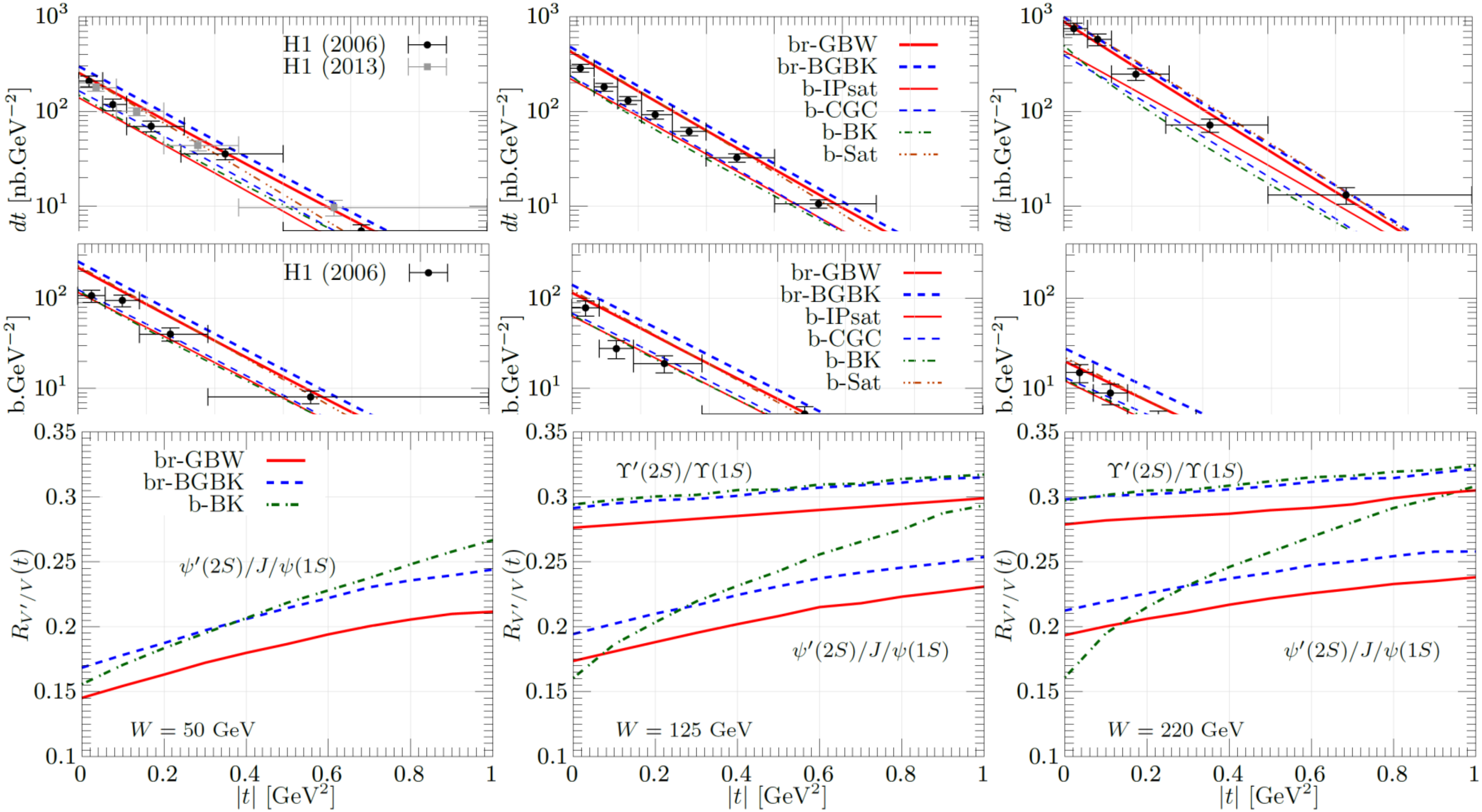

H1 (2006)
H1 (2013)
br-GBW
br-BGBK
b-IPsat
b-CGC
b-BK
b-Sat
$\gamma\,p \to J/\psi\,p$
$W = 55$ GeV
$W = 100$ GeV
$W = 251$ GeV
$d\sigma/dt$ [nb.GeV^{-2}]
$|t|$ [GeV^2]

Results



H1 (2006)
H1 (2013)
br-GBW
br-BGBK
b-IPsat
b-CGC
b-BK
b-Sat
$d\sigma/dt$ [nb.GeV^{-2}]
$|t|$ [GeV^2]
$\gamma\,p \to J/\psi\,p$
$W = 100$ GeV
$Q^2 = 3.2$ GeV^2
$Q^2 = 7$ GeV^2
$Q^2 = 22.4$ GeV^2

Results



$R_{V'/V}$ is a good way of how to exclude some models.

Conclusions

- t-dependent differential cross sections of diffractive production of various quarkonium states with $\vec{b}$-$\vec{r}$ correlation
  - vs models without $\vec{b}$-$\vec{r}$ correlation (using the same wave function)
- **br-GBW, br-BGBK** exhibit a better description of HERA data then b-IPsat, b-CGC, b-BK and b-Sat
- $R_{V'/V}$ is a good way of how to exclude some models.
- t-dependent differential cross sections for quarkonium electroproduction on protons can be generalized for nuclear targets
- $\vec{b}$-$\vec{r}$ correlation leads to a specific polarization of the produced quarkonia, which can be observed in the polar angle asymmetry of the dileptons from quarkonium decays

  -> in progress

Thank you for your attention!

B slope

- Notes: scanning radius
  - Scanning radius corresponds to the typical size of the system
  - i.e. dipole sizes where the VM is scanned predominantly
  - The fixed values of r_S at different Q^2 lead to very similar cross sections in electroproduction of different vector mesons – universality in production

$$r_S \approx \frac{Y}{\sqrt{Q^2 + M_V^2}} \qquad Q^2(J/\psi) = M_\Upsilon^2 \frac{Y_{J/\psi}^2}{Y_\Upsilon^2} - M_{J/\psi}^2$$

- Notes: full B slope calculations

$$B_{T,L}(\gamma^* \to V, s, Q^2) \cdot \mathcal{A}_{T,L}^{\gamma^* p \to V p}(s, Q^2, \vec{q}=0) = \int d^2r \int_0^1 d\alpha \, B_{Q\bar{Q}}(r,s) \, \sigma_{Q\bar{Q}}(r,s) \, \Psi_V^*(\vec{r},\alpha)_{T,L} \, \Psi_{\gamma^*}(\vec{r},\alpha,Q^2)_{T,L}$$

$$B_T(\gamma^* \to V, s, Q^2) \cdot \mathcal{A}_T^{\gamma^* p \to V p}(s, Q^2, \vec{q}=0) = \frac{N_p}{2} \int d^2r \int_0^1 d\alpha \int d^2b \, b^2 \, \mathrm{Im}\mathcal{A}_{Q\bar{Q}}^N(\vec{r}, s, \alpha, \vec{b}) \times \left[\Sigma_T^{(1)}(r,\alpha,Q^2) + \Sigma_T^{(2)}(r,\alpha,Q^2)\right],$$

$$B_L(\gamma^* \to V, s, Q^2) \cdot \mathcal{A}_L^{\gamma^* p \to V p}(s, Q^2, \vec{q}=0) = \frac{N_p}{2} \int d^2r \int_0^1 d\alpha \int d^2b \, b^2 \, \mathrm{Im}\mathcal{A}_{Q\bar{Q}}^N(\vec{r}, s, \alpha, \vec{b}) \, \Sigma_L(r,\alpha,Q^2), \quad (4$$